Discrete Mathematics

Class: MATH 2410 ©Maximilien Notz 2025

Mathematical Statement

Statement	is any declarative sentence which is either true or false.
Atomic	if it cannot be divided into smaller statements.
Molecular	if it can be divided into smaller statements.
conjunction	$p \wedge q$ equivalent to "p and q".
disjunction	$p \vee q$ equivalent to "p or q".
	where p is the hypothesis and q the conclusion.
Implication	$p \to q$ equivalent to "if p then q".
Biconditional	$p \leftrightarrow q$ equivalent to "if and only if p then q".
Negation	$\neg p$ equivalent to "not p".
Converse	
Contrapositive	
There is a x	$\exists x$
For all x	$\forall x$

Naive Set Theory

Set Notation

Universal set	$\mathbb{U}$
Empty set	$\emptyset = \{\}$, Remember: $\forall A(\emptyset \subset A)$
Power set	$\mathcal{P}(A)$ is the set of all the subsets of A.
Partition of A	A collection of nonempty, pairwise-disjoint subsets whose union is A.
Element of	$\in$. Example:$2 \in \{1, 2, 3\}$
Subset of	$\subseteq$. Example: $\{A, B, C\} \subseteq \{B, C, D\}$ $A \subseteq B \Leftrightarrow \forall x$
Proper subset of	$\subset$. Example: $\{A, B, C\} \subset \{A, B, C, D\}$
Intersection	$\bigcap_{i \in I} A_i = \{x \in \mathbb{U} \mid \forall i \in I, x \in A_i\}$ $A \cap B = \{x \in \mathbb{U} \mid x \in A \wedge x \in B\}$
Union	$\bigcup_{i \in I} A_i = \{x \in \mathbb{U} \mid \exists i \in I, x \in A_i\}$ $A \cup B = \{x \in \mathbb{U} \mid x \in A \vee x \in B\}$
Difference	$A \backslash B = \{x \in A \mid x \notin B\}$
Symmetric difference	$A \Delta B = (A \backslash B) \cup (B \backslash A)$
Cartesian Product	$A \times B = \{(x, y) \mid x \in A \wedge y \in B\}$
Complement of	$\bar{A} = \{x \in \mathbb{U} \mid x \notin A\}$
Cardinality	$\lvert A \rvert$

Cardinality

finite set	Let X be a finite set then $\lvert X \rvert \in \mathbb{N}$
countable set	A set S is countable if and only if that is finit or $\lvert S \rvert = \lvert \mathbb{N} \rvert$.
aleph null.	$\aleph_0 = \lvert \mathbb{N} \rvert$

Theorem 1 *Let A and B be sets, then $|A| = |B|$ if and only if there is a one-to-one correspondence from A to B.*

Theorem 2 *If A and B are countable, then $A \cup B$ is countable.*

Theorem 3 (Cantor's Theorem) *For every set A, $|A| < |\mathcal{P}(A)|$.*

Theorem 4 (Schröder–Bernstein) *If there are injective function(one-to-one) functions $f : A \to B$ and $g : B \to A$, then there is a one-to-one correspondence between A and B. In other words If A and B are set with $|A| \neq |B|$ and $|B| \neq |A|$, then $|A| = |B|$.*

Functions

Functions	A rule that assigns each input exactly one output.
Domain	The set of all input of a function.(X in $f : X \to Y$)
Codomain	The set of all output a function.(Y in $f : X \to Y$)
Range	Is the subset of Y of elements that have an antecedent in X by f
$f : x \to y$	a function f with a domain x and a codomain y.
Recursive f.	
Injective	every element of the codomain is the image of $f(a) = f(b) \Rightarrow a = b$ **at most** one element from the domain.
Surjective	every element of the codomain is the image of **at least** one element from the domain.
Bijection	A function that is **Injective** and **Surjective**.
Image	$f(A) = \{f(a) \in Y : a \in A\}$, where $A \subset$ domain.
Inverse Image	$f^{-1}(B) = \{f(b) \in X : b \in B\}$, where $B \subset$ codomain.

Counting

power set cardinality	$\lvert \mathcal{P}(A) \rvert = 2^{\lvert A \rvert}$
n-bit string	
bit string weight	the number of **1** in a bit string.
B^n_k	the set of all **n-bit strings** of weight k.

Additive Principle

General Definition: if event A can occur in m ways, and even B can occur in n **disjoint** (A and B can't apen at the same time.) ways, then A and B can occur in $m + n$ ways.
Set Definition: Given 2 sets A and B, if $A \cap B = \emptyset$, then $|A \cap B| = |A| + |B|$.

Multiplicative Principle

General Definition: if event A can occur m ways, and each possibility for A allows for exactly n ways for event B, then the event "A and B" can occur $m \cdot n$ ways.
Set Definition: Given 2 sets A and B, we have $|A \times B| = |A| \cdot |B|$.

Binomial coefficient

Sequences

Symbolic Logic

deMorganLaws

- $\neg \forall x P(x) = \exists x p(x)$
- $\neg \exists x P(x) = \forall x p(x)$
- $\neg(a_1 \wedge a_2 \wedge \cdots \wedge a_n) \equiv \neg a_1 \vee \neg a_2 \vee \cdots \vee \neg a_n$
- $\neg(a_1 \vee a_2 \vee \cdots \vee a_n) \equiv \neg a_1 \wedge \neg a_2 \wedge \cdots \wedge \neg a_n$

Proofs

Graph Theory