Image Compression

Lossless and Lossy Grayscale Image Compression

A3 - Image and Signal Processing

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Goal: Compress images using bit-plane coding (lossless) and low-rank approximation (lossy), and determine the achieved compression rate!
Input: a grayscale image $I \in \{0, ..., 255\}^{m \times n}$
Output:

1. **Lossless:** an exact reconstruction $\hat{I} = I$
2. **Lossy:** an approximate reconstruction $\hat{I} \approx I$ with controlled quality

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Lossless Compression

Mathematical Model

- Gray coding: bitwise XOR and right shift

$$G = I \oplus (I >> 1)$$

- Bit-plane extraction:

$$B_k = bitget(G, k), \qquad k = 1, ..., 8$$

- Run-length encoding (RLE):
  - Convert B_k to a 1D sequence
  - Store runs as pairs (v_i, l_i),
    where $v_i \in \{0, 1\}$ and l_i is the run length

Reconstruction

- Decode each B_k from (v_i, l_i)
- Rebuild gray-coded image:

$$\hat{G} = \sum_{k=1}^{8} \hat{B}_k \cdot 2^{k-1}$$

- Undo gray-coding:

$$\hat{I} = \hat{G} \oplus (\hat{G} >> 1) \oplus (\hat{G} >> 2) \oplus (\hat{G} >> 4)$$

Metrics

- Losslessness check: $isequal(I, \hat{I})$
- Compression ratio (CR):

$$CR = \frac{m \cdot n \cdot 8}{\text{stored bits for all planes}}$$

Lossy Compression

Mathematical Model

- Treating image as a matrix $A \in \mathbb{R}^{m \times n}$
- SVD:

$$A = U \Sigma V^T$$

- Truncated rank-k approximation:

$$A_k = U_k \Sigma_k V_k^T$$

- Choosing k to achieve 'quality' via retained energy:

$$\frac{\sum_{i=1}^{k} \sigma_i^2}{\sum_{i=1}^{r} \sigma_i^2} \geq q \qquad (1)$$

Metrics

- Quality achieved: fraction (1)
- PSNR:

$$PSNR = 10 \cdot \log_{10} \left(\frac{255^2}{MSE} \right)$$

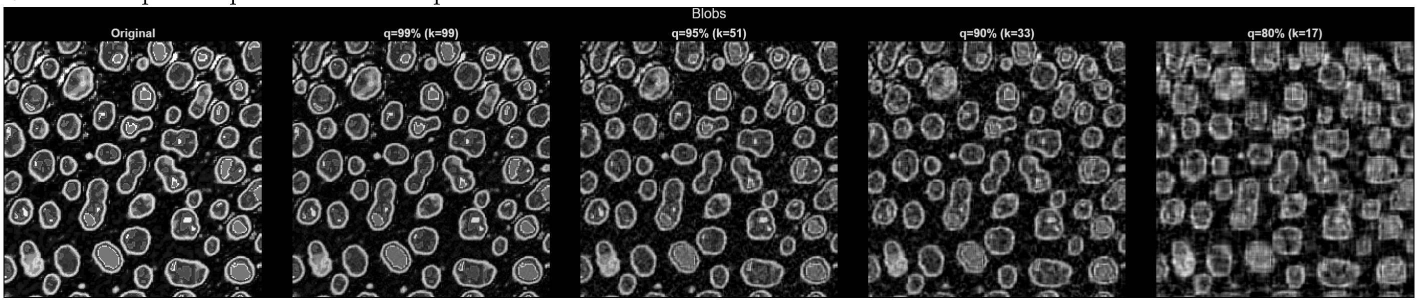
- CR estimate:

$$\text{bits} \approx (mk + nk + k) \cdot 32$$

Interesting Test Cases

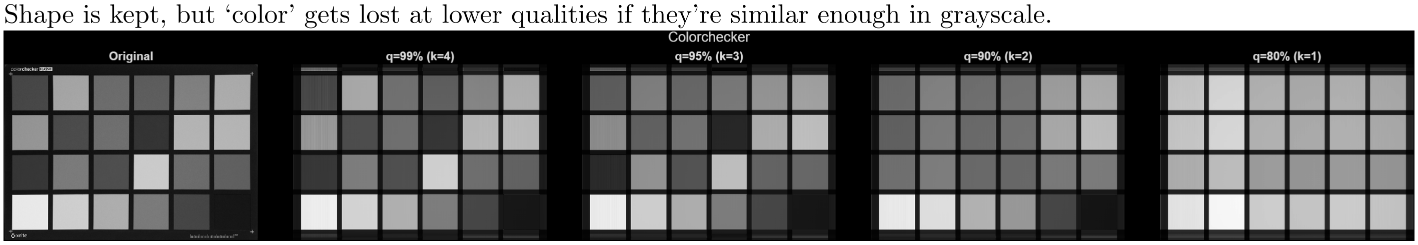
Blobs

General shape is kept even at lower qualities.



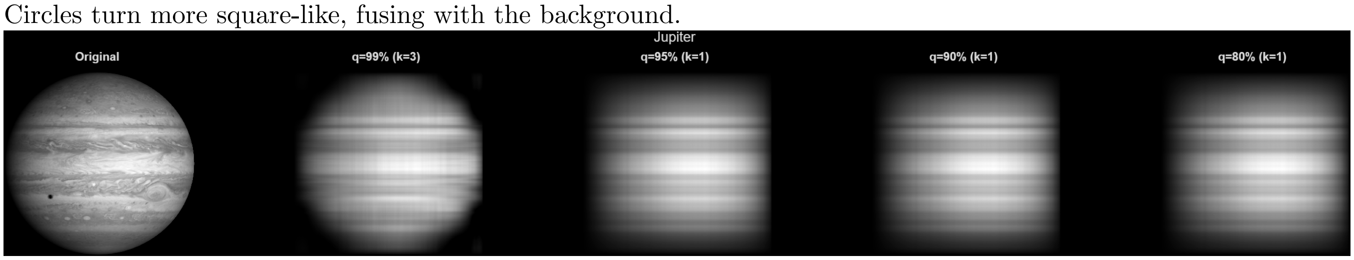
Colorchecker

Shape is kept, but ‘color’ gets lost at lower qualities if they’re similar enough in grayscale.



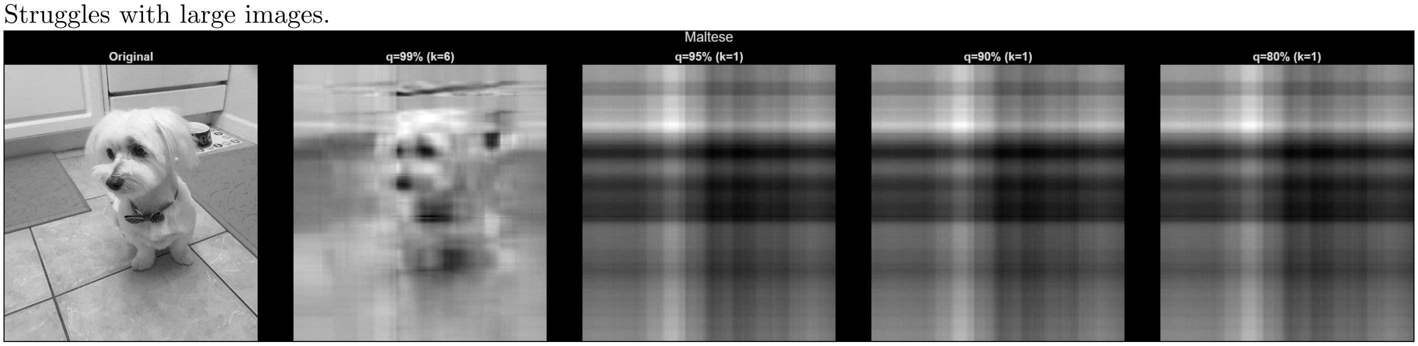
Jupiter

Circles turn more square-like, fusing with the background.



Maltese

Struggles with large images.



Conclusions

Lossless compression achieves perfect reconstruction on all tests, but often expands natural images ($CR < 1$) and is slow on run-heavy content.
Lossy compression can achieve massive compression for images that are effectively low-rank (very smooth / structured), but struggles on textures / noise (needs larke k, CR can fall below 1), and SVD runtime can be very large on big images.