Quantum Information

Linear Algebra

Complexe conjugate	$(a+ib)^* = a - ib$ $(a-ib)^* = a + ib$ where $a \in \mathbb{R}$, $b \in \mathbb{R}$

Properties 1 (Complexe conjugate)
• $(Z^*)^* = Z$ • $(Z+W)^* = Z^* + W^*$
• $(Z-W)^* = Z^* - W^*$ • $(ZW)^* = Z^*W^*$
• $Z^*Z = |Z|^2$ • $(Z^n)^* = (Z^*)^n$, *for* $n \in \mathbb{Z}$
• $ln(Z^*) = (ln(Z))^*$ *if* Z *is not 0 or a negative real number.*

Linear Operator

A **linear operator** between the vector spaces V and W is define to be any function $A : V \to W$ which is linear in its input.

Properties 2 *Let* $\hat{A}$ *be a linear operator on* $V \to W$ *and* A *be the matrix representation of* $\hat{A}$. • $\hat{A}(\sum_i a_i |v_i\rangle) = \sum_i a_i \hat{A} |v_i\rangle$
• $\hat{A} |v_j\rangle = \sum_i A_{ij} |w_i\rangle$

Inner product

Quantum mecanics

Pauli Matrices

• $\sigma_0 = I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ • $\sigma_x = X = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$
• $\sigma_y = Y = \begin{bmatrix} 0 & -i \\ i & 0 \end{bmatrix}$ • $\sigma_z = Z = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$

Mathematical Concepts

Dirac Notation(or Bra-Ket Notation)

Terminology: ket of A is $|A\rangle$ and bra of A is $\langle A|$. Example, let $\Sigma = \{A, B, C\}$ then $|A\rangle = \begin{bmatrix}1\\0\\0\end{bmatrix}$, $|B\rangle = \begin{bmatrix}0\\1\\0\end{bmatrix}$ and $\langle A| = (1,0,0)$, $\langle B| = (0,1,0)$ Note that $|\Psi\rangle$ then $\langle \Psi| = |\Psi\rangle^T$.
bra-kets: we denote $\langle a|b\rangle$ the matrice product of

Cartesian product

Let $y = \{0,1\}$ and $x = \{a,b\}$. Then,
$x \times y = \{(0,a),(0,b),(1,a),(1,b)\}$ and
$y \times x = \{(a,0),(a,1),(b,0),(b,1)\}$.

Tensor Product of vectors

$$\begin{bmatrix}a_0\\ \vdots \\ a_n\end{bmatrix} \otimes \begin{bmatrix}b_0\\ \vdots \\ b_n\end{bmatrix} = \begin{bmatrix}a_0 b_0\\ \vdots \\ a_n b_n\end{bmatrix}$$

Tensor Product Properties

1. $(|\phi_1\rangle + |\phi_2\rangle) \otimes |\psi\rangle = |\phi_1\rangle \otimes |\psi\rangle + |\phi_2\rangle \otimes |\psi\rangle$
2. $(a\,|\phi\rangle) \otimes |\psi\rangle = a(|\phi\rangle \otimes |\psi\rangle)$
3. $|a\rangle \otimes |b\rangle = |b\rangle \otimes |a\rangle$

Quantum Information Systems

State Vector

Quantum State of a sytem is represented by a complex column vector. Let the quantum state vector v be equal to $\begin{bmatrix}a_0\\ \vdots \\ a_n\end{bmatrix}$, where $\sum_{i=0}^{n} |a_i|^2 = 1$ The **euclidean norm** of the $||v|| = \sqrt{\sum_{i=0}^{n} |a_i|^2}$

Common Quantum States

Plus State	$\|+\rangle = \frac{1}{\sqrt{2}}\|0\rangle + \frac{1}{\sqrt{2}}\|1\rangle$
Minus State	$\|-\rangle = \frac{1}{\sqrt{2}}\|0\rangle - \frac{1}{\sqrt{2}}\|1\rangle$
Other State	$\frac{1+2i}{3}\|0\rangle - \frac{2}{3}\|1\rangle$

Standart Basis Measurement

Let a quantum system be in the state $|\psi\rangle$, then the probability for the measure outcome to be a is $Pr(\text{outcome} = a) = |\langle a|\psi\rangle|^2$ If U is an unary matrice then the following Propertie hold, $||U\psi|| = ||\psi||$

Unary Operations

Unary Matrice

A squared matrix U having complex number entries is unitary if it satisfies the equations, $UU^\dagger = U^\dagger U = \mathbb{I}$ where $\mathbb{I}$ is the identity matrix.

Some unitary operations on qubits

Pauli operations:
$\mathbb{I} = \begin{pmatrix}1 & 0\\0 & 1\end{pmatrix}$ $\sigma_x = \begin{pmatrix}0 & 1\\1 & 0\end{pmatrix}$ $\sigma_y = \begin{pmatrix}0 & -i\\i & 0\end{pmatrix}$
$\sigma_z = \begin{pmatrix}-1 & 0\\0 & 1\end{pmatrix}$

Hadamard operation: $H = \begin{pmatrix}\frac{1}{\sqrt{2}} & 0\\0 & \frac{1}{\sqrt{2}}\end{pmatrix}$

Phase operations: $P_\theta = \begin{pmatrix}1 & 0\\0 & e^{i\theta}\end{pmatrix}$

Quantum circtuit