

The Exercise of Chap 2

Yanwei Fu

September 13, 2016

1 Simple Linear Regression

We have a quantitative response Y on the basis of a single regression predictor variable X . It assumes that there is approximately a linear relationship between X and Y ,

$$Y \approx \beta_0 + \beta_1 X$$

In practice, β_0 and β_1 are unknown; and to estimate the coefficients. We have n observations

$$(x_1, y_1), (x_2, y_2), \dots, (x_n, y_n)$$

Suppose we define the loss function as the *residual sum of squares (RSS)*,

$$RSS = \sum_{i=1}^n \left(y_i - \hat{\beta}_0 - \hat{\beta}_1 x_i \right)^2$$

Prove that the minimizers are

$$\hat{\beta}_1 = \frac{\sum_{i=1}^n (x_i - \bar{x})(y_i - \bar{y})}{\sum_{i=1}^n (x_i - \bar{x})^2}$$

$$\hat{\beta}_0 = \bar{y} - \hat{\beta}_1 \bar{x}$$

where $\bar{y} \equiv \frac{1}{n} \sum_{i=1}^n y_i$, and $\bar{x} \equiv \frac{1}{n} \sum_{i=1}^n x_i$ are the sample means. (hint: $\sum_{i=1}^n (x_i - \bar{x}) = 0$).

2 Regularized Least Squares (prove Eq (3.28), Page 145, Bishop book)

Given the dataset with n observations

$$(x_1, t_1), (x_2, t_2), \dots, (x_n, t_n)$$

and the loss function is

$$\frac{1}{2} \sum_{n=1}^n \{t_n - w^T \phi(x_n)\}^2 + \frac{\lambda}{2} w^T w$$

please prove that $w = (\lambda I + \Phi^T \Phi)^{-1} \Phi^T t$

$$\Phi = \begin{bmatrix} \phi_0(x_1) & \phi_0(x_1) & \cdots & \phi_0(x_1) \\ \phi_0(x_1) & \phi_0(x_1) & \cdots & \phi_0(x_1) \\ \vdots & \vdots & \ddots & \vdots \\ \phi_0(x_1) & \phi_0(x_1) & \cdots & \phi_0(x_1) \end{bmatrix}$$

More details, please refer to “Sec. 3.1.4 Regularized least squares”(Bishop book);