Shock-dependent Cognitive Discounting and the Forward Guidance Puzzle

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Abstract

We introduce cognitive discounting into a standard DSGE model to address the forward guidance puzzle and propose an estimation strategy that employs system priors to ensure data-consistent responses to monetary policy. We show that implementing cognitive discounting as in Gabaix (2020) to replicate empirically observed forward guidance effects results in a substantial dampening of conventional monetary policy's effectiveness, which contradicts empirical evidence. Our findings reveal that the coexistence of empirically plausible effects of both conventional monetary policy and forward guidance requires a degree of cognitive discounting that is specific to announcements of future policy shocks. This idiosyncratic discounting may stem from factors such as credibility constraints in forward guidance communication.

Keywords: forward guidance; expectations formation; cognitive discounting; behavioral DSGE; monetary policy; system priors.

JEL classification: C11; E7; E32; E37; E52; E58; F41

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1 Introduction

The Great Financial Crises and its aftermath underscored the importance of Forward Guidance (FG) as a key policy instrument during episodes in which the effective lower bound on monetary policy rates becomes binding. By announcing a commitment to keep future rates low, central banks were able to provide further accommodation despite constrained monetary policy rates. This initiated a line of research that revealed that standard monetary DSGE models were not suited to study anticipated monetary policy because they generated unrealistic FG effects. This shortcoming, which Del Negro et al. (2023) named the *forward guidance puzzle*, continues to pose a challenge today. Now, as the ongoing debate on natural interest rates suggest a potential return to a low-rate environment—where effective lower bound episodes are likely to occur more frequently—developing appropriate frameworks to study the impact of forward guidance regained its importance. We contribute in this respect.[1]

In recent years, an increasing number of studies started implementing the bounded rationality hypothesis of Gabaix (2020) to solve the forward guidance puzzle.[2] A behavioral hypothesis that relies on cognitively biased agents to obtain, as most proposed solutions in the literature, a heightened discounting of the future. The set-up, referred to as cognitive discounting (CD), is particularly appealing because of its relative simplicity. However, as we show in the context of the well-known Smets and Wouters (2007) model (SW), an off-the-shelf implementation of Gabaix (2020) to replicate empirical evidence on forward guidance effects leads to a significant attenuation of conventional monetary policy's effectiveness. This dampening effect substantially underestimates the impact of monetary policy reported in other studies.

We find that the coexistence of empirically plausible effects of both conventional monetary policy and forward guidance requires a degree of cognitive discounting that is specific to announcements of future policy shocks.[3] The additional discounting on unrealized future monetary actions could be attributed, for example, to credibility constraints in forward guidance commu-

[1] This view, where pre-pandemic factors—such as sluggish productivity growth, high savings due to ageing populations and rapidly growing emerging markets, and a higher demand for safe and liquid assets amid higher risk aversion—become dominant again is, nonetheless, not uncontested. New technologies, such as AI and green transition solutions, could increase private investment and even potentially rise secular productivity growth. Also, public investment could rise to meet the needs of climate change and to address geopolitical tensions through higher defense spending. These forces would pressure natural rates upwards, potentially off setting the pre-pandemic trends. For a detailed discussion of this topic see Benigno et al. (2024), Fernandez-Villaverde et al. (2024), Schnabel (2024), Obstfeld (2023), and the references within.

[2] See Erceg et al. (2021), Pfäuti and Seyrich (1995), and Hohberger et al. (2024) among others.

[3] In addition to allowing for a proper study of forward guidance, a successful resolution of the puzzle is also important for the study of conventional monetary policy effects. The reason is that the latter can be significantly affected by ignoring forward guidance shocks in a model; see Milani and Treadwell (2012) and English et al. (2003).

nication, in the line, for example, of Campbell et al. (2019) and Bodenstein et al. (2012).

The intuition for why an announcement-specific discounting is needed goes as follows. In empirically relevant monetary models, changes in the nominal interest rate mainly operate through the expectations channel. This can be readily seen by noticing the auto regressive term present in most estimated Taylor rules. It not only captures past influences on current outcomes but also the impact of contemporary shocks on future trajectories. In such a context, a monetary policy rate change induces persistent deviations of future interest rates from the steady state, which, being anticipated by agents in the model, drive most of the effect.[4] At the same time, CD helps to solve the forward guidance puzzle because it tapers the expectations channel through which forward guidance affects the economy. The problem stands because the standard specification for CD entails a common discount factor for all future events, regardless of the shocks that caused them. Then, when trying to moderate forward guidance shocks, the common cognitive discount factor mutes all expectations channels simultaneously, including those crucial for conventional monetary policy. By distinguishing cognitive discount factors by shock type we can resolve this issue.

Our proposed specification also allows for a natural interpretation of the cognitive discount parameter that agents use to discount future events triggered by a forward guidance announcements. Namely, we argue that it captures agents' credibility concerns about the central bank's promises. Indeed, during ELB episodes, as we already mentioned, central banks can stimulate the economy by promising to keep future interest rates low beyond the period in which the policy rate is constraint. This promotes higher current and expected demand and inflation, which, in turn, lowers real interest rates, further stimulating the economy. However, as the economy recovers, central banks have an incentive to renege their prior commitment and prioritize inflation control. As agents understand this dilemma, the monetary authority's credibility becomes a crucial factor.

In this respect, our specification is closely related to the loose commitment or imperfect credibility literature that addresses this type of time inconsistency problems. In that literature agents are assumed to form expectations by weighing two extreme scenarios, one where promises are kept and one where promises are re-evaluated and changed; see Debortoli and Nunes (2010). The weight, given by the probability that promises are kept, captures how credible the promise is. Under CD expectations are formed in a similar way. Agents weigh rational expectations with the steady state, where the weight is given by the CD factor.[5] While in our model the former can

[4]This is precisely the argument in Woodford (1999), who showed that a gradual adjustment of interest rates can be optimal, as it amplifies the effect of monetary policy on long rates and that, therefore, a generalization of the Taylor rule that incorporates inertia in the form of an auto regressive component is desirable.

[5]To be more precise, the weighted average is between rational expectations and the agent's *focal point*, which,

be understood as capturing a scenario with full credibility, the latter reflects a scenario where a promise is not credible at all and is then not expected to affect the economy. Implementing shock-specific cognitive discount factors helps us isolate the credibility concerns agents have about FG. The fact that we only find evidence for CD of forward guidance and not of other future events, reinforces our interpretation.

Bodenstein et al. (2012) analyze policymakers' statements in the context of a simple NK model with optimal monetary policy under imperfect credibility and a zero lower bound. They find evidence suggesting that the Federal Reserve was indeed far from being fully credible in the aftermath of the great financial crises. For this, they depart from statements produced by chairman Ben Bernanke and vice chairman Kohn in 2009, where they emphasized how important not letting inflation increase beyond its long-run target was. Campbell et al. (2019) in a context of noisy signals also provides evidence suggesting the Fed's communicates imperfectly.[6]

To obtain our results, we first develop a strategy for incorporating shock-specific cognitive discount factors into the model. Our approach is intentionally simple and generic, facilitating easy extensions to various settings. Notably, it preserves the model's linearity, enabling the use of efficient linear solution and estimation methods. This retains significant computational advantages, particularly for medium- and large-scale models.

For the estimation of the model, we further propose a strategy based on system priors, building on Andrade et al. (2019). It is a Bayesian approach that allows us to condition our CD estimates—and the whole estimation—on empirical forward guidance impulse responses, in addition to standard macro time series. For this, we rely on Brubakk et al. (2022) and Ferreira (2022), who find that conventional monetary policy and forward guidance shocks have similar—or not significantly different—effects on inflation. Results that are in line with Swanson (2021).

Our estimated rationally bounded model successfully resolves the forward guidance puzzle and delivers data-consistent monetary policy responses, while achieving a better fit to the data than its rational expectations and regular CD counterparts and relying on only slightly modified estimates of structural parameters. Therefore, as such, this strategy offers a largely non-disruptive alternative for implementing FG in standard monetary DSGE models, making it an attractive option, specially for policy institutions.

Observing professional forecasters' expectations about future interest rates, we find that they are close to the average ones formed within the model by the behavioral agents; sharing a similar degree of cognitive discounting when they are the result of future monetary policy

usually, is assumed to be the steady state. We explain this set-up in more detail below.

[6] Goy et al. (2022) studies Delphic and Odyssean forward guidance and their effects during liquidity traps in the context of heterogeneous rationally bounded agents.

announcements. Finally, the model performs well in terms of identifying accommodative forward guidance during the effective lower bound episode that followed the financial crises. Before and after this period, monetary policy announcements tend to remain relatively neutral.

Since the identification of the *puzzle*, the literature has been proposing alternative modifications to the standard NK DSGE model to cope with this shortcoming. For example, McKay et al. (2016) consider incomplete markets and borrowing constraints. The risk of hitting a borrowing constraint in the future effectively shortens the planning horizon, simply because agents cannot react to interest rate changes that take place after the borrowing constraint becomes binding. Additionally, agents' precautionary motives will reduce their response to future interest rate shocks, as running down their assets becomes costly. Del Negro et al. (2023), in turn, propose to introduce perpetual youth as in Blanchard (1985) and Yaari (1965), a set-up that has agents dying with a given probability. This induces a greater discounting and prevents a fraction of the population—the ones that have not yet been born—from reacting to future interest rate changes announcements and so containing the response of consumption decisions. Other proposed solutions include Michaillat and Saez (2021), who introduce wealth in the utility function of households to limit the sensibility of consumption; Cochrane (2017), who argues that the puzzle can be averted by the appropriate equilibrium selection strategy; and Mankiw and Reis (2002), who's sticky information has also been proven successful to mitigate the problem as they render the Phillips curve less forward looking and, hence, make the front loading of it less of an issue; see Carlstrom et al. (2015) and Kiley (2016).[7]

Another important strand of literature resorted to bounded rationality to solve the puzzle. Gertler (2017) adopts a hybrid adaptive/rational expectations belief mechanism that helps cope with the puzzle, adaptive on macro aggregates and rational on policy. Angeletos and Lian (2018) show that relaxing the standard assumption that agents have common knowledge of news and face no uncertainty about how others will respond gives rise to a form of aggregate myopia which resolves the forward guidance puzzle. Farhi and Werning (2019) introduce heterogeneous agents and incomplete markets together with level-k thinking, and argue that the interaction of both frictions is necessary to successfully answer the forward guidance puzzle; and Beqiraj et al. (2019) tappers the effects of forward guidance by considering heterogeneous agents of which a part remains rational while the rest features bounded rationality and forecasts future variables as a fraction of their past value. Also in this category falls the CD approach of Gabaix (2020) that we consider.[8]

[7] In a recent paper Rannenberg (2024) shows that by introducing preferences over safe assets in a New Keynesian model, consumption responsiveness to future interest rates diminishes and a real wealth effect from real government bond holdings is generated that attenuates the forward guidance puzzle.

[8] General note. The fact that in a given model forward guidance does not generate unrealistically large re-

More closely related to our work Ilabaca et al. (2020), Andrade et al. (2019), Hirose et al. (2022) estimate simple three-equations behavioral New Keynesian (NK) models under CD. In particular, Hirose et al. (2022) incorporate a zero lower bound constraint on monetary policy and estimate the nonlinear model. They find that their model fits the data better than its rational expectations counterpart and find a cognitive discount parameter of about 0.85; on the upper side of the estimates found by the other two previous papers, which range from 0.4 to 0.85. Kolasa et al. (2022), Brzoza-Brzezina et al. (2022), in turn, introduce CD into small open economy models and study its effect on their dynamics, finding that it improves the model's ability to account for certain phenomena, for example, related to the UIP. Kolasa et al. (2022) estimate a cognitive discount parameter of about 0.5, whereas Brzoza-Brzezina et al. (2022) find a value of about 0.7. Moreover, Brzoza-Brzezina et al. (2022) claim that monetary policy is less powerful when agents are behavioral. A similar result that is found by Hohberger et al. (2024) in the context of a SW type of model. As we already argued, these results are to be expected under the standard CD specification and are at odds with the empirical evidence. Erceg et al. (2021) and Meggiorini (2023) are two additional examples of estimations of the SW model under CD. The former, estimate the model for Europe and, despite a relatively high cognitive discount factor estimate of ca 0.95, also find that conventional monetary policy is significantly less effective than under rational expectations. The latter, emphasizes that the behavioral assumption dominates rational expectations in terms of data fit.

Finally, Andrade and Ferroni (2021), Brubakk et al. (2022) and Ferreira (2022) are three exponents of the type of empirical evidence we use to discipline our model.[9] They use high frequency data together with sign and zero restrictions to identify forward guidance shocks, which are then included in VARs to obtain their effect on aggregate macro variables.[10] In particular, we will rely on the last to papers, as they provide estimates of the impact on inflation that forward guidance and surprise monetary policy shocks have. More concretely, they find that an Odyssean forward guidance shock has similar or not significantly different effects on inflation compared to an unanticipated monetary policy shock of the same magnitude.[11]

sponses, does not necessarily mean that the causes of the problem have been resolved. The underlying transmission mechanisms of the economy may still very well affect or distort other dimensions of a model and, ultimately, prescribe ill policy recommendations. Basically, the Lucas critique.

[9]Other papers studying the effect of forward guidance - in a low interest rate context - include Swanson (2021), Del Negro et al. (2023), Kiley and Roberts (2017), Chung et al. (2019) and Coenen et al. (2021).

[10]Whereas Andrade and Ferroni (2021) impose sign and zero restrictions on interest rates and inflation expectations and finds that the ECB was able to stimulate the economy with Odyssean forward guidance policy; the other two papers impose, as Jarociński and Karadi (2020), restrictions on stock market price variations and interest rates. Closely related to D'Amico and King (2015), these papers try also to differentiate between Delphic and Odyssean—using the terminology of Campbell et al. (2012)—forward guidance; that is shocks that convey information about the future outlook of the economy and shocks which inform about future deviations from the policy rule. They add survey forecasts to an otherwise standard VAR for the US and find quite sizable effects of forward guidance, and that the effects increase with the horizon.

[11]Smith and Becker (2015) find that forward guidance yields quantitatively similar effects on US employment

The paper proceeds as follows. Section 2 examines the forward guidance puzzle. Section 3 briefly outlines the Smets and Wouters (2007) model and introduces cognitive discounting, both as proposed by Gabaix (2020) and our shock-specific framework. Section 4 and 5 discuss estimation strategy and results, respectively. Section 6 concludes.

2 The Forward Guidance Puzzle

As it has now been well established in the literature, standard NK DSGE models are not well suited for the study of forward guidance policy. Even though they are built upon the precondition of forward-looking agents, they deliver unrealistically large responses of prices and activity, which, additionally, tend to increase as the announced future interest rate policy change moves further away in time; even reaching explosive dynamics in some cases.[12] Del Negro et al. (2023) named this limitation *the forward guidance puzzle*. The main culprits for these shortcomings are an implausible high sensitivity of consumption to future interest rate changes and the front-loading associated with the NK Phillips curve.

Borrowing from McKay et al. (2016), let us look a little bit deeper into this issue. Consider the Euler equation of the canonical NK monetary policy model:

$$c_t = E_t(c_{t+1}) - \sigma^{-1} E_t(i_t - E_t[\pi_{t+1}] - r_t^n) \tag{1}$$

where c_t stands for consumption, i_t for the short-term nominal interest rate, π_t for price inflation, r_t^n for the natural real interest rate, σ^{-1} is the intertemporal elasticity of substitution and $E_t[\cdot]$ denotes the rational expectations operator. Iterating (1) forward we can explicitly see how, in this model, current consumption depends on the entire future real interest rate path, and that a change, say, in period $t+j$, has the same direct effect as a change in period t,

$$c_t = -\sigma^{-1} \sum_{j=0}^{\infty} E_t(i_{t+j} - \pi_{t+1+j} - r_{t+j}^n) \tag{2}$$

This constitutes the first mechanism behind the puzzle, the excessive sensitivity of consumption to interest rate changes. It is worth noticing that, as Walsh (2009) points out, this equation makes explicit a potential overstatement of the control central banks have on agents' expectations over future nominal interest rates. Lack of credibility, most likely, would prevent central banks from manipulating those expectations. Our interpretation of the shock-specific CD factor

and inflation as conventional policy shocks.

[12] See Del Negro et al. (2023), McKay et al. (2016) and Carlstrom et al. (2015).

for forward guidance-triggered future events builds on this idea.

Let us now consider the second mechanism behind the puzzle, namely, the Phillips curve front-loading. For this let us also iterating the Phillips curve of the same canonical NK model forward, i.e.,

$$\pi_t = \beta E_t[\pi_{t+1}] + \kappa c_t \implies \pi_t = \kappa \sum_{j=0}^{\infty} \beta^j E_t[c_{t+j}] \tag{3}$$

Equation (3) shows how current inflation in the model depends on the whole expected future consumption sequence. Consider now an expected change, say a cut, in the future nominal interest rate of period $t+k$, with $k > 0$. Noticing that equation (2) also holds in expectations for all t, we can see that the expected interest rate cut not only increases current consumption but also all expected consumptions between period t and $t+k$. Consequently, the sum in equation (3), and therefore the effect on current (and expected) inflation, grows with the horizon of the announced future interest rate cut. This effect on inflation (current and expected) further reduces the real interest rate upon which consumption depends—equation (2)—further increasing consumption between periods t and $t+k$, further increasing inflation and so on and so forth. A feedback process that amplifies the initial direct effects. These are the main mechanisms behind the *forward guidance puzzle*: the high sensitivity of consumption to expected interest rates - which does not diminish with the horizon - and the amplification mechanism triggered by the front-loading of the Phillips curve.

The literature has proposed to taper this mechanism by introducing frictions that limit how forward-looking agents form expectations, e.g., bounded rationality, by proposing alternative preferences, e.g., for safe assets or wealth, or by preventing agents from reacting to expected future monetary policy changes, e.g., using binding borrowing constraints. We adopt the bounded rationality approach proposed by Gabaix (2020).

3 A standard DSGE model

To illustrate our findings, we employ the well-known medium-scale DSGE model developed by Smets and Wouters (2007). It is a New Keynesian model that incorporates various real and nominal frictions to simulate the US economy and was originally estimated using data from 1966 to 2004; it builds on Christiano, Eichenbaum, and Evans (2005) and Smets and Wouters (2003) and we refer to the published paper for details.[13] To this model we add forward guidance shocks.

[13] Pfeifer (2015) provides codes that correct some minor issues of the original paper version; we base ourselves on his version.

By building on a familiar model, we want to make our results more understandable, transparent, and to facilitate comparisons with already established results and with other proposed solutions for the *forward guidance puzzle.*[14]

The new Taylor rule in the model is the same as the original except that it now features forward guidance shocks. This are introduced as usual, i.e.,

$$r_t = \rho r_{t-1} + (1-\rho)[r^{\pi}\pi_t + r^{y}(y_t - y_t^f)] + r^{\delta y}[(y_t - y_t^f) - (y_{t-1} - y_{t-1}^f)] + \varepsilon_t + \varepsilon_t^{fg} \tag{4}$$

where

$$\varepsilon_t^{fg} = \rho_{FG}\varepsilon_{t-1}^{fg} + e_{t-1}^{fg,1} + e_{t-2}^{fg,2} + e_{t-3}^{fg,3} + e_{t-4}^{fg,4} + e_{t-8}^{fg,8} \tag{5}$$

and $e_t^{fg,i}$ are iid Gaussian shocks with mean 0 and standard deviation σ. We consider forward guidance shocks 1, 2, 3, 4 and 8 periods ahead. The first 4 horizons are needed to construct the IRFs of inflation to a one-year-ahead announcement of a policy change, while keeping the nominal interest rate unchanged before that. Having a maximum horizon of two years, i.e., 8 periods ahead, allows us to capture most future monetary policy announcements post financial crises, while retaining parsimony.[15]

We begin by showing that the SW model indeed suffers from the *forward guidance puzzle.* Figure (1), following McKay et al. (2016) and Gabaix (2020), depicts the response of current inflation to an announced one period real interest rate cut h periods ahead relative to its response to a rate cut today of the same magnitude; blue line.[16] So, for example, announcing a real rate cut three years from now - for one quarter - induces an increase in current inflation that is about 50 times larger than the increase resulting from a real rate cut today of the same magnitude. As the horizon of the announced cut moves later in time, the response of current inflation increases relative to the response to the current cut. These implied dynamics are unrealistic and, at least partially, product of extreme assumptions on rationality, credibility and forward-lookingness in the model. Figure (1) also plots the same ratios for output and inflation when forward guidance is about the nominal rate. In that case, after an initial growth, the ratio reverses, and changes sign. Then, it continues to increase in magnitude. This is due to the so called *reversal puzzle*, that tends to emerge in models featuring inflation indexation as SW; see Carlstrom et al. (2015)

[14]We thank an anonymous referee for this suggestion. In a previous version of the paper we used the main DSGE monetary model of the Central Bank of Chile to illustrate our proposed framework, where we obtained similar findings; see Arias et al. (2023).

[15]For example, in August 2011 the fed announced that it was likely to keep the federal funds rate at exceptionally low levels "at least through mid-2013".

[16]The forward guidance puzzle is not straight forward to elicit. Looking directly at forward guidance shocks realization hides the endogenous response of monetary policy. The latter is the reason why announcements include keeping the rate unchanged until the promised cur.

for an explanation.[17] As it turns out, our shock-specific cognitive discounting approach, also resolves this problem.

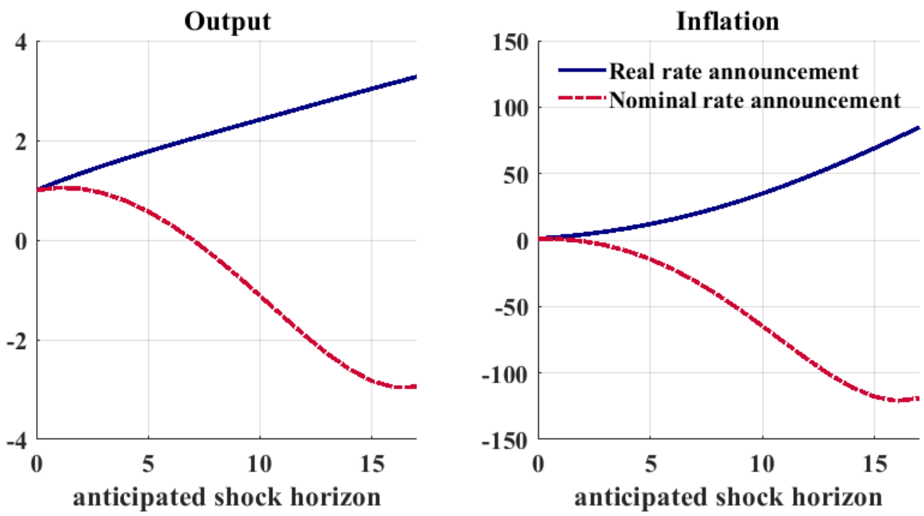

Figure 1: Forward guidance puzzle in the Smets and Wouters (2007) model. Curves depict the ratio between the on-impact effect on output (left) and inflation (right) of an announced t quarters ahead interest rate cut relative to the effect of a contemporary cut of the same magnitude. The interest rate is assumed to remain at its steady state value before and after the cut. Solid blue lines denote the effects of an announced real rate trajectory. Red dashed lines denote the effects of an announced nominal rate trajectory.

3.1 Introducing behavioral features

Following Gabaix (2020), we deviate from the rational expectations (RE) hypothesis assumed in the original Smets and Wouters (2007) model and implement CD in order to solve the forward guidance puzzle. This behavioral approach for imputing agents' expectations amounts to introducing a type of myopia which increases exponentially with the horizon and results in a decreasing sensitivity to future events. The original idea seeks to capture a limited understanding of the world, which further diminishes as events take place further in the future. As such, a proposed—illustrative—micro-foundation of this expectations formation mechanism departs from agents that receive noisy signals about the economy. Gabaix (2020) provides a series of other possible, though not universal, micro-foundations for this set-up, including how other specifications, even under rational expectations, can give rise to a similar discount factor. Our interpretation of this set-up, when we limit CD to forward guidance shocks, as capturing credibility concerns falls into this last category.

According to Lemma 1 in Gabaix (2020), we can write agents' expectations in the model

[17] This is also the reason why usually the real rate is used in these exercises.

under this bounded rationality hypothesis (BR) as a shrinkage of rational expectations towards a focal point, where the degree of shrinkage is governed by the cognitive discount parameter M,

$$E_t^{BR}[x_{t+k}] = E_t^{BR}[x_{t+k}^{fp} + \hat{x}_{t+k}] = E_t[x_{t+k}^{fp}] + M^k E_t[\hat{x}_{t+k}] = \bar{x} + M^k E_t[\hat{x}_{t+k}] \quad (6)$$

where the variable x_{t+k} is decomposed as the sum of its associated focal point, x_{t+k}^{fp}, and its deviation from it, $\hat{x}_{t+k} = x_{t+k} - x_{t+k}^{fp}$. E_t^{BR} denotes rationally bounded expectations and E_t rational ones. The last equality follows from assuming, as most of the literature, the steady state, $\bar{x}$, as the focal point. We present a slightly more general specification because this will prove useful when we show how to introduce shock-specific cognitive discount factors.[18] This rationally bounded expectations can also be written as a weighted average between the focal point x_{t+k}^{fp} and the rational expectation of the variable, $E_t[x_{t+k}]$, i.e.,

$$E_t^{BR}[x_{t+k}] = E_t[x_{t+k}^{fp}] + M^k E_t[\hat{x}_{t+k}] = (1 - M^k)E_t[x_{t+k}^{fp}] + M^k E_t[x_{t+k}] \quad (7)$$

Then, if $|M| < 1$, as agents forecast events further away in time, their projections tend to increasingly concentrate around the variable's steady state. Sufficiently distant events, i.e., deviations from the steady state, become negligible and so do their direct effects on current decisions. Note that this specification nests the rational expectations hypothesis, which holds when $M = 1$.

The implementation of CD as originally proposed and applied so far assumes that all expected future events, regardless of how they are generated, are discounted with the same factor. This can be directly seen from the last term in equation (6), i.e., $M^k E_t[\hat{x}_{t+k}]$. The expected future deviation of a given variable x_{t+k} from its focal point is discounted by a factor M^k irrespective of the shock that induced that deviation. This is reasonable if one interprets CD as capturing a diminished understanding of the future. Where the crucial dimension is the distance with the observable present and not the source of the event being projected. However, this specification becomes a problem in our NK model. Figure (2) shows conventional monetary policy impulse responses for the SW model under rational expectations and under this original bounded rationality specification of Gabaix (2020), which assumes a single common cognitive discount parameter.[19] Even though a conventional monetary policy shock induces a substantially more tight policy under BR than under RE, the responses of output, inflation and consumption are exceedingly smaller than under RE.[20] The reason is that in empirically relevant monetary models, both

[18] As Gabaix (2020) explains, the focal point is, however, not limited to the steady state.

[19] All model specifications are estimated using a sample that extends the original one until 2019.

[20] This result is similar to the ones in Hohberger et al. (2024), Brzoza-Brzezina et al. (2022), and Erceg et al. (2021).

conventional monetary policy and forward guidance operate through the expectations channel.[21] Since, CD, helps solve the forward guidance puzzle by tapering the expectations channel, a problem stands when trying to moderate the effect that forward guidance has on the economy, as the common cognitive discount factor mutes all expectations channels simultaneously, including those crucial for conventional monetary policy.

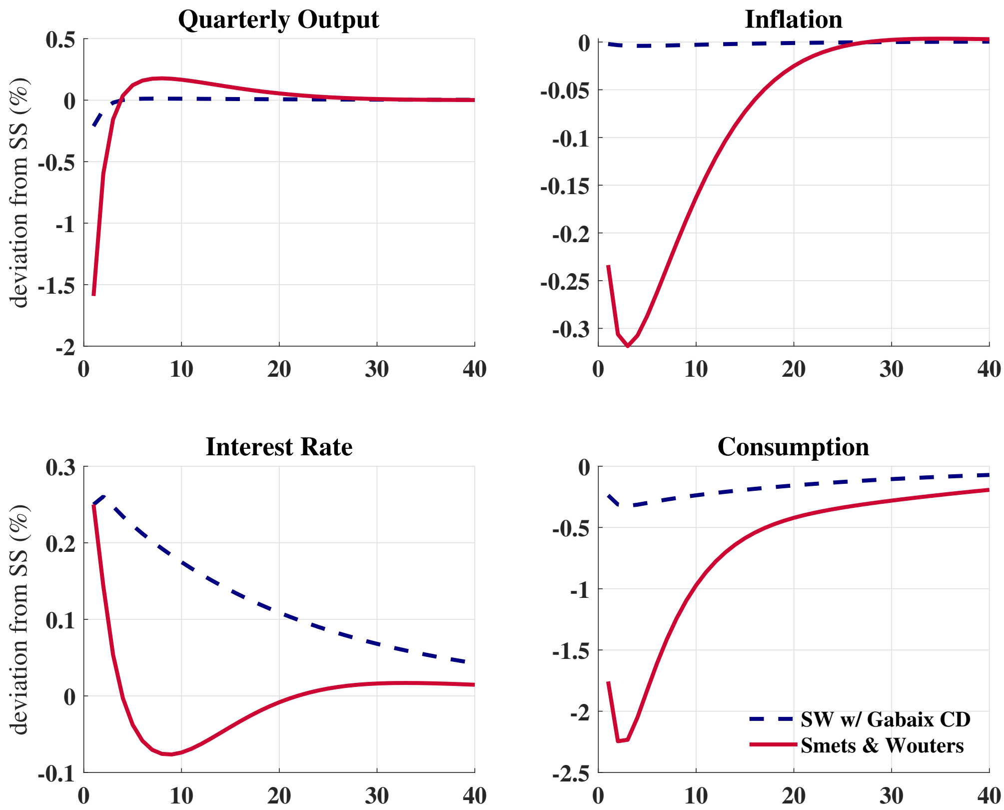

Figure 2: Impulse response functions of selected variables to a conventional monetary policy shock. The red solid line corresponds to the SW model. The blue dashed line corresponds to the same model under the standard CD specification introduced in Gabaix (2020) and estimated for the same sample.

Consequently, we propose allowing for different cognitive discount factors depending on the source of the future event that agents observe. Our findings support this specification. In particular, we find that agents only discount expected future deviations triggered by forward guidance shocks. This, and the close relation our shock-specific CD framework has with the loose commitment or imperfect credibility literature, as mentioned in the introduction, lead us to interpret the CD factor as a measure of forward guidance credibility.

In what follows, we explain how we implement our shock-specific discount parameters into the SW model. We will show how to introduce agents that cognitively discount future events

[21] See our discussion in the introduction about the role of the auto regressive term in Taylor rules.

generated by forward guidance shocks, while entertaining rational expectations about all other future events. The generalization of this scheme, where each future event is discounted differently depending on the shock that produced it, is left for the appendix; see section (C.1).

Given that the model is linear we introduce shock-specific factors by means of an auxiliary model. Consider two sub-models, the first one, $\mathcal{M}$, equal to the SW model except it features forward guidance shocks and a different expectations formation mechanism; the second one, $\mathcal{M}^{aux}$, exactly the same as the first one, except that it does not feature forward guidance shocks. Variables in the auxiliary sub-model are denoted with the superscript fp, which stands for focal point; variable sin the main sub-model are also the variables in our economy, the ones we are interested in (e.g., consumption is denoted as c_t in sub-model $\mathcal{M}$ and c_t^{fp} in sub-model $\mathcal{M}^{aux}$; when we want to see the effect of any shock on consumption, we look at c_t). We will refer to sub-model $\mathcal{M}$ as the main sub-model and to sub-model $\mathcal{M}^{aux}$ as the auxiliary sub-model. Our economy under shock-specific CD - or final model - will comprise both sub-models. In line with equation (7), agents in the main sub-model form expectations of forward-looking variables, say x_{t+k}, according to,

$$E_t^{BR}[x_{t+k}] = (1 - M^k)E_t[x_{t+k}^{fp}] + M^k E_t[x_{t+k}] \tag{8}$$

where x_{t+k}^{fp} is fully determined in sub-model $\mathcal{M}^{aux}$. The auxiliary model features rational expectations, i.e., $E_t[x_{t+k}^{fp}]$. [22]

Consider any non-forward guidance shock hitting our economy. Since both sub-models are linear and exactly the same - except for the presence of forward guidance shocks - such a non-forward guidance shock will induce the same responses in both sub-models.[23] The idea is that if $x_{t+k}^{fp} = x_{t+k}$ for all x and for all $k \geq 0$ when a non-forward guidance shock hits, expectations in both sub-models coincide, i.e.,

$$\begin{aligned} E_t^{BR}[x_{t+k}] &= (1 - M^k)E_t[x_{t+k}^{fp}] + M^k E_t[x_{t+k}] \\ &= (1 - M^k)E_t[x_{t+k}] + M^k E_t[x_{t+k}] = E_t[x_{t+k}] \end{aligned} \tag{9}$$

Consider now a forward guidance shock hitting our economy, which starts from its steady state. Since this shock is not present in the auxiliary sub-model, variables x_{t+k}^{fp} are not affected and remain in the steady state. One can thing of x_{t+k}^{fp} as an exogenous variable from the perspective of sub-model $\mathcal{M}$. In turn, since forward guidance shocks are present in the main sub-model,

[22] Remember both sub-models are exactly the same except for forward guidance shocks, since we want the main model to feature RE about all future events not induced by forward guidance shocks, the auxiliary model features rational expectations. We relax this RE assumption later.

[23] Notice that in a linear model the effects of different shocks are additive.

variables x_{t+k} will be affected. Then, expectations in the main sub-model - and our economy - become,

$$\begin{aligned} E_t^{BR}[x_{t+k}] &= (1-M^k)x_{t+k}^{fp} + M^k E_t[x_{t+k}] \\ &= (1-M^k)\bar{x} + M^k E_t[x_{t+k}] = \bar{x} + M^k E_t[\hat{x}_{t+k}] \end{aligned} \tag{10}$$

notice that we used the fact that because both sub-models are equal (except for the forward guidance shocks) they have the same steady state, i.e., $\bar{x}^{fp} = \bar{x}$. That is, when a forward guidance shock induces a deviation in a given variable x_{t+k}, agents cognitively discount this deviation.

This can be seen graphically in figure (3). On the left we can see the impulse responses of inflation to a conventional monetary policy shock and on the right the impulse responses for a forward guidance shock. While the impulse responses to the conventional monetary policy shock are the same, they are, as constructed, different for the forward guidance shock. Indeed, inflation in the main sub-model π_{t+k} clearly responds to the FG shock, but inflation in the auxiliary sub-model, π_{t+k}^{fp}, does not and stays at its steady state value.

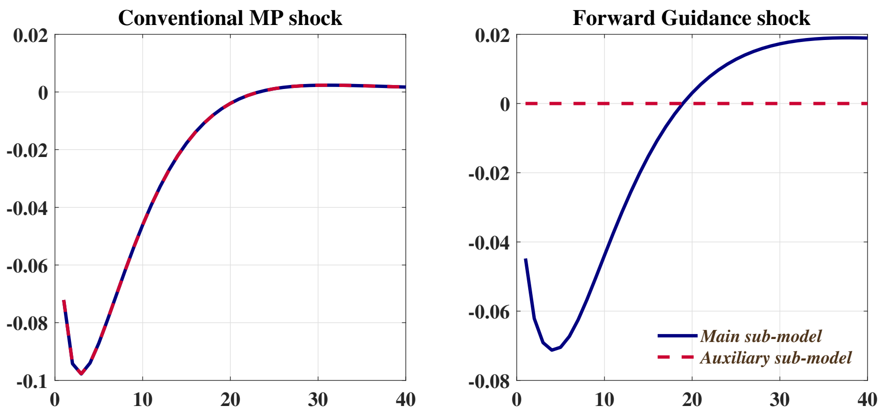

Figure 3: Impulse response functions of inflation to a 25bp conventional monetary policy increase (left) and to a 25bp FG shock (right). Blue line corresponds to the main sub-model $\mathcal{M}$, red dashed line to the axuiliary sub-model $\mathcal{M}^{aux}$.

This strategy for introducing shock-specific CD is intentionally simple and generic, facilitating easy extensions to various settings. Notably, it preserves the model's linearity, enabling the use of efficient linear solution and estimation methods. This retains significant computational advantages, particularly for medium- and large-scale models. In section (C) of the Appendix we briefly explain how to implement this set-up in the context of Dynare and present a generalization that allows for different CD factors depending on the shock that induced a future event.

4 Estimation

For the estimation of the model we further propose a strategy based on system priors, building on Andrade et al. (2019).[24] It is a Bayesian approach that allows us to condition our CD estimates—and the whole estimation—on empirical forward guidance impulse responses functions (IRFs), in addition to standard macro time series.

Succinctly, *system priors* use ex ante probability distributions on certain desired model properties—e.g., dynamics, moments, etc.—to guide the model into satisfying them. Working as standard priors on model parameters, they constitute a simple and transparent way to impose intuitive and economic priors about the dynamics of the model, which can still be redirected by the evidence contained in, say, macro time series. We will use a system prior to induce our behavioral model to deliver IRFs for conventional monetary policy and forward guidance shocks that are similar to each other, in accordance with the results of Brubakk et al. (2022) and Ferreira (2022). Regarding the size of the effects and the precise trajectory they follow, we will take an agnostic view, and rely on the data and the model to determine them.

Brubakk et al. (2022) and Ferreira (2022) find that conventional monetary policy and forward guidance shocks have similar, or not significantly different, effects on inflation.[25] To arrive to this results they use a similar methodology to Jarociński and Karadi (2020), Andrade and Ferroni (2021), and D'Amico and King (2015). Using high frequency data on both interest rates and stock prices and sing and zero restrictions they identify three different types of monetary policy shocks. A conventional surprise shocks, which captures contemporaneous deviations from the policy rule, a *Delphic* shock, which reveals information about the central bank's assessment of the economic outlook (also known as informational shocks), and an *Odyssean* shock, which expresses a commitment to a certain future policy path.[26] The identified monetary shocks are then included as exogenous variables in a VAR to obtain IRFs for, among other variables, inflation.[27]

Consider a model of the economy, $\mathcal{E}$. The standard Bayesian approach for estimating its parameters, θ, consists in updating the prior beliefs about those parameters, $p(\theta \mid \mathcal{E})$, with the evidence stemming from the data, Y, as seen through the model, i.e., by means of the model's likelihood, $L(Y \mid \theta, \mathcal{M})$. We can then obtain the posterior of the parameters applying Bayes

[24]It constitutes a generalization of the *endogenous priors* concept of Christiano et al. (2011) and the work on how to set priors for DSGE models of Del Negro and Schorfheide (2008), among others.

[25]Results that are in line with, for example, Swanson (2021)

[26]As earlier work has shown, it is very important to account for this information channel when identifying the macroeconomic effects of policy surprises extracted from financial data, since not doing so could lead to a biased evaluation of monetary policy shocks (Campbell et al. (2012); Nakamura and Steinsson (2012); Andrade and Ferroni (2021); Jarociński and Karadi (2020); Miranda-Agrippino and Ricco (2021); Bauer and Swanson (2023) argue that the so called information channel can alternatively be explained by Fed response to news.

[27]Whereas Brubakk et al. (2022) use data for Norway and Sweden, Ferreira (2022) uses data for the US.

rule,

$$p(\theta \mid Y, \mathcal{E}) = \frac{L(Y \mid \theta, \mathcal{E}) \times p(\theta \mid \mathcal{E})}{p(Y \mid \mathcal{E})} \tag{11}$$

the prior specification in the previous rule is not restricted to the marginal priors of parameters contained in θ. Andrle and Benes (2012) propose, instead, to employ a composite prior built as the combination of the standard marginal distributions for individual parameters and a system prior on the desired properties of the model. For this, in addition to the - usually - independent marginal priors on parameters, i.e., $p(\theta \mid \mathcal{E}) = p(\theta_1) \times p(\theta_2) \times ... \times p(\theta_n)$, one specifies a distribution on a set o model properties, $Z = h(\theta)$. These properties are endowed with a proper probabilistic model $Z \sim S(Z^s)$. Then, the likelihood of the non-sample information, say $Z = z$, is denoted by $p_s(z \mid \theta, h, \mathcal{E})$. Then, combining the marginal priors on the parameters with the system prior on the desired properties, the composite prior takes the following form,

$$p_c(\theta \mid z, h, \mathcal{E}) \propto p_s(z \mid \theta, h, \mathcal{E}) \times p(\theta \mid \mathcal{E}) \tag{12}$$

where $p_c(\cdot)$ and $p_s(\cdot)$ denote the composite and system prior respectively. The posterior distribution of the model's parameters can be, then, obtained from

$$\begin{aligned} p(\theta \mid Y, z, \mathcal{E}) &\propto L(Y \mid \theta, \mathcal{E}) \times p_c(\theta \mid z, h, \mathcal{E}) \\ &\propto L(Y \mid \theta, \mathcal{E}) \times p_s(z \mid \theta, h, \mathcal{E}) \times p(\theta \mid \mathcal{E}) \end{aligned} \tag{13}$$

notice that the likelihood does not depend on the properties z, since conditional on the parameters the model is fully defined as a distribution for the data. This posterior can, then, be simply explored and constructed using, for instance, a standard Metropolis-Hastings algorithm.

Given that we want to incorporate information about the similarity between the IRFs of inflation to a conventional monetary policy shock and to a forward guidance shock of the same magnitude, we choose $h(\cdot)$ to be the Euclidean distance between to vectors. For the distribution of this measures, i.e., $S(\cdot)$, we assume a normal distribution with mean 0 and standard deviation 0.0675. Concretely, the algorithm will draw from the parameter space, compute the IRFs conditional on that draw—both for a conventional contemporaneous monetary policy and for a forward guidance shock—, calculate the Euclidean distance and evaluate in the normal distribution.[28]

Regarding the priors for the parameters, $p(\theta_i)$, we keep the ones in the original paper. We add the prior for the cognitive discount factor, which we set as a $beta(0.5, 0.2)$, a rather agnostic prior. Additionally, we set the priors for the forward guidance shocks equal to the one of the

[28]The forward guidance shock corresponds to a promise to keep the nominal interest rate unchanged for one year and to change it at the end of that year by the same magnitude as the conventional shock.

conventional monetary shock (see table 2). Since we care, particularly, about forward guidance policy during effective lower bound episodes, we extend the original Smets and Wouters (2007) sample until 2019; just before the pandemic. This amounts to 15 more years of data relative to the original paper, and includes, of course, the aftermath of the financial crises. We use the same standard macro time-series as the original paper. Importantly, as Nelson (2021) shows, the Federal Reserve did not start resorting to announcements about their future monetary policy stance until the 2000s.[29] Consequently, we limit forward guidance shocks only to the post 2000 period. Finally, we compute the posterior estimates by means of a random walk Metropolis-Hastings algorithm, with one million draws and five thousand burn-in. Conventional convergence tests are checked.

5 Results

Tables (1) and (2) present the estimation results of the non-cognitive discounting parameters, together with the assumed priors. The estimation of the original model under rational expectations, RE, is shown as a reference side-by-side our *baseline* behavioral model, BR.

The estimation results show that both structural and exogenous shocks parameters remain largely unchanged. This has to do with the non-disruptive way in which cognitive discounting is introduced, namely, only affecting the expected future deviations from steady state generated by future monetary policy announcements.

Table (3), in turn, shows the posterior estimates for the cognitive discounting parameters and the marginal data density (MDD) for different model versions. First, we consider the SW model under rational expectations and forward guidance shocks in column RE. This specification, as in the previous tables, is used as a reference.[30] Second, we consider the SW model under cognitive discounting as it was originally proposed by Gabaix (2020), i.e., where all future events are discounted equally by a common factor, regardless of the shock that caused them. We denote this second version BR^{std}.

The third version we consider is our baseline specification, featuring shock-specific CD factors only for future events generated by forward guidance shocks and estimated with system priors, BR. The fifth version help us understand the role of system priors. We denote it BR w/o SP and it is the same as the BR specification but estimated without system priors. Finally, the last

[29]This shift in views is in line with, for example, Eggertsson and Woodford (2003), who showed how a commitment to future low policy rates can help stimulate activity when a lower bound on interest rates is binding; in line with Krugman (1998) proposals for Japan.

[30]The original SW model with RE is equivalent to a CD specification that imposes a cognitive discounting factor equal to one.

Parameter	Description	Initial Prior			Posterior RE				Posterior BR			
		distr.	mean	s.d.	mode	mean	pct. 5	pct. 95	mode	mean	pct. 5	pct. 95
ϕ	Adjustment costs	N	4	1.5	4.14	4.32	3.01	5.61	4.50	4.71	3.29	6.12
σ^c	risk aversion	N	1.5	0.375	1.31	1.45	1.05	1.86	1.67	1.60	1.06	2.09
h	external habit degree	B	0.7	0.1	0.46	0.45	0.35	0.55	0.41	0.44	0.32	0.56
χ^w	Calvo parameter wages	B	0.5	0.1	0.78	0.77	0.71	0.84	0.77	0.76	0.70	0.82
σ^L	Frisch elasticity	N	2.0	0.75	1.41	1.52	0.78	2.27	1.56	1.51	0.75	2.24
χ^p	Calvo parameter	B	0.5	0.1	0.64	0.65	0.57	0.72	0.65	0.63	0.56	0.71
ι^w	wages indexation	B	0.5	0.15	0.49	0.49	0.32	0.67	0.50	0.50	0.32	0.67
ι^p	prices indexation	B	0.5	0.15	0.38	0.36	0.22	0.49	0.37	0.34	0.20	0.47
ψ	Capacity utilization cost	B	0.5	0.15	0.38	0.45	0.27	0.63	0.39	0.48	0.28	0.69
Φ	fixed cost share	N	1.25	0.125	1.53	1.54	1.41	1.66	1.50	1.54	1.43	1.66
r^π	Taylor rule inflation feedback	N	1.5	0.25	2.03	2.09	1.81	2.35	2.02	2.06	1.79	2.33
ρ	interest rate persistence	B	0.75	0.10	0.88	0.88	0.86	0.91	0.88	0.88	0.86	0.91
r^y	Taylor rule output level feedback	N	0.125	0.05	0.03	0.05	0.01	0.09	0.07	0.06	0.01	0.10
$r^{\delta y}$	Taylor rule output growth feedback	N	0.125	0.05	0.20	0.20	0.16	0.23	0.18	0.19	0.15	0.22
$\bar{\pi}$	Steady state inflation rate	G	0.625	0.1	0.83	0.84	0.66	1.01	0.82	0.85	0.66	1.03
β	time preference rate in percent	G	0.25	0.1	0.10	0.12	0.05	0.19	0.10	0.12	0.05	0.19
$\bar{L}$	steady state hours	N	0.0	2.0	-2.34	-1.14	-4.71	2.31	1.01	-0.27	-3.92	3.13
$\bar{\gamma}$	net growth rate in percent	N	0.4	0.1	0.42	0.41	0.38	0.45	0.40	0.41	0.36	0.45
α	capital share	N	0.3	0.05	0.17	0.17	0.14	0.20	0.16	0.17	0.14	0.20

Table 1: Prior and posterior distributions - Structural parameters. The prior distributions are: beta (B), inverse gamma (IG), gamma (G), and normal (N). Both models are estimated with forward guidance shocks, as modeled in equation (5) and with a sample that extend until the fourth quarter of 2019. Column RE refers to the model under rational expectations, while BR refers to our baseline version (with shock-specific CD parameter for FG shocks and estimated with system priors (SP).

version, helps us justify our assumption that agents hold rational expectations on events that are not generated by forward guidance shocks. We denote this version, featuring two shock-specific CD factors, one for forward guidance-triggered events and one for all other future events, BR^{2CD}.

We begin with the BR^{std} model, which yields a CD factor estimate of 0.6. Interestingly, this value, is very similar to the one found by Meggiorini (2023). That paper, as mentioned in the introduction, studies CD in its standard form in the context of the SW model, although it presents two main differences with the BR^{std} model. First our model is estimated using a longer sample that extends until 2019; instead, she uses the original Smets and Wouters (2007) sample that extends until 2004. Second, our model features forward guidance shocks. Also, we too find that deviating from RE in this way improves the model's fit to the data. However, as we showed in Figure (2) this specification significantly mutes conventional monetary policy, result that is at odds with empirical evidence (see Romer and Romer, 2004 and Miranda-Agrippino and Ricco, 2021).

Implementing shock-specific CD as in BRw/oSP, instead of the standard form of CD, yields a smaller CD factor estimate of 0.53. This shows that most of the reason behind the larger CD factor in our baseline model is due to the adoption of a system prior. In addition, while the CD factor in the BR version is estimated very precisely around 0.94, the two models under bounded rationality estimated without SP yield much less precise estimates. The baseline model, BR, also is the version with the best fit to the data, having a marginal data density of -2823.6, at least 5 log points better than the second best. More importantly, as we later show, the baseline model achieves this better fit, while delivering empirically plausible conventional monetary policy

Parameter	Description	Initial Prior			Posterior RE				Posterior BR			
		distr.	mean	s.d.	mode	mean	pct. 5	pct. 95	mode	mean	pct. 5	pct. 95
ρ_a		B	0.5	0.2	0.98	0.98	0.98	0.99	0.99	0.99	0.98	0.99
ρ_b		B	0.5	0.2	0.89	0.88	0.84	0.92	0.89	0.87	0.83	0.92
ρ_g		B	0.5	0.2	0.98	0.98	0.96	0.99	0.99	0.98	0.97	0.99
ρ_I		B	0.5	0.2	0.64	0.71	0.58	0.83	0.67	0.71	0.59	0.83
ρ_r		B	0.5	0.2	0.14	0.15	0.06	0.23	0.12	0.14	0.06	0.22
ρ_p		B	0.5	0.2	0.99	0.97	0.99	0.96	0.99	0.98	0.97	0.99
ρ_w		B	0.5	0.2	0.99	0.97	0.99	0.94	0.99	0.98	0.97	0.99
μ_p		B	0.5	0.2	0.92	0.90	0.84	0.95	0.91	0.88	0.82	0.95
μ_w		B	0.5	0.2	0.96	0.95	0.92	0.97	0.96	0.95	0.92	0.97
ρ_{ga}		B	0.5	0.25	0.59	0.59	0.50	0.70	0.57	0.58	0.46	0.69
ρ_{FG}		B	0.5	0.2	0.40	0.37	0.16	0.57	0.20	0.25	0.08	0.41
σ_a		IG	0.1	2.0	0.52	0.53	0.48	0.57	0.54	0.53	0.48	0.57
σ_b		IG	0.1	2.0	0.10	0.10	0.08	0.12	0.09	0.10	0.08	0.12
σ_g		IG	0.1	2.0	0.60	0.61	0.56	0.65	0.60	0.61	0.56	0.65
σ_I		IG	0.1	2.0	0.45	0.43	0.36	0.51	0.44	0.43	0.36	0.50
σ_r		IG	0.1	2.0	0.22	0.22	0.20	0.24	0.21	0.22	0.20	0.24
σ_p		IG	0.1	2.0	0.22	0.21	0.19	0.24	0.21	0.21	0.18	0.24
σ_w		IG	0.1	2.0	0.34	0.34	0.31	0.37	0.34	0.34	0.31	0.37
σ_{FG_1}		IG	0.1	2.0	0.03	0.04	0.02	0.06	0.04	0.04	0.02	0.06
σ_{FG_2}		IG	0.1	2.0	0.04	0.04	0.02	0.06	0.04	0.04	0.02	0.06
σ_{FG_3}		IG	0.1	2.0	0.04	0.04	0.02	0.06	0.04	0.04	0.02	0.06
σ_{FG_4}		IG	0.1	2.0	0.04	0.04	0.02	0.06	0.04	0.04	0.02	0.06
σ_{FG_8}		IG	0.1	2.0	0.04	0.04	0.02	0.06	0.04	0.04	0.02	0.06

Table 2: Prior and posterior distributions - Exogenous parameters. The prior distributions are: beta (B), inverse gamma (IG), gamma (G), and normal (N). Both models are estimated with forward guidance shocks, as modeled in equation (5) and with a sample that extend until the fourth quarter of 2019. Column RE refers to the model under rational expectations, while BR refers to our baseline version (with shock-specific CD parameter for FG shocks and estimated with system priors (SP).

CD factor	RE	BR^{std}	BR w/o SP	BR	BR^{2CD}
M_{FG}	1*	**0.60**	**0.53**	**0.94**	**0.93**
		(0.44, 0.76)	(0.2, 0.88)	(0.91,0.97)	(0.90,0.96)
M_{rest}	1*	0.60**	1*	1*	**0.993**
		(0.44, 0.76)			(0.99,0.996)
MDD	-2842.2	-2828.4	-2839.7	**-2823.6**	-2866.3

Table 3: Models' fit and cognitive discount factors' mean estimates. MDD: Marginal data density. In round brackets the 5^{th} and 95^{th} percentiles of the posterior distributions. M_{FG}: Forward Guidance shock discount factor; M_{rest}: CD factor associated to all remaining shocks. Versions: RE, rational expectations; BR^{std} standard cognitive discounting specification with a common universal CD factor; BR, baseline model, estimated with system priors; $BR\, w/o\, SP$, same as previous version but without SP; BR^{2CD}, version with two CD factors. *: Calibrated under the assumption of no cognitive discounting. **: under this specification we impose $M_{FG} = M_{rest}$; only one parameter is estimated.

and forward guidance IRFs of inflation. Finally, the last column of table (3) shows that our assumption in the baseline model that agents form rational expectations about all future events not generated by forward guidance shocks is appropriate. Indeed, that CD factor is estimated to be virtually 1.

5.1 The Forward Guidance Puzzle revisited

The estimated behavioral model, with shocks-specific CD factor and estimated with system priors, successfully resolves the *forward guidance puzzle*. As we did before for the Smets and Wouters (2007) original model, figure (4) depicts the response of current inflation to an announced one-period real interest rate cut h periods ahead relative to the response to a rate cut today of the same magnitude.

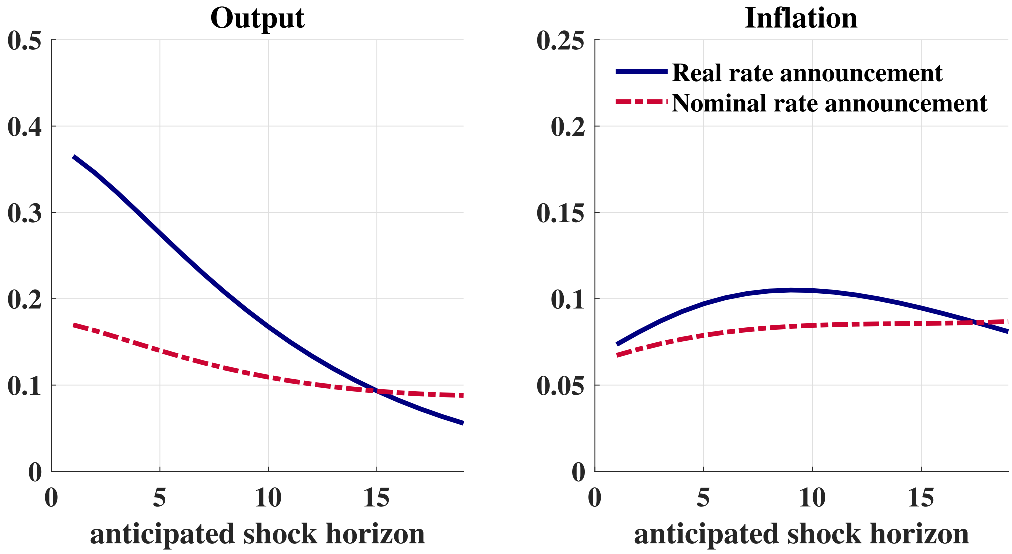

Figure 4: Forward guidance puzzle in the SW under shock-specific CD, model BR. Curves depict the ratio between the on-impact effect on output (left) and inflation (right) of an announced t quarters ahead interest rate cut relative to the effect of a contemporary cut of the same magnitude. The interest rate is assumed to remain at its steady state value before and after the cut. Solid blue lines denote the effects of an announced real rate trajectory. Red dashed line denotes the effects of an announced nominal rate trajectory.

The effect of a forward guidance shock relative to a monetary policy one decreases monotonically on activity as the horizon of the announcement moves further back in time. This relative effect presents a hump-shaped pattern on inflation.[31] In both cases, however, the effects no longer display the increasing dynamics of the rational expectations model.

To look a bit deeper into why this happens, let us go back to the iterated Euler and Phillip's curve equations of the canonical New Keynesian model (2) and (3). Introducing CD as its standard, see Gabaix (2020), we get

$$c_t = -\sum_{j=0}^{\infty} M^j E_t \left[r_{t+j} - r^n_{t+j} \right] \qquad \pi_t = \sum_{j=0}^{\infty} \beta^j M^j E_t \left[c_{t+j} \right]$$

where, for simplicity, we have set $\kappa = \sigma = 1$ and, since we are considering forward guidance

[31] We removed from these graphs the initial period, i.e., 0, where the ratio is one, to avoid showing a large drop that distorts the scale.

about the real rate, we have defined $r_{t+j} = i_{t+j} - \pi_{t+j+1}$. Let us first consider a current increase of the real interest rate of Δr units. As shown in the first row of Table (4), consequence of that increase, both consumption and inflation will decrease - on impact - by the same amount, namely Δr. Moreover, the effect under rational expectations, $M = 1$, coincides with the effect under CD, $M < 1$. Consider next a forward guidance announcement stating a real interest rate increase n periods from now. While the impact on current consumption under the RE version remains Δr as n increases, under CD the effect is moderated by M^n, making the ratio of the impacts to be constant under RE - and equal to one in this simple model - and to decrease monotonically under cognitive discounting. For inflation, the dynamics are a bit more complex. As n increases, the consumption of an increasing number of periods becomes affected. Then, because of the front-loading of the Phillips curve, the accumulated effect on inflation increases steadily, i.e., $-\Delta r \sum_{i=0}^{n} \beta^i$. While that is the only force at play under RE, under CD, as the period of the announced real rate change increases, agents CD factor decreases, M^n. While at the beginning the first force dominates, eventually the second one does, and the inflation impact ratio decreases monotonically.

	Effect on Consumption		Effect on Inflation	
T	M=1	M<1	M=1	M<1
0	$-\Delta r$	$-\Delta r$	$-\Delta r$	$-\Delta r$
1	$-\Delta r$	$-M\Delta r$	$-(\Delta r + \beta\Delta r)$	$-M(\Delta r + \beta\Delta r)$
2	$-\Delta r$	$-M^2\Delta r$	$-(\Delta r + \beta\Delta r + \beta^2\Delta r)$	$-M^2(\Delta r + \beta\Delta r + \beta^2\Delta r)$
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$
n	$-\Delta r$	$-M^n\Delta r$	$-\Delta r\sum_{i=0}^{n}\beta^i$	$-M^n\Delta r\sum_{i=0}^{n}\beta^i$
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$
∞	$-\Delta r$	0	$-\Delta r/(1-\beta)$	0

Table 4: Intuition of how cognitive discounting affects the forward guidance puzzle.

5.2 Dynamics

In this section we compare the dynamics induced by monetary policy under different specifications. Figure (5) plots the IRFs of output growth and inflation to both a conventional monetary policy shock (solid lines) and to a two-quarters-ahead forward guidance shock (dashed lines).[32] Both shocks entail an increase of the nominal interest rate of 25 basis points. Whereas blue lines correspond to the SW model under rational expectations, red lines correspond to our baseline model, BR (with shock-specific CD factor for forward guidance-triggered future events and esti-

[32]FG shocks beyond two quarters yield responses under rational expectations that are too large to plot alongside the responses under CD.

mated with system priors). The first thing to notice is that the responses of activity and inflation to the FG shock under RE are excessive. Compared to the response the same model produces to a conventional monetary policy shock, the drop of output growth is about four times larger; for inflation is around seven times. On the other hand, under our proposed framework the responses to the forward guidance shock are similar to the responses to the conventional monetary policy shock, and there is no sign of a forward guidance puzzle. Furthermore, they are comparable to the responses of inflation and activity to a conventional monetary policy shock under RE. Figure (8) in the appendix plots the response of the SW model estimated with the sample until 2019Q4 (without forward guidance shocks). Both the IRFs of inflation to a conventional monetary policy shock in that model and in our baseline version, BR, are very similar.

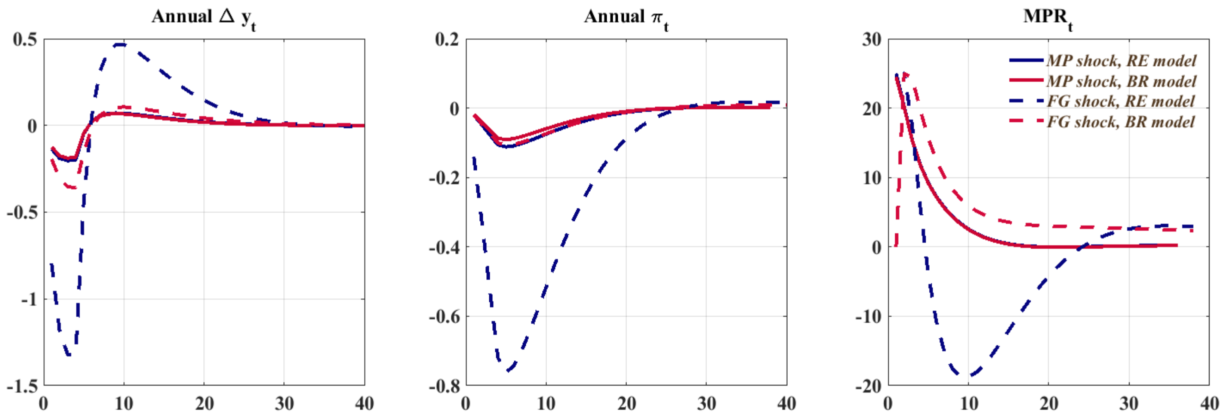

Figure 5: Bounded rationality effect on output and inflation.

The fact that the responses of inflation to the conventional monetary policy shock and the FG shock are similar should not be surprising. It is the result of using system priors on the similarity of those same IRFs in the estimation of the model. To see this more clearly figure(6) plots the same IRFs for our baseline version (blue) and the same specification but estimated without system priors.

5.3 Observing expectations data

In this section we present a simple exercise in which we re-estimate our baseline version of the model while additionally observing data on one-year-ahead expectations for the 3 months treasury bill rate from the Survey of Professional Forecasters (SPF).[33] Furthermore, since our model specification, as the original SW model, observes the fed funds rate, which do not always coincides with the 3 months treasury bill rate, we adjust the expectations time series by subtracting the

[33]While the time series for the survey (coded as TBILL6), is available since 1981Q4, we only consider the period starting in 2000Q1. We do this to be consistent with our assumptions on the beginning of forward guidance as a policy tool, discussed in section 4.

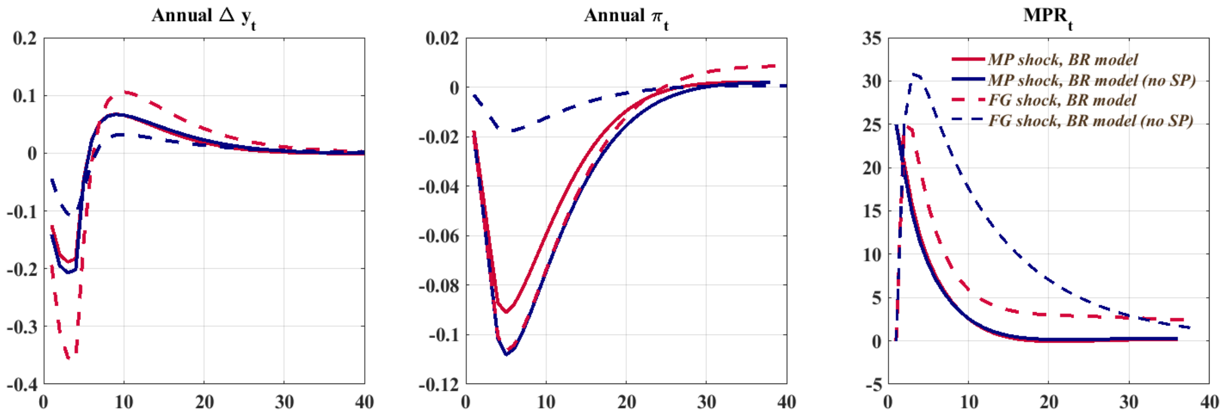

Figure 6: The role of the system prior on the BR model.

premium observed each period. That is, for each period t, our observed variable is $E_t(FFR_{t+4}) = TBILL6 - (3tbill_t - FFR_t)$, where $3tbill_t$ denotes the actual 3 months treasury bill rate in period t and FFR_t denotes the federal funds rate of period t.

For the estimation, we take an agnostic view and create a new agent in the model who only forecasts. Her forecasts are the same as the ones of the other agents in the model, except that we allow them to have their own cognitive discount factor. The measurement equation, then, matches the observed survey data to the expectations of the model's forecaster agent.[34] Table (5) shows the estimates for the CD factors[35]; M_{FG}, continues to denote the CD factor of households, firms, etc., while, M_{FG}^{PF}, denotes the CD factor of the professional forecaster agent. M_{FG} is estimated to be 0.93, practically the same value of our baseline model. The estimates suggest that the professional forecasters data is generated with a degree if rationality similar to that of the average agent in the economy.

CD factor	Prior	mean	5%	95%
M_{FG}	$beta(0.5, 0.20)$	0.93	0.91	0.95
M_{FG}^{PF}	$beta(0.5, 0.20)$	0.94	0.91	0.97

Table 5: Cognitive discount factors estimating observing expectations on future interest rates from professional forecasters (median), observing Exp. data from year 2000.

Finally, we take a look at how the model performs since 2000Q1, date that Nelson (2021) identifies as when the Federal Reserve started to use forward guidance as a policy tool. We are particularly interested in the period when the effective lower bound became binding following the

[34]We include a measurement error, with a standard deviation calibrated to be 10% of the time series standard deviation. Given our model specification, this is equivalent to introducing a shock to the agent expectation's equation to account for the survey's non-modeled variance.

[35]We show in the appendix that estimating the model with a restricted sample starting in 2000Q1 yields similar results for the estimated cognitive discount factors.

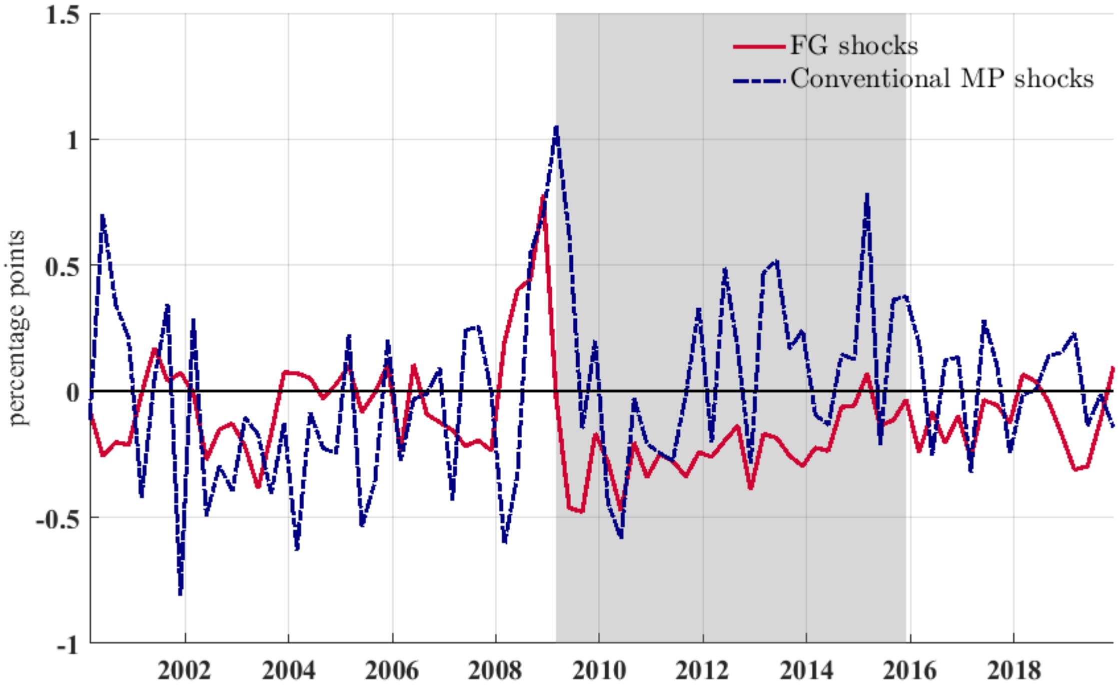

Figure 7: The dashed blue line corresponds to conventional monetary policy shocks. The red solid line corresponds to the inferred change in the average two-year rate induced by forward guidance shocks. The shaded area marks the periods in which the federal funds rate was constrained by its effective lower bound.

great financial crises. That episode extends form late 2008 until late 2016. Figure (7) plots two lines, in blue the inferred conventional monetary policy shocks and in red the inferred change in the average two year rate implied by the forward guidance shocks each period.[36]

The model shows that forward guidance shocks systematically provided further accommodation during the effective lower bound, that only started to recede as the lift-off date of the FFR approached. Prior to the financial crises, FG shocks move significantly closer to 0, being moderately negative following the early 2000s recession.[37] Then, after the financial crises, FG remained expansionary through mid-2015, to start reverting thereafter.

[36]That is, a value of, say, -0.5 in period t means that the model estimates that in period t the Fed announced a 50bp cut of the average annualized two years interest rate.

[37]One exception is the pick in late 2008, which we see as a mechanical response of the model to the required large conventional monetary policy shock needed to explain the large drop in inflation and activity product of the crises. Instead of inferring an even larger conventional monetary policy shock, which would be very unlikely, it gives part of the contractive role to the FG shock of the period.

6 Conclusions

The effective lower bound episode following the financial crisis highlighted the importance of forward guidance as a relevant policy tool to stimulate the economy when monetary policy rates become constrained. As the economy might be potentially returning to a world of low natural interest rates, developing appropriate frameworks where the effects of forward guidance can be studied becomes increasingly important.

We contribute in this respect by expanding on previous work on the role of cognitive discounting as an hypothesis capable of resolving the forward guidance puzzle. Concretely, we show in the context of the well-known Smets and Wouters (2007) model, that an off-the-shelf implementation of Gabaix (2020) to replicate empirical evidence on forward guidance effects leads to a significant attenuation of conventional monetary policy's effectiveness. This diminished effectiveness substantially underestimates the impact of monetary policy reported in other studies. To address this problem, we propose a framework featuring shock-specific cognitive discount factors and develop a strategy for incorporating them into our model. Our approach is simple and straight-forward, facilitating easy extensions to various settings. Importantly, it preserves the model's linearity, enabling the use of efficient linear solution and estimation methods. Additionally, we propose the use of system priors in the estimation, which allows to directly incorporate external empirical evidence on the effects of FG shocks.

We find that the coexistence of empirically plausible effects of both conventional monetary policy and forward guidance requires a degree of cognitive discounting that is specific to announcements of future policy shocks. Our estimated rationally bounded model, featuring a CD factor specific for future events triggered by FG shocks, successfully resolves the forward guidance puzzle and delivers data-consistent monetary policy responses. At the same time, it achieves a better fit to the data than its rational expectations and regular CD counterparts, while relying on only slightly modified estimates of structural parameters. The model does well identifying forward guidance during the post-financial crisis ELB episode.

We argue that in this context the CD parameter has a natural interpretation as a measure of the credibility of monetary policy announcements. In fact, our specification closely resembles the one found in the loose commitment or imperfect credibility literature.

Perhaps an important message of this paper is that there is still much to learn about how to implement cognitive discounting in practice. Though a very promising hypothesis, as most deviations from rational expectations, it allows for some flexibility in its formulation that needs to be systematically and carefully explored.

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Appendix

A Effects of 1-year ahead rise of MPR

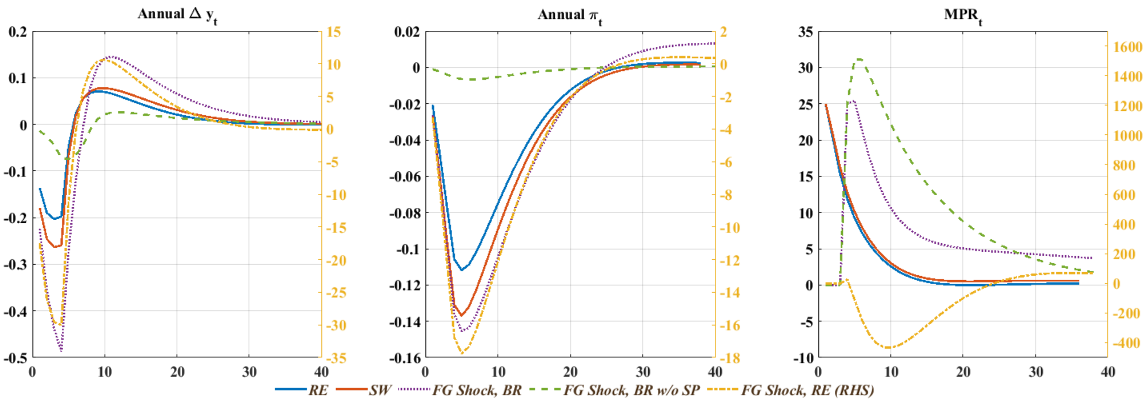

Figure 8: Monetary Policy surprise vs forward guidance announcement. The graphs plot the responses of output growth and inflation to a conventional monetary policy shock of 25bp and to a forward guidance announcement that the nominal interest rate will be increased in one year 25 bp. SW denotes the original Smets and Wouters (2007) model estimated, as all other version, with a sample that extends until the fourth quarter of 2019. RE is the same as SW except that it features FG shocks. BR denotes our baseline specification with a FG-specific CD factor estimated with system priors. BR w/o SP same BR but estimated without system priors. Solid lines correspond to conventional monetary policy shocks, dashed lines to FG shocks.

B Robustness estimation for Professional Forecasters CD

To evaluate the robustness of the Professional Forecasters' CD factor the sample can be important to properly identify this parameter. The Expectations data is available from Q4 1981. Given that we considered the period from 1982 to 2019. Table 6 shows that both the FG specific shocks CD as well as the Professional Forecasters CD factor are stable in their respective estimations. An additional robustness check to carry out correspond to the data used as the observed Expectations. We estimated the model using the mean of the expectations survey's data. The results can be seen in table 7, which shows that the parameters estimation are still robust to the expectations' data used.

CD factor	Details	Prior	full-sample			Since 1982		
			mean	5%	95%	mean	5%	95%
M^{FG}	Households BR	$beta(0.5, 0.20)$	0.93	0.91	0.95	0.93	0.91	0.95
M^{FG}_{PF}	PF BR	$beta(0.5, 0.20)$	0.94	0.91	0.97	0.94	0.91	0.97

Table 6: Cognitive discount factors estimating observing expectations on future interest rates from professional forecasters (median), observing Exp. data from year 2000.

CD factor	Details	Prior	full-sample			Since 1982		
			mean	5%	95%	mean	5%	95%
M^{FG}	Households BR	$beta(0.5, 0.20)$	0.93	0.91	0.95	0.93	0.91	0.95
M^{FG}_{PF}	PF BR	$beta(0.5, 0.20)$	0.93	0.89	0.96	0.92	0.88	0.96

Table 7: Cognitive discount for professional forecasters (mean), observing Exp. data from year 2000.

C Introducing Cognitive Discounting in Dynare

Cognitive discounting can be easily be implemented using Dynare. It suffices to replace forward looking variables, $x(+1)$, by auxiliary variables, say $x^{BR}(+1)$, and, then, to define that variable in a new equation as,

$$x^{BR}_{t+1} \equiv x^{M_{fg}}_{t+1} + m\left(E_t\left(x_{t+1}\right) - x^{M_{fg}}_{t+1}\right) \tag{14}$$

where m stands for cognitive discount factor and $x^{M_{fg}}_t$ is the corresponding focal point; generally, the variable's steady state value. In our proposed set-up, as discussed in section 3.1, we introduced cognitive discounting for deviations from the steady state induced by forward guidance shocks. To do this, we define $x^{M_{fg}}_t$ in an auxiliary model equal to the base one except that it does not feature forward guidance shocks. Dynare will, then, linearize this equation together with the rest of the model.

C.1 Generalizing shock-specific cognitive discount factors

When we assume the steady state as the focal point for all variables and shocks, we can also easily introduce shock-specific cognitive discount factors in Dynare for all shocks in the model. Consider a non-linear model with three shocks, such as Gali's three equations model with a monetary policy shock (e), a preference shock (g), and a cost-push shock (u). We want to write the model such that when one of this shocks hits and triggers an expected future deviation of a variables from its steady state, that deviation will be cognitively discounted with a shock-specific factor. In other words, we want to have the following specification for any given variable x_{t+1},

$$x^{BR}_{t+1} \equiv x_{ss} + m_e\left(E_t\left(x_{t+1}\right) - x_{ss}\right)|_{shock=e} + m_g\left(E_t\left(x_{t+1}\right) - x_{ss}\right)|_{shock=g} + m_u\left(E_t\left(x_{t+1}\right) - x_{ss}\right)|_{shock=u} \tag{15}$$

To implement such a framework in a linearized model, one can proceed as follows,

- write down the original model - call it M - and three auxiliary models, one for each shock in M - call them M_e, M_g, and M_u respectively - which are equal to M except that they only feature one shock (the one they are indexed with).
- then, set the expectations in each of these auxiliary models to be equal to the ones in model M whenever the only shock they feature hits. In our example we write expectations in model M as,

$$x^{BR}_{t+1} \equiv x_{ss} + m_e\left(E_t\left(x^{M_e}_{t+1}\right) - x_{ss}\right) + m_g\left(E_t\left(x^{M_g}_{t+1}\right) - x_{ss}\right) + m_u\left(E_t\left(x^{M_u}_{t+1}\right) - x_{ss}\right) \tag{16}$$

and expectations in model M_k with $k = e, g, u$ as

$$x_{t+1}^{BR,k} \equiv x_{ss} + m_k \left(E_t \left(x_{t+1}^{M_k} \right) - x_{ss} \right) \tag{17}$$

Notice that the auxiliary models no longer feature variables x_t but $x_t^{M_k}$, whereas model M features both variables, x_t for lag and present variables and $x_{t+1}^{M_k}$ for forward-looking variables.

Let us see what happens to this new model - consisting of M, M_e, M_g, and M_u. Consider a monetary policy shock, e, that hits the economy. Since the shock is not present in models M_g and M_u, $x_t^{M_g}$ and $x_t^{M_u}$ remain at their steady state values. Therefore, expectations in model M collapse to

$$x_{t+1}^{BR} \equiv x_{ss} + m_e \left(E_t \left(x_{t+1}^{M_e} \right) - x_{ss} \right)$$

which coincide with the ones in model M_e given by equation (17). Then, since expectations in model M and model M_e are equal, and the two models are exactly equal (except for the latter not featuring shocks g and u which are not active), $x_t = x_t^{M_e}$ and we obtain our shock-specific cognitive discount factor, m_e. Analogously, when the other shocks, g and u, hit we get $x_{t+1}^{BR} \equiv x_{ss} + m_g \left(E_t \left(x_{t+1}^{M_g} \right) - x_{ss} \right)$ and $x_{t+1}^{BR} \equiv x_{ss} + m_u \left(E_t \left(x_{t+1}^{M_u} \right) - x_{ss} \right)$, respectively.