Calculus 2 For Engineers

Class: APPM 1360

Trig. Formula

Integral's

$\int \sec x \, dx = ln|\sec x + \tan x| + c$
$\int \csc x \, dx = ln|\csc x + \cot x| + c$

Identity's

$\sin^2 x + \cos^2 x = 1$	$\sin^2 x = \frac{1}{2}(1 - \cos 2x)$
$\sin 2x = 2 \sin x \cos x$	$\sec^2 x = 1 + \tan^2 x$
$\cos^2 x = \frac{1}{2}(1 + \cos 2x)$	

Integration technique

Integration by part	$\int udv = uv - \int vdu$
Trig. Integration method	$\int f^3(x)dx = \int f^2(x)f(xt)dx$

Trig. Substitution:

Integrand	Substitution	Bound	Triangle
$\sqrt{a^2 - x^2}$	$x = a \sin \theta$	$-\frac{\pi}{2} \leq \theta \leq \frac{\pi}{2}$	fig. 1
$\sqrt{a^2 + x^2}$	$x = a \tan \theta$	$-\frac{\pi}{2} \leq \theta \leq \frac{\pi}{2}$	fig. 2
$\sqrt{x^2 - a^2}$	$x = a \sec \theta$	...	fig. 3

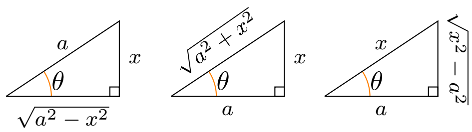

fig. 1 fig. 2 fig. 1

Partial Fraction

$$\frac{N(x)}{(ax + b)^2} = \frac{A}{ax + b} + \frac{B}{(ax + b)^2}$$
$$\frac{N(x)}{(ax + b)(cx + d)} = \frac{A}{ax + b} + \frac{B}{cx + d}$$
$$\frac{N(x)}{(ax + b)(cx + d)^2} = \frac{A}{ax + b} + \frac{B}{(cx + d)^2}$$

Integral Approximation

Riemann Sum Rules

Delta X	$\Delta x = \frac{b - a}{n}$
Right Endpoint	$R_n = \sum_{i=1}^{n} \Delta x f(x_i)$ $x_i = a + i\Delta x$
Left Endpoint	$L_n = \sum_{i=0}^{n-1} \Delta x f(x_i)$ $x_i = a + i\Delta x$
Midpoint	$M_n = \sum_{i=0}^{n} \Delta x f(\overline{x}_i)$ $\overline{x}_i = \frac{x_{i-1} + x_i}{2}$
Trapezoidal	$T_n = \frac{\Delta x}{2}(f(a) + 2f(x_1) + ... + 2f(x_{n-1}) + f(b))$

Short Formula

$L_n = [f(a) + f(a + \Delta x) + f(a + 2\Delta x) + ... + +f(b - \Delta x)]$
$R_n = [f(a + \Delta x) + f(a + \Delta x) + f(a + 2\Delta x) + ... + +f(b)]$
$M_n = [f(a + \frac{\Delta x}{2}) + f(a + \frac{3\Delta x}{2}) + ... + f(b - \frac{\Delta x}{2})]$
$T_n = \dfrac{\Delta x}{2}[f(a) + 2f(a + \Delta x) + 2f(a + 2\Delta x) + ... + f(b)]$

Error

Exact Error

Trapezoidal Exact Er.	$E_T = T_N - \int_a^b f(x)dx$
Midpoint Exact Er.	$E_M = T_M - \int_a^b f(x)dx$

Bound Error

Suppose that $|f''(x)| \leq K$ for $a \leq x \leq a$ and let N be the amount of iteration.

Trapezoidal Er.	$E_T \leq \dfrac{K(b - a)^3}{12N^2}$
Midpoint Er.	$E_M \leq \dfrac{K(b - a)^3}{24N^2}$

Comparison Thm

Supose that $f(x)$ and $g(x)$ are continuous function with $f(x) \geq g(x) \geq 0$ for $x \geq a$.
1. If $\int_a^\infty f(x)dx$ is convergent, then $\int_a^\infty g(x)dx$ is convergent.
2. If $\int_a^\infty g(x)dx$ is divergent, then $\int_a^\infty g(x)dx$ is divergent.

p-series $\int_1^\infty \frac{1}{x^p} dx$ is convergent if $p > 1$ and divergent if $p \leq 1$.

Volumes

General Formula	$V = \int_a^b A(x)\,dx$
Disc. Method	$V = \int_a^b \pi R^2(x)\,dx$
Washer Method	$V = \int_a^b \pi R^2 - \pi r^2$
Cylindrical Shell	$V = \int_a^b 2\pi R(x)h(x)\,dx$

Surface area

Arc-length	$L = \int_a^b ds = \int_a^b \sqrt{1 + [f'(x)]^2}\,dx$
Surface	$SA = \int_a^b dA = \int_a^b 2\pi r ds$ $= \int_a^b 2\pi r\sqrt{1 + (y')^2}dx$

Center of mass

$(\overline{x}, \overline{y})$	$\overline{x} = \frac{M_y}{M} = \frac{\sum_{i=1}^n m_i\,x_i}{M}$, $\overline{y} = \frac{M_x}{M} = \frac{\sum_{i=1}^n m_i\,y_i}{M}$
density	$\rho = \frac{m}{A}$ $m = \rho \int_a^b [f(x) - g(x)]dx$

Differential Equation

Separable Equation

1. $\frac{dy}{dx} = g(x)f(y)$

2. $\int f(y)dy = \int \frac{1}{g(x)}dx$

Sequence & Series

Sequence

Factoriel	$n! = 1 \cdot 2 \cdot 3 \cdot ... \cdot n$
	$n! = n(n-1)(n-2)...2 \cdot 1$
	$0! = 1$
Geometric	$\{r^N\}_{n=0}^{\infty} = \{1, r^1, r^2, ...\}$
Increasing	If $a_n < a_{n+1}$
Convergence	$\lim_{x\to\infty} a_n$ exist and is ∞ then we say the sequence **converges** and **diverges** otherwise.
Limit	$\lim_{x\to\infty} a_n = L$
Monotonic	If it is **increasing** or **decreasing**.
Bounded	If it is bounded **above** or **below**.
Bounded above	If there is a number m such that $a_n \leq m, \forall n \geq 1$
Recursive	Ex: Let $a_1 = 1, a_n = 2a_{N-1} + 1$

Series convergence tests

Geometric serie

for $\sum a_n = \sum Ar^{n-1}$, if $|r| < 1$ then the series is convergent and if $1 \leq |r|$ then the series is divergent.

Direct computation

Compute $s_\infty = \sum_{i=1}^{\infty} a_n$. If $s_\infty \neq \infty$ then the series is convergent, other wise the serie is divergent.

Divergence test

$\lim_{n\to\infty} a_n \neq 0$ Divergent.
$\lim_{n\to\infty} a_n = 0$ Convergent or Divergent.

Integral Test

For $\sum_{n=1}^{\infty} a_n \leq \int_1^{\infty} a_x \ dx$, if $\int_1^{\infty} a_x \ dx$ is convergent then $\sum_{n=1}^{\infty} a_n$ is also convergent. The opposite is also true.

Direct Comparison test (DCT)

Let $b_n \geq a_n < 0$:

1. if $\sum b_n$ is convergent than $\sum a_n$ is also convergent.
2. if $\sum a_n$ is divergent than $\sum b_n$ is also divergent.

Limit comparison test (LCT)

Alternating test

Ration & Root

Absolute convergence	$\sum \lvert a_n \rvert$ is convergent.
Ratio test	$L = \lim_{n\to\infty} \lvert \frac{a_{n+1}}{a_n} \rvert$
Root test	$L = \lim_{n\to\infty} \sqrt[n]{\lvert a_n \rvert} = \lim_{n\to\infty} \lvert a_n \rvert^{\frac{1}{n}}$
Tests Conclusion:	$L < 1 \Rightarrow \sum a_n$ is absolute convergent.
	$L > 1 \Rightarrow \sum a_n$ divergent.
	$L = 1 \Rightarrow$ The test is inconclusive.

note: if $L = 0$ then $R = \infty, I(-\infty, \infty)$

Monotone convergence Theorem(MCT)

Taylor Series & Maclaurin Series

Taylor	$f(x) = \sum_{n=0}^{\infty} \frac{f^n(a)}{n!}(x-a)^n, \ \lvert x-a \rvert < R$
Maclaurin	$f(x) = \sum_{n=0}^{\infty} \frac{f^n(0)}{n!} x^n$

parametric & polar equations

parametric equations

Def.	$x = f(t)$
	$y = g(t)$
Derivative	$\frac{dy}{dx} = \frac{dy\frac{1}{dt}}{dx\frac{1}{dt}} = \frac{y'}{x'}$
	$\frac{d^2y}{dx^2} = \frac{\frac{d}{dt}(\frac{dy}{dx})}{\frac{dx}{dt}}$
Area	$A = \int_a^b y\frac{dx}{dt}dt$
Arc length	$ds = \sqrt{(x')^2 + (y')^2}dt$
	$L = \int_a^b ds$

polar equations

polar notation	$r = f(\theta)$
	$f(r, \theta) = 0$
polart→carteian	$x = r\cos\theta$
	$y = r\sin\theta$
carteian→polart	$r^2 = x^2 + y^2$
	$\tan\theta = \frac{y}{x}$

Other

Hooke's law	$F = k(x - x_0)$
Work	$W = \int_a^b F(x)dx$
Triangle inequality	$\lvert a+b \rvert \leq \lvert a \rvert + \lvert b \rvert$
	$\lvert \sin x \rvert \leq 1$
	$\lvert \cos x \rvert \leq 1$