

## Complex Analysis Short recap

Let  $a \in \mathbb{R}$ ,  $b \in \mathbb{R}$  and  $z \in \mathbb{C}$ .

Real part  $\Re(a + ib) = a$

Real part  $\Im(a + ib) = b$

Absolute Values  $|z| = \sqrt{zz^*} = \|(a \ b)\|_2$

Complex conjugate  $(a + ib)^* = a - ib$

$(a - ib)^* = a + ib$

### Properties 1 (Complex conjugate)

- $(Z^*)^* = Z$
- $(Z + W)^* = Z^* + W^*$
- $(Z - W)^* = Z^* - W^*$
- $(ZW)^* = Z^*W^*$
- $Z^*Z = |Z|^2$
- $(Z^n)^* = (Z^*)^n$ , for  $n \in \mathbb{Z}$
- $\ln(Z^*) = (\ln(Z))^*$  if  $Z$  is not 0 or a negative real number.

### Properties 2 (Complex Absolute Values)

- $|z_1 z_2| = |z_1| |z_2|$
- $|z_1 + z_2|^2 = |z_1|^2 + |z_2|^2 +$

## Linear Algebra

### Linear Operator

A **linear operator** between the vector spaces  $V$  and  $W$  is defined to be any function  $A : V \rightarrow W$  which is linear in its input.

**Properties 3** Let  $\hat{A}$  be a linear operator on  $V \rightarrow W$  and  $A$  be the matrix representation of  $\hat{A}$ .

- $\hat{A}(\sum_i a_i |v_i\rangle) = \sum_i a_i \hat{A}|v_i\rangle$
- $\hat{A}|v_j\rangle = \sum_i A_{ij} |w_i\rangle$

### Inner product

**Properties 4**  $\langle A, A \rangle = \|A\|^2$

## Quantum mechanics

### Pauli Matrices

- $\sigma_0 = I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$
- $\sigma_x = X = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$
- $\sigma_y = Y = \begin{bmatrix} 0 & -i \\ i & 0 \end{bmatrix}$
- $\sigma_z = Z = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$

## Mathematical Concepts

### Dirac Notation(or Bra-Ket Notation)

Terminology: ket of  $A$  is  $|A\rangle$  and bra of  $A$  is  $\langle A|$ . Example, let  $\Sigma = \{A, B, C\}$  then

$$|A\rangle = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}, |B\rangle = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} \text{ and } \langle A| = (1, 0, 0),$$

$\langle B| = (0, 1, 0)$  Note that  $|\Psi\rangle$  then  $\langle\Psi| = |\Psi\rangle^T$ .

**bra-kets:** we denote  $\langle a|b\rangle$  the matrix product of

### Cartesian product

Let  $y = \{0, 1\}$  and  $x = \{a, b\}$ . Then,  
 $x \times y = \{(0, a), (0, b), (1, a), (1, b)\}$  and  
 $y \times x = \{(a, 0), (a, 1), (b, 0), (b, 1)\}$ .

### Tensor Product of vectors

$$\begin{bmatrix} a_0 \\ \vdots \\ a_n \end{bmatrix} \otimes \begin{bmatrix} b_0 \\ \vdots \\ b_n \end{bmatrix} = \begin{bmatrix} a_0 b_0 \\ \vdots \\ a_n b_n \end{bmatrix}$$

### Tensor Product Properties

1.  $(|\phi_1\rangle + |\phi_2\rangle) \otimes |\psi\rangle = |\phi_1\rangle \otimes |\psi\rangle + |\phi_2\rangle \otimes |\psi\rangle$
2.  $(a|\phi\rangle) \otimes |\psi\rangle = a(|\phi\rangle \otimes |\psi\rangle)$
3.  $|a\rangle \otimes |b\rangle = |b\rangle \otimes |a\rangle$

## Quantum Information Systems

### State Vector

Quantum State of a system is represented by a complex column vector. Let the quantum

state vector  $v$  be equal to  $\begin{bmatrix} a_0 \\ \vdots \\ a_n \end{bmatrix}$ , where

$\sum_{i=0}^n |a_i|^2 = 1$  The **euclidean norm** of the  $\|v\| = \sqrt{\sum_{i=0}^n |a_i|^2}$

### Common Quantum States

Plus State  $|+\rangle = \frac{1}{\sqrt{2}}|0\rangle + \frac{1}{\sqrt{2}}|1\rangle$   
Minus State  $|-\rangle = \frac{1}{\sqrt{2}}|0\rangle - \frac{1}{\sqrt{2}}|1\rangle$   
Other State  $\frac{1+2i}{3}|0\rangle - \frac{2}{3}|1\rangle$

### Standard Basis Measurement

Let a quantum system be in the state  $|\psi\rangle$ , then the probability for the measure

outcome to be  $a$  is  $Pr(\text{outcome} = a) = |\langle a|\psi\rangle|^2$  If  $U$  is a unitary matrix then the following Properties hold,  $\|U\psi\| = \|\psi\|$

## Unary Operations

### Unary Matrices

A square matrix  $U$  having complex number entries is unitary if it satisfies the equations,  $UU^\dagger = U^\dagger U = \mathbb{I}$  where  $\mathbb{I}$  is the identity matrix.

### Some unitary operations on qubits

Pauli operations:

$$\mathbb{I} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \sigma_x = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \sigma_y = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} \sigma_z = \begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix}$$

$$\text{Hadamard operation: } H = \begin{pmatrix} \frac{1}{\sqrt{2}} & 0 \\ 0 & \frac{1}{\sqrt{2}} \end{pmatrix}$$

$$\text{Phase operations: } P_\theta = \begin{pmatrix} 1 & 0 \\ 0 & e^{i\theta} \end{pmatrix}$$

## Quantum circuit