

# **Capítulo 2. - Tecnologías Básicas de Radio**

## **Chapter 2. - Radio Basics**

### **2. Kapituluua.- Oinarrizko Irrati Teknologiak**



Bsc Degree on Telecommunication  
Dpt. Communication Engineering  
Bilbao Faculty of Engineering

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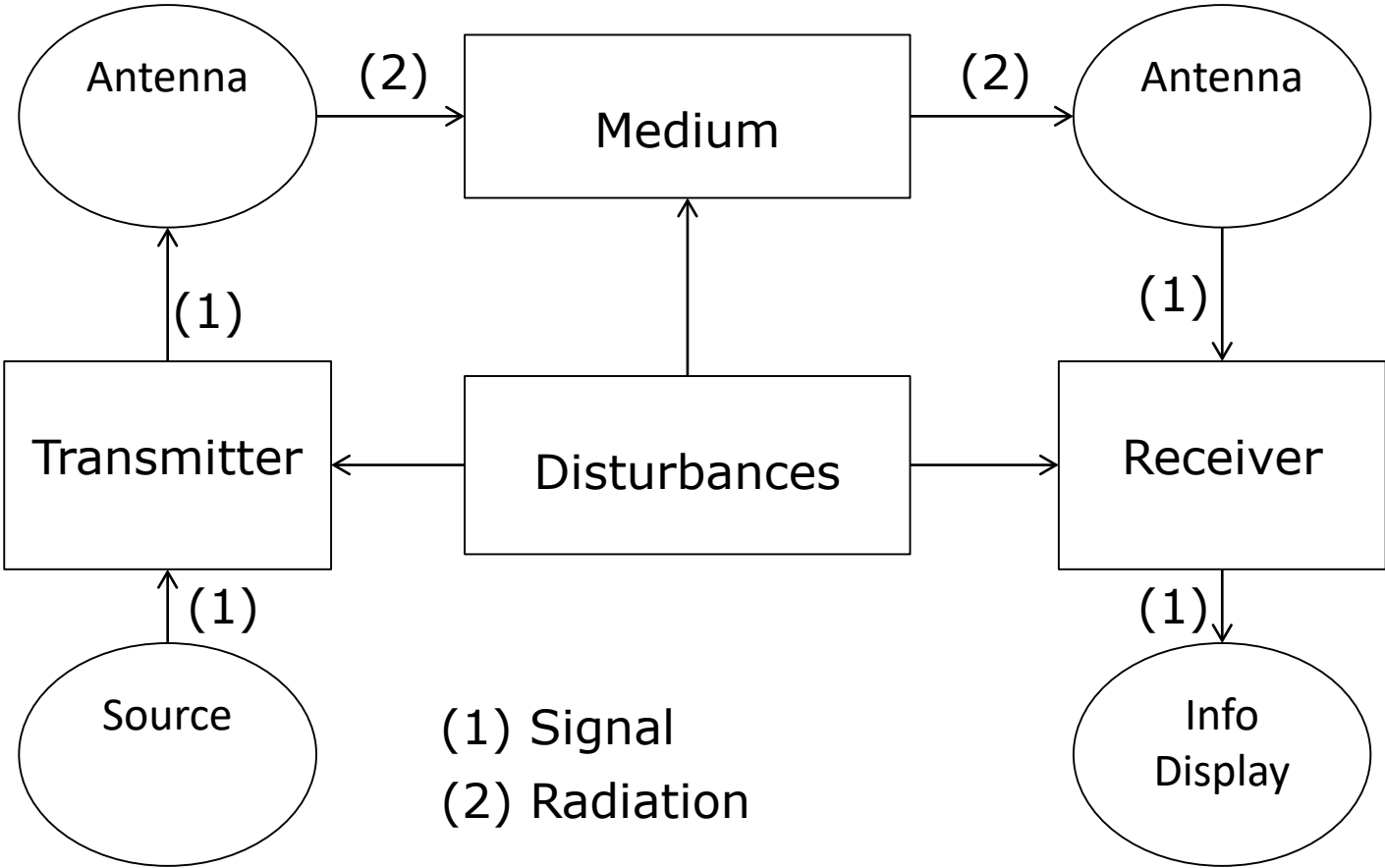
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5. Quality measurements. Alternatives to  $C/N$

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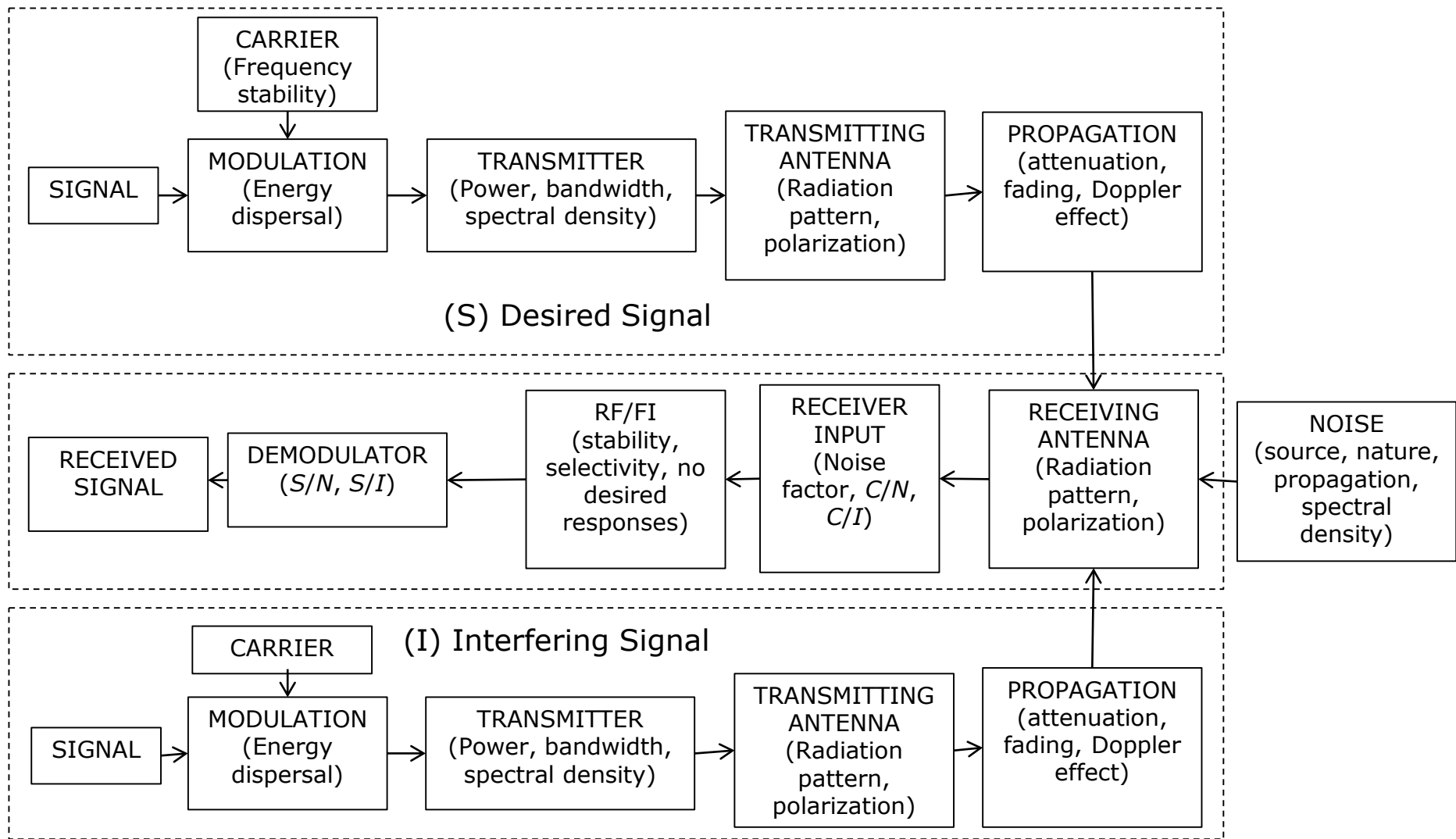
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1. Blocks of a generic radio communication system:
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# Blocks: General scheme



# Blocks: General scheme. Desired Signal, Interference and Noise



# Tx block. Signal processing: Channel Coding

## Coding

- ❑ In any digital communication errors occur when some of the transmitted bits are received with the wrong value. In Radiocommunication systems errors occur very often, because of the low power level of the received signal (C).
- ❑ **Channel Coding or Forward Error Correction (FEC)** is a technique that allows **detection and correction of erroneous bits** by adding **redundant bits** to the data bits.
- ❑ The ratio of data bits to total bits (data bits plus redundant bits) is named **code rate** (code rate < 1). Small code rates mean that the data bits are well protected but at expenses of effective bit rate reduction. Usual code rates are: 1/4, 1/3, 1/2, 2/3, 3/4, 4/5, 5/6 or 7/8, being **1/4 the most protected one and 7/8 the less** protected.
- ❑ The useful bitrate (or net bitrate) is obtained multiplying the gross bit rate by the code rate.
- ❑ There are **several Coding types**. The **best** are the ones that **maximize** the number of errors that can be **corrected for a given code rate**, but normally at expenses of **increasing** the processing **complexity**, which means more processing time, more hardware needed, **and** more power **consumption**.

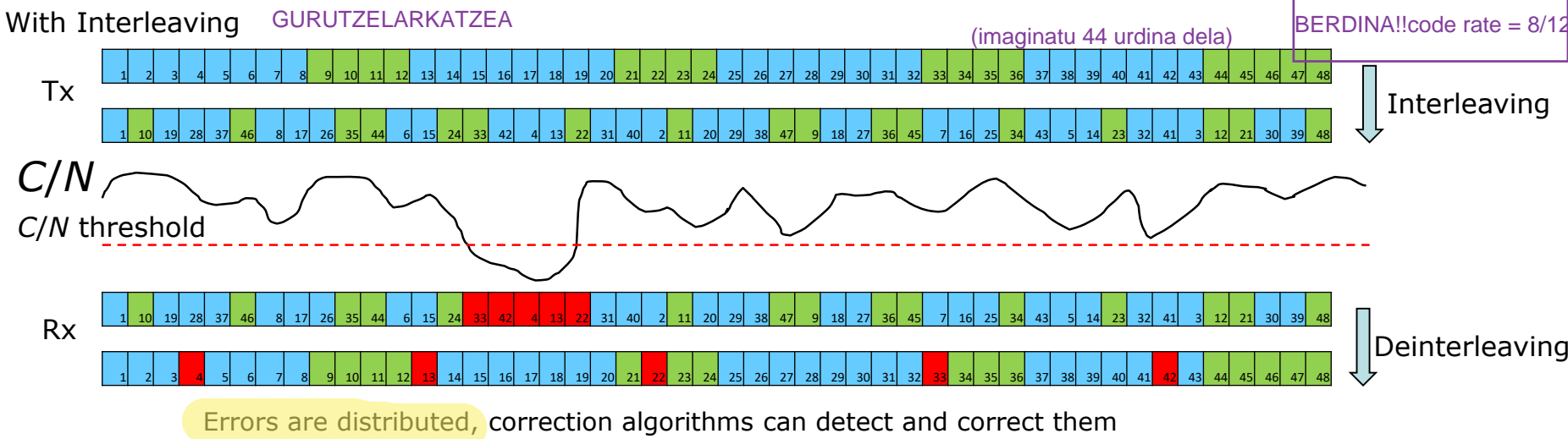
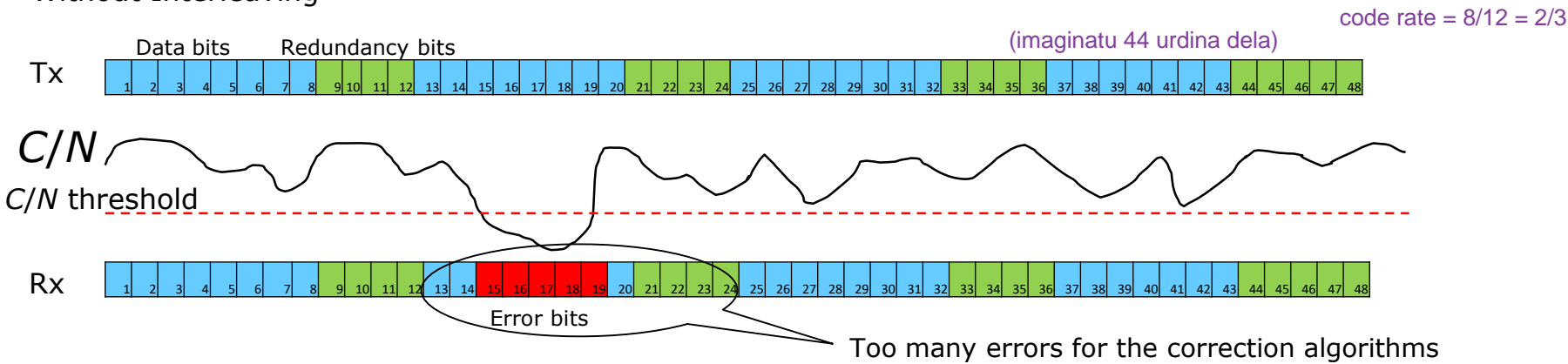
# Tx block. Signal processing: Interleaving

Gurutzelarkatzea

- ❑ In some systems to improve the performance of the channel coding an interleaving stage is added after coding.
- ❑ The interleaving **mixes the coded bits in the transmitter and rearrange them in the receiver.**
- ❑ The effect is that **errors occurred during short period of time are distributed along longer time**, and erroneous bits are more uniformly distributed, which helps to error detection and correction algorithms in the receiver.
- ❑ **The drawback is a latency time**, because the receiver has to **wait until** all the bits needed for **rearranging** are received.
- ❑ Interleaving is one of the reasons for the delay of the voice in GSM, and for long zapping times in some digital radio and TV systems.

# Tx block. Signal processing: Interleaving

Without Interleaving





# Tx block. Signal processing: Filtering

- Assigned RF spectrum: a central frequency ( $CF$ ) and a **bandwidth** ( $BW_{RF}$ ).

$$V_{bit} \geq V_{Symbol}$$

$$BW_{min} = V_{Symbol}$$

$$V_{bit} = V_{Symbol} \cdot M$$

$M$ ? M-QAM  $\rightarrow$  Zenbat bit sinbolo bakoitzean

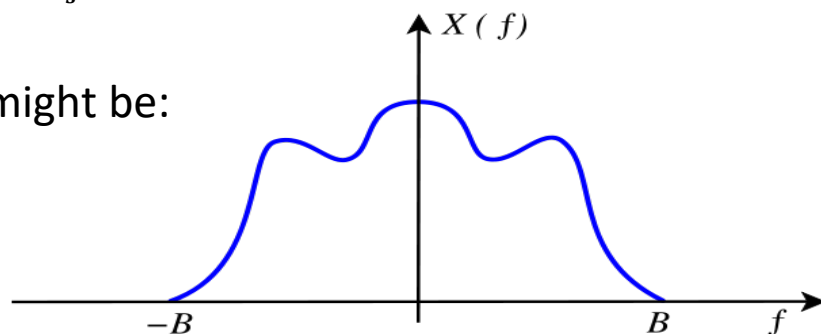
- However, circuits such as amplifiers are **not linear**: harmonics and spurious. Unwanted components out of the assigned RF spectrum  $\rightarrow$
- They may cause harmful interference to services occupying adjacent channels or even further away frequencies.
- In order to **minimize RF emissions out of the  $BW_{RF}$** , a **RF filter** is needed.
- The aim of the channel filter is to **limit the overlap of RF adjacent spectra**, thus reducing interference. **What is the right filter BW?**

# Tx block. Signal processing: Filtering

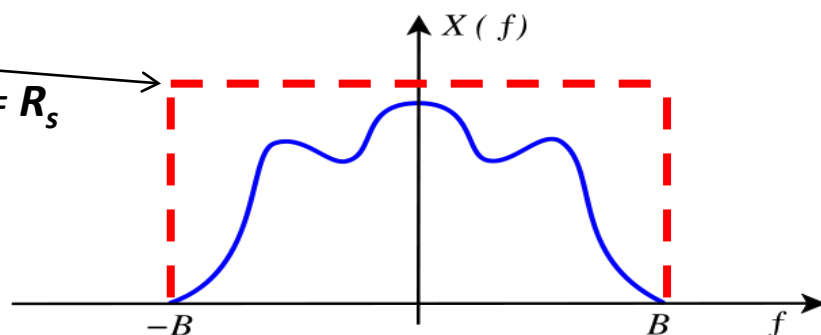
## □ Ideal Nyquist BW.

- Nyquist rate ( $R_s$ ): if the maximum frequency of a baseband signal is  $f_{max}$ , the sampling frequency for avoiding aliasing is:  $\frac{1}{T_s} = f_s = R_s = 2 \cdot f_{max}$

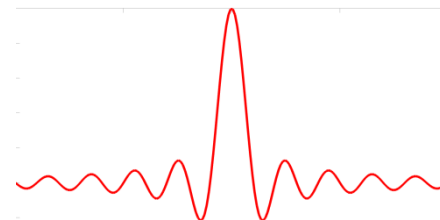
- The Fourier transform of any given signal might be:



- The ideal Nyquist filter would have a bandwidth of:  $BW_{Nyq} = 2 \cdot B = 2 \cdot f_{max} = R_s$



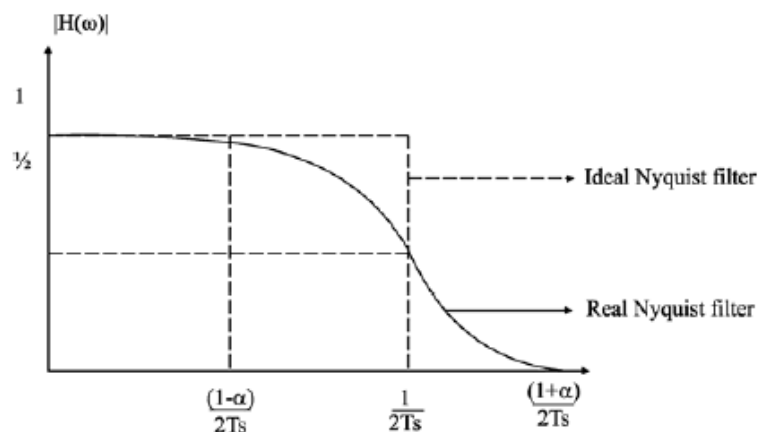
- Such a filter is not realizable as it is infinite in time:



# Tx block. Signal processing: Filtering

## □ Raised cosine filter or how to relax Nyquist BW with a roll-off factor ( $\alpha$ ):

- $\alpha_{min} = 0 \quad \Leftrightarrow \quad BW_{RF} = BW_{Nyq}$
- $\alpha_{MAX} = 1 \quad \Leftrightarrow \quad BW_{RF} = 2 \cdot BW_{Nyq}$



## □ The RF filter BW will then be: $BW_{RF} = (\alpha + 1) \cdot BW_{Nyq} = (\alpha + 1) \cdot R_s$

## □ Actual $\alpha$ values range from 15% to 40%.

# Tx block. Signal processing: Modulation

- ❑ Modulation is the process of **transferring the information** of the baseband modulating signal **to** any of the characteristics (amplitude, frequency, phase) of **a carrier signal** generated by a local oscillator.
- ❑ The **digital modulation** process associates the information sequence to a set of discrete amplitudes, or carrier frequencies or phases.
- ❑ Some examples of digital modulations:

Type	
PSK, Phase-shift keying	Quadrature PSK (QPSK - 4PSK)
	Differential QPSK (DQPSK),
	Offset QPSK (O-QPSK)
QAM, Quadrature amplitude modulation	16, 64, 256, and 1024

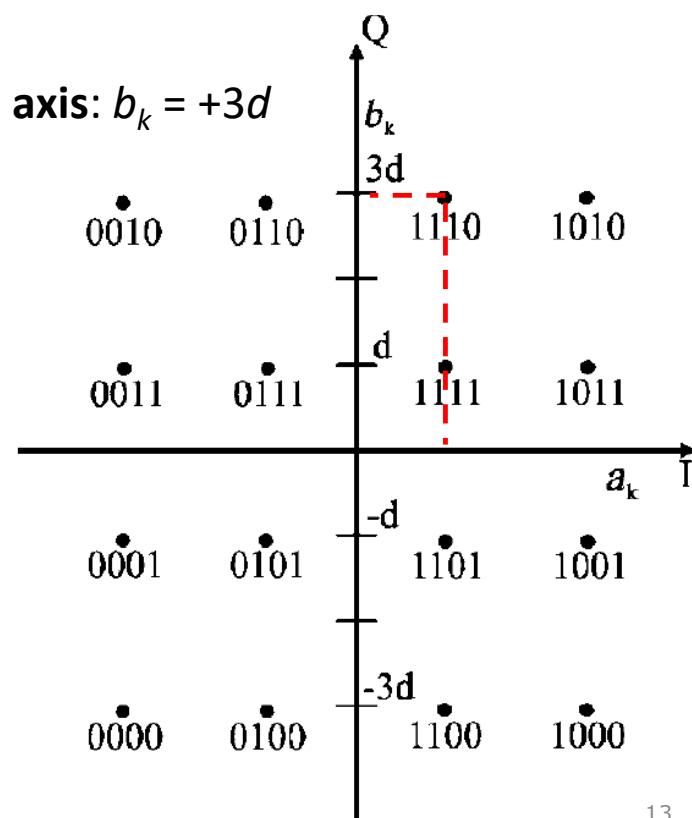
# Tx block. Signal processing: Modulation

## Example: 16 QAM Constellation points

□ The binary sequence “1110” is **mapped** on the point  $(d, 3d)$ :

- Coordinate in the  $0^\circ$  axis or *In-phase axis*:  $a_k = +d$
- Coordinate in the  $90^\circ$  axis or *Quadrature-phase axis*:  $b_k = +3d$
- If we consider the complex plane:

$$d + j \cdot 3d$$



# Tx block. Signal processing: Modulation

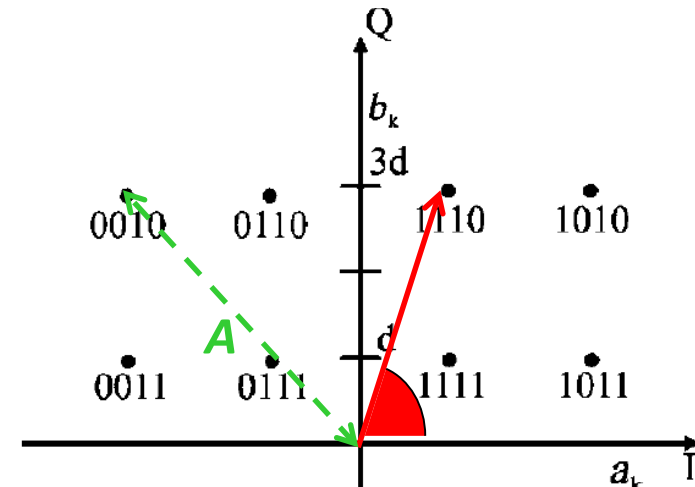
## Example: 16 QAM Constellation points

□ To transmit “1110” in RF we use a carrier with:

- Amplitude =  $\sqrt{d^2 + (3d)^2} = d\sqrt{10}$
- Phase =  $\text{atan}(3d/d) = 1.2490 \text{ rad} = 71.05^\circ$

In complex format:

$$d\sqrt{10} \cdot e^{j1.2490}$$



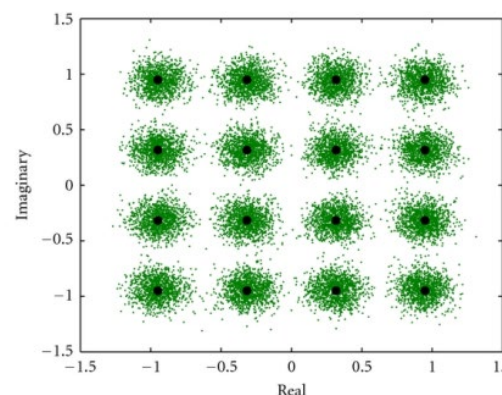
- The RMS power of this carrier would be:  $P_{1110} = (d\sqrt{10})^2/2 = 5 \cdot d^2$
- The amplitude is always normalized by the maximum amplitude of the constellation  $A$  ( $\sqrt{2 \cdot (3d)^2} = d\sqrt{18}$ )  $\rightarrow$  “1110” at the transmitter is:  $\sqrt{5/9} \cdot e^{j1.2490}$
- The **frequency** of the carrier can be placed within the ideal assigned bandwidth, that is, **within**  $BW_{Nyq} \leftarrow$  The remaining  $BW_{RF}$  is a safety margin not to be occupied

# Tx block. Signal processing: Modulation

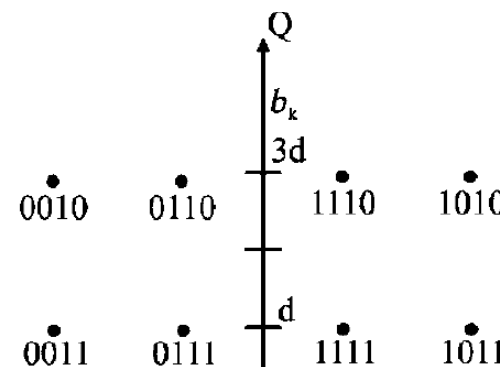
## Example: 16 QAM Decision distance

- ❑ Propagation degrades RF carriers  $\rightarrow$  at Rx “1110” will not be:  $A \cdot \sqrt{5/9} \cdot e^{j1.2490}$   
Distortion in amplitude and phase

- ❑ Signal equalization improves the signal to provide a cloud of points:



- ❑ After equalization. When do we have an error?
  - If the point is displaced, horizontally or vertically, more than “ $d$ ”: the decision distance.
  - “ $d$ ” is half the length of the segment that joins two consecutive points of the constellation.



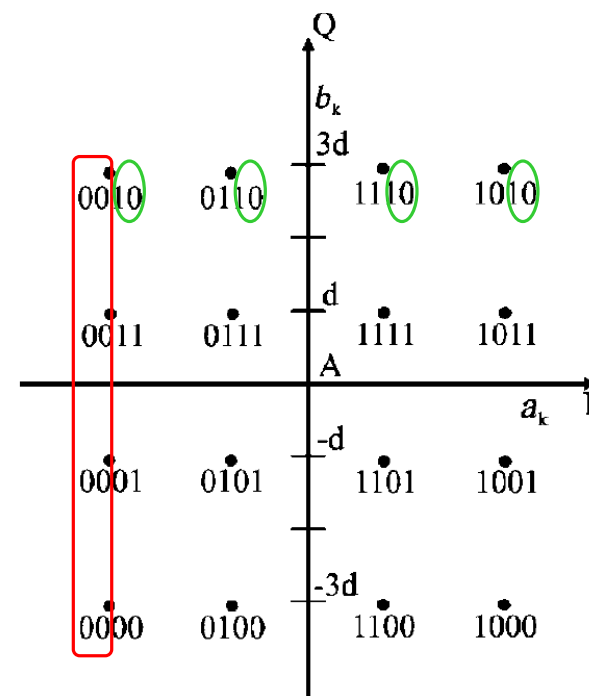
# Tx block. Signal processing: Modulation

## Example: 16 QAM Assignment of codes to constellation points

- The assignment is usually done in accordance to the Gray code:
  - Adjacent constellation states or points only differ in one bit.  
→ bit error rate is minimized when a demodulation error occurs.

- Codes are assigned in order to align bit patterns:

- 00XX points are located at  $a_k = -3d$
- XX10 points are located at  $b_k = +3d$
- 11XX points are located at  $a_k = +d$
- .....





# Tx block. Signal processing: Modulation

## Example: 16 QAM Average power

- $A = d\sqrt{18}$

- Power in one quadrant:

- $A_{1010} = A$

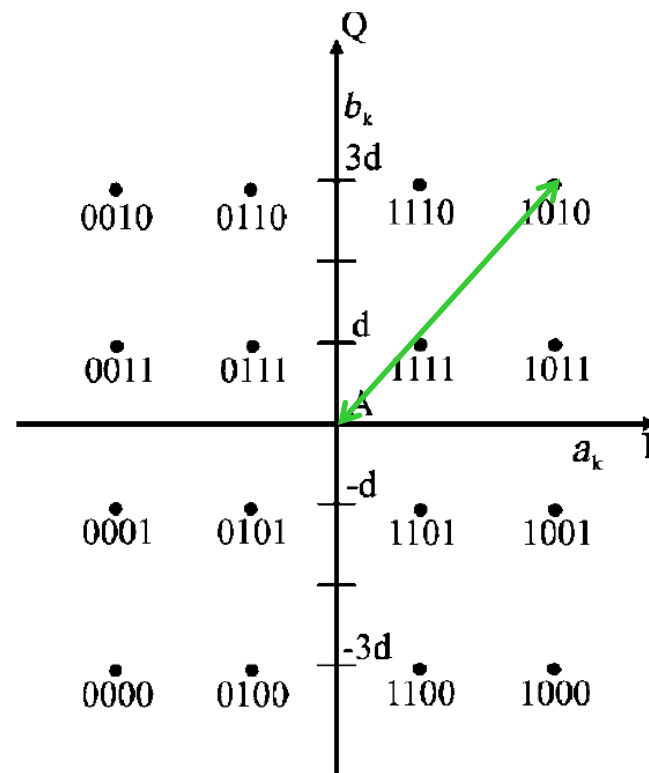
- $A_{1110} = A_{1011} = d\sqrt{10} = A \cdot \sqrt{5/9} = A \sqrt{10} / 3\sqrt{2}$

- $A_{1111} = d\sqrt{2} = A / 3$

$$P_c = \frac{1}{2}A^2 + 2\frac{1}{2}\left(\frac{A\sqrt{10}}{3\sqrt{2}}\right)^2 + \frac{1}{2}\left(\frac{A}{3}\right)^2 = \frac{10A^2}{9}$$

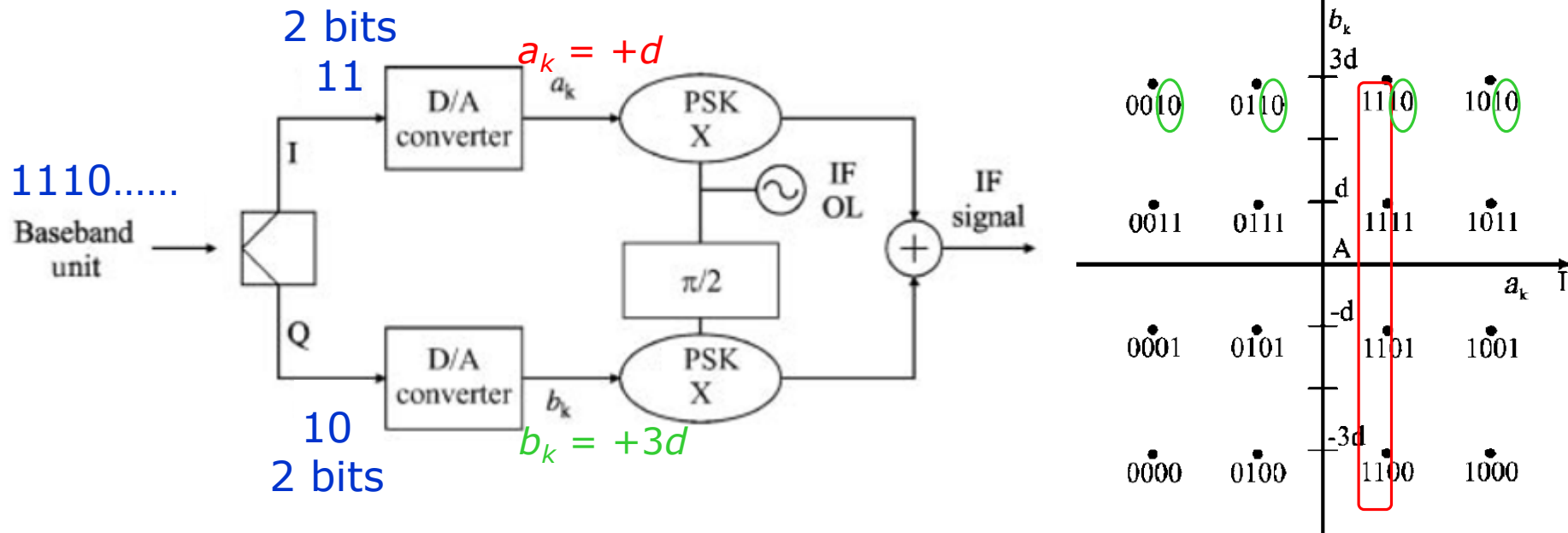
- Average modulation power:

$$P_m = \frac{P_c}{4} = \frac{5A^2}{18}$$



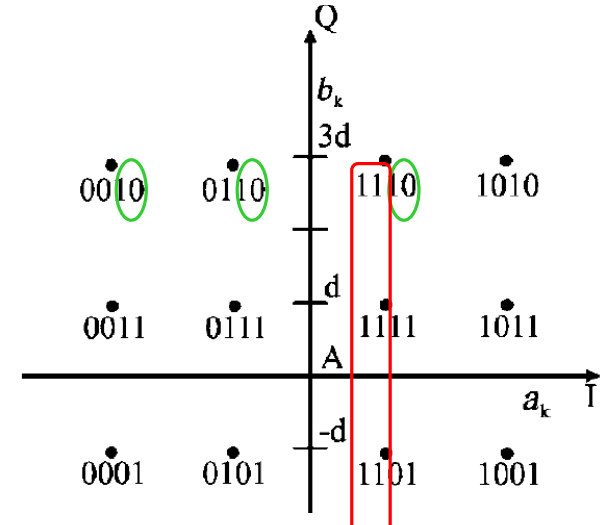
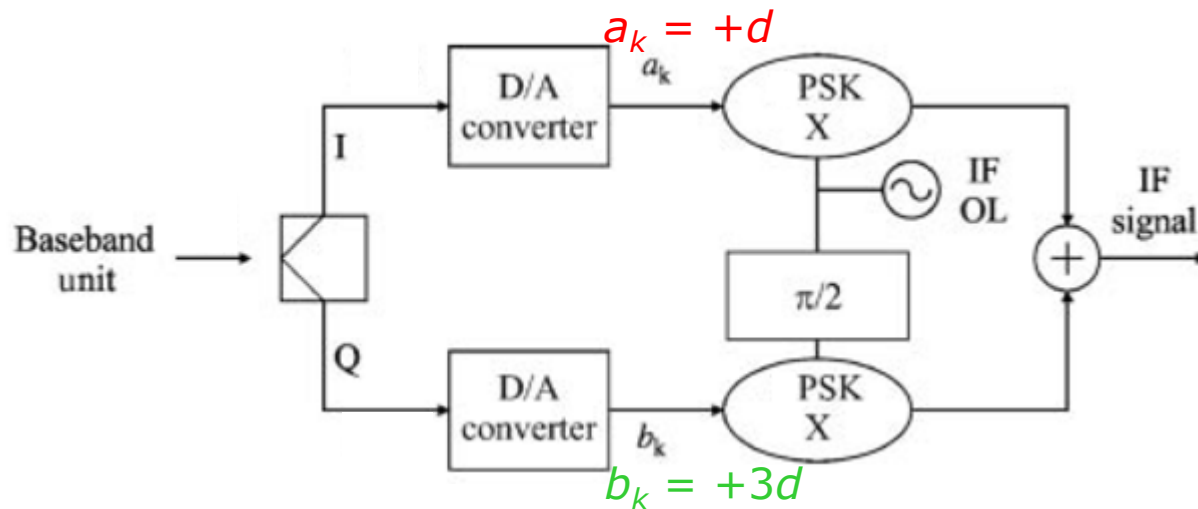
# Tx block. Signal processing: Modulation

## Block diagram of a typical quadrature **modulator** operating in IF



- The Digital/Analogue converters do the mapping by assigning values to  $a_k$  and  $b_k$
- The Oscillator (OL) generates the IF carrier,  $\cos(\omega_c t)$ , which passes through a first IF filtering stage (with a raised cosine filter for one carrier)

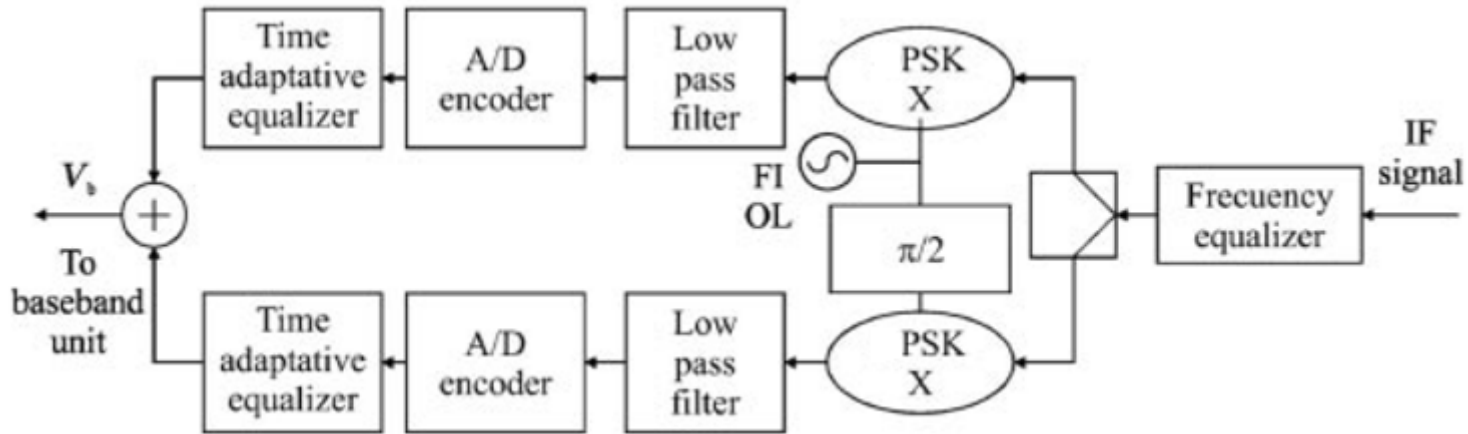
# Tx block. Signal processing: Modulation



- The IF carrier from the Oscillator (OL),  $\cos(\omega_c t)$  :
  - Gets to the **I** component (the upper one) mixer:  $a_k \cdot \cos(\omega_c t)$
  - Gets to the **Q** component (the lower one) mixer after a  $\pi/2$  phase shift:  $b_k \cdot \sin(\omega_c t)$
  - The resulting carrier is:  $d \cdot \cos(\omega_c t) + 3d \cdot \sin(\omega_c t) \iff d + j \cdot 3d = A \cdot \sqrt{5/9} \cdot e^{j1.2490}$
- A number of IF carriers is distributed in the bandwidth. They will be sent simultaneously
- When the group of carriers is ready, the whole bandwidth is shifted to RF
- The RF filter (a raised cosine filter for the whole RF bandwidth) is applied and the RF signal is sent to the power stage of the Tx and then to the air.

# Tx block. Signal processing: Modulation

## Block diagram of a typical quadrature demodulator



■ At the Rx, after RF filtering and once the Rx is synchronized in frequency and time, the whole process is reversed:

- RF to IF
- Equalization for improving the signal
- Recovery of components **I** and **Q** of the received signal
- A/D implements a decoder that provides the  $a_k$  and  $b_k$  estimations of the transmitted symbols  $\rightarrow$  1110, if everything is OK. Otherwise FEC (slide 6)

# Tx block. Signal processing: Transmission configuration

- The configuration of the transmitted signal is given by the parameters that characterize the radiocommunication standard:
  - Assigned RF spectrum: central frequency ( $CF$ ) and a bandwidth ( $BW_{RF}$ )
  - Symbol rate (related to the number of carriers within  $BW_{RF}$ ):  
 **$V_S$  (bauds or symbols/s)**  
Sampling frequency  $R_S = V_S \rightarrow BW_{RF} = (\alpha + 1) \cdot BW_{Nyq} = (\alpha + 1) \cdot R_S = (\alpha + 1) \cdot V_S$
  - Constellation of the modulation:  
**number of states or points or modulation levels is  $M=2^m$**   
16QAM:  $M=16$  points, each point mapping  $m=4$  **bits/symbol**
  - Raw or total bit rate:  **$V_b$  (bits/s or bps) =  $V_S \cdot m$**
  - Code Rate (1-CR is the fraction of redundant bits used for FEC)  
Net or useful bit rate:  **$V_{bN} = V_b \cdot CR$**
  - Average energy per bit:  **$e_b$  (J/b) =  $P_m / V_b$**  ( $P_m$  = avg modulation power, see slide 17)

# Tx block. Signal processing: Transmission Configuration

- Each signal configuration of a standard requires a specific  $C/N_{min}$  threshold value.

Rec. ITU-R BT.1368-10			
TABLE 1			
Proposed preferable DVB-T mode types for measurements on protection ratios			
Modulation	Code rate	$C/N^{(1)}$ (dB)	Bit rate <sup>(2)</sup> (Mbit/s)
QPSK	2/3	6.9	$\approx 7$
16-QAM	2/3	13.1	$\approx 13$
64-QAM	2/3	18.7	$\approx 20$

<sup>(1)</sup> The figures are given for a Gaussian channel (including a typical implementation margin) for a BER  $< 1 \times 10^{-11}$ .

- BER: Bit Error Rate (or Ratio) of  $10^{-11}$  or  $1 \times 10^{-11}$  means that there is 1 erroneous bit in  $10^{11}$  bits. A rule of three can be applied to calculate how many erroneous bits would be in X bits.
- A certain standard can cope with a certain maximum number of erroneous baseband bits before failing:  $C/N_{min}$  threshold values ensure:  $BER < BER_{MAX}$

# Tx block. Power

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- ❑ The transmitter takes the IF modulated signal to the RF channel by means of a mixer and amplifies it by means of Power Amplifiers (PA) to achieve the required power level (from some watts to some kW)
- ❑ Normally another **RF channel filter** is added as the **final stage** to avoid unwanted signal to be radiated.
- ❑ **PAs are very sensitive to reflected power**. In order to avoid reflections the impedance of the Tx and the one of the load (the antenna) should match.
- ❑ Also the PAs are responsible of some **degradation** of the transmitted signal **due to nonlinearities**.
- ❑ In the transmitters, a trade off is needed between maximum power and operating power for making the Tx operate in its more linear region (efficiency < 50%).
- ❑ **Transmitters datasheets** provide the **nominal** operating **power** and the impedance.

# Tx block. Power

What are we talking about when we say “power” or “nominal power” of a Tx?

- For **voltage and current calculation** purposes it is necessary to use **circuit theory**.

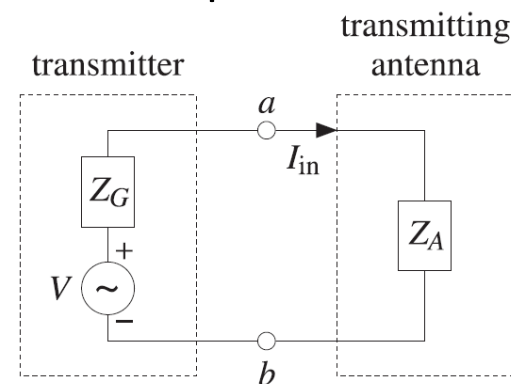
The Tx block prior to the antenna can be modeled by its Thevenin equivalent:

- Let's suppose the following data in a Tx datasheet:

- $V = 300 \text{ V}$
- $Z_G = 50 \Omega$

- How much power delivers this Tx to the load?

- If terminated with an open circuit ( $Z_A = \infty \Omega$ ):  $I_{in} = 0 \text{ A} \rightarrow P_{in} = 0 \text{ W}$
- If terminated with a short circuit ( $Z_A = 0 \Omega$ ):  $I_{in} = 300/50 = 6 \text{ A} \rightarrow P_{in} = 6^2 \cdot 0\Omega = 0 \text{ W}$
- If terminated with a load  $Z_A = 100 \Omega$ :  $I_{in} = 300/150 = 2 \text{ A} \rightarrow P_{in} = 2^2 \cdot 100 = 400 \text{ W}$
- **These power values depend on the load, they are not fixed values**
- But: If terminated **with a matched impedance** ( $Z_A = Z_G^* = 50 \Omega$ ), the **power** delivered to the load will **always** be the **maximum**:  $I_{in} = 300/100 = 3 \text{ A} \rightarrow P_{in} = 3^2 \cdot 50 = 450 \text{ W}$



- The **nominal power** ( $P_{Tx}$ ) must be the only reference value: the maximum power providable by the Tx:  $P_{Tx} = 450 \text{ W}$



# Tx block. Power

- ❑ Mismatch losses
- ❑ **Circuit theory model**: For calculation purposes we need  $V$  of the Thevenin equivalent circuit.
- ❑ Let's suppose the previous Tx with  $Z_A = 350 \Omega$ .  
How much power ( $P_{in}$ ) is delivered to the antenna?
  - $P_{Tx} = 450 \text{ W}$
  - $Z_G = 50 \Omega$

Nominal power (in this case: 450W) is calculated with  $Z_G = Z_A$

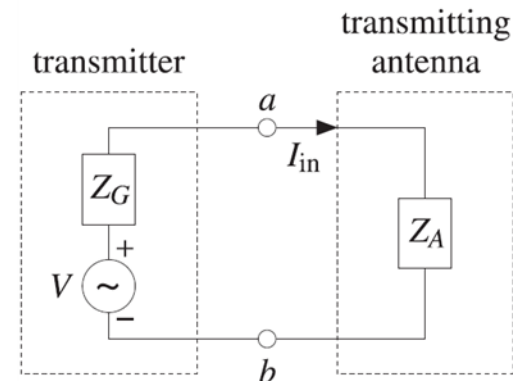
1) Calculate  $V$  supposing  $Z_A = Z_G^* = 50 \Omega$ :

$$\begin{aligned} \circ P_{in} &= I_{in}^2 \cdot Z_A = 450 \text{ W} \quad \rightarrow \quad I_{in}^2 \cdot 50 = 450 \text{ W} \quad \rightarrow \quad I_{in} = \sqrt{\frac{450}{50}} = 3 \text{ A} \\ \circ V &= (Z_A + Z_G) \cdot I_{in} = 100 \cdot 3 = 300 \text{ V} \end{aligned}$$

2) Calculate  $P_{in}$  with the actual  $Z_A$  value (350  $\Omega$ ):

$$\begin{aligned} \circ I_{in} &= V / (Z_A + Z_G) = 300 / 400 = 0.75 \text{ A} \\ \circ P_{in} &= 0.75^2 \cdot 350 = 196.88 \text{ W} \end{aligned}$$

■ The **remaining power**,  $450 - 196.88 = 253.12 \text{ W}$  is **reflected back** to the Tx



# Tx block. Power

- ❑ **Mismatch losses:**  $L_{Mis\ Tx} (dB) = -10 \cdot \log(1 - |\rho_{Tx}|^2)$
- ❑ We can measure that reflected power in the lab.
- ❑ However, the concept of **reflected power is not part of circuit theory** →
- ❑ **Transmission lines theory:** we work directly with power.

How much power ( $P_{in}$ ) is delivered to the antenna?

$$\blacksquare P_{Tx} = 450\text{ W} \qquad Z_G = 50\ \Omega \qquad Z_A = 350\ \Omega$$

1) We calculate the reflection coefficient as:  $\rho = \frac{Z_2 - Z_1}{Z_2 + Z_1} = \frac{Z_A - Z_G}{Z_A + Z_G} = \frac{350 - 50}{350 + 50} = 0.75$

2)  $P_{in} = P_{Tx} \cdot (1 - |\rho|^2) = 450 \cdot (1 - 0.75^2) = 196.88\text{ W}$

The **reflected power** at the entrance of the antenna is defined:  $P_{ref} = P_{Tx} \cdot |\rho|^2 = 253.12\text{ W}$ , that is, the remaining power:  $450 - 196.88 = 253.12\text{ W}$  is **reflected back** to the Tx.

- ❑ In fact, the Thevenin equivalent of **circuit theory is only a model** we use for calculating voltages and currents and then power values.
- ❑ **RF Tx**s do not behave as voltage sources, but as **power sources** of value  $P_{Tx}$

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  - b. Types
  - c. Parameters: Impedance, Power density and radiated power, Directivity, Ohmic Losses, Gain, Radiation pattern, EIRP (and ERP), Polarization, Effective Area, Antenna Factor
3. Link budget. C calculation
4. Noise. C/N calculation
5. Quality measurements. Alternatives to  $C/N$

# Antennas. Introduction

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- ❑ Antennas are the devices that **adapt** the **conducted or guided** waves, which are transmitted by wire or guides **to radio waves** propagating in free space, while **adding certain** features of **directionality**.
- ❑ So they are the parts of telecommunication systems in charge of radiate or receive electromagnetic radio waves.
- ❑ Depending on the communication system, antennas are used for point to point links, for broadcasting television or radio signals, for portable equipment etc. In each case, the antennas should have specific characteristics, of radiation, size, etc.

# Antennas. Principle of reciprocity

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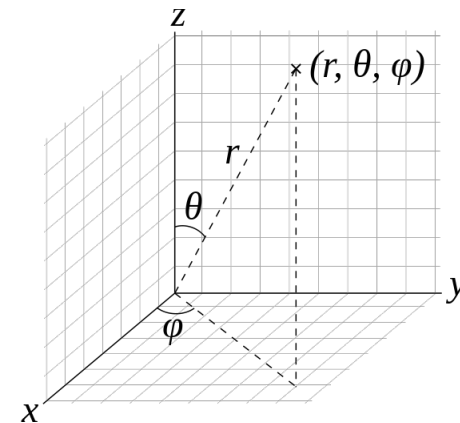
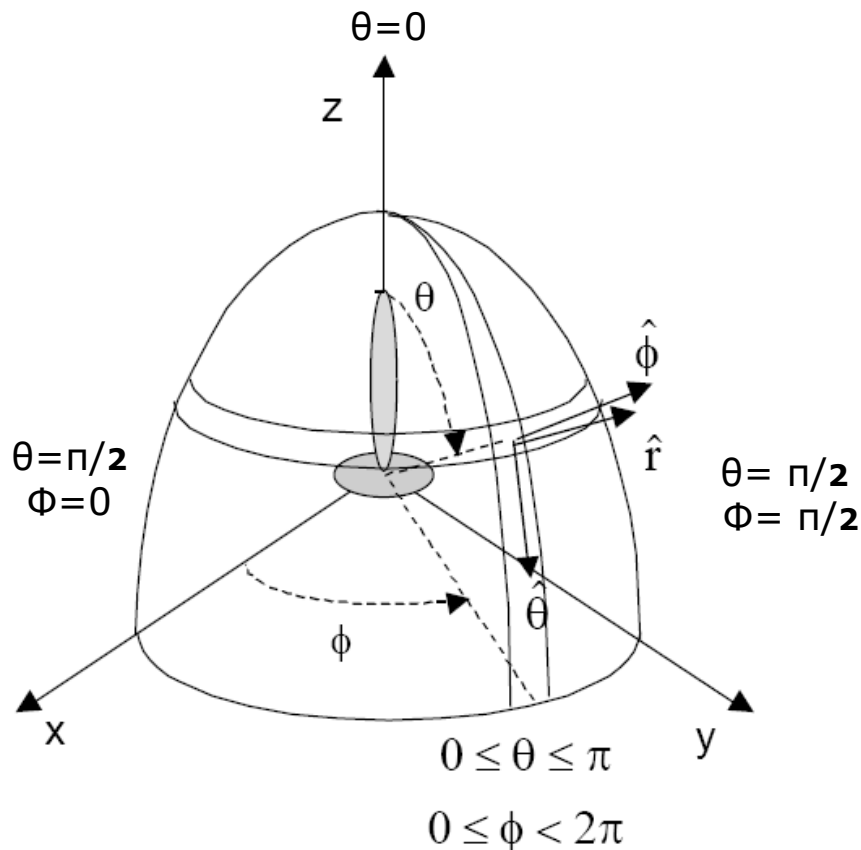
- ❑ The reciprocity theorem states that **antennas have the same characteristics when used in transmission and reception.**
- ❑ It applies to antennas without non linear components (amplifiers, diodes...).  
So it applies exclusively to the antenna device, not to the stuff connected to it no matter how close it is.
- ❑ It means that all the parameters are the same in an antenna if we use it in the transmitter or in the receiver: impedance, radiation pattern, directivity, polarization, ...
- ❑ Nevertheless, there are parameters that are only useful in a receiving antenna.
- ❑ For instance: the effective area or aperture,  $A_{ef}$  [m<sup>2</sup>]
  - It is related to the amount of power in the air that the receiving antenna can capture.
  - It is not equal to the physical area of the antenna.

# Antennas.

## Coordinate System Review

- Cartesian coordinate system  $(x, y, z)$  and spherical coordinate system  $(r, \theta, \phi)$  or  $(r, \theta, \varphi)$ .

$\varphi$

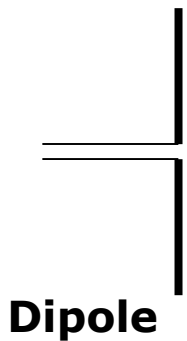


- Spherical coordinate system  $(r, \theta, \phi)$  is more suitable to study the radiation characteristics of the antennas, as only two parameters  $(\theta, \phi)$  are needed to define one **radial direction**.

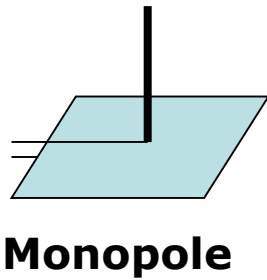
# Antennas. Types

Usually antennas are classified according to the geometry in the following types:

- ❑ **Linear or Wire Antennas:** Formed by rod, or wire elements. The knowledge of the currents flowing through the wires are used to obtain the radiated fields.



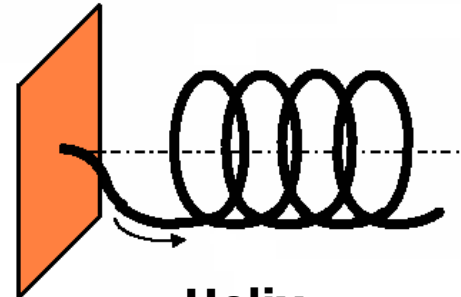
**Dipole**



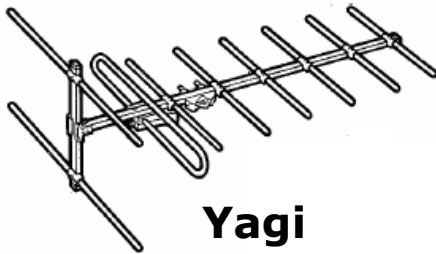
**Monopole**



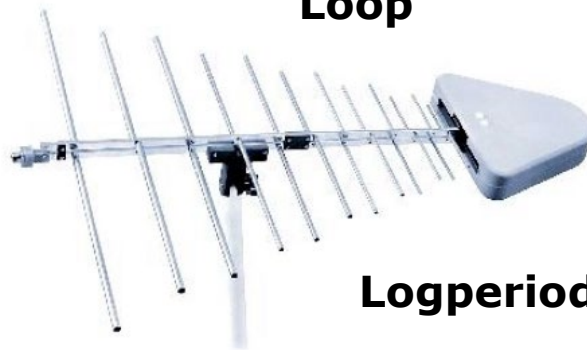
**Loop**



**Helix**



**Yagi**

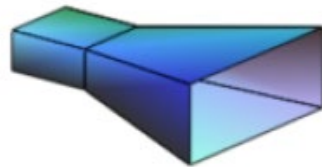


**Logperiodic**

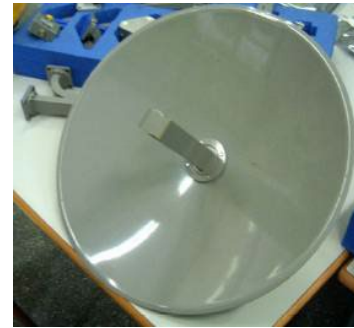
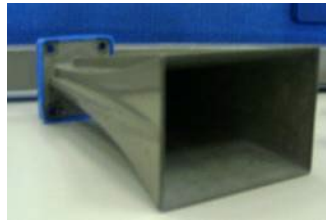


# Antennas. Types

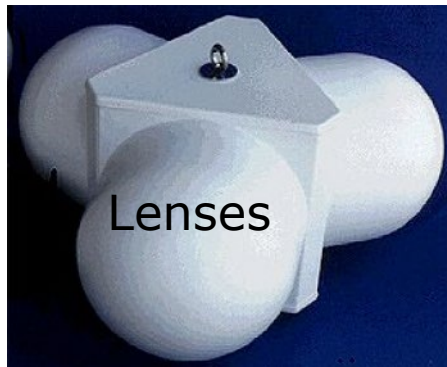
- **Aperture Antennas:** Formed by an open surface on which the field distribution is produced. This distribution generates radiation throughout the space. To study the radiated fields is necessary to know the fields in the aperture.
- Depending on how the fields are generated in the aperture we can group them into 5 types of aperture antennas :
  - Horns
  - Reflectors
  - Lenses
  - Slots
  - Patches



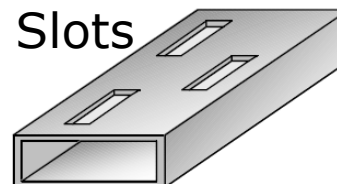
Horns



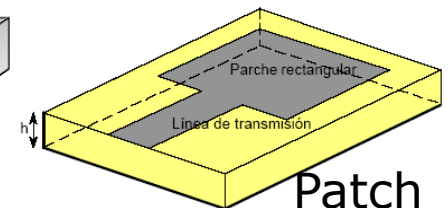
Reflectors



Lenses



Slots

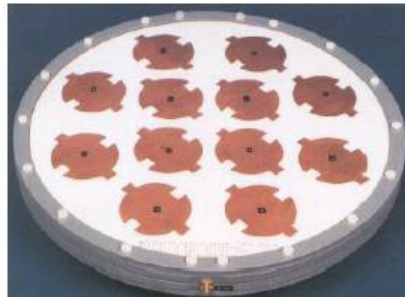
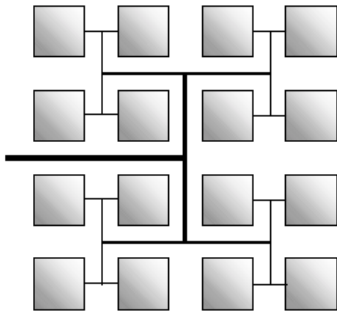
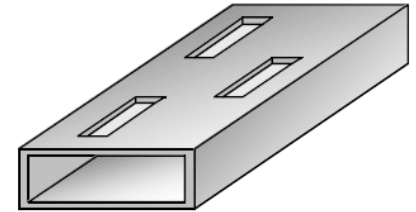
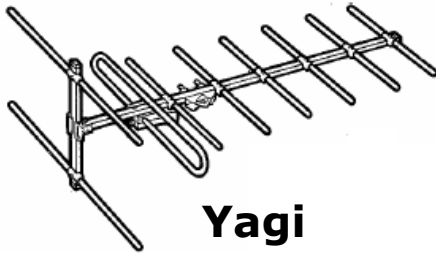


Patch



# Antennas. Types

- ❑ **Array Antennas:** Formed by a group of antennas operating as if it were a single antenna. The radiation pattern depends principally on the array and less on the diagram of each individual antenna.
- ❑ Normally all antennas are equal but in some cases may be different (e.g. Yagi or Logperiodic)



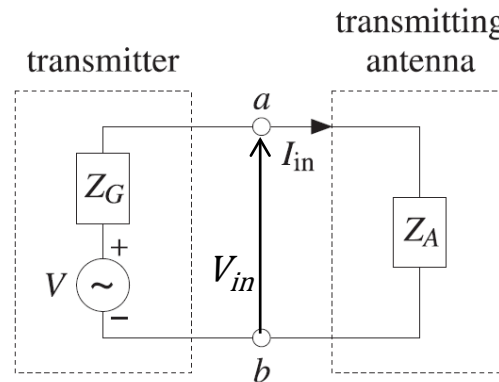
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    5. Gain
    6. Radiation pattern
    7. EIRP (and ERP)
    8. Polarization
    9. Effective Area
    10. Antenna Factor

# Antennas. Parameters: Impedance

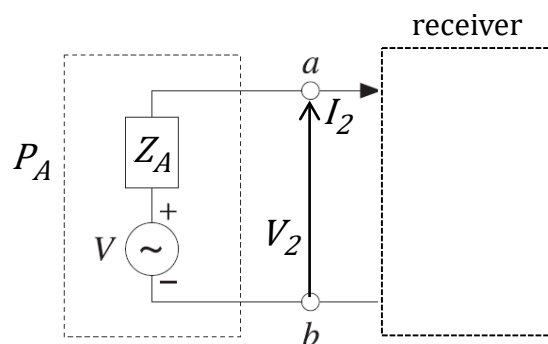
- The **antenna input impedance ( $Z_{in}$  or  $Z_A$ )** is defined at the antenna terminals as the ratio voltage/current.”
- **The transmitting antenna is the load** for all the previous parts of the transmitter (modelled by the Thevenin equivalent circuit):  $Z_{in} = Z_A = V_{in} / I_{in}$



- In general:  $Z_A(f) = R_A + j X_A(f) \rightarrow V_{in} = (R_A + j X_A) \cdot I_{in}$ 
  - If at a certain frequency  $X_A(f) = 0$ , the antenna is said to be resonant antenna at that frequency.
- For power calculations we are NOT interested on the reactive power:  $P_{in} = R_A \cdot (I_{in})^2$

# Antennas. Parameters: Impedance

- At the Rx, the antenna is the first sub-block.
- The power to the following parts of the Rx comes from the antenna → it acts as a **generator** that can be analyzed by means of its Thevenin equivalent circuit.
- As it happened in the Thevenin of the Tx ( $P_{Tx}$ ), **the reference** power of the antenna  $P_A$ , will be the **maximum power that it could ideally provide** to the receiver.



- If we have **the same antenna** both in the Tx and in the Rx, **the impedance** of the Thevenin **at Rx is the same as in Tx**, and it has the same properties: Reciprocity.

# Antennas. Parameters: Power density and radiated power

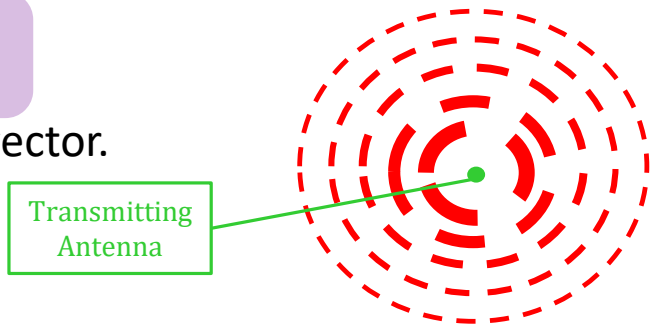
- An **isotropic antenna** would be an antenna that would radiate its power **uniformly in all directions**:
  - It is an ideal antenna that can't be manufactured.
  - It is the main reference antenna.

- The **power density** around an isotropic antenna that radiates a power  $P_{rad}$  W is:

$$S_{iso} = \frac{P_{rad}}{\text{Area of the sphere at "r"}} = \frac{P_{rad}}{4 \cdot \pi \cdot r^2} \text{ [W/m}^2\text{]}$$

Why "S"? Because it is the module of Poynting vector.

$$W_{rad\_r} = P_{rad} / (4 \cdot \pi \cdot r^2)$$



- The **total radiated power** can be obtained by integrating the power flux through a surface (sphere is the simplest one) enclosing the antenna:

$$P_r = \iint_s \vec{S}(\theta, \phi) \cdot d\vec{s} = \int_0^{2\pi} \int_{\theta=0}^{\theta=\pi} \vec{S}(\theta, \phi) r^2 \cdot \sin \theta \cdot d\theta \cdot d\phi$$

# Antennas. Parameters: Directivity

- The **Directivity, D**, of an antenna is defined as **the ratio of the power density in the directions of maximum radiation with respect to the power density of an isotropic antenna** under the same conditions (equal distance and total radiated power).
- However directivity only gives information about the directions of maximum radiation.
- The **Directive gain** is defined as the ratio of the radiated power density in each and every direction and the power density that an isotropic antenna would radiate under the same conditions (equal distance and total radiated power):  $D(\theta, \phi)$
- **Directivity is the maximum of the directive gain**

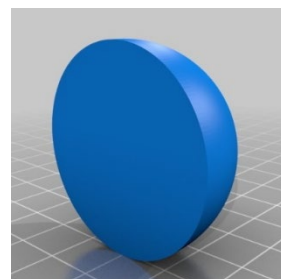
$$D(\theta, \phi) = \frac{S(\theta, \phi)}{S(\theta, \phi)|_{isotropic}} = \frac{S(\theta, \phi)}{\frac{P_{rad}}{4 \cdot \pi \cdot r^2}} \quad D = \frac{S(\theta, \phi)_{max}}{S(\theta, \phi)|_{isotropic}} = \frac{S(\theta, \phi)_{max}}{\frac{P_{rad}}{4 \cdot \pi \cdot r^2}}$$

# Antennas. Parameters: Directivity

## Examples:

- Directivity of an ideal antenna with an hemispheric radiation pattern

Directions of no radiation



Esfera hemisferio bat da!

Directions of maximum radiation

- We can use linear units:  $D = 2$  [dimensionless]

- Or logarithmic units (dB):

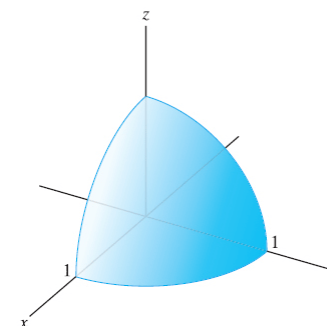
$$10 \cdot \log(S_{HEM}) = 10 \cdot \log(2 \cdot S_{ISO}) = 10 \cdot \log(2) + 10 \cdot \log(S_{ISO}) \rightarrow S_{HEM} [\text{dB(W/m}^2\text{)}] \approx 3 [\text{dB}] + S_{ISO} [\text{dB(W/m}^2\text{)}]$$

- In order to state that **the reference antenna is the isotropic** one we write:  **$D = 3\text{dB}$**

## Directive gain of the octant antenna:

- Numerically: with a function of  $(\theta, \phi)$

$$D(\theta, \phi) = \begin{cases} D & 0 < \theta < \pi/2; 0 < \phi < \pi/2 \\ 0 & \text{Other cases} \end{cases}$$



- Graphically: with a graphical representation of  $D(\theta, \phi)$ , that is, the radiation pattern

# Antennas. Parameters: Ohmic losses (efficiency)

- $P_{rad}$  is the power actually radiated to the air by the antenna.
 

Baina gogoratu: Konplexua da: Atal irudikaria falta da! (jXa)
- In order to model  $P_{rad}$ ,  $R_A = Re(Z_A)$  is divided into two components:  $R_A = R_{rad} + R_{\Omega}$ 
  - The total power consumed by the antenna is:  $P_{in} = I_{in}^2 \cdot R_A$ 

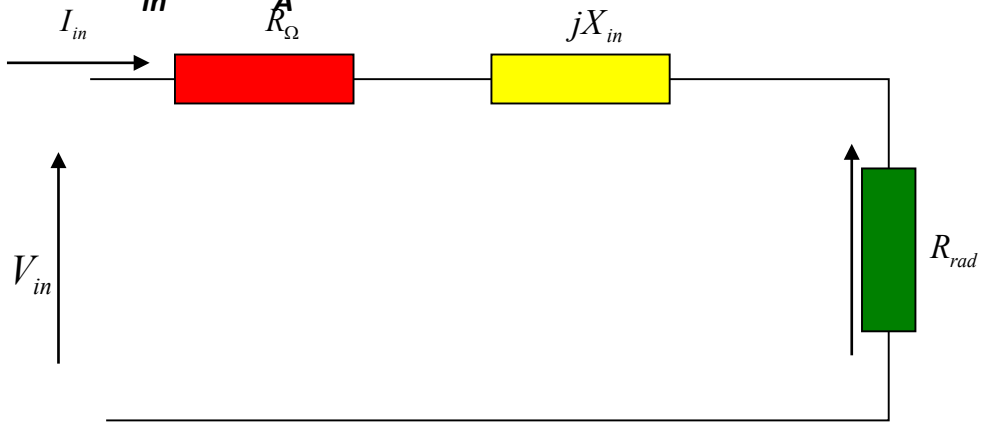
Unitate linealetan!!
  - The radiated power is:  $P_{rad} = I_{in}^2 \cdot R_{rad}$ 

<--- Hau maximizatu nahi dugu!!
  - The power dissipated as heat due to the currents in the antenna:  $P_{\Omega} = I_{in}^2 \cdot R_{\Omega}$

Obviously:  $P_{in} = P_{rad} + P_{\Omega}$

- We define the **antenna efficiency** as:  $\eta = \frac{P_{rad}}{P_{in}} = \frac{R_{rad}}{R_A} \rightarrow \eta \leq 1$

- Ohmic losses:  $L_{\Omega} = P_{in} / P_{rad} = 1/\eta$   
 $P_{rad} = P_{in} / L_{\Omega}$   
 In dB:  $L_{\Omega} (dB) = -10 \cdot \log(\eta)$





# Antennas. Parameters: Gain

- The **Gain, G**, of an antenna is defined as **the ratio of the power density in the direction of maximum radiation with respect to the power density of an isotropic antenna (without ohmic losses)** under the same conditions (equal distance and same  $P_{in}$  ).

$$G = \frac{S(\theta, \varphi)_{max}}{S(\theta, \varphi)|_{isotropic, without ohmic losses}} = \frac{S(\theta, \varphi)_{max}}{\frac{P_{in}}{4 \cdot \pi \cdot r^2}} \qquad D = \frac{S(\theta, \varphi)_{max}}{\frac{P_{rad}}{4 \cdot \pi \cdot r^2}}$$

- Gain** is defined as:  $G = D \cdot \eta \qquad \rightarrow \qquad S_{MAX} = G_{Tx} \cdot \frac{P_{in}}{4 \cdot \pi \cdot r^2} \text{ [W/m}^2\text{]}$

- $D_{Tx} \cdot P_{rad} = G_{Tx} \cdot P_{in}$  as  $P_{in} \geq P_{rad} \qquad \Leftrightarrow \qquad G \leq D \qquad \Leftrightarrow \qquad \eta \leq 1$

$G(\text{dBi}) = D(\text{dBi}) + 10\log(\eta)$

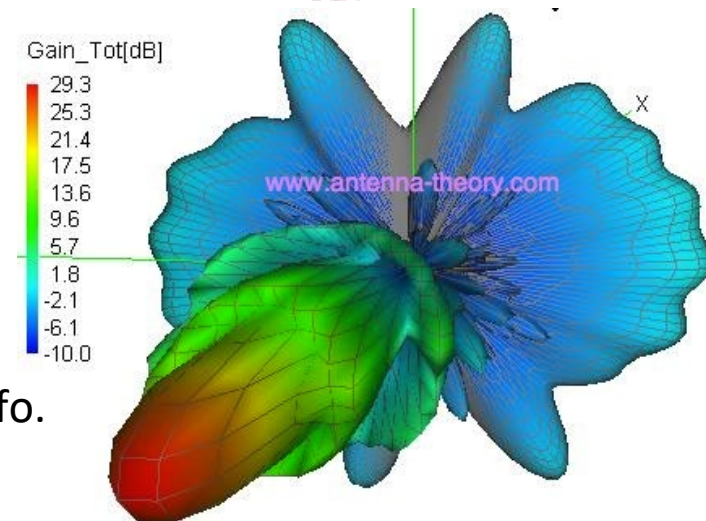
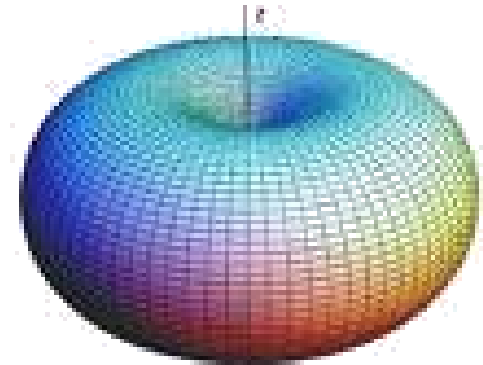
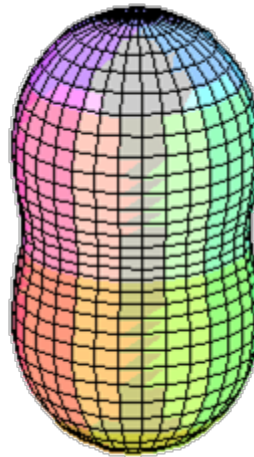
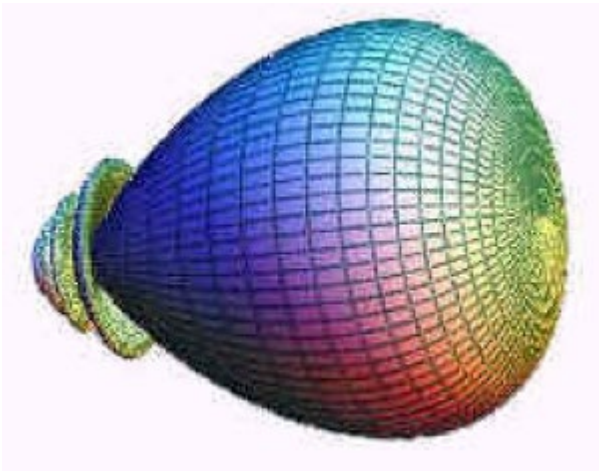
$\eta \leq 1$  da, beraz,  $10\log(\eta)$  NEGATIBOA (edo zero) IZANGO DA

$Pin(\text{dBm}) - Prad(\text{dBm}) = D(\text{dBi}) - G(\text{dBi})$   
 $Pin(\text{dBm}) + G(\text{dBi}) = Prad(\text{dBm}) + D(\text{dBi})$   
 $Prad(\text{dBm}) = Pin(\text{dBm}) + 10\log(\eta)$

Edo erabiltzen dugu Pin eta irabazia  
 Edo erabiltzen dugu Prad eta zuzenkortasuna

# Antennas. Parameters: Radiation pattern

- However, with real antennas such as:



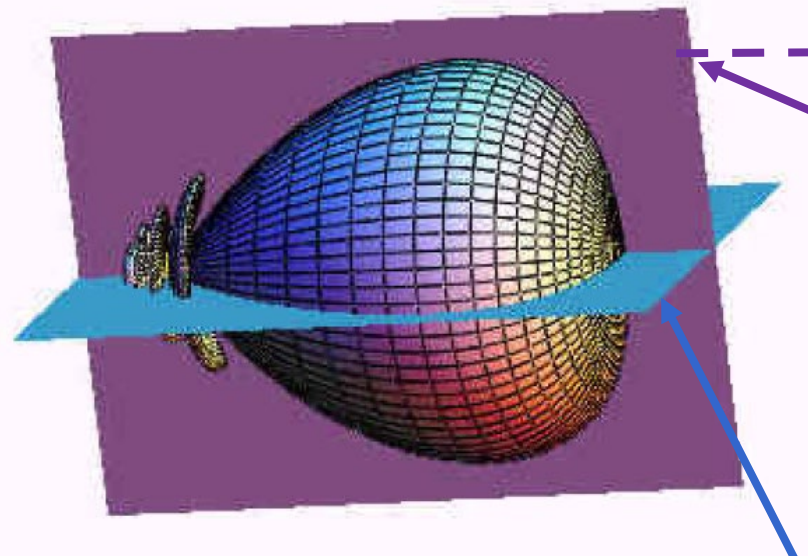
- The 3D plot
- The function  $D(\theta, \phi)$

Are very difficult to handle in order to extract useful info.

- **2D representations** are used instead → **Planar sections:**
  - Constant  $\theta$  (parallels or latitude lines of the sphere)
  - Constant  $\Phi$  (meridians or longitude lines of the sphere).

# Antennas. Parameters: Radiation pattern

## 2D Radiation Pattern or Antenna Pattern when $\Phi = 0$

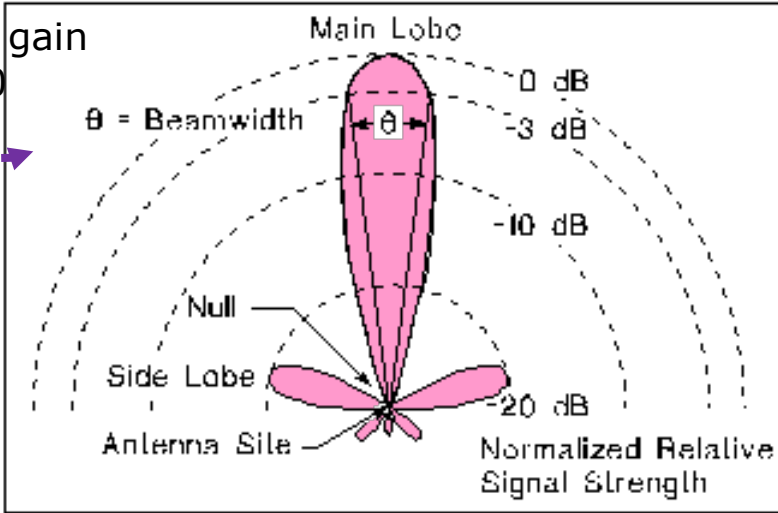


$D(\theta, 0)$  directive gain

$\Phi = 0$

$D(\pi/2, \Phi)$

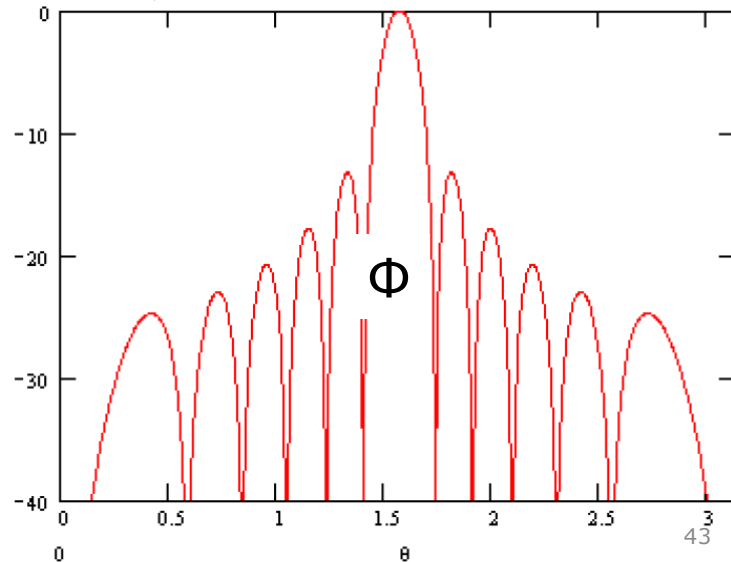
$\theta = \pi/2$



side lobe  
2D Cartesian plots

In 2D we visualize more clearly the directions where the antenna transmits (or receives) power

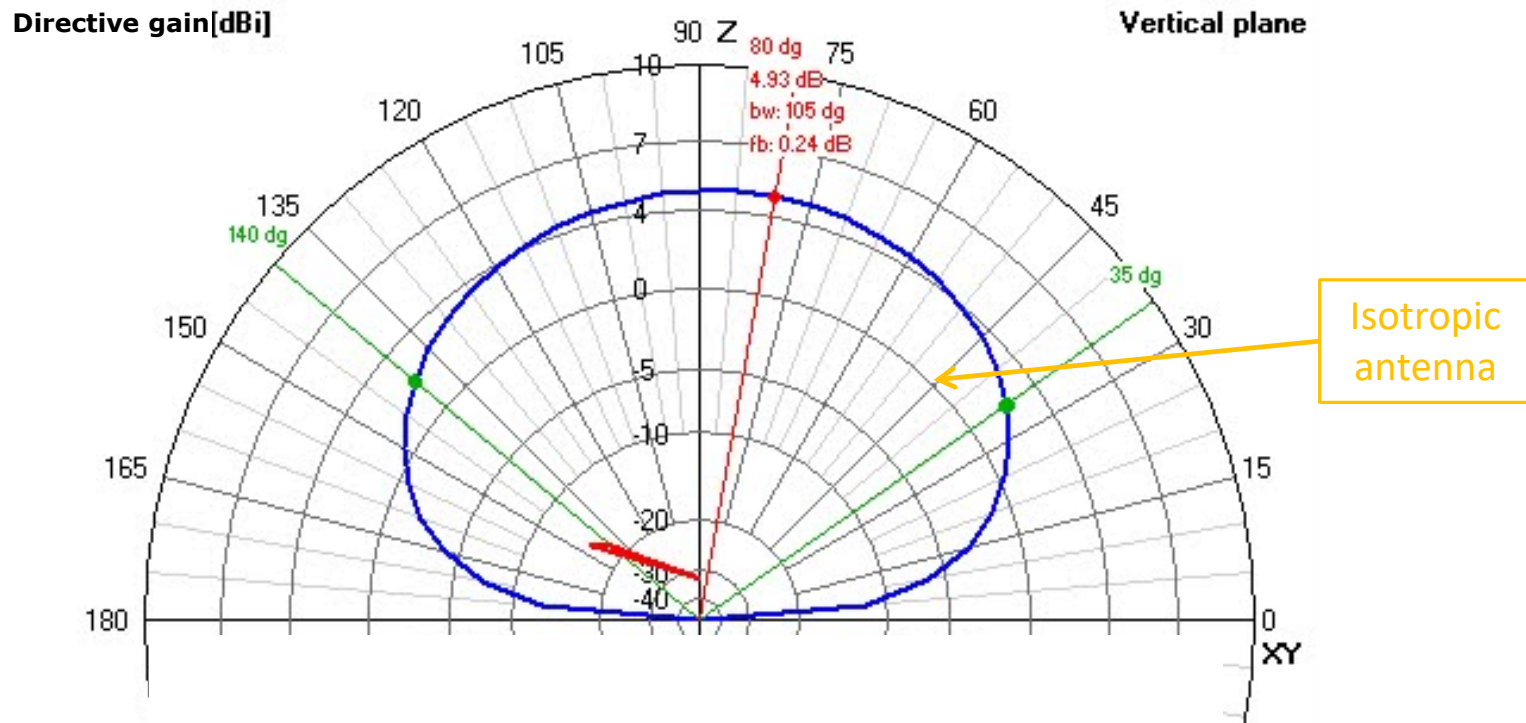
The 2D cartesian plot is less intuitive but eas



# Antennas. Parameters: Radiation pattern

## Normalization

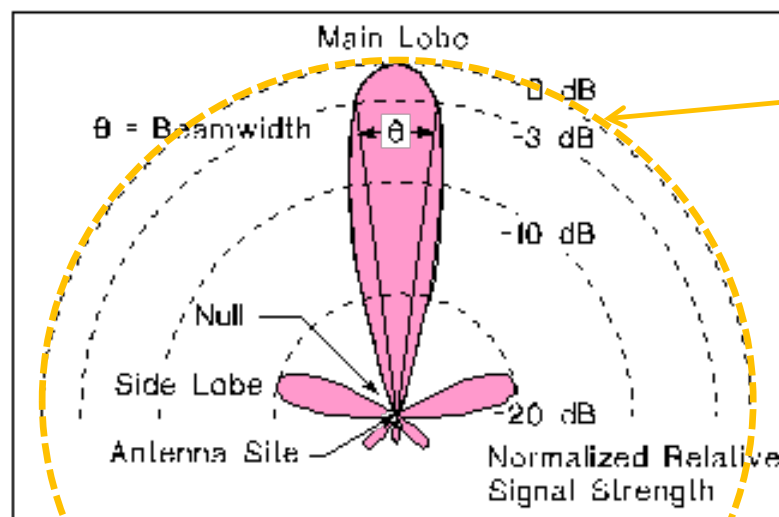
- We can display 2D antenna patterns in two ways:
  1. It is possible to “read” the actual directive gain value from the graph.



# Antennas. Parameters: Radiation pattern

## Normalization

- We can display 2D antenna patterns in two ways:
  2. **Normalized to directivity.** All patterns have the same maximum size.



Isotropic  
antenna

In addition, we need to specify the directivity of the antenna:

$D = 14.4 \text{ dBi}$  (for instance)

$D = 0 \text{ dBi}$

**2D normalized polar plots in dB**, are the **most common format** of radiation patterns.

As the maximum is always 0, we have to **watch for the right directivity value**.

# Antennas. Parameters: Radiation pattern

## The MATH

- Three dimensional graph of one of the following magnitudes:

- Normalized electric field intensity:  $20 \cdot \log \frac{|\vec{E}(\theta, \varphi)|}{|\vec{E}_{\max}|}$

- Normalized radiated power density:  $10 \cdot \log \frac{S_{\text{rad}}(\theta, \varphi)}{S_{\text{rad max}}}$

- Due to the plane waves properties (far field region), both diagrams are exactly the same, also the magnetic field could be used with the same result.

- Some times (quite unusual) linear units are used to plot the graph:

$$\frac{|\vec{E}(\theta, \varphi)|}{|\vec{E}_{\max}|} \qquad \frac{S_{\text{rad}}(\theta, \varphi)}{S_{\text{rad max}}}$$

- Planar sections just giving constant values to  $\theta$  or constant values to  $\Phi$ .

# Antennas. Parameters: Radiation pattern

## Parametrization

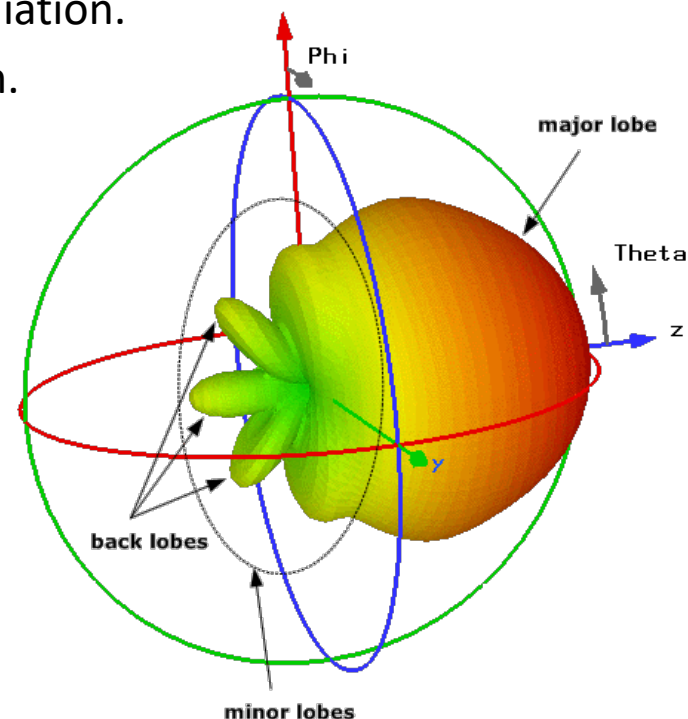
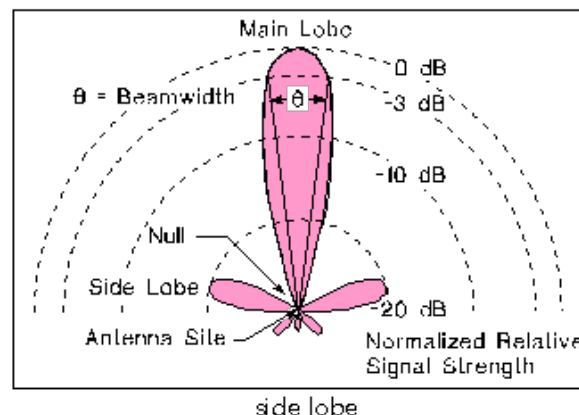
□ **Lobe:** Diagram portion bounded by regions of weaker radiation.

- **Main Lobe** contains the maximum radiation direction.

- **Sidelobes:** The others.

- **Lateral lobes:** Lobes adjacent to the main lobe.
- **Back lobe:** Opposite the main radiation direction.

$D = 14.4 \text{ dBi}$



□ **Main lobe parameters:**

- **Half Power Beamwidth (HPBW)** or 3dB Beamwidth (3dB BW): Angle between half-power (-3 dB) points. It indicates **how wide the directivity keeps more or less constant**.
- **Null to Null Beamwidth:** Null-Null BW  $\sim 2.25 \cdot \text{HPBW}$ .

□ **Sidelobe level (SSL):** The highest side lobe level relative to the main lobe level.

□ **Front to back ratio:** Ratio between the main lobe and the back lobe.

# Antennas. Parameters: Radiation pattern

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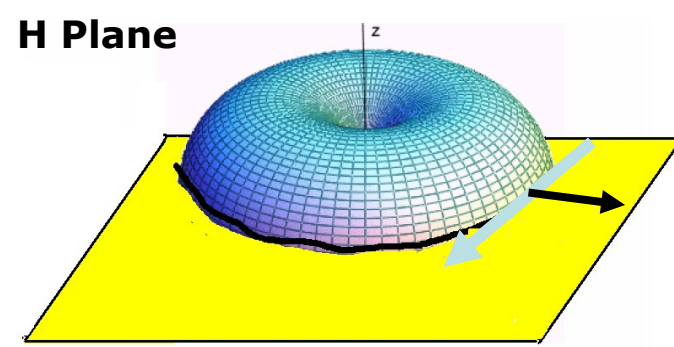
## Antenna classification:

- ❑ **Isotropic Antenna:** Ideal antenna that radiates the same way in all directions. Its radiation pattern is a sphere and the sections are all circles.
- ❑ **Directional antenna:** An antenna that is not Isotropic or Omnidirectional.
- ❑ **Omnidirectional Antenna:** That antenna whose radiation pattern has symmetry of revolution around one axis. To define the 2D diagram is sufficient to draw a single cut containing that axis (the 3D diagram can be obtained by revolution)
  - Unlike the isotropic antenna, omnidirectional antennas can be manufactured
  - The most known example is the **half wave dipole** ( $D=2.15$  dBi)

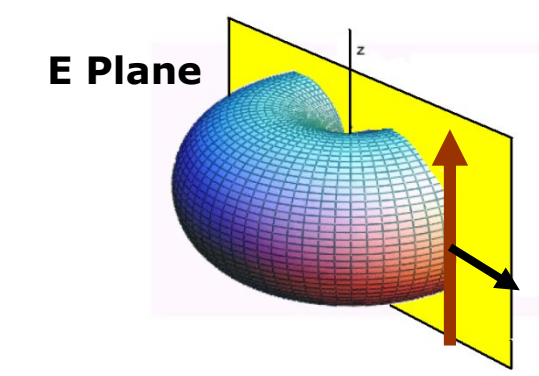
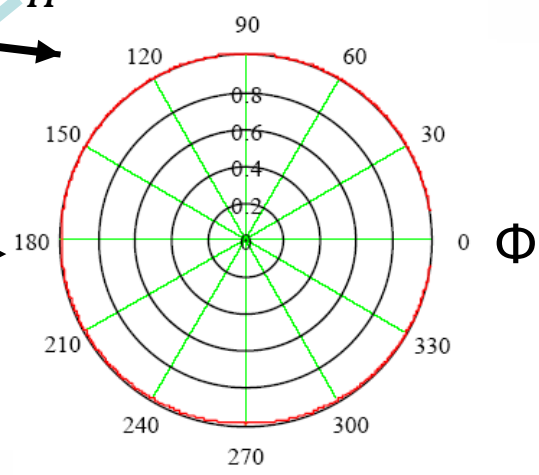
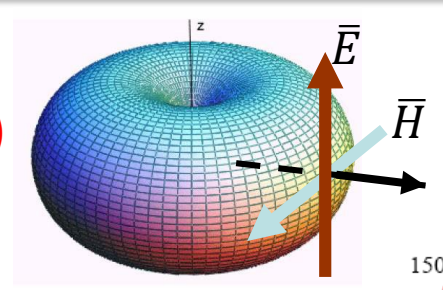


# Antennas. Parameters: Radiation pattern

Example of omnidirectional antenna: Dipole  
**E and H planes in a plane wave (far field region)**

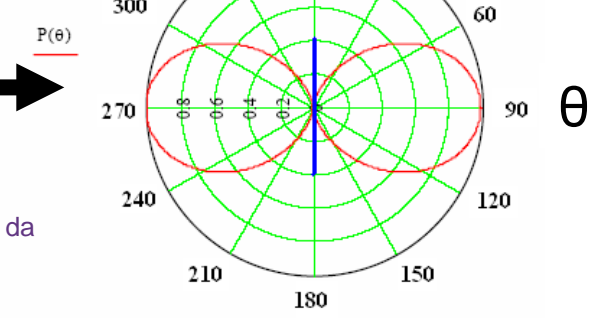


$$\theta = \frac{\pi}{2}$$



$$\varphi = Cte$$

Plano horizontalean beraz potentzia maximoa dago.  
Hurrengo diapositiban dioen moduan, maximoa 2.15dB(i) da



The two planes are perpendicular and their intersection is the direction of maximum gain.

# Antennas. Parameters: Radiation pattern

The **half wave dipole** ( $D = 2.15$  dBi) is the second **reference antenna** → Units: **dBd**

- $D = 14.4$  dBi  $\leftrightarrow$   $D = 12.25$  dBd (where  $12.25 = 14.4 - 2.15$ )
- For **obtaining the radiation pattern** ( $D(\theta, \phi)$ ) of an **unknown antenna**, we can measure in all directions the power emitted by both antennas (unknown and half wave dipole) in open field and then do the subtraction.
- Sometimes antenna radiation pattern values are given only with “dB” as units:
  - They will usually be dBi.
  - It is necessary to check it.
- 2.15 dB is 1.64 in linear units



# Antennas. Parameters: *EIRP* (and *ERP*)

- ❑ ***EIRP* (Effective Isotropic Radiated Power)** or *PIRE* (Potencia Isotrópica Radiada Equivalente) in Spanish is the power that would have to radiate an isotropic antenna to get the same power density which produces a directional antenna at the maximum radiation direction.

The english here is a bit wonky, assume spanish sentence structure (not mine)

- ❑ ***EIRP* is defined as:**  $EIRP = D_{Tx} \cdot P_{rad} = G_{Tx} \cdot P_{in}$  [W]  $\rightarrow S_{MAX} = \frac{EIRP}{4 \cdot \pi \cdot r^2}$  [W/m²]

LINEAR!!

- $EIRP(dBm) = D_{Tx}(dBi) + P_{rad}(dBm) = G_{Tx}(dBi) + P_{in}(dBm)$  <----- The good shit

- If instead of an antenna with directivity  $D$  and power  $P_{rad}$ , we had an isotropic antenna ( $D=1$ ) with power  $EIRP$
- What would be the difference? Exactly: none

Poynting

$$S = \frac{|E|^2}{\eta}$$

$$S_{MAX} = D_{Tx} \cdot \frac{P_{rad}}{4 \cdot \pi \cdot r^2} = 1 \cdot \frac{EIRP}{4 \cdot \pi \cdot r^2}$$

- ❑ ***ERP* (Effective or Equivalent Radiated Power)** or *PRA* (Potencia Radiada Aparente) in Spanish is is the power that would have to radiate an half wave dipole to get the same power density which produces a directional antenna at the maximum radiation direction. The same as *EIRP* but the reference antenna is the half wave dipole.

$ERP = EIRP - 2.15$  (remember the example  $D = 14.4$  dBi  $\leftrightarrow D = 12.25$  dBd)

In linear units  $ERP = EIRP / 1.64$

Same idea as EIRP but replacing the antenna for a half wavelength dipole

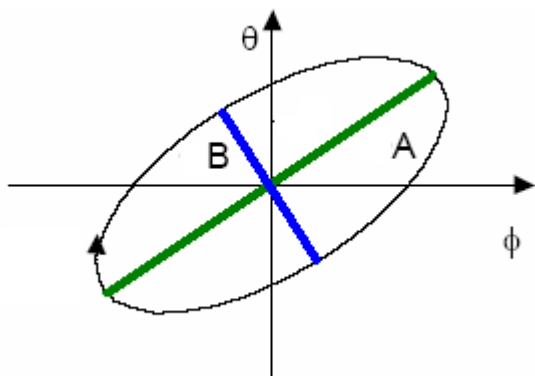
# Antennas. Parameters: Polarization

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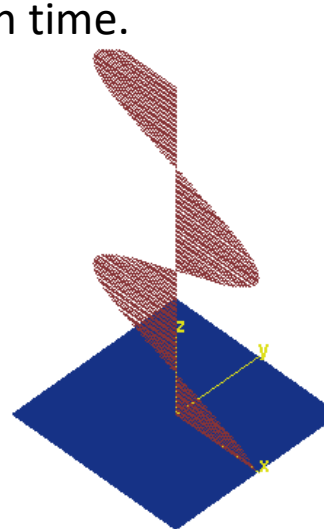
- ❑ Polarization is the parameter that enables us to determine how the direction of  $E$  and  $H$  fields vary along the propagation direction and with time.
- ❑ Actually just the direction of one of the vectors (the  $E$  field ) needs to be defined because the other ( $H$  field) is perpendicular in an electromagnetic plane wave (far field region).
- ❑ In general, if we watch how  $\vec{E}$  evolves with time from a fixed observation point, we will see that it describes an ellipse.
- ❑ Two particular cases:
  - Linear polarization (the case of the half wave dipole): one of the ellipse semi axis is zero.
  - Circular polarization: the two axes of the ellipse are equal (circular).
- ❑ The polarization of an antenna is the polarization of the waves that it radiates .

# Antennas. Parameters: Polarization

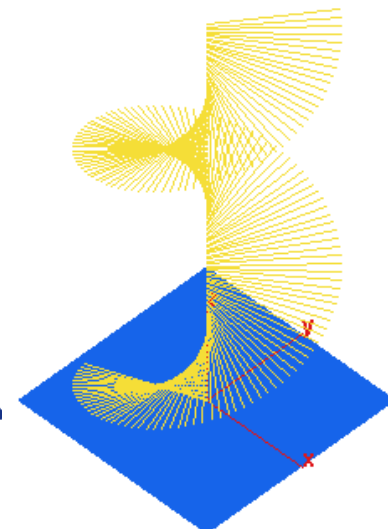
- Polarization is the parameter that enables us to determine how the direction of  $E$  and  $H$  fields vary along the propagation direction and with time.



General case. Ellipse with axes of length A and B. Elliptical polarization



Particular case  $B=0$ . Linear Polarization



Particular case  $A=B$ . Circular Polarization

- Usually antennas are designed to have linear or circular polarization.
  - Linear polarization (the case of the half wave dipole): one of the ellipse semi axis is zero.
  - Circular polarization: the two axes of the ellipse are equal (circular).
- For elliptical or circular polarization the direction of rotation has to be defined.
  - Right handed: Looking towards the direction of propagation the wave rotates clockwise.
  - Left handed: anticlockwise.
- Transmitter and receiver antennas should have the same polarization, if not some losses occur.

# Antennas. Parameters: Effective Area

## Effective Area or Effective Aperture (used along with power)

- The effective area or effective aperture  $A_{ef}$  of the antenna is defined as the area which when intercepted by the incident power density  $S$  gives the amount of received power:

$$P = S \cdot A_{ef} \quad \rightarrow \quad A_{ef} = P / S$$

- The effective area is not equal to the physical area of an antenna:
  - Linear antennas do not even have any characteristic physical area.
  - Dish or horn antennas: the effective area is typically a fraction of the physical area (about 55–65 percent for dishes and 60–80 percent for horns).
- Most commonly  $P = P_A$ , the **maximum power that the antenna could ideally provide** to the receiver → **If so**, we have **optimal conditions and** effective area is related to directivity:

$$A_{ef} = \frac{\lambda^2}{4\pi} \cdot D = P_A / S$$

Buruz ikasi! Ez dugu demostratuko!

D = zuzenkortasuna

# Antennas. Parameters: Antenna Factor

$$\left. \begin{aligned} S &= \frac{\text{EIRP}}{4\pi r^2} \\ S &= \frac{|E|^2}{\eta_0} \end{aligned} \right\} |E|$$

**Antenna Factor or K Factor (used along with electric field strength in the far field region)**

- Far field region: **D is not directivity**. It is the largest dimension of the antenna.
  - The wave fronts are locally plane:
    - $\vec{E} \perp \vec{H}$  and both are perpendicular to the propagation direction.
  - Far field region starts at a distance which depends on the antenna and on frequency:
 
$$r > \frac{2D^2}{\lambda}$$
- K factor** provides the value of the incident  $E$  field from the received voltage level, measured in volts in the load in **optimal conditions**: K txikiagoa: Antena hobea

$$|E| = V_L \cdot K \Rightarrow \boxed{AF = K = \frac{|E|}{V_L}}$$

$$\boxed{K(dB) = 20 \log \frac{|E|}{V_L} dBm^{-1}}$$

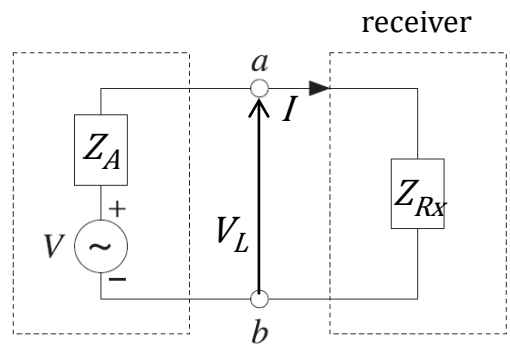
- In **optimal conditions ( $Z_A = Z_{Rx}$ )**: the **maximum power** that the receiving antenna could **ideally provide** to the receiver,  $P_A$ , will be provided to the Rx:  **$P_{Rx} = P_A$**

$$P_{Rx} = P_A \Rightarrow \frac{V_L^2}{R_{Rx}} = \frac{V_L^2}{R_A} = S_{MAX Rx} \cdot A_{ef} = \frac{|\vec{E}|^2}{\eta} \cdot A_{ef} \Rightarrow \frac{|\vec{E}|^2}{V_L^2} = \frac{\eta}{R_A \cdot A_{ef}}$$

$$\Rightarrow K = \sqrt{\frac{\eta}{R_A \cdot A_{ef}}}$$

$$S = |E|^2 / \eta = |H|^2 \cdot \eta$$

**$\eta$ : intrinsic impedance ( $\eta = 120\pi \Omega$  in vacuum or air)**



Considering  $A_{ef} = \frac{\lambda^2}{4 \cdot \pi} \cdot D_{Rx}$  if  $R_A = 50 \Omega \Rightarrow K = \frac{9.73}{\lambda \cdot \sqrt{D_{Rx}}}$

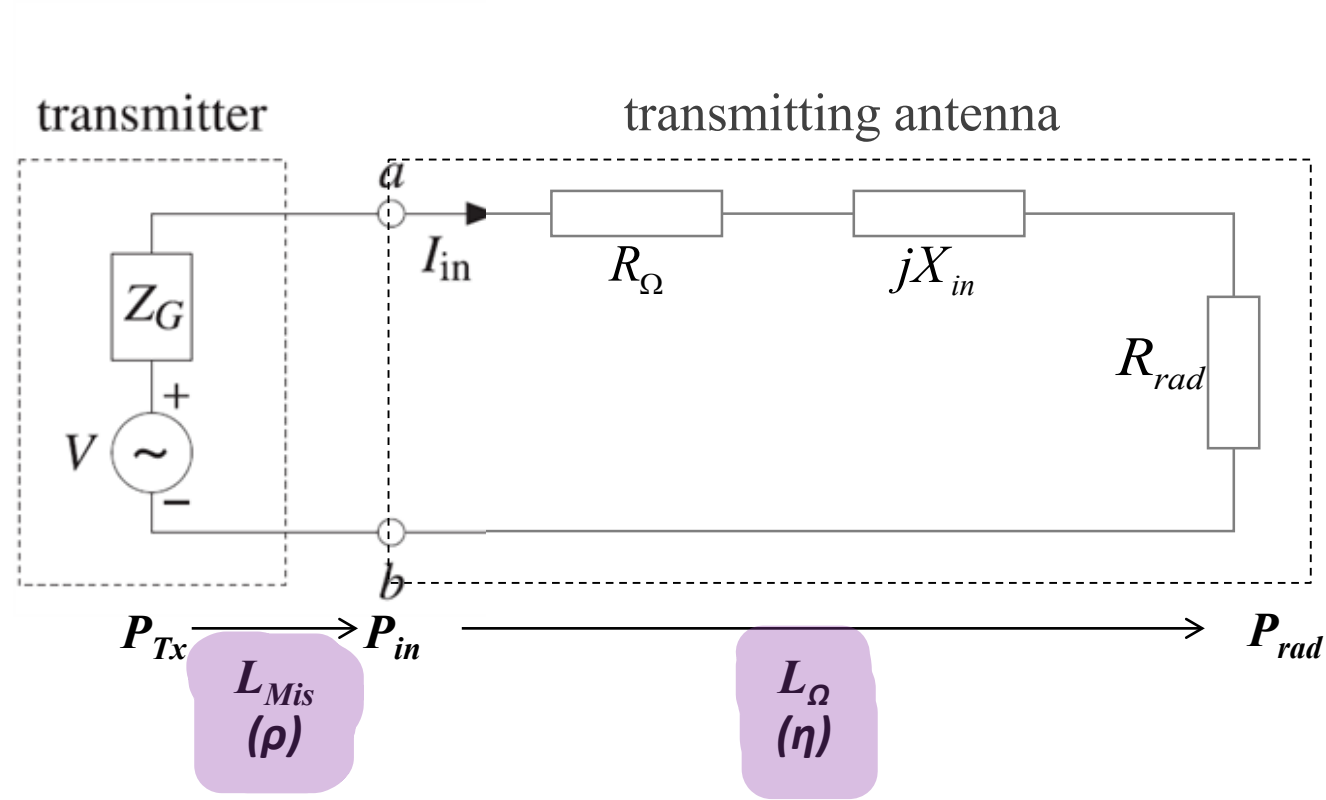
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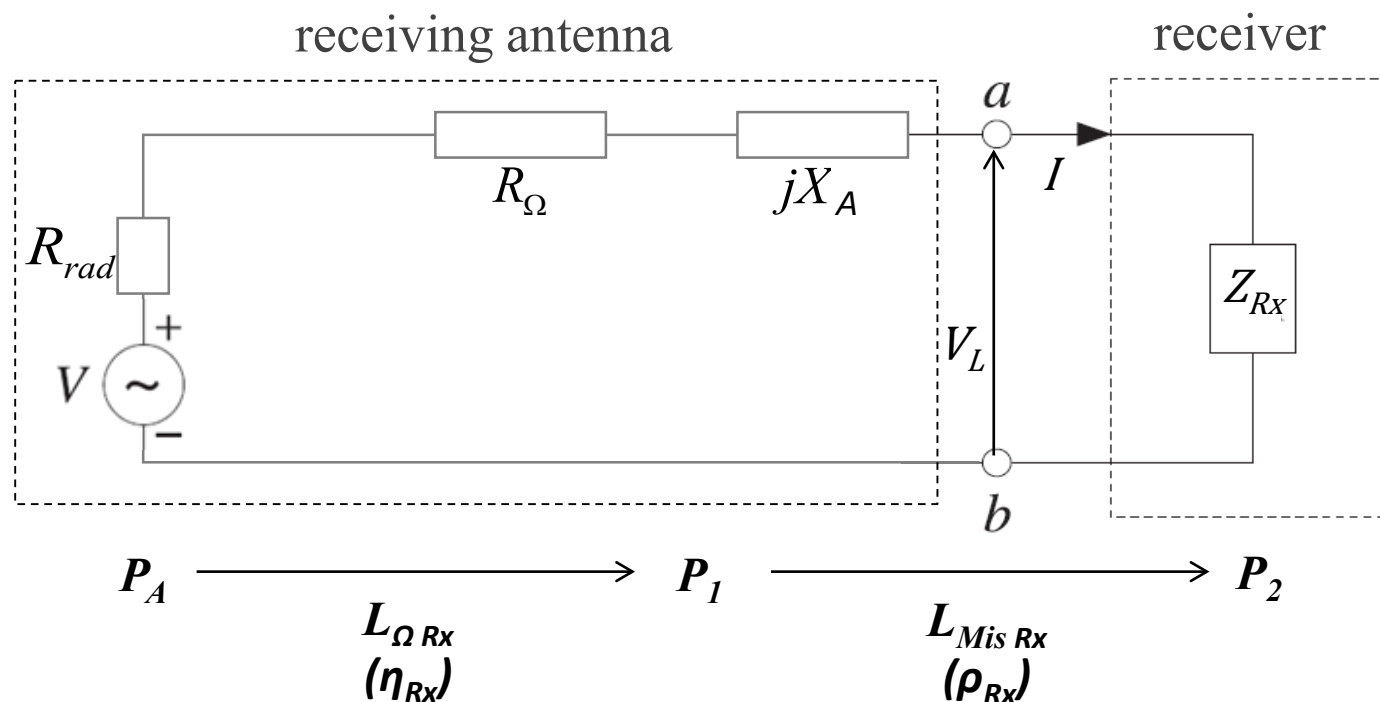
# Link Budget: losses in transmitter side



$$P_{Tx} \geq P_{in} \geq P_{rad}$$

Mismatch losses and ohmic losses are completely different!!!

# Link Budget: losses in receiver side

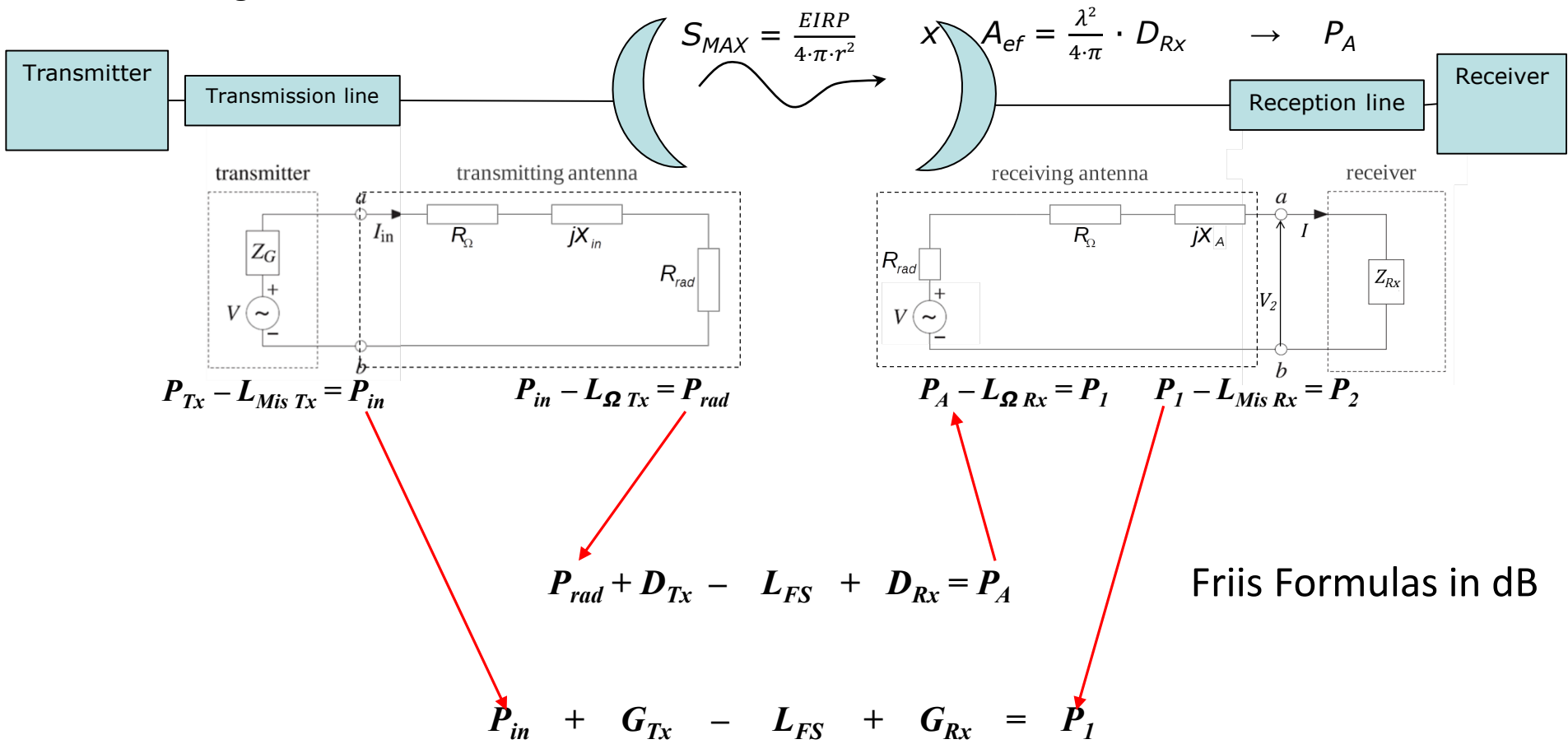


$$P_A \geq P_1 \geq P_2$$

$P_A$  is the maximum power (optimal conditions), like  $P_{Tx}$  was at Tx

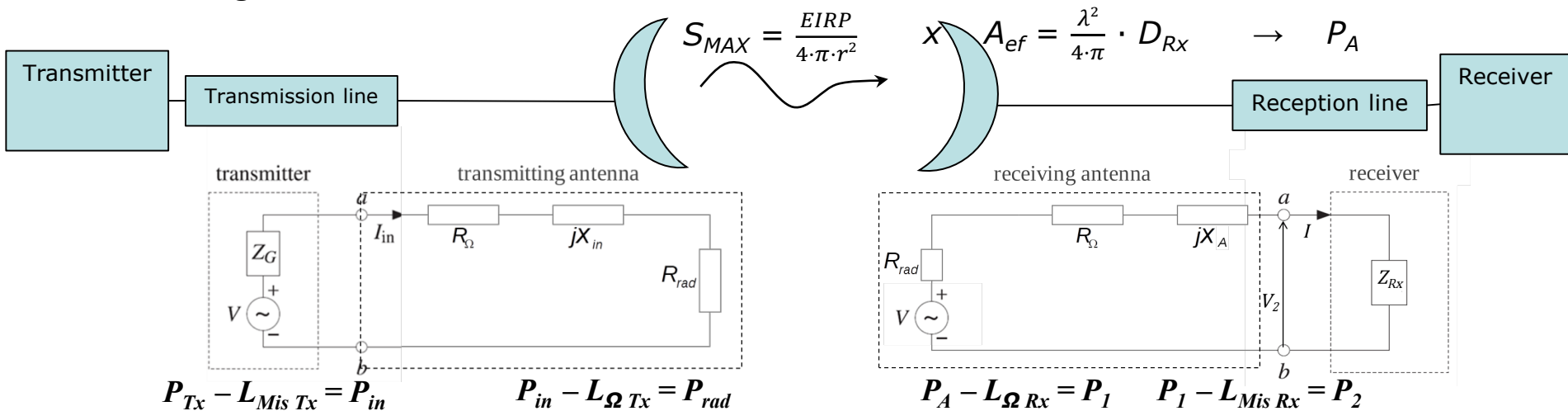
# Link Budget

- The Link Budget is the accounting of all of the gains and losses from the transmitter, through the medium to the receiver.



# Link Budget

- The Link Budget is the accounting of all of the gains and losses from the transmitter, through the medium to the receiver.



$$P_{rad} + D_{Tx} - L_{FS} + D_{Rx} = P_A$$

Friis Formulas in dB

$$P_{in} + G_{Tx} - L_{FS} + G_{Rx} = P_1$$

$$\text{PowerRad} + \text{GainTx} - \text{LossesFreeSpace} + \text{GainRx} = P_1$$

- Remember:  $D_{Tx} \cdot P_{rad} = G_{Tx} \cdot P_{in} \iff \text{dB (Friis): } P_{rad} + D_{Tx} = P_{in} + G_{Tx}$
- Conversely in the Rx**, in dB the Friis Formula:  $P_A - D_{Rx} = P_1 - G_{Rx} \iff \frac{P_A}{D_{Rx}} = \frac{P_1}{G_{Rx}}$

# Link Budget

It is possible to complete the Friis Formula of Propagation:

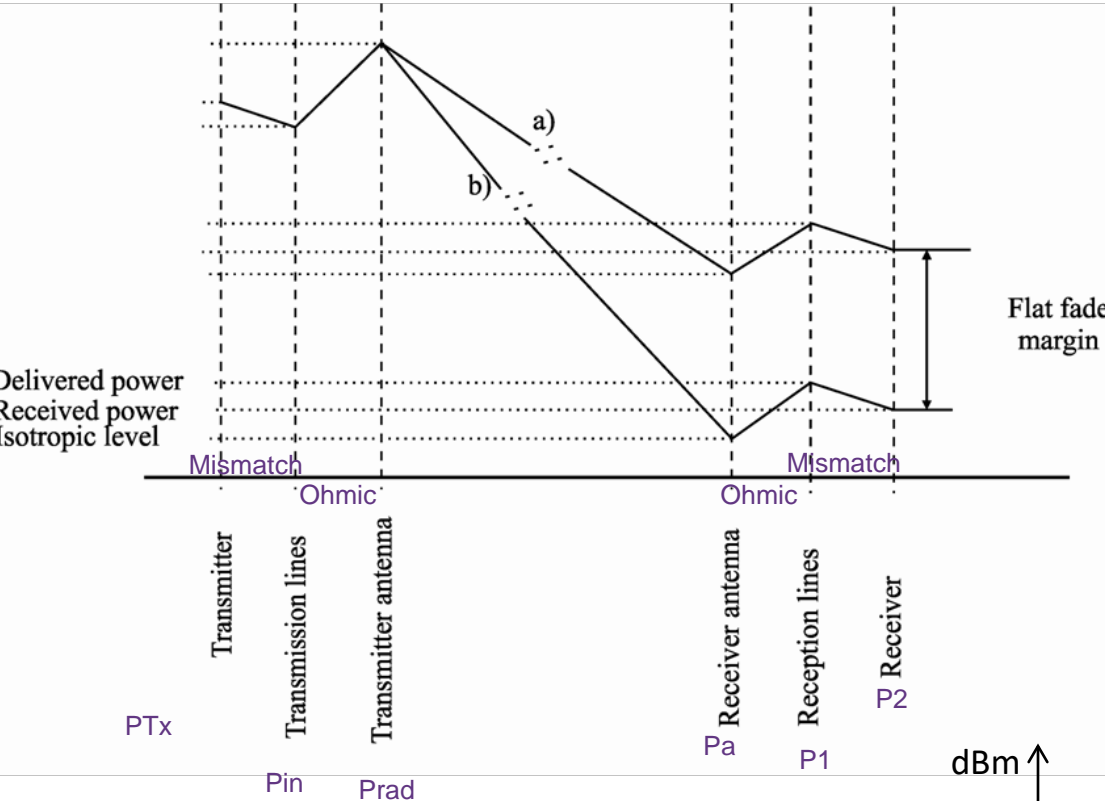
$$P_{rad} + D_{Tx} - L_{FS} + G_{Rx} + OTHER = P_1$$

□ The *OTHER* factors can be related to:

- Transmission line from the **Tx** block to the transmitting antenna:  $OTHER_{Tx} = -L_{Cables\ Tx}$
- Other factors of the **Rx**:
  - To calculate  $P_2$  instead of  $P_1$ :  $P_1 - L_{Mis\ Rx} = P_{rad} + D_{Tx} - L_{FS} + G_{Rx} - L_{Mis\ Rx} = P_2$
  - To calculate  $C$  (power at the detector):  $P_{rad} + D_{Tx} - L_{FS} + G_{Rx} - L_{Mis\ Rx} - L_{Cables\ Rx} + G_{AMP} = C$
  - Altogether:  $OTHER_{Rx} = -L_{Mis\ Rx} - L_{cables\ Rx} + G_{AMP}$
- Other factors of the **propagation path** (additional to  $L_{FS}$ ):
  - Fixed losses: Gases absorption and fixed diffraction and reflection (to be seen in Chapter 3); misalignment losses, cross polarization (*XPD*) losses; vegetation attenuation.
  - Time-varying losses: rain (to be seen in Chapter 3), anomalous diffraction, multipath, atmospheric scintillation, ...

**Randomly varying factors are taken into account by calculating some safety margin to be added to  $P_{Tx}$**

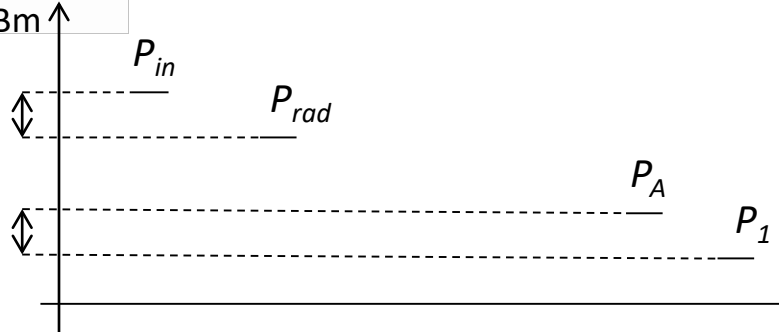
# Link Budget



- Transmission medium
- a) Fixed basic losses
- $L_{fs}$  : Propagation basic losses (free space)
  - $L_{gases}$  : Gases and water vapor absorption
  - $L_{vegetation}$  : Vegetation attenuation
- b) Time-varying losses in excess
- $L_{diffraction}$  : Diffraction losses (anomalous diffraction)
  - $L_{scintillation}$  : Atmospheric scintillation losses
  - $L_{rain}$  : Losses due to rain and other hydrometeors
  - $L_{multipath}$  : Losses due to multipath
  - $L_{beam\ spreading}$  : Fading due to beam spreading

Ohmic losses  
in Tx Antenna

Ohmic losses  
in Rx Antenna



# C calculation

## Summary procedure for calculating C

### □ In dB:

1.  $\rho_{Tx} = \frac{Z_A - ZG}{Z_A + ZG}$
2.  $L_{Mis\ Tx} = -10 \cdot \log(1 - |\rho_{Tx}|^2)$
3.  $L_{\Omega\ Tx} = -10 \cdot \log(\eta_{Tx})$
4.  $P_{rad} = P_{in} - L_{\Omega\ Tx} = (P_{Tx} - L_{Mis\ Tx}) - L_{\Omega\ Tx}$
5.  $L_{FS} = -20 \cdot \log\left(\frac{\lambda}{4 \cdot \pi \cdot d}\right)$
6. Friis:  $P_A = P_{rad} + D_{Tx} - L_{FS} + D_{Rx}$
7.  $L_{\Omega\ Rx} = -10 \cdot \log(\eta_{Rx})$
8.  $\rho_{Rx} = \frac{Z_{Rx} - ZA}{Z_{Rx} + ZA}$
9.  $L_{Mis\ Rx} = -10 \cdot \log(1 - |\rho_{Rx}|^2)$
10.  $P_2 = P_1 - L_{Mis\ Rx} = P_A - L_{\Omega\ Rx} - L_{Mis\ Rx}$
11. Other stages in the Rx (+ G, -L)

### □ Other magnitudes:

- $EIRP = P_{rad} + D_{Tx}$
- $G_{Tx} = EIRP - P_{in}$

In linear units:

- $G_{Rx} = D_{Rx} \cdot \eta_{Rx}$
- $|I|^2 = P_{rad} / R_{rad}$
- $|V_G| = |I| \cdot |Z_A|$
- $P_A = \frac{V_{L\ ideal}^2}{R_{Rx}} \rightarrow |V_{L\ ideal}| = \sqrt{P_A \cdot R_{Rx}}$
- $|V_{L\ ideal}| = |E| / K$
- $|E| = \sqrt{S_{MAX} \cdot \eta} = \left(\frac{EIRP}{4 \cdot \pi \cdot r^2} \cdot 120\pi\right)^{1/2}$   
 $\eta = 120\pi$  in vacuum or air

# C calculation

## Summary procedure for calculating C

### □ In linear units:

1.  $\rho_{Tx} = \frac{Z_A - ZG}{Z_A + ZG}$

2.  $P_{in} = P_{Tx} \cdot (1 - |\rho_{Tx}|^2)$

3.  $P_{rad} = P_{in} \cdot \eta_{Tx}$

4.  $EIRP = P_{rad} \cdot D_{Tx}$

5.  $S_{MAX} = \frac{EIRP}{4 \cdot \pi \cdot r^2}$

6.  $A_{ef} = \frac{\lambda^2}{4 \cdot \pi} \cdot D_{Rx}$

7.  $P_A = S_{MAX Rx} \cdot A_{ef} = \frac{EIRP}{4 \cdot \pi \cdot d^2} \cdot \frac{\lambda^2}{4 \cdot \pi} \cdot D_{Rx}$

8.  $P_1 = P_A \cdot \eta_{Rx}$

9.  $\rho_{Rx} = \frac{Z_{Rx} - Z_A}{Z_{Rx} + Z_A}$

10.  $P_2 = P_1 \cdot (1 - |\rho_{Rx}|^2)$

11. Other stages in the Rx ( $\cdot g$ ,  $/I$ )

### □ Other magnitudes:

■  $G_{Tx} = EIRP / P_{in}$

■  $G_{Rx} = D_{Rx} \cdot \eta_{Rx}$

■  $|I|^2 = P_{rad} / R_{rad}$

■  $|V_G| = |I| \cdot |Z_A|$

■  $P_A = \frac{V_{L ideal}^2}{R_{Rx}} \rightarrow |V_{L ideal}| = \sqrt{P_A \cdot R_{Rx}}$

■  $|V_{L ideal}| = |E| / K$

■  $|E| = \sqrt{S_{MAX} \cdot \eta} = \left( \frac{EIRP}{4 \cdot \pi \cdot r^2} \cdot 120\pi \right)^{1/2}$   
 $\eta = 120\pi$  in vacuum or air



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5. Quality measurements. Alternatives to  $C/N$

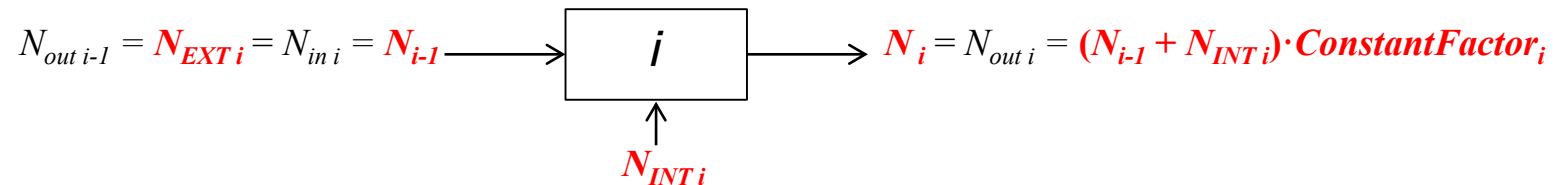
# Noise: Definition and types

---

- ❑ Radio Noise is defined by ITU-R as a time-varying electromagnetic phenomenon having components in the radio-frequency range, apparently not conveying information and which may be superimposed on, or combined with, a wanted signal.
- ❑ Noise is present in all the equipment and communication systems.
- ❑ It is a critical factor for the receiver as part of the  $C/N$  ratio.
- ❑ The most common type is **thermal** (due to the chaotic motion of electrons) **AWGN** (Additive, White and Gaussian Noise).
  - Additive: Power from different noise sources are directly added.
  - White: Its power spectral density is flat in the frequency band of interest.
  - Gaussian: Its instant value is a random variable that follows a Gaussian distribution with zero mean and variance  $\sigma^2$
- ❑ If the spectral density is not uniform in the band of interest: colored noise.
- ❑ According to its distribution in time: burst noise, flicker noise, continuous noise.
- ❑ According to its origin: cosmic noise, man made noise, atmospheric noise.

# Noise: General modeling principles

- There are two contributors to the noise coming out of any stage  $i$  of a Rx:
  - External noise or noise coming from the previous stage ( $i-1$ )
  - Internal noise or noise produced within stage  $i$ , **always considered as input noise**.



There are two types of output *ConstantFactor* depending on the characteristics of stage  $i$ :

Type 1. Active stage

Type 2. Passive stage

- The formula used to **model every type of noise** is directly based on the RMS thermal noise power produced by a resistor at temperature  $T$  (K):  **$N = k \cdot T \cdot B$** 
  - $k$ : Boltzmann constant =  $1.38064852 \cdot 10^{-23} \left[ \frac{W}{HzK} \right]$
  - $T$ : Temperature in Kelvin [K]
  - $B$ : Bandwidth of interest [Hz], that is, where the information is:  **$B = BW_{Nyq}$**

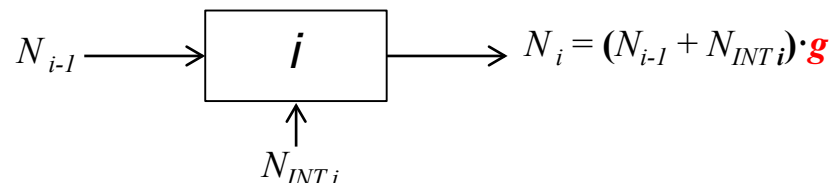
# Noise: General modeling principles

- It is possible to find a value of the temperature so that every type of noise is expressed using the expression of the RMS thermal noise power by a resistor:

- $N_{INT i} = k \cdot T_{eq} \cdot B$
- $N_{EXT i} = N_{i-1} = k \cdot T_{i-1} \cdot B$

- As for the output *ConstantFactor*:

- Active stage (amplifier, detector, mixer,...)



$$T_{eq} = T_0 \cdot (f - 1)$$

- $T_0 = 290 \text{ K}$
- $f$ : Noise factor  $\geq 1 \rightarrow 10 \cdot \log(f) = F$ : Noise figure (more at: [en.wikipedia.org/wiki/Noise\\_figure](https://en.wikipedia.org/wiki/Noise_figure))
- If  $f \gg 1$  (at least  $f = 10$ )  $\rightarrow T_{eq} \approx T_0 \cdot f$

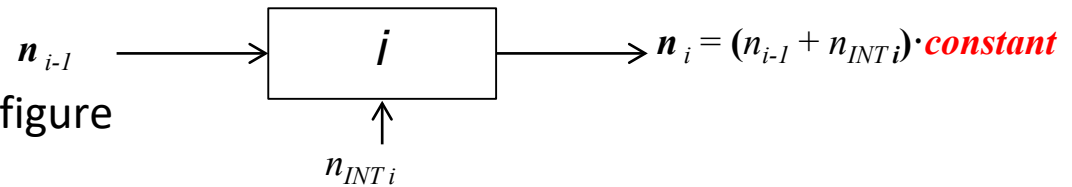
$$N_{INT i} = k \cdot T_0 \cdot (f - 1) \cdot B$$

- Passive stage (cable, attenuator, antenna...)

- $g = 1 / l \rightarrow N_i = (N_{i-1} + N_{INT i}) \cdot 1 / l$
- $T_{eq} = T_{amb} \cdot (l - 1) \rightarrow N_{INT i} = k \cdot T_{amb} \cdot (l - 1) \cdot B$
- It is possible to provide a noise factor for passive elements. If  $T_{amb} = T_0 \rightarrow f = l$

# Noise: Basics

AMPLIFIER, G(dB) gain, F(dB) noise figure



$$T_{eq} = T_o (f - 1)$$

$$n_{INT i} = k \cdot T_o (f - 1) \cdot B$$

$$n_i = (n_{i-1} + n_{INT i}) \cdot g = [n_{i-1} + k \cdot T_o (f - 1) \cdot B] \cdot g$$

ATTENUATOR, L(dB) attenuation,

$$\text{Attenuation } l = 10^{L(dB)/10}$$

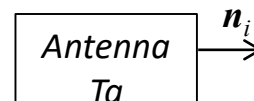
$$T_{eq} = T_{amb} (l - 1)$$

$$n_{INT i} = k \cdot T_{amb} (l - 1) \cdot B$$

$$n_i = (n_{i-1} + n_{INT i}) \cdot 1/l = [n_{i-1} + k \cdot T_{amb} (l - 1) \cdot B] \cdot 1/l$$

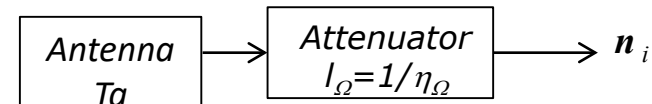
Lossless matched ANTENNA,  $T_a$

$$n_i = k \cdot T_a B$$



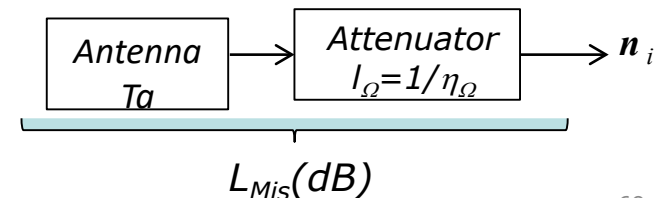
ANTENNA + ohmic losses,  $T_a$ ,  $l_\Omega = 1/\eta_\Omega$

$$n_i = k \cdot T_a B / l_\Omega + k \cdot T_{amb} (l_\Omega - 1) \cdot B / l_\Omega$$

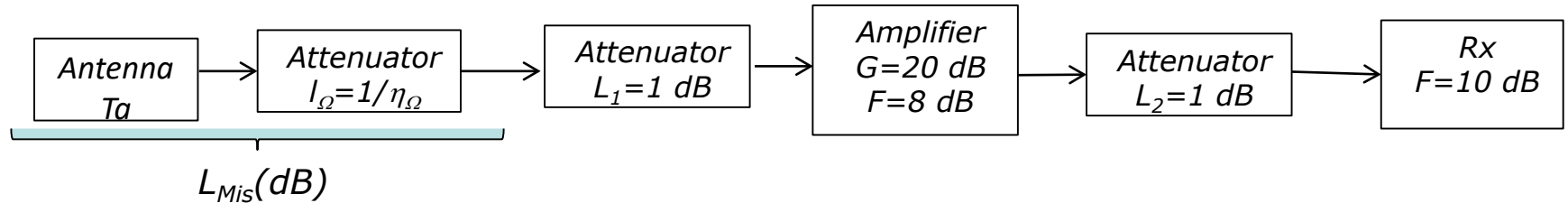


ANTENNA + ohmic losses + mismatch losses,  $T_a$ ,  $l_\Omega = 1/\eta_\Omega$ ,  $L_{Mis}(dB)$

$$n_i = [k \cdot T_a B / l_\Omega + k \cdot T_{amb} (l_\Omega - 1) \cdot B / l_\Omega] \cdot 1/10^{L_{Mis}/10}$$

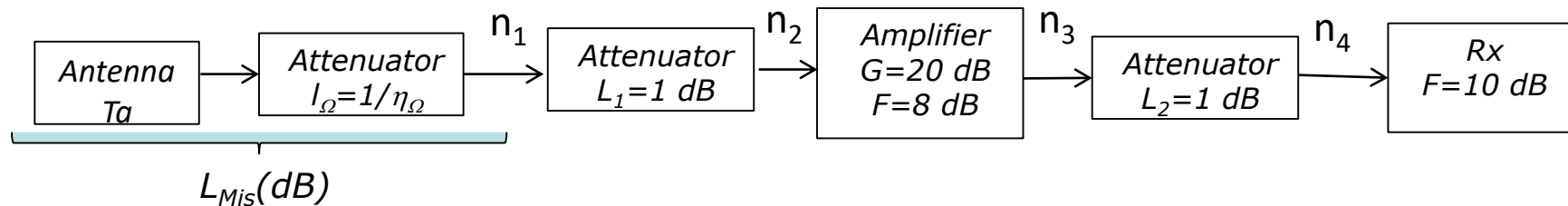


# Noise: Basics



$$\begin{aligned}
 n_{Rx} = & \left[ \frac{k \cdot T_a B}{l_\Omega} + \frac{k \cdot T_{amb} (l_\Omega - 1) B}{l_\Omega} \right] \frac{1}{l_{Mis}} \frac{g}{l_1 l_2} + \frac{k \cdot T_{amb} (l_1 - 1) B}{l_1} \frac{g}{l_2} + \\
 & + k \cdot T_o \left[ 10^{\frac{F}{10}} - 1 \right] B \frac{g}{l_2} + \frac{k \cdot T_{amb} (l_2 - 1) B}{l_2} + k \cdot T_o \left[ 10^{\frac{F_{Rx}}{10}} - 1 \right] B
 \end{aligned}$$

# Noise: Basics



$$n_1 = \left[ \frac{k \cdot T_a B}{l_\Omega} + \frac{k \cdot T_{amb} (l_\Omega - 1) B}{l_\Omega} \right] \frac{1}{l_{Mis}}$$

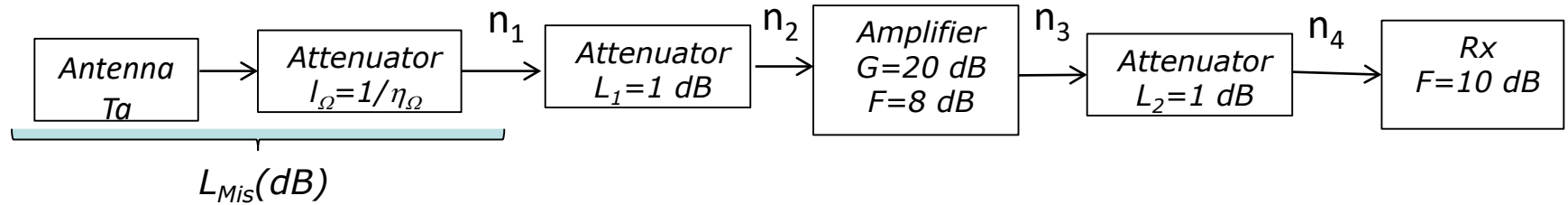
$$n_2 = \left[ \frac{k \cdot T_a B}{l_\Omega} + \frac{k \cdot T_{amb} (l_\Omega - 1) B}{l_\Omega} \right] \frac{1}{l_{Mis}} \frac{1}{l_1} + \frac{k \cdot T_{amb} (l_1 - 1) B}{l_1}$$

$$n_3 = \left[ \frac{k \cdot T_a B}{l_\Omega} + \frac{k \cdot T_{amb} (l_\Omega - 1) B}{l_\Omega} \right] \frac{1}{l_{Mis}} \frac{g}{l_1} + \frac{k \cdot T_{amb} (l_1 - 1) B}{l_1} g + k \cdot T_o \left[ 10^{\frac{F}{10}} - 1 \right] B g$$

$$n_4 = \left[ \frac{k \cdot T_a B}{l_\Omega} + \frac{k \cdot T_{amb} (l_\Omega - 1) B}{l_\Omega} \right] \frac{1}{l_{Mis}} \frac{g}{l_1 l_2} + \frac{k \cdot T_{amb} (l_1 - 1) B}{l_1} \frac{g}{l_2} + k \cdot T_o \left[ 10^{\frac{F}{10}} - 1 \right] B \frac{g}{l_2} + \frac{k \cdot T_{amb} (l_2 - 1) B}{l_2}$$

$$n_{Rx} = n_4 + k \cdot T_o \left[ 10^{\frac{F_{Rx}}{10}} - 1 \right] B$$

# Noise: Basics



$$n_1 = \left[ \frac{k \cdot T_a B}{l_\Omega} + \frac{k \cdot T_{amb} (l_\Omega - 1) B}{l_\Omega} \right] \frac{1}{l_{Mis}}$$

$$n_2 = n_1 \frac{1}{l_1} + \frac{k \cdot T_{amb} (l_1 - 1) B}{l_1}$$

$$n_3 = n_2 g + k \cdot T_o \left[ 10^{\frac{F}{10}} - 1 \right] B g$$

$$n_4 = \frac{n_3}{l_2} + \frac{k \cdot T_{amb} (l_2 - 1) B}{l_2}$$

$$n_{Rx} = n_4 + k \cdot T_o \left[ 10^{\frac{F_{Rx}}{10}} - 1 \right] B$$



# N calculation

## Summary procedure for calculating $N$ of stage $i$

### □ In linear units:

#### 1. $N_{EXT}$

- The stage is an antenna:  $N_{EXT} = k \cdot T_a \cdot B$
- The stage is not an antenna:  $N_{EXT} = N_{i-1}$

#### 2. $T_{eq}$

- Passive stage:  $T_{eq} = T_{amb} \cdot (l - 1)$
- Active stage:  $T_{eq} = T_0 \cdot (f - 1)$

If an antenna:  $l = 1 / \eta_{Rx}$

#### 3. $N_{INT} = k \cdot T_{eq} \cdot B$

#### 4. Output *ConstantFactor*

- Passive stage:  $N_i = (N_{EXT} + N_{INT}) \cdot 1 / l$
- Active stage:  $N_i = (N_{EXT} + N_{INT}) \cdot g$

If an antenna:  $N_i = (N_{EXT} + N_{INT}) \cdot \eta_{Rx}$

If the last stage:  $N = N_{EXT} + N_{INT}$

#### 5. $N_i$ will be $N_{EXT}$ of the next stage

If the last stage → Calculate  $C/N$ . Or in dB:  $C - N$  (10 · log ( $N$  linear))

# N calculation

## Summary procedure for calculating $N$ of stage $i$

### □ In linear units:

#### 1. $N_{EXT}$

- The stage is an antenna:  $N_{EXT} = k \cdot T_a \cdot B$
- The stage is not an antenna:  $N_{EXT} = N_{i-1}$

#### 2. $T_{eq}$

- Passive stage:  $T_{eq} = T_{amb} \cdot (l - 1)$
- Active stage:  $T_{eq} = T_0 \cdot (f - 1)$

If an antenna:  $l = 1 / \eta_{Rx}$

#### 3. $N_{INT} = k \cdot T_{eq} \cdot B$

#### 4. Output *ConstantFactor*

- Passive stage:  $N_i = (N_{EXT} + N_{INT}) \cdot 1 / l$
- Active stage:  $N_i = (N_{EXT} + N_{INT}) \cdot g$

If an antenna:  $N_i = (N_{EXT} + N_{INT}) \cdot \eta_{Rx}$

If the last stage:  $N = N_{EXT} + N_{INT}$

#### 5. $N_i$ will be $N_{EXT}$ of the next stage

If the last stage → Calculate  $C/N$ . Or in dB:  $C - N$  (10 · log ( $N$  linear))

# Interference and final C/N calculation

---

- ❑ Interference is defined by the ITU as the effect of unwanted energy due to one or a combination of emissions, radiations, or inductions upon reception in a radiocommunication system, manifested by any performance degradation, misinterpretation, or loss of information which could be extracted in the absence of such unwanted energy.
- ❑ In many aspects is similar to noise, but noise is always uncorrelated to the signal.
  - If noise is the dominant effect the  $C/N$  ratio can be used.
  - If interference is the main effect the  $C/I$  ratio is used.
  - In many cases the interference is considered as noise and the  $C/(N+I)$  is used.

$$\frac{C}{N+I} (dB) = C(dBm) - (N + I)(dBm) = C(dBm) - 10 \cdot \log[n(mW) + i(mW)]$$

- ❑ To decrease the interference level: change antenna polarization or antenna diagram, input filters, etc.
- ❑ Example: We consider interference negligible

$$C = -12.6 \text{ dBm}; N = -76.99 \text{ dBm}$$

$$C/N = -12.6 - (-76.99) = 64.39 \text{ dB}$$

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    - ii.  $EVM$
  - c. Error causes and error probability

# Quality measurements. Alternatives to $C/N$

## Energy per bit, noise per Hertz ratio

- As  $C$ , the energy per bit ( $e_b$ ) is referred to the maximum amplitude of the modulated carrier,  $A$  (see slide 23 for an example)  $\rightarrow$   $RMS\ power = \frac{A^2}{2}$ 
  - $e_b = \frac{A^2}{2V_b}$      $[\frac{W}{b/s}] = [\frac{J/s}{b/s}] = [J/b] = [J]$     where  $V_b$  is the gross bit rate.
  - $n_0 = n / B$      $[\frac{W}{Hz}] = [\frac{J/s}{1/s}] = [J]$

**Lowercase** letters can be used to distinguish magnitudes in **linear units**

- Relation to  $c/n$ :

- $c = \frac{A^2}{2}$      $\rightarrow$      $c = e_b \cdot V_b$
- $B = BW_{Nyq} = V_s$     (see slide 30)     $\rightarrow$      $n = n_0 \cdot B = n_0 \cdot V_s$

$$\frac{c}{n} = \frac{e_b}{n_0} \cdot \frac{V_b}{V_s} = \frac{e_b}{n_0} \cdot m \quad m \text{ [bits/symbol]}$$

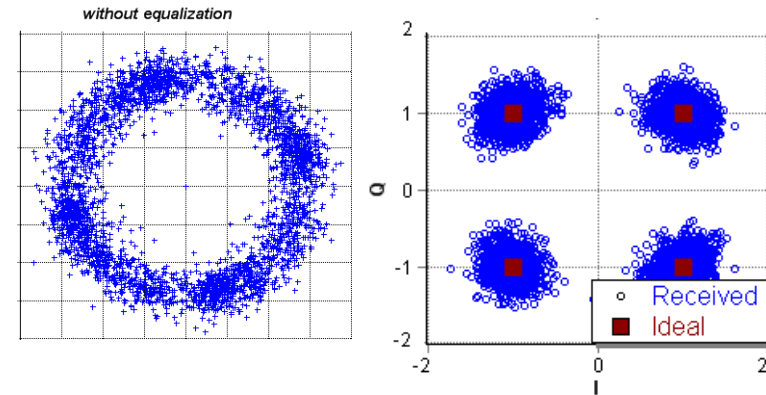
If  $B_{eq} = BW_{RF}$  is considered:  $w = \frac{e_b}{n_0} = \left(\frac{c}{n}\right)_{eq} \cdot \frac{B_{eq}}{V_b}$

# Quality measurements. Alternatives to $C/N$

## Other

### Intro. Signal processing at the Rx:

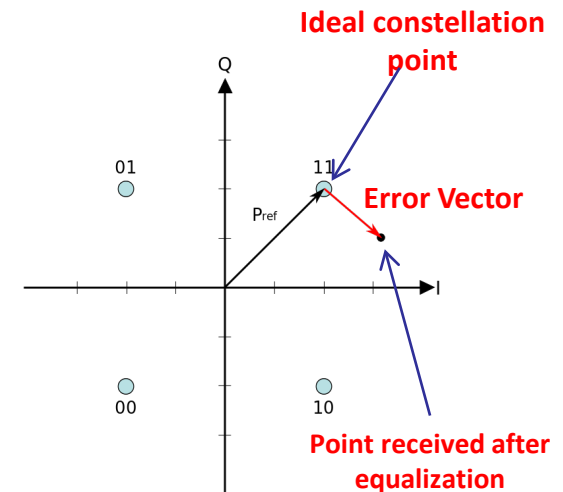
- After time and frequency synchronization:  
Equalization using pilot symbols (constant values)



- Then an stochastic (statistics applied to pseudo-random variables) process estimates the position of the ideal constellation points

For a certain point  $j$ :

- Ideal position vector:  $(I_j, Q_j)$
- Error vector:  $(\delta I_j, \delta Q_j)$



# Quality measurements. Alternatives to C/N

## Other. MER and EVM

- **MER.** Modulation Error Ratio.

$$MER(dB) = 10 \cdot \log \left\{ \frac{\sum_{j=1}^N (I_j^2 + Q_j^2)}{\sum_{j=1}^N (\delta I_j^2 + \delta Q_j^2)} \right\}$$
  - $MER(dB) = 10 \cdot \log \frac{\text{Average symbol power}}{\text{Average error power}}$
  - It is **related to S/N or SNR** → Ideally **similar but lower than C/N or CNR**
  - It can be calculated **before or after equalization**. As the equalization and the stochastic estimation process **depends on** the implementation of **each Rx**:  
There can be **variations** even **> 10 dB** between *MER* values of the **same signal**
  - In the case of **MER**, the **higher** the number, the better.
  - If  $C/N < C/N_{min}$  → Wrong demodulation: *MER* values are not well calculated

- **EVM.** Error Vector Magnitude.

$$EVM_{RMS}(\%) = \sqrt{\frac{\frac{1}{N} \sum_{j=1}^N (\delta I_j^2 + \delta Q_j^2)}{S_{max}^2}} \cdot 100$$
  - $EVM(\%) = \frac{\text{RMS error magnitude}}{\text{Maximum symbol magnitude}} \cdot 100$
  - $S_{max}^2 = \text{RMS power} = \frac{A^2}{2}$
  - In the case of **EVM**, the **lower** the number, the better.
  - It can also be calculated using the average modulation power  $P_m$  (see slide 26)

# Quality measurements. Alternatives to $C/N$

## Error causes and error probability

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- ❑ Reception impairments can be caused by:
  - External and internal noise
  - External RF interference: co-channel or of adjacent channel
  - **ISI**: Inter-symbol interference caused by imperfect filtering or by multi-path propagation. The word “**symbol**” does not refer to a constellation QAM point (point modulated in one carrier or subcarrier) but to a modulation symbol, that is, **a complete set of carriers transmitted with a certain information for a certain period of time.**
  - **ICI**: Inter-carrier interference (with adjacent carriers in multicarrier signals like OFDM) caused by imperfect filtering of each carrier or by Doppler effect.
  - Mutual Interference between I and Q channels, due to an asymmetric transfer function of the radio-frequency channel and selective fading
  - System nonlinearities



# Quality measurements. Alternatives to C/N

## Error causes and error probability

- ❑ Measured error: Bit Error Ratio **BER**. Ratio of erroneous bits to total bits.  
**If the number of received bits is large enough BER is also the error probability.**
- ❑ Alternatives to BER:
  - FER (Frame Error Ratio). Ratio of erroneous frames (a frame is a certain group of modulation symbols)
  - ESR (Errored Second Ratio). Ratio of erroneous seconds
  - BLER (Block Error Ratio). Ratio of erroneous blocks (a block can be one frame or can be a group of frames depending on the standard)
  - Theoretical error probability. Assuming Gray coding for modulation:  $P_{eb} = k \cdot G\left(\frac{d}{\sigma}\right)$ 
    - $d$  is decision distance in constellation points
    - $k$  is a constant that depends on type of modulation
    - $\sigma$  is the normalized additive white Gaussian noise power (AWGN) in the receiver
    - $G(t)$  is the Complementary Gaussian distribution function:

$$G(t) = \frac{1}{\sqrt{2\pi}} \int_t^{\infty} \exp\left(-\frac{u^2}{2}\right) du \quad G(t) \approx \frac{1}{\sqrt{2\pi t}} \exp\left(-\frac{t^2}{2}\right)$$

The  $d/\sigma$  ratio can be normalized in terms of the bit energy to noise density ratio,  $e_b/n_o$

As stated,  $P_{eb}$  and  $BER$  have the same value is  $BER$  measurement considered enough bits

# Quality measurements. Alternatives to $C/N$

## Error causes and error probability

Example of  $SNR$ (dB) values for different modulations and code rates.

SNR Requirements Versus Coding Rate and Modulation Scheme		
Modulation	Code Rate	SNR [dB]
QPSK	1/8	-5.1
	1/5	-2.9
	1/4	-1.7
	1/3	-1.0
	1/2	2.0
	2/3	4.3
	3/4	5.5
	4/5	6.2
16 QAM	1/2	7.9
	2/3	11.3
	3/4	12.2
	4/5	12.8
64 QAM	2/3	15.3
	3/4	17.5
	4/5	18.6

# Quality measurements. Alternatives to $C/N$

## Error causes and error probability

Example of BER versus  $SNR(dB)$  for different modulations

