Tanya Qu, Stokes Danell, Divya Aikat, Evan Zhang

Hello boss,

Our team has investigated your problem and would like to present potential solutions to the problem. Since this problem is most applicable during rush hour, we assume that each elevator carries the maximum amount of people possible (10). We also assume that the elevators won't reopen for additional people, and that all workers go immediately to their designated floor.

Before proposing our solutions, we needed to figure out what is happening now to help us establish a goal for us to beat. We designed a mathematical model which uses the probability that an elevator skips a floor given the number of workers on each floor. For example, a floor with less workers (3,5,6) would be skipped relatively more than the floors with more workers (1,2,4). In this probability model, we saw that the process of taking every worker to their respective floors currently takes around 22 minutes. Taking into account the 110 people late, or 11 elevator trips (2-3 trips per each of the elevators), we were able to calculate ~17 minutes elevator cycle as the ideal time to beat with our solutions. Additionally, we designed a computer simulation that uses the given variables and runs multiple simulations to converge to an average.

Our perhaps more realistic solution, would be to assign each worker to an elevator, and the elevator they are assigned is dependent on what floor they work on. We can optimize this solution by grouping certain groups of workers; we figured out that assigning floors 2 and 4 to separate elevators, and grouping floors 1, 3 and 5, 6 would lead to the fastest solution. Floor 2, 4, and 1&6 all have 120 people total, while floor 3&5 will serve 140.

Our next solution is to create 6 waiting stations, one per floor. Each worker would go to their respective waiting station and then get on an elevator when there are more than 10 people at the waiting station. This solution would be effective if plausible, but is rather unfeasible.

Here is a list of our assumptions:

1. There will be no reopening of the elevator doors once they have hit each floor it is going to.
    a. People will exit the elevator the first time the elevator lands on their floor, meaning there is no need to stop on the way down from.
2. There is no movement within the floors (i.e. no worker already on floor 6 will go down to floor 3 for fun)
3. Each elevator starts at Ground Zero
4. Every elevator will contain 10 people every time it operates

Tanya Qu, Stokes Danell, Divya Aikat, Evan Zhang

Current situation

Seconds it takes for an elevator trip:

$$15 + 8(a) + 10(b)$$

15 =	how long it takes to load from the ground floor
8 =	4 seconds between the floors, the elevators must go up and down so this is doubled.
a =	highest floor the elevator reaches
10 =	time it takes to unload the elevator on each floor
b =	number of floors the elevator actually stops at instead of skipping

The number of trips total should be 50, as each can take 10 max. Our calculations are done for 1 elevator because each elevator should have the same actions. An elevator carries around 13 trips.

Calculations of the probability of a floor being skipped based on the number of workers on each:

f6

$$\binom{480}{10} / \binom{500}{10} = .662$$

$.662(13) = 8–9$

8–9 trips per 13 skip f6

f3

$$\binom{440}{10} / \binom{500}{10} = .275$$

$.275(13) = 3–4$

3–4 trips per 13 skip.

f5

$$\binom{420}{10} / \binom{500}{10} = .17$$

$.17(13) = 2–3,$

2–3 trips skip f5

f5 + 6

$$\binom{500-20-80}{10} / \binom{600}{10} = .1049$$

$.1049(13) = 1–2$

1–2 trips skip floors 5+ 6.

- The probability of skipping floors 1, 2, 4, 3&6, 3&5 are so little (not even 1 trip) they're negligible.
- F6: 8-9 trips skipped, 3-4 trips hit; F5: 3-4 trips skipped, 9-10 hit;
- F6: 4/13 times hit; F5: 10/13 times hit.
- 1 trip skips floors 5+6, meaning the highest floor it'll go to is 4 ∴ **12 trips must go to either / both floor 5 and floor 6**
- Out of the 12 trips that go to either/both F5 & F6, using our probability calculations, 4 trips must go to floor 6, and 10 trips must go to floor 5 ∴ **2 trips must go to both f5 & f6**

- After these 2 trips are accounted for, 2 more trips must go to floor 6 to complete the 4/13 trips needed for floor 6, and 8 more trips must go floor 5 to complete the 10/13 trips needed for floor 5

Highest floor each elevator trip goes to. (T = trip number), 6(5) = hits both floor 6 and 5

T1	T2	T3	T4	T5	T6	T7	T8	T9	T10	T11	T12	T13
4	6 (5)	6 (5)	6	6	5	5	5	5	5	5	5	5

Out of all 13 trips, floor 3 is skipped 3-4 times, therefore, we skipped floor 3 for each of the trips that go to floor 6 as its highest floor. This was arbitrarily chosen for simplicity, as it doesn't matter which trips we use to account for the skipping of floor 3.

Calculations

4 different types of trips per elevator:

A) Going to floors 1, 2, 3, and 4
B) Going to floors 1, 2, 4, 5, and 6
C) Going to floors 1, 2, 4, and 6
D) Going to floors 1, 2, 3, 4, and 5

Time for 1 cycle of each trip (using previous equation):

A) 87 sec
B) 113 sec
C) 103 sec
D) 105 sec

Trip cycles needed:

A) 1 cycle
B) 2 cycles
C) 2 cycles
D) 8 cycles

Therefore total time for each trip type = Cycles x Time per cycle

Total time for each trip:

A) 87 sec
B) 226 sec
C) 206 sec
D) 840 sec

Total time for one elevator to complete its required trips:

- 1359 sec OR 22.65 min

To find what time the elevators began moving, we use the total time for an elevator's full trips, the total people using elevators, and the amount of people late to make a proportion.

$$\frac{110\ people}{500\ people} = \frac{X\ min}{22.65\ min}$$

This proportion finds the time required for 110 people to use the elevators by relating it to the total use time and the total number of people using the elevators. Using this, we find that

$$x = 4.97\ min$$

This gives us the amount of time after the office opens that people were late. We can subtract this time from the total elevator use time to find the time the elevator was running before the office opened:

$$22.65\ min\ -\ 4.97\ min\ =\ 17.68\ min$$

This means the elevators were running for 17.68 min before the office opens, so the solution we come up with must have an elevator use time shorter than 17.68 min so that everyone arrives on schedule.

Proposed Solutions

To beat the current situation, we decided to assign each calculator a specific floor set it could serve. Elevator 1 will serve floors 1&6, elevator 2 will serve floors 3&5, elevator 3 will serve floor 2, and elevator 4 will serve floor 4. Under this grouping, 3 elevators will take 120 workers in the morning and 1 elevator (serving 3&5) will take 140 workers.

Time for all workers to be served using our solution:

Floors 1&6:

- Floor 6 & 1 only need to serve 120 workers, meaning only 12 trips are necessary
- Probability of skipping floor 6 from just floors 1 and 6, with 100 people in floor 1 and 20 people in floor 6:

$$\frac{\binom{100}{10}}{\binom{120}{10}} = .149$$

- The probability of skipping floor 6, is around 0.15. 15(13) = ~2, 2/12 trips for this elevator are likely to skip floor 6
- ∴ 2/12 trips will only serve floor 1

15 + 8(1) + 10(1) = 33

33 x 2 (two trips) = 66

66 seconds total for the 2/12 trips that only serve floor 1

- 10/12 trips must serve both floors

$$15 + 8(6) + 10(2) = 83$$
$$83 \times 10 = 830$$

830 seconds total for the other 10 trips this elevator

- 66 s + 830 s = **14.93 minutes for the elevator that only serves floor 1 and 6.**

Floors 5&3

- Floor 3 (60 people) and Floor 5 (80 people) have 140 workers in total, so 14 elevator trips will be necessary.
- Probability to skip floor 3:

$$\frac{80\ choose\ 10}{140\ choose\ 10} = .014$$
$$.014 \times 13 = .182$$

Less than one time skipped = Negligible, floor 3 never skipped

- Probability to skip floor 5:

$$\frac{60\ choose\ 10}{140\ choose\ 10} = .00064$$
$$.00064 \times 13 = .00832$$

Less than one time skipped = Negligible, floor 5 never skipped

- Therefore, every cycle will stop at both floor 3 and floor 5

$$15 + 8(5) + 10(2) = 75\ sec/cycle$$
$$75 \times 14 = 1050\ sec\ OR\ 17.5\ min$$

- **17.5** min is less than our maximum time of 17.68 sec, therefore this elevator grouping works and delivers everyone on time.

Floor 2:

- To calculate the time it takes for all 120 workers to be brought to floor 2, we can use the earlier equation for the seconds that each full elevator trip takes: 15 + 8a + 10b, where a is the highest floor reached and b is the number of floors stopped at. Each elevator will be at maximum capacity, carrying 10 workers, meaning that there will be 12 elevator trips necessary to accommodate all 120 workers.
- With 15 seconds of loading time at the ground floor, 8*2 = 16 seconds to go up to floor 2, and 10 seconds since only one floor stopped at, we can calculate each elevator trip to take 41 total seconds. With 41 seconds per trip and 12 necessary trips, it will take 41*12 = 492 total seconds, **8 minutes and 12 seconds,** to deliver all floor 2 workers.

Floor 4:

- To find the time it takes 120 floor 4 workers to reach their destination, we can solve for the value in a similar manner as the floor 2 solution. Once again, there are 12 elevator trips needed with 10 workers per elevator.
- 15 seconds of loading time + 8*4 seconds to go up to floor 4, and 10 seconds since only one floor is stopped at = 57 seconds per elevator trip. 57 seconds/trip * 12 trips = 684 total seconds, or **11 minutes and 24 seconds**, to reach all floor 4 workers.

Elevator 1 (floors 1&6) : 14.93 minutes
Elevator 2 (floors 3&5) : 17.5 minutes
Elevator 3 (floor 2) : 8.2 minutes
Elevator 4 (floor 4) : 11.4 minutes

Conclusion:

Since each of the groups is designated a separate elevator, the total time necessary to reach all of the workers at their designated floor is limited by the group that takes the longest, floors 3 and 5. It takes the elevator 17.5 minutes to deliver all the workers from floors 3 and 5, which is lower than 17.68 minutes, the amount of time that the elevators were running before the office opened. Since the estimated time is shorter than the time to beat, our plan successfully delivers all workers to their floors in a timely manner and should mitigate the issue of workers arriving late.

Though other groupings of elevators may yield a better result, we combined the elevators taking into account the likelihood of when workers came to work. For example, if we grouped 5&6, we risk a longer wait time for these elevators to fill to ten people who work on these floors. Grouping floors 1 & 6, though it has a longer time in between the floors, it’s more likely that these elevators will fill and reduce the wait time. As well as this, each elevator will serve a similar amount of people.