Quantum Information

Class: IBM Quantum Learning

Mathematical Concepts

Dirac Notation(or Bra-Ket Notation)

Terminology: ket of A is $|A\rangle$ and bra of A is $\langle A|$. Example, let $\Sigma = \{A, B, C\}$ then $|A\rangle = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$, $|B\rangle = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$ and $\langle A| = (1, 0, 0)$, $\langle B| = (0, 1, 0)$ Note that $|\Psi\rangle$ then $\langle \Psi| = |\Psi\rangle^T$.

bra-kets: we denote $\langle a|b\rangle$ the matrice product of

Cartesian product

Let $y = \{0, 1\}$ and $x = \{a, b\}$. Then,
$x \times y = \{(0, a), (0, b), (1, a), (1, b)\}$ and
$y \times x = \{(a, 0), (a, 1), (b, 0), (b, 1)\}$.

Tensor Product of vectors

$$\begin{bmatrix} a_0 \\ \vdots \\ a_n \end{bmatrix} \otimes \begin{bmatrix} b_0 \\ \vdots \\ b_n \end{bmatrix} = \begin{bmatrix} a_0 b_0 \\ \vdots \\ a_n b_n \end{bmatrix}$$

Tensor Product Properties

1. $(|\phi_1\rangle + |\phi_2\rangle) \otimes |\psi\rangle = |\phi_1\rangle \otimes |\psi\rangle + |\phi_2\rangle \otimes |\psi\rangle$
2. $(a\,|\phi\rangle) \otimes |\psi\rangle = a(|\phi\rangle \otimes |\psi\rangle)$
3. $|a\rangle \otimes |b\rangle = |b\rangle \otimes |a\rangle$

Quantum Information Systems

State Vector

Quantum State of a sytem is represented by a complex column vector. Let the quantum state vector v be equal to $\begin{bmatrix} a_0 \\ \vdots \\ a_n \end{bmatrix}$, where $\sum_{i=0}^{n} |a_i|^2 = 1$ The **euclidean norm** of the $||v|| = \sqrt{\sum_{i=0}^{n} |a_i|^2}$

Common Quantum States

Plus State	$\|+\rangle = \frac{1}{\sqrt{2}}\|0\rangle + \frac{1}{\sqrt{2}}\|1\rangle$
Minus State	$\|-\rangle = \frac{1}{\sqrt{2}}\|0\rangle - \frac{1}{\sqrt{2}}\|1\rangle$
Other State	$\frac{1+2i}{3}\|0\rangle - \frac{2}{3}\|1\rangle$

Standart Basis Measurement

Let a quantum system be in the state $|\psi\rangle$, then the probability for the measure outcome to be a is $Pr(\text{outcome} = a) = |\langle a|\psi\rangle|^2$ If U is an unary matrice then the following Propertie hold, $||U\psi|| = ||\psi||$

Unary Operations

Unary Matrice

A squared matrix U having complex number entries is unitary if it satisfies the equations, $UU^\dagger = U^\dagger U = \mathbb{I}$ where $\mathbb{I}$ is the identity matrix.

Some unitary operations on qubits

Pauli operations:
$\mathbb{I} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$ $\sigma_x = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$ $\sigma_y = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}$ $\sigma_z = \begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix}$

Hadamard operation: $H = \begin{pmatrix} \frac{1}{\sqrt{2}} & 0 \\ 0 & \frac{1}{\sqrt{2}} \end{pmatrix}$

Phase operations: $P_\theta = \begin{pmatrix} 1 & 0 \\ 0 & e^{i\theta} \end{pmatrix}$

Quantum circtcuit