Hiring New Teachers

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9/25/2020

SUMMARY

In this paper, we propose a method of fairness for hiring 7 additional teachers at MST. MST is a magnet school named Mathematics, Science, and Technology, and has added a new wing to increase its school size from 490 to 630, therefore requiring more teachers to handle the additional workload. Our method first calculates the student to teacher ratios in the 2021-22, 2022-2023, 2023-2024 school years, and then employs a greedy/minimax algorithm to minimize the maximum student per teacher ratio with a focus on the major disciplines. The major disciplines are defined as English, Social Studies, Mathematics, Physics, Biology, Chemistry, and Foreign Language. We believe this algorithm is the most fair method of hiring teachers, as it ensures no teacher has too large of a workload. Our solution found that demand for certain departments were higher than others to a degree that one department should receive two new teachers while some departments shouldn't receive new teachers at all.

THE PROBLEM

We are being asked by MST to propose a method of fairness for hiring 7 additional teachers to incorporate the fact 140 additional students are being accepted in the 2021-22 class. We are given the enrollment totals of August 2020 and the number of teachers present for each department before the addition of the 7 teachers. The number of teachers for each department are listed below.

- 6 mathematics
- 5 English faculty
- 3 chemistry faculty
- 3 foreign language faculty
- 3 physics faculty
- 1 music instructor
- 4 biology faculty
- 1 art instructor
- 5 social studies faculty

ASSUMPTIONS

- For the sake of simplicity, we are assuming that students will not drop out (therefore ignoring the 5% dropout rate that the Registrar's office suggests). This means that each grade's class size is modulo 3, repeating in size for every 3 years.
    - Based on a separate simulation, a dropout rate of 5% will not even the classes out for 100+ years. Therefore, we can assume that in the short run, the aggregate of each class's size will be similar to the initial values for the class size.

- The school will not hire more teachers for a couple more years. This means that when calculating the student to teacher ratio, we have to average one cycle of class sizes (modulo 3).
    - This is a reasonable assumption as most teachers will stay with an institution for several years, and even if a teacher leaves, MST is likely to replace the teacher with a teacher in the same department.
- The proportions of the new classes with extra people are the same as in the year before extra students were added
    - The school does not state that the curriculum or placement systems are changing and interest in departments are assumed to stay the same.
- The previous student-to-teacher ratio is not necessarily fair or unfair.
    - Thus, this ratio is not used in our definition of fairness.
- A foreign language teacher can teach 2 classes; however, if a teacher does, then their workload will be split between the two classes according to class size (the number of students in each class) in order to maximize fairness.
- Every student takes only one English class during each grade, and so we can keep track of class sizes using the number of students taking English in every year.
    - This assumption was made through the observation that the sum of students across all English classes added up to the total number of students enrolled in the school.
- The maximum capacity of the school is held constant at 630 students after the 2021-22 year, with no additional students being added in any subsequent year.
    - This is reasonable as MST is not likely to make a huge renovation that increases the number of students soon after adding a new wing.
- The workload for each teacher per student is constant throughout all the major disciplines.

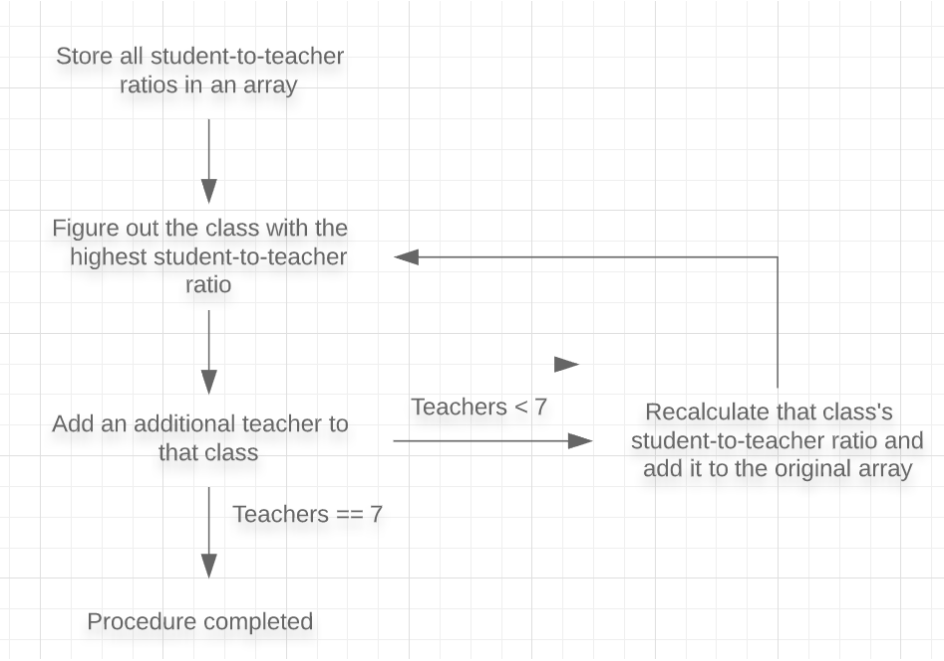
PROBLEM ANALYSIS AND MODEL JUSTIFICATION

We were presented with devising an algorithm to fairly hire 7 additional teachers at MST. In order to fairly calculate which subjects would need these new teachers, we devised a definition of fairness that we directly apply to this problem. Our definition for *fairness* was to minimize the highest student-to-teacher ratio out of all the classes using a greedy/minimax algorithm, and we gave priority to the major disciplines defined by the problem: English, Social Studies, Mathematics, Physics, Biology, Chemistry, and Foreign Languages. We also defined another term, *need*, as being the greatest for the classes with the maximum student-to-teacher ratio amongst all the disciplines (giving priority to the major disciplines), since a class with a higher student-to-teacher ratio would necessitate the addition of another teacher to lighten the workload for each teacher. We believe our minimax procedure is the most accurate measure of fairness because the method would allow us to allocate teachers to disciplines with the highest amount of *need* without basing our results on previous years.

STEPS AND MODEL DESIGN

Step 1: Add 140 students to the incoming class and calculate the total enrollment for each grade accordingly.
Step 2: Calculate the enrollment of each class for one cycle of 3 years, since the enrollment repeats modulo 3.
Step 3: Calculate the average enrollment for each class over the cycle of 3 years.
Step 4: Calculate the average student-to-teacher ratio over the cycle.
Step 5: Use the greedy/minimax procedure to figure out what disciplines should have more teachers.

At each iteration of our model, we deployed the minimax function, f(x) = min(max(student-to-teacher ratio)), to determine the most fair method for hiring new teachers.



SOLUTION

We prepared our data using the aforementioned steps and then used our model to calculate what disciplines require more teachers.

Initially, we noticed that French had the highest student-to-teacher ratio out of all major disciplinary subjects (therefore excluding Music). We also noticed that the student-to-teacher ratio for Spanish was relatively high, and therefore allocated half a teacher to both French and Spanish (which is allowed under our assumption that foreign language teachers can teach up to 2 languages at once).

Department	AVG students	Initial Teachers	Students-per-teacher Ratio	Teachers Added
Art	130.6666667	1	130.6666667	
Biology	338.6666667	4	84.66666667	
Chemistry	412.6666667	3	137.5555556	
English	630	5	126	
French	161	1	161	
German	62.66666667	1	62.66666667	
Spanish	133.3333333	1	133.3333333	
Mathematics	884	6	147.3333333	
Music	202.6666667	1	202.6666667	
Physics	436.3333333	3	145.4444444	
Social Studies	435	5	87	

We allocated 60 percent of that teacher's time to French and 40 percent to Spanish in order to even out the student-to-teacher ratios.

Department	AVG students	Initial Teachers	Students-per-teacher Ratio	Teachers Added
Art	130.6666667	1	130.6666667	
Biology	338.6666667	4	84.66666667	
Chemistry	412.6666667	3	137.5555556	
English	630	5	126	
French	161	1	100.625	0.6
German	62.66666667	1	62.66666667	
Spanish	133.3333333	1	95.23809524	0.4
Mathematics	884	6	147.3333333	
Music	202.6666667	1	202.6666667	
Physics	436.3333333	3	145.4444444	
Social Studies	435	5	87	

The next highest student-to-teacher ratio is Mathematics, so we add one teacher there.

Department	AVG students	Initial Teachers	Students-per-teacher Ratio	Teachers Added
Art	130.6666667	1	130.6666667	
Biology	338.6666667	4	84.66666667	
Chemistry	412.6666667	3	137.5555556	
English	630	5	126	
French	161	1	100.625	0.6
German	62.66666667	1	62.66666667	
Spanish	133.3333333	1	95.23809524	0.4
Mathematics	884	6	126.2857143	1
Music	202.6666667	1	202.6666667	
Physics	436.3333333	3	145.4444444	
Social Studies	435	5	87	

We continued this minimax procedure until we hired 7 additional teachers.

Department	AVG students	Initial Teachers	Students-per-teacher Ratio	Teachers Added
Art	130.6666667	1	130.6666667	
Biology	338.6666667	4	84.66666667	
Chemistry	412.6666667	3	103.1666667	1
English	630	5	105	1
French	161	1	100.625	0.6
German	62.66666667	1	62.66666667	
Spanish	133.3333333	1	95.23809524	0.4
Mathematics	884	6	110.5	2
Music	202.6666667	1	101.3333333	1
Physics	436.3333333	3	109.0833333	1
Social Studies	435	5	87	
Total		31		7

Because the major disciplinary subjects have similar student-to-teacher ratios (with the maximum out of all them being Mathematics with a student-to-teacher ratio of 110.5), we decided to add a teacher to Music to appropriately even the student-per-teacher ratio for that class. **This is the division of hiring teachers that we propose to MST.**

STRENGTHS AND WEAKNESSES

Some of the strengths for our model include the fact that we don't rely on previous years' student-to-teacher ratios to decide what is fair. If we relied on previous data, we would be forced to assume that the teachers allotted to each discipline in previous years was fair, which would make our model much less stable and our error analysis much more complicated if it wasn't true. Rather than depending on previous data, we chose to focus on equity between the student-to-teacher ratios of each discipline, giving priority to the given major disciplines in the problem.

A weakness in our model is our assumption that each of the major disciplines has the same workload for teachers. This assumption is the basis for fairness in our model and if it is untrue, it may result in some teachers having a significantly higher workload than others. We could, however, perform a sensitivity analysis on this assumption and calculate how much more workload (defining *workload* by a set student-to-teacher ratio) could be put on one or more disciplines before the difference between the highest student-to-teacher ratios amongst all the disciplines becomes too high. Another weakness of our model is that we are unable to reach a perfectly equal student-to-teacher ratio between each discipline with our current procedure and the number of additional teachers to be hired.

TEAM MEMBER CONTRIBUTIONS

Sean Jung - Worked on gathering data, discussed fairness, wrote summary and problem restatement, and helped out with assumptions.
Carina Lei - Worked on gathering data, discussed fairness, wrote problem analysis and model justification.
Grace Sullivan - Worked on gathering data, analyzing strengths and weaknesses, discussed and justified fairness, and created google sheets formulas.
Evan Zhang - Worked on gathering data, discussed fairness, and worked on the modeling procedure.

REFERENCES

1. Posner, R., & Sala, M. (2019, August 01). Romance languages. Retrieved September 25, 2020, from https://www.britannica.com/topic/Romance-languages

A special thanks to Ms. Belledin for helping us with the modeling procedure and proactively answering our questions.