Discrete Mathematics

Class: MATH 2410

Mathematical Statement

Statement	is any declarative sentence which is either true or false.
Atomic	if it cannot be divided into smaller statements.
Molecular	if it can be divided into smaller statements.
conjunction	$p \wedge q$ equivalent to "p and q".
disjunction	$p \vee q$ equivalent to "p or q".
	where p is the hypothesis and q the conclusion.
Implication	$p \rightarrow q$ equivalent to "if p then q".
Biconditional	$p \leftrightarrow q$ equivalent to "if and only if p then q".
Negation	$\neg p$ equivalent to "not p".
Converse	
Contrapositive	
There is a x	$\exists x$
For all x	$\forall x$

Naive Set Theory

Set Notation

Universal set	$\mathbb{U}$
Empty set	$\emptyset = \{\}$
Power set	$\mathcal{P}(A)$ is the set of all the subsets of A.
Element of	$\in$. Example:$2 \in \{1, 2, 3\}$
Subset of	$\subseteq$. Example: $\{A, B, C\} \subseteq \{B, C, D\}$
Proper subset of	$\subset$. Example: $\{A, B, C\} \subset \{A, B, C, D\}$
Intersection	$\bigcap_{i \in I} A_i = \{x \in \mathbb{U} \mid \forall i \in I, x \in A_i\}$ $A \cap B = \{x \in \mathbb{U} \mid x \in A \wedge x \in B\}$
Union	$\bigcup_{i \in I} A_i = \{x \in \mathbb{U} \mid \exists i \in I, x \in A_i\}$ $A \cup B = \{x \in \mathbb{U} \mid x \in A \vee x \in B\}$
Difference	$A \backslash B = \{x \in A \mid x \notin B\}$
Symmetric difference	$A \Delta B = (A \backslash B) \cup (B \backslash A)$
Cartesian Product	$A \times B = \{(x, y) \mid x \in A \wedge y \in B\}$
Complement of	$\bar{A} = \{x \in \mathbb{U} \mid x \notin A\}$
Cardinality	$\lvert A \rvert$

Functions

Functions	A rule that assigns each input exactly one output.
Range	The set of all elements which are assigned to at least one element of the domain by the function.
Domain	The set of all input of a function.
Codomain	The set of all allowable output a function.
$f : x \rightarrow y$	a function f with a domain x and a codomain y.
Recursive f.	
Injectiuve	every element of the codomain is the image of **at most** one element from the domain.
Surjective	every element of the codomain is the image of **at least** one element from the domain.
Bijection	A function that is **Injective** and **Surjective**.
Image	$f(A) = \{f(a) \in Y : a \in A\}$, where $A \subset$ domain.
Inverse Image	$f^{-1}(B) = \{f(b) \in X : b \in B\}$, where $B \subset$ codomain.

Counting

Additive Principle

General Definition: if event A can occur in m ways, and even B can occur in n **disjoint** (A and B can't apen at the same time.) ways, then A and B can occur in $m + n$ ways.
Set Definition: Given 2 sets A and B, if $A \cap B = \emptyset$, then $|A \cap B| = |A| + |B|$.

Multiplicative Principle

General Definition: if event A can occur m ways, and each possibility for A allows for exactly n ways for event B, then the event "A and B" can occur $m \cdot n$ ways.
Set Definition: Given 2 sets A and B, we have $|A \times B| = |A| \cdot |B|$.

Sequences

Symbolic Logic

deMorganLaws

- $\neg \forall x P(x) = \exists x p(x)$
- $\neg \exists x P(x) = \forall x p(x)$
- $\neg(a_1 \wedge a_2 \wedge \cdots \wedge a_n) \equiv \neg a_1 \vee \neg a_2 \vee \cdots \vee \neg a_n$
- $\neg(a_1 \vee a_2 \vee \cdots \vee a_n) \equiv \neg a_1 \wedge \neg a_2 \wedge \cdots \wedge \neg a_n$

Proofs

Graph Theory