Associated Legendre Polynomials

October 28, 2016

1 General

Assocoiated Legendre Polynomials are the canonical solutions to the General Legendre differential equation 1.1. The term canonical may refer in this context to a natural representation. For example canonical coordinates are a set of coordinates which describe a system properly at any given time. But canonical may also refer to a unique representation of a given mathematical problem like for example the solutions to the generalized Legendre differential equation.

$$\frac{d}{dx}\left[\left(1-x^2\right)\frac{d}{dx}P_l^m\left(x\right)\right]+\left[l\left(l+1\right)-\frac{m^2}{1-x^2}\right]P_l^m\left(x\right)=0 \tag{1.1}$$

In this equation $P_l^m\left(x\right)$ denotes the associated Legendre polynomial and m and l are integers which refer to the degree and order of the considered polynomial respectively. The equation has a finite number of solutions in the intervall $[-1:1]$ only if $0 \leq m \leq l$.
If m is even $P_l^m\left(x\right)$ is a polynomial. If $m=0$ and $l \in N$ then one receives the Legendre Polynomials. Moreovoer it should be mentioned that the Legendre polynomials play an mportant role when determining the so called spherical harmonics.

2 Definition