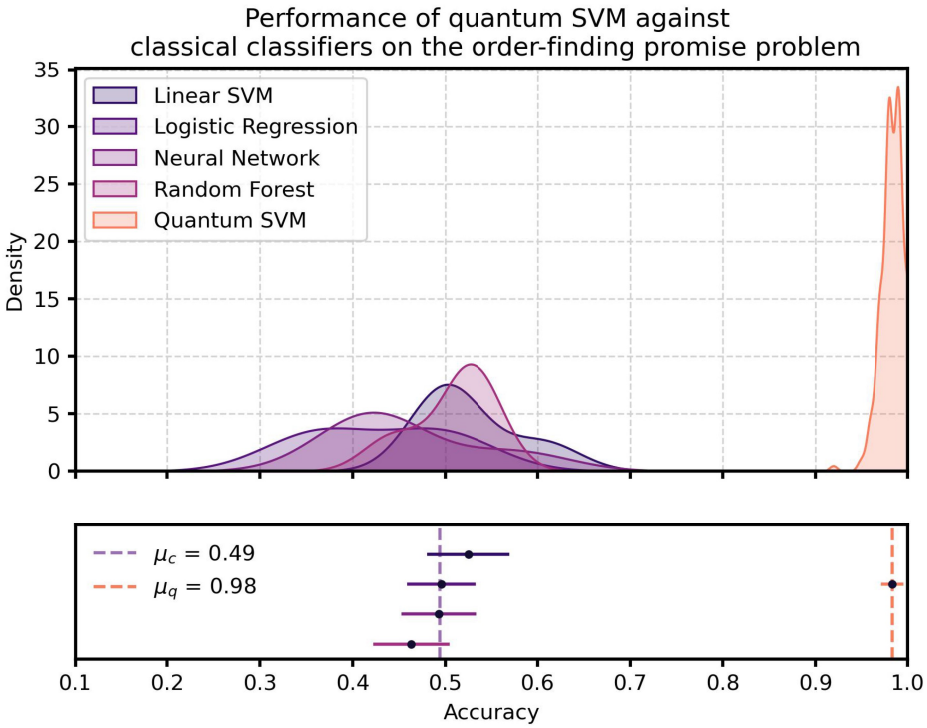
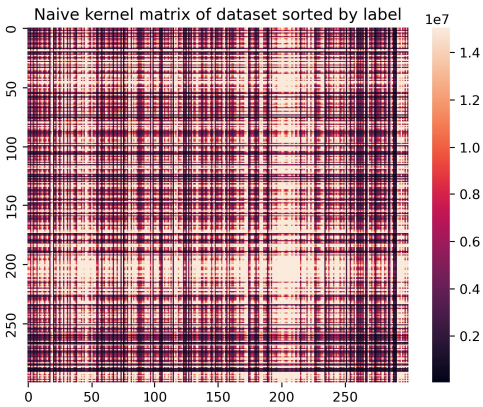
Abstract

In response to the many computational bottlenecks faced by classical machine learning, the field of quantum machine learning has arisen over the past decade to perform certain tasks comparatively faster. By leveraging quantum phenomena such as entanglement and superposition, the field of quantum machine learning has recently shown promise in claiming quantum speedup, that is, performing some tasks exponentially faster than their classical counterparts. However, to date, most of these algorithms rely on quantum RAM — a speculative data loading model whose experimental realisation has proved to be very difficult due to hardware constraints. It was not until 2021 when Liu *et al.* demonstrated the first quantum machine learning model without quantum RAM, called the `QKE-SVM`, that obtains provable quantum speedup in performing a specific $+1/-1$ binary classification task. In particular, they showed that only the `QKE-SVM` can embed the data points provided in their specific task into a *quantum feature space* wherein the $+1$ and -1 labels become largely separated and distinguishable. In this thesis we deconstruct the classification task by Liu *et al.* to see if there are other examples that admit the same exponential quantum speedup. We derive three mathematical properties of Liu *et al.'s* binary classification task that enabled it to be solved exponentially faster by the `QKE-SVM`. We then devise two new binary classification problems, OF_P^1, and QR_P, with the former being based on the encryption protocol of order finding, and the latter that of quadratic residuosity. We show that QR_P and OF_P^1 ultimately fail to admit quantum speedup with a `QKE-SVM` because they satisfy some but not all of the three mathematical properties exhibited by Liu *et al.'s* task. By combining the best aspects of these two problems, we design a third and final classification task OF_P^2 based also on order finding, which we prove does in fact admit quantum speedup as it satisfies all three properties. Together with Liu *et al.'s* result, the OF_P^2 serves as the only other instance of a classification task that a quantum machine learning model can complete exponentially faster than all classical counterparts. Our framework represents a new tool by which classically intractable classification tasks can be discovered that are efficiently solvable using a quantum machine learning model, which is the primary goal of quantum machine learning.

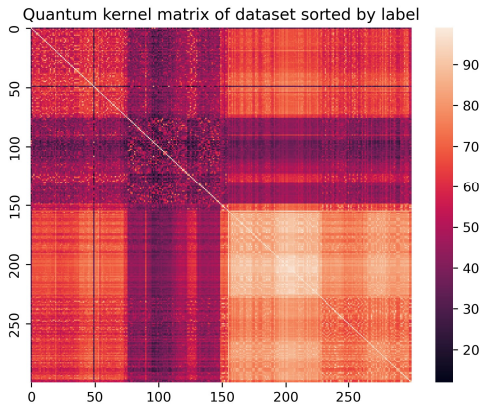
5.2.3 Results and simulations



(a)



(b)



(c)

Figure 5.2: **Smoothed histograms displaying several classical machine learning models being drastically outperformed by the `QKE-SVM` at the task of OF_P^2 (5.2a). 5.2b and 5.2c respectively show kernel matrices corresponding to the classical and quantum feature maps that were used, revealing that the latter clearly separates instances based on their binary label unlike the former.** The bottom panel of 5.2a summarises the average accuracy of each model flanked by a σ error bar. The dataset on which the models were trained had an even distribution of $+1$ and -1 instances, thus the classical models on average performed no better than random guessing ($\mu_c = 49\%$) suggesting the classical intractability of the OF_P^2.