Linear Algebra

Class: ...

Definition

Linear Functions	All terms are of degree 0 or 1. A solution of a system of linear equation is set of points that makes the equation system true.
Consistent	lin. systems is consistent if either 1 or ∞ solutions exist else inconsistent.
Conist	

Coefficient Matrix

$$\begin{cases} A_1x_1 + A_2x_2 + A_3x_3 = \alpha \\ B_1x_1 + B_2x_2 + B_3x_3 = \beta \\ C_1x_1 + C_2x_2 + C_3x_3 = \gamma \end{cases} \Leftrightarrow \begin{bmatrix} A_1 & A_2 & A_3 \\ B_1 & B_2 & B_3 \\ C_1 & C_2 & C_3 \end{bmatrix} \tag{1}$$

Augmented Matrix

$$\begin{cases} A_1x_1 + A_2x_2 + A_3x_3 = \alpha \\ B_1x_1 + B_2x_2 + B_3x_3 = \beta \\ C_1x_1 + C_2x_2 + C_3x_3 = \gamma \end{cases} \Leftrightarrow \left[\begin{array}{ccc|c} A_1 & A_2 & A_3 & \alpha \\ B_1 & B_2 & B_3 & \beta \\ C_1 & C_2 & C_3 & \gamma \end{array}\right] \tag{2}$$

Row-Equivalence

Two matrce are row-equivalent if there is a sequence of **EROS** that transforms one into the other.

Elementary Row Operations (EROS)

1. [**Replacement**] Replace one row by sum of itself.
2. [**Interchange**] Swap position of 2 rows.
3. [**Scaling**] Multiply all entries in row by non-zero constant.

Echelon Form (ef)

1. All non-zero rows are above any rows of all-zero.
2. Each leading entry of a row is in a column to the right of the roe above it.
3. All entries in a column below a leading entry are 0.

Reduced Row Echelon Form (rref)

1. As to be in echelon form.
2. Leading entry in each row is 1.
3. Each leading 1 is the only non-zero entry in its column.

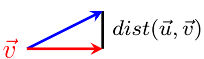
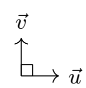
Theorems

Theorem 1 *Every matrix is row equivalent to a unique row echelon form.*

Theorem 2 *Every matrix is row equivalent to a unique row echelon form.*

Orthogonality and Diagonalization

Definition

Inner Product	$\vec{v} \cdot \vec{u} = \vec{v}^T\vec{u} = u_1v_1 + ... + u_nv_n$ (Also called dot product or scalar product)
Length of $\vec{x}$	$\|\|\vec{x}\|\| = \sqrt{\vec{x} \cdot \vec{x}} = \sqrt{x_1^2 + ... + x_n^2}$ (Also called Norm or Magnitude)
Unit vectore	A vector with $\|\|\vec{x}\|\| = 1$
Normalization	The formula $\vec{u} = \frac{\vec{x}}{\|\|\vec{x}\|\|}$ creat a unit vector in the same direction as $\vec{x}$.
Distance	$dist(\vec{u}, \vec{v}) = \vec{u} - \vec{v}$ 
Orthogonality	$\vec{u} \cdot \vec{v} = 0$ 
Orthogonal Complements	Let W be a subspace of $\mathbb{R}^n$. The orthogonal complement of W is: $W^\perp = \{\vec{x} \in \mathbb{R}^n \| \vec{x} \cdot \vec{w} = \vec{0}, \forall \vec{w} \in W\}$

Fact about the Orthogonal Complements

- $\vec{0} \in W^\perp$ since $\vec{0} \cdot \vec{w} = 0$
- If $W \in W^\perp, c \in \mathbb{R}$ then $cW \in W^\perp$
- If $\vec{w}_1, \vec{w}_2 \in W^\perp$ so $(\vec{w}_1 + \vec{w}_2) \cdot \vec{x} = \vec{w}_1 \cdot \vec{x} + \vec{w}_2 \cdot \vec{x} \in W$

Theorems

Properties 1 *Let $\vec{u}, \vec{v}, \vec{w} \in \mathbb{R}^n$ and $c \in \mathbb{R}^n$ then:*

- $\vec{u} \cdot \vec{v} = \vec{v} \cdot \vec{u}$
- $(\vec{u} + \vec{v}) \cdot \vec{w} = \vec{u} \cdot \vec{w} + \vec{v} \cdot \vec{w}$
- $(c\vec{u}) \cdot \vec{v} = c(\vec{u} \cdot \vec{v}) = (c\vec{v}) \cdot \vec{u}$
- $\vec{u} \cdot \vec{u} = u_1^2 + ... + u_n^2 \geq 0$

Theorem 3 (Fundamental Subspaces Theorem) *Let $A_{M \times N}$*

- $(row(A))^\perp = nul(A)$
- $(COl(A))^\perp = nul(a^\perp)$