Do prudent consumers save against inflation risk?*

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Abstract

The main result in this note is that prudence (convex marginal utility of consumption) is not a sufficient condition for price risk to generate precautionary savings. With a logarithmic utility function, the consumer spends a constant fraction of total (nominal) wealth on nominal consumption. As a result, an unexpected change in the price level is absorbed by real consumption, leaving nominal consumption and savings unchanged.

1 Introduction

Research in macroeconomics has put significantly more attention on the relation between inflation and economic performance in a context where agents are subject to risk and save for precautionary reasons since the seminal work by Kaplan et al. (2018). This line of research requires to improve our understanding of precautionary savings in an inflationary environment. In this note, we focus on the relation between precautionary savings and inflation risk. Two traditional assumptions that generate precautionary savings in macro models are prudence (i.e. convex marginal utility of consumption)

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and financial frictions. We show in an example that prudence is not a sufficient condition for inflation risk to produce precautionary savings. This comes as a surprise because risky real wages typically imply precautionary savings in models where consumers are prudent. In our example, inflation risk implies real wage risk. Yet the representative consumer does not increase savings in the comparative static exercise where one increases the standard deviation of the stochastic component of the price level. The intuition for this result relies on a well-known property of logarithmic utility: the representative consumer spends a constant fraction of wealth on each type of goods when preferences are Cobb-Douglas. In a dynamic context, this property implies that the representative consumer spends a constant fraction of the present value of total wealth on the nominal consumption of a given period. Hence, consumption is adjusted each period to keep total consumption spending constant over time. As a consequence, savings do not depend on price volatility in the example since consumption absorbs all the fluctuation in prices.

2 Model

Time is discrete and discounted by a factor $\beta \in (0, 1)$. An infinitely-lived representative consumer maximizes the present discounted value of utility, which depends on the consumption level c chosen in each period. The utility function is logarithmic $u(c) = \ln c$. Its concavity implies that the consumer is risk averse and the convexity of its marginal utility implies that the consumer is also prudent. Consumption goods cost P each period. This price is stochastic and depends on two components : i) an aggregate component $\Gamma > 0$ that deterministically grows at a gross rate Π and ii) a stochastic component $\tilde{p}$ that is drawn each period from a log normal distribution with mean zero and variance σ^2, i.e. $\ln \tilde{p} \sim \mathcal{N}(0, \sigma^2)$. Hence,

$$P = \Gamma \tilde{p}.$$

In each each period, the consumer receives an exogenous deterministic nominal wage W that depends on a parameter $\tilde{w} > 0$ (which can be interpreted as a constant marginal product of labor) and the aggregate component of the price level Γ, that is

$$W = \Gamma \tilde{w}.$$

An asset owned in quantities A is used to save nominal resources across periods. Its gross nominal interest rate I is known before the realization of the $\tilde{p}$ shock and it is fixed by a monetary policy rule at a given level each period.[1] Notice that the asset does not provide insurance against the risk associated with the stochastic component of the price level $\tilde{p}$. We can thus write the budget constraint in nominal terms as

$$A' = IA + W - Pc,$$

where the prime notation refers to next period variables while variables without a prime are associated with current period. Below we will sometimes use the subscript notation t when more than two periods will be involved.

In order to be able to write a Bellman equation, we need to rescale this budget constraint. By dividing both sides of the equation by Γ, one obtains

$$a' = Ra + w - pc, \tag{1}$$

with $a = \frac{A}{\Gamma}$, $R = \frac{I}{\Pi}$, $w = \frac{\tilde{w}}{\Pi}$ and $p = \frac{\tilde{p}}{\Pi}$. We assume that the standard absolute impatience assumption $R\beta < 1$ holds as well as the finite human wealth assumption $R > 1$ (Carroll, 2024).

The following Bellman equation describes the program of the consumer:

$$V(a, p) = \max_{c,a'} u(c) + \beta E_{p'} \left[V(a', p') \right] \tag{2}$$

subject to the budget constraint (1) as well as standard non negativity constraints and the no-Ponzi game condition.

3 Equilibrium

Given the program in (2), one can derive the following Euler equation:

$$\frac{1}{pc} = \beta R E_{p'} \left[\frac{1}{p'c'} \right] \tag{3}$$

Notice that, according to (3), the representative consumer chooses to smooth total spending on consumption over time. This property is a consequence of the logarithmic assumption for the utility function. Log utility is Cobb Douglas utility and one of its well-known properties is that the consumer spends a fixed fraction of her wealth on a given good. Even though this property has been given more attention in a static context, it still applies in a dynamic context, where a fixed fraction of the present value of total wealth is spent on the consumption of a given period. As a consequence,

[1] In Section 4.4, we explore an alternative monetary policy rule that fixes the real interest rate.

one can conjecture that the consumer chooses a high level of consumption when the realization of $\tilde{p}$ is low and, vice versa, a low level of consumption is chosen when the realization of $\tilde{p}$ is high, so that the total amount spent on consumption pc follows a flat trajectory over time. This conjecture is confirmed by the following proposition that gives the optimal decision rule for consumption spending:

Proposition 1. *The following decision rule for spending on consumption is consistent with the Euler equation (3):*

$$pc = (1-\beta)R\left(a + \frac{w}{R-1}\right) \tag{4}$$

Moreover, the decision described in (4) is also consistent with the following transversality condition:

$$\lim_{k\to\infty} \beta^k E_t\left[\frac{1}{p_{t+k}} u'(c_{t+k})a_{t+k}\right] = 0$$

The proof of Proposition 1 can be found in the Appendix A.1 and Appendix A.2. In economic terms, the consumer in (4) simply chooses to spend a fraction $(1-\beta)$ of total wealth on consumption[2], while leaving a fraction β for savings. Notice that the decision rule implies that total spending is independent of the realization of the stochastic component of the price level $\tilde{p}$, nor it depends on the variability σ^2 of this component. A corollary of this proposition is that next period assets (i.e. savings) do not depend on σ^2 either:

$$a' = \beta Ra + w\frac{R\beta - 1}{R-1} \tag{5}$$

4 Further discussion

4.1 On duality

The result about inflation risk not affecting precautionary savings may come as a surprise because one may think it should have effects similar to the impact of real wage risk. However, the price of goods in the model affects other components of the budget constraint than the real wage. To see it, one simply needs to devide each side of equation (1) by p to get

$$\frac{a'}{p} = \frac{Ra}{p} + \frac{w}{p} - c.$$

[2] a are the accumulated assets while $\frac{w}{R-1}$ is the human wealth derived from the present value of current and future wages. Given the timing of earned interests and consumption spending in equation (1), total wealth needs to take into account the interests earned during the period, implying the presence of R in front of the sum of these two terms.

One can see from the equation above that a change in p has consequences for purchasing power as both w and a are divided by it. But p also affects the consumer's ability to build assets for the future since a' on the left hand side is also divided by p. This difference explains why the risk associated with p has different consequences than a potential risk associated with w.

4.2 Multiple goods

Our Cobb-Douglas framework can be easily extended to multiple goods. Consider now that c in the utility function is a composite good that depends on the consumption of N other goods x_i:

$$c = \Pi_{i=1}^{N} {x_i}^{\theta_i}$$

with $\sum_{i=1}^{N} \theta_i = 1$, and the following formulation of the budget constraint:

$$a' = Ra + w - s. \tag{6}$$

where $s = \sum_{i=1}^{N} x_i p_i$ is total consumption spending: each one of the new introduced goods is priced p_i, which is stochastic and drawn from a log normal distribution. We store the p_i's in a vector $\bar{p}$ of prices.

Intratemporal first-order conditions allow to establish that

$$x_i p_i = \theta_i s,$$

This relation allows us to write the Bellman equation in terms of total spending as

$$\hat{V}(a, \bar{p}) = \max_{s,a'} v(s, \bar{p}) + \beta E_{\bar{p}'} \left[\hat{V}(a', \bar{p}') \right], \tag{7}$$

subject to (6) and with the pseudo indirect utility $v(s, \bar{p}) = \ln s + \sum_{i=1}^{N} \theta_i \ln \left(\frac{\theta_i}{p_i} \right)$.

With this formulation, one can show that the corollary result of equation (5) also applies.

4.3 Risk aversion

Proposition 1 and the corollary decision rule for next-period assets in equation (5) illustrate that savings do not depend on the variance σ^2. It is possible to show that the value V is also independent of price volatility, even though it is decreasing in p. The proof can be found in Appendix A.3 and this property is summarized in the next proposition:

Proposition 2. *The value $V(a,p)$ does not depend on σ^2 and $V(a,p)$ is strictly decreasing in p.*

The upshot here is there is no risk associated with total spending. Given that the representative agent chooses to smooth total spending over time, larger price variability does not affect expected utility.

4.4 Monetary policy rule

The monetary policy rule in Section 2 is such that the nominal interest rate is constant over time. Such assumption typically describes a rather passive central bank or a central bank worried about the zero lower bound or within a monetary union. We now consider an alternative rule that keeps the real interest rate constant. Given a target $\bar{R}$ for the real interest rate, the monetary policy rule rescales the nominal interest to compensate for expected inflation such that $R(p) = \frac{\bar{R}P}{p}$ with $P = E_{p'}\left(\frac{1}{p'}\right)^{-1}$.

Under the monetary policy rule, the Euler equation can be written as follows:

$$\frac{1}{c} = \beta \bar{R} P E_{p'}\left[\frac{1}{p'c'}\right]. \tag{8}$$

One can understand from this condition that the growth rate of real consumption is deterministic and subject to the following evolution:

$$c' = gc, \tag{9}$$

with $g = \beta\bar{R}$.

In the Appendix A.4, we show that this deterministic trajectory implies that spending on consumption is independent of price risk in the case of this monetary policy rule as it satisfies

$$c_t p_t = \frac{\bar{R}(1-\beta)\left(a_t + \frac{w}{R-1}\right)}{(1-\beta) + \beta\frac{P}{p_t}}. \tag{10}$$

Notice that, when $p_t = P$, consumption spending is as in (4), where the monetary policy rule targets a constant nominal interest rate. But, when $p_t \neq P$, nominal spending deviates from (4) and it is now increasing in p_t to keep the trajectory of real consumption as in (9), but price risk *per se* does not influence savings.[3]

[3]Given the lognormal assumption for p_t, an increase in σ^2 should also increase the expected price P, but it would also increase the average realization of p_t. Hence, σ^2 influences consumption spending in (10) not for precautionary reasons but because it increases expected future prices for a given p_t.

4.5 Borrowing constraint

The two standard assumptions that yield precautionary savings are prudence and an occasionally binding borrowing constraint. In the example of Section 2, we consider the former assumption. We now investigate how the latter assumption shapes the relation between inflation risk and savings. We can only present numerical results, but the upshot is that an increase in the variance of the stochastic component of the price level generates precautionary savings in this case but the impact is lower than an increase in the variance of nominal wages, though the difference is not very large. Because we want to produce results that are comparable from a quantitative perspective, we also consider stochastic nominal wages in the model of Section 2, that is, the parameter w now becomes a random variable and we fix its stochastic properties so that we obtain the same distribution for the real wage (i.e, the variable w/p) as in the case where p is stochastic. We find that an increase in the variance of w has a larger impact on savings than in the case of an increase in the variance of p.

In Table 1, we consider four sets of simulations.[4] In the first row, we simulate 100,000 paths of 50 time periods for nominal assets and consider shocks to p, while the second row considers shocks to w instead. The first column considers a standard deviation for the real wage w/p of 0.17, while the second column has a standard deviation of 0.25. Additionally, the table reports in parentheses the share of observations where the borrowing constraint is binding for each set of simulations. One notice that moving from the low variance to the high variance framework doubles the average amount of assets that the agent owns. However the impact is larger in the case where wages are stochastic. This suggests that the results in Proposition 1 only extend partially to a context with an occasionally binding borrowing constraint.

5 Conclusion

In this note, we have illustrated that prudent consumers do not necessarily save more against an increase in inflation risk when they are prudent. The intuition for the result relies on logarithmic utility: the consumer spends a constant fraction of total nominal wealth on nominal consumption. As a consequence, a shock to prices is absorbed by consumption to smooth nominal spending over time, leaving savings unaffected by the shock.

[4] Policy functions are approximated numerically by using the endogenous gridpoint method described in Carroll (2006) while recentering the grid in each iteration as in Auclert (2024). We consider grids of 11 points for each simulation.

Table 1: Nominal assets across four sets of simulations

	Low variance	High variance
Stochastic price	1	2.07
	(10.9%)	(8.24%)
Stochastic wage	0.99	2.12
	(10.6%)	(7.4%)

Notes: the table compares average nominal assets across four sets of simulations. The variable p is the one that is stochastic under the "Stochastic price" simulations, while w is the one that is stochastic under the "Stochastic wage" simulations. Under "Low variance" the real wage (i.e. the variable w/p) has a mean of 1 and a standard deviation of 0.17, while the standard deviation is 0.25 under "High variance" (with a mean of 1 too). Each set includes 100,000 simulated paths of 50 time periods. Initial assets are fixed to 0.5 and the debt limit is 0. Numbers are normalized by the level of the upper left case to ease comparison. We include in parentheses the share of time periods where the borrowing constraint is binding.

A Appendix: Proofs

A.1 Policy function

We show that the decision rule (4) is consistent with the Euler equation (3). We rely on a guess and verify procedure that relies on the following guess structure:

$$pc = \alpha a + \delta,$$

where α and δ are parameters to be determined. By replacing this guess in the Euler equation we obtain:

$$\frac{1}{\alpha a + \delta} = \beta R E_{\tilde{p}'} \left[\frac{1}{\alpha a' + \delta} \right].$$

Given that a' is determined before the realization of the $\tilde{p}$ shock, we can ignore the expectation on the right hand side of the equation as follows:

$$\alpha a' + \delta = \beta R \left[\alpha a + \delta \right],$$

which, by using the budget constraint (1), can be rewritten as

$$\alpha[Ra + w - (\alpha a + \delta)] + \delta = \beta R \left[\alpha a + \delta \right].$$

By rearranging terms in the equation above, we obtain:

$$\left[(1-\beta)R - \alpha \right] \alpha a + \alpha w + (1 - \alpha - \beta R)\delta = 0$$

The equation above can be zero if the parameters α and δ take the following values:

$$\alpha = (1-\beta)R \quad \text{and} \quad \delta = (1-\beta)R\frac{w}{R-1}$$

These parameter values yield the decision given by equation (4).

A.2 Transversality condition

We now show if the transversality condition given in Proposition 1 holds for the decision rule (4). This is a necessary step to show that (4) is an optimal decision.

Given the law of motion for assets in (5), one can obtain the level of assets k periods ahead:

$$a_{t+k} = (\beta R)^k a_t + w\frac{R\beta - 1}{R-1}\left(\sum_{i=0}^{k-1}(\beta R)^i\right) \tag{11}$$

Given the absolute impatience assumption $\beta R < 1$, we can derive the following limit:

$$\lim_{k\to\infty} a_{t+k} = -\frac{w}{R-1},$$

implying also that

$$\lim_{k\to\infty} c_{t+k}p_{t+k} = 0.$$

Unfortunately, the latter two results imply that the limit $\lim_{k\to\infty} E_t\left[\frac{\beta^k a_{t+k}}{p_{t+k}c_{t+k}}\right]$ is indeterminate as both the numerator and the denominator converge towards the value zero. But one can solve this indeterminacy by applying l'Hôpital's rule. Notice that the derivative of the numerator with respect to k is equal to $\left[\beta^k \ln(\beta)a_{t+k} + \beta^k\frac{da_{t+k}}{dk}\right]$, while the derivative of the denominator is equal to $(1-\beta)R\frac{da_{t+k}}{dk}$. We thus have that

$$\lim_{k\to\infty} E_t\left[\frac{\beta^k a_{t+k}}{p_{t+k}c_{t+k}}\right] = \frac{\beta^k\ln(\beta)a_{t+k} + \beta^k\frac{da_{t+k}}{dk}}{(1-\beta)R\frac{da_{t+k}}{dk}}$$

Now notice from (11) that $\lim_{k\to\infty}\frac{da_{t+k}}{dk} = a_t(\beta R)^k\ln(\beta R)$. Hence,

$$\lim_{k\to\infty} E_t\left[\frac{\beta^k a_{t+k}}{p_{t+k}c_{t+k}}\right] = \frac{1}{(1-\beta)R}\left[\ln(\beta)\frac{(\beta R)^k a_t - \frac{w}{R-1}}{a_t R^k \ln(\beta R)} + \beta^k\right] = 0$$

given the restrictions on β and R that were assumed in Section 2.

A.3 Risk aversion

By using the information in (7) for $N = 1$ together with decision rules (4) and (5), we can write the value function as

$$V(a,p) = \ln\left[(1-\beta)R\left(a+\frac{w}{R-1}\right)\right] - \ln p + \beta E_{p'}\left[V\left(\beta Ra + w\frac{R\beta-1}{R-1}, p'\right)\right].$$

Given the log normal assumption for the stochastic component of the price level, the expectation can be written as follows:

$$E_p\left[V(a,p)\right] = \ln\left[(1-\beta)R\left(a+\frac{w}{R-1}\right)\right] + \beta E_{p'}\left[V\left(\beta Ra + w\frac{R\beta-1}{R-1}, p'\right)\right].$$

One can calculate the expectation on the continuation value above in a similar way and obtain recursively that

$$E_p\left[V(a,p)\right] = \frac{1}{1-\beta}\ln\left[(1-\beta)R\left(a+\frac{w}{R-1}\right)\right] + \ln\left(\beta R\right)\left[\sum_{i=0}^{\infty} i\beta^i\right],$$

after noticing that next period spending is related to current period spending as $s' = \beta Rs$. The equation above can be rewritten as

$$E_p\left[V(a,p)\right] = \frac{1}{1-\beta}\ln\left[(1-\beta)R\left(a+\frac{w}{R-1}\right)\right] + \ln\left(\beta R\right)\frac{\beta}{(\beta-1)^2}, \tag{12}$$

given that $\sum_{i=0}^{\infty} i\beta^i = \frac{\beta}{(\beta-1)^2}$.

Hence, the value does not depend on σ^2, though it is strictly decreasing in p as

$$V(a,p) = E_p\left[V(a,p)\right] - \ln p,$$

with $E_p\left[V(a,p)\right]$ as in (12).

A.4 Monetary policy rule

According to the budget constraint (1), we can write that

$$p_t c_t = R_t a_t + w - a_{t+1}.$$

Given that, according to the same budget constraint, we have that

$$a_{t+1} = \frac{a_{t+2}}{R_{t+1}} - \frac{w}{R_{t+1}} + \frac{p_{t+1}}{R_{t+1}}c_{t+1},$$

we can also write it as

$$p_t c_t + \frac{p_{t+1}}{R_{t+1}} c_{t+1} = R_t a_t + w + \frac{w}{R_{t+1}} - \frac{a_{t+2}}{R_{t+1}}.$$

Following a similar reasoning, by recursive substitution, one can get for any $k \geq 0$ that

$$\sum_{i=0}^{k} \frac{p_{t+i}}{\left(\Pi_{j=0}^{i} R_{t+j}\right)} c_{t+i} = a_t + \sum_{i=0}^{k} \frac{w}{\left(\Pi_{j=0}^{i} R_{t+j}\right)} - \frac{a_{t+k+1}}{\left(\Pi_{j=0}^{k} R_{t+j}\right)}.$$

Given the deterministic growth rate established in (9), it follows that

$$c_t \left[\sum_{i=0}^{k} \frac{g^i p_{t+i}}{\left(\Pi_{j=0}^{i} R_{t+j}\right)} \right] = a_t + \sum_{i=0}^{k} \frac{w}{\left(\Pi_{j=0}^{i} R_{t+j}\right)} - \frac{a_{t+k+1}}{\left(\Pi_{j=0}^{k} R_{t+j}\right)}.$$

Taking expectations conditional on the information set in period t for the limit when $k \to \infty$ and considering the no-Ponzi game condition yields

$$c_t E_t \left[\sum_{i=0}^{\infty} \frac{g^i p_{t+i}}{\left(\Pi_{j=0}^{i} R_{t+j}\right)} \right] = a_t + w E_t \left[\sum_{i=0}^{\infty} \frac{1}{\left(\Pi_{j=0}^{i} R_{t+j}\right)} \right].$$

Considering now the monetary policy rule introduced in Section 4.4, notice that $E_t \left[\frac{\bar{R} P}{p_{t+1}} \right] = \bar{R}$. Hence, after considering the law of iterated expectations, one gets that

$$c_t E_t \left[\sum_{i=0}^{\infty} \frac{g^i p_{t+i}}{\bar{R}^{i+1}} \right] = a_t + w E_t \left[\sum_{i=0}^{\infty} \frac{1}{\bar{R}^{i+1}} \right],$$

which can be rewritten as

$$c_t p_t \left((1-\beta) + \beta \frac{P}{p_t} \right) = \bar{R}(1-\beta) \left(a_t + \frac{w}{\bar{R}-1} \right),$$

yielding the decision rule (10).

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