# Linear Algebra

#### Definition

Linear Functions  $\,$  All terms are of degree 0 or 1.

A solution of a system of linear equation is set of points that makes

the equation system true.

Consistent lin. systems is consistent if either 1 or  $\infty$ 

solutions exist else inconsistent.

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### Coefficient Matrix

$$\begin{cases} A_1x_1 + A_2x_2 + A_3x_3 = \alpha \\ B_1x_1 + B_2x_2 + B_3x_3 = \beta \Leftrightarrow \begin{bmatrix} A_1 & A_2 & A_3 \\ B_1 & B_2 & B_3 \\ C_1x_1 + C_2x_2 + C_3x_3 = \gamma \end{cases}$$

### **Augmented Matrix**

$$\begin{cases} A_1 x_1 + A_2 x_2 + A_3 x_3 = \alpha \\ B_1 x_1 + B_2 x_2 + B_3 x_3 = \beta \Leftrightarrow \begin{bmatrix} A_1 & A_2 & A_3 & \alpha \\ B_1 & B_2 & B_3 & \beta \\ C_1 & C_2 & C_3 & \gamma \end{bmatrix} \\ C_1 & C_2 & C_3 & \gamma \end{cases}$$

### Row-Equivalence

Two matrice are row-equivalent if there is a sequence of **EROS** that transforms one into the other.

### Elementary Row Operations (EROS)

- 1. [Replacement] Replace one row by sum of itself.
- 2. [Interchange] Swap position of 2 rows.

3. [Scaling] Multiply all entries in row by non-zero constant.

### Echelon Form (ef)

- 1. All non-zero rows are above any rows of all-zero.
- 2. Each leading entry of a row is in a column to the right of the roe above it.
- 3. All entries in a column below a leading entry are 0.

## (1) Reduced Row Echelon Form (rref)

- 1. As to be in echelon form.
- 2. Leading entry in each row is 1.
- 3. Each leading 1 is the only non-zero entry in its column.

#### **Pivot**

(2)

A pivot position

#### Theorems

**Theorem 1** Every matrix is row equivalent to a unique row echelon form.

**Theorem 2** Every matrix is row equivalent to a unique row echelon form.