Calculus 1 For Engineers

Class: APPM 1350

Geometry (Pre-Calculus)

Circle Cir.	$2\pi r$
circle area	$A = \pi r^2$
Sphere area	$A = 4\pi r^2$
Sphere Vol.	$V = \frac{4}{3}\pi r^3$
Pyramid Vol.	$V = \frac{1}{3}a_{base}h$

Trigonometric (Pre-Calculus)

General Trigonometric

$\cos(a-b) = \cos a \cos b - \sin a \sin b$
$cos^2\theta + sin^2\theta = 1$
$\sec^2 x = 1 + \tan^2 x$
$\tan\theta = \dfrac{\sin\theta}{\cos\theta}$

Double-Angle

$cos(2\theta) = \cos^2\theta - \sin^2\theta$
$cos(2\theta) = 1 - 2\sin^2\theta$
$cos(2\theta) = 2\cos^2\theta - 1$
$sin(2\theta) = 2\cos\theta \cdot \sin\theta$
$\tan(2\theta) = \dfrac{2\tan\theta}{1-\tan^2\theta}$

Half-Angle

$\sin(\frac{\theta}{2}) = \pm\sqrt{\dfrac{1-\cos\theta}{2}}$
$\cos(\frac{\theta}{2}) = \pm\sqrt{\dfrac{1+\cos\theta}{2}}$
$\tan(\frac{\theta}{2}) = \pm\sqrt{\dfrac{1-\cos\theta}{1+\cos\theta}}$
$\tan(\frac{\theta}{2}) = \dfrac{\sin\theta}{1+\cos\theta}$
$\tan(\frac{\theta}{2}) = \dfrac{1-\cos\theta}{\sin\theta}$

Algebra (Pre-Calculus)

Identités Remarquables

Second Degrées

$(a+b)^2 = a^2 + 2ab + b^2$
$(a-b)^2 = a^2 - 2ab + b^2$
$a^2 - b^2 = (a+b)(a-b)$
$a^2 + b^2 = (a+ib)(a-ib)$

Troisiéme Degrées

$(a+b)^3 = a^3 + 3a^2b + 3ab^2 + b^3$
$(a-b)^3 = a^3 - 3a^2b + 3ab^2 - b^3$
$a^3 + b^3 = (a+b)(a^2 - 2ab + b^2)$
$a^3 - b^3 = (a-b)(a^2 + 2ab + b^2)$

Logarithm

$\log_b(a) = c \Leftrightarrow a = b^c$
$\log(a \cdot b) = \log a + \log b$
$\log(\frac{a}{b}) = \log a - \log b$
$\log(a^b) = b \cdot \log(a)$
$\log_a x = \forall n \dfrac{\log_n x}{\log_n a}$

Calculus

Limits

Properties

$\lim\limits_{x\to a} c \cdot f(x) = c \cdot \lim_{x\to a} f(x)$
$\lim\limits_{x\to 0} \dfrac{\sin(\alpha x)}{\alpha x} = 1, \alpha \in \mathbb{R}$

Theorems

Limits Simplified Theorem

$$\lim_{x\to a} f(x) = L \tag{1a}$$

$$\Rightarrow \lim_{x\to a^+} f(x) = \lim_{x\to a^-} f(x) = L \tag{1b}$$

Limits Formal Theorem

Let $\epsilon \in \mathbb{R}$ and $\delta \in \mathbb{R}$

$$\lim_{x\to a} f(x) = L \Rightarrow \forall\epsilon\exists\delta(\epsilon > 0 \wedge \delta > 0) \tag{2a}$$

such that:

$$\forall x(\mid f(x) - L \mid < \epsilon \wedge \mid x - a \mid < \delta) \tag{2b}$$

Squeeze Theorem

$$g(x) \geq f(x) \geq h(x) \tag{3a}$$

if $\lim\limits_{x\to a} g(x) = \lim\limits_{x\to a} h(x) = L$ then:

$$f(x) = L \tag{3b}$$

Let $I = [A, B]$ be an interval of $\mathbb{R}$
If $f(A) < u < f(B)$
Then $\exists c \in (A, B) \mid f(c) = u$

L'hopital theorem

Let $g(x)$ and $f(x)$ be some function differentiable on $x = a$.
If $(\lim\limits_{x\to a} f(x) = 0 \wedge \lim\limits_{x\to a} g(x) = 0) \vee (\lim\limits_{x\to a} f(x) = \pm\infty \wedge \lim\limits_{x\to a} g(x) = \pm\infty)$ then:

$$\lim_{x\to a} \frac{f(x)}{g(x)} \overset{\text{LH}}{=} \lim_{x\to a} \frac{f'(x)}{g'(x)}$$

Mean Value Theorem

1) $f(x)$ is continuious on $[A, B]$
2) $f(x)$ is differentialble (A, B)
Then $\exists c \in (A, B)\ f'(c) = \dfrac{f(B) - f(A)}{B - A}$

Rolle's Theorem

1) $f(x)$ is continuious on $[A, B]$
2) $f(x)$ is differentialble (A, B)
3) $f(A) = f(B)$
Then $\exists c \in (A, B)\ f'(c) = 0$

Extreme value theorem

If f is continuous on $[a, b]$ then f attain an abs max $f(c)$ and abs min $f(d)$ for some c,d in $[a, b]$.

$$f(d) \leq f(x) \leq f(c) \tag{4}$$

Derivative

Definition

$$\frac{d}{dx} f(x) = \lim_{h\to 0} \frac{f(x+h) - f(x)}{h}$$

Formulas

Constante	$\frac{d}{dx} a = 0$
Constante	$\frac{d}{dx} a \cdot f(x) = a \cdot \frac{d}{dx} f(x)$
Sum	$\frac{d}{dx} f(x) + g(x) = f'(x) + g'(x)$
Power	$\frac{d}{dx} a \cdot x^n = a \cdot n \cdot x^{n-1}$
Square root	$\frac{d}{dx} \sqrt{x} = -\frac{1}{2\sqrt{x}}$
Product	$\frac{d}{dx} f \cdot g = f' \cdot g + f \cdot g'$
Quotient	$\frac{d}{dx} \frac{f}{g} = \dfrac{f' \cdot g - f \cdot g'}{g^2}$
Logarithm	$\frac{d}{dx} \log_a x = \dfrac{1}{x \ln a}$
Natural log	$\frac{d}{dx} \ln x = \dfrac{1}{x}$
Chain	$\frac{d}{dx} f(g(x)) = f'(g(x)) \cdot g'(x)$

Trigonometric Formula

Sinus	$\frac{d}{dx}\sin x = \cos x$
Cosinus	$\frac{d}{dx}\cos x = -\sin x$
Tangent	$\frac{d}{dx}\tan x = \sec^2 x$
Cotangent	$\frac{d}{dx}\cot x = -\csc^2 x$
Second	$\frac{d}{dx}\sec x = \sec x \tan x$
Cosecant	$\frac{d}{dx}\csc x = -\csc x \cot x$
Arc Sinus	$\frac{d}{dx}\arcsin x = \frac{1}{\sqrt{1-x^2}}$
Arc Cousins	$\frac{d}{dx}\arccos x = -\frac{1}{\sqrt{1-x^2}}$
Arc Tangent	$\frac{d}{dx}\arctan x = \frac{1}{x^2+1}$
Arc Cotangent	$\frac{d}{dx} arccot\, x = -\frac{1}{x^2+1}$
Arc Second	$\frac{d}{dx} arcsec\, x = \frac{1}{\lvert x\rvert\sqrt{1-x^2}}$
Arc Cosecant	$\frac{d}{dx} arccsc\, x = -\frac{1}{\lvert x\rvert\sqrt{1-x^2}}$

Linearization and differential

Linearization

GOAL: Simplify relatively complicated function by approximation with a linear function. $L(x) = F'(a)(x - a+) + F(a)$

Sum

propertie 1	$\sum_{i=1}^{n} a = an$
propertie 2	$\sum_{i=1}^{n} ca = c\sum_{i=1}^{n} a$
Expantion 1	$\sum_{i=1}^{n} i = \frac{n(n+1)}{2}$
Expantion 2	$\sum_{i=1}^{n} i^2 = \frac{n(n+1)(2n+1)}{6}$
Expantion 3	$\sum_{i=1}^{n} i^3 = \frac{n^2(n+1)^2}{4}$
Rieman sum	$\int_a^b f(x)\,dx$
	$= \lim_{n\to\infty} \sum_{i=1}^{n} f(x_i)\Delta x$
Where	$\Delta x = \frac{b-a}{n}$
	$x_i = a + i\Delta x$

Integrals

Properties of the Definite Integral

$\int_a^a f(x)\,dx = 0$
$\int_b^a f(x)\,dx = -\int_a^b f(x)\,dx$
$\int_b^a cf(x)\,dx = c\int_b^a f(x)\,dx$, where c is a const.
$\int_a^c f(x)\,dx + \int_c^b f(x)\,dx = \int_a^b f(x)\,dx$

Even-Odd Symmetry

Suppose f is continuous on $[-a, a]$.

If f is even	$\int_{-a}^{a} f(x)\,dx = 2\int_0^a f(x)\,dx$
If f is odd	$\int_{-a}^{a} f(x)\,dx = 0$

Fundamental Thm Of Calculus

If f is cont on $[a, b]$ then
Part: 1

$$f(x) = \frac{d}{dx}\int_a^x f(t)\,dt \tag{5}$$

Where $a \le x \le b, a \le t \le x$. Then
Part: 2

$$\int_a^b f(x)\,dx = F(x)\Big|_a^b = F(b) - F(a) \tag{6}$$

Hyperbolic functions

Definition

$\cosh x = \frac{e^x + e^{-x}}{2}$

$\sinh x = \frac{e^x - e^{-x}}{2}$

$\tanh x = \frac{\sinh x}{\cosh x} = \frac{e^x - e^{-x}}{e^x + e^{-x}}$

Derivative

$\frac{d}{dx}\cosh x = \sinh x$
$\frac{d}{dx}\sinh x = \cosh x$
$\frac{d}{dx}\tanh x = sech^2\, x$
$\frac{d}{dx}\coth x = -csch^2\, x$
$\frac{d}{dx} csch\, x = -csch\, x \coth x$
$\frac{d}{dx} sech\, x = sech\, x \coth x$