ASSIGNMENT 01

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QUESTION 01

position vectors

$A(4i+4j+k)$

$B(-4i+3j-4k)$

$C(4i-j-2k)$

(a)

Find equation of plane ABC

$AB = (-4-4,\ 3-4,\ -4-1)$

$= (-8, -1, -5)$

$BC = (4+4, -1-3, -2+4)$

$= (8, -4, 2)$

$$= \begin{vmatrix} i & j & k \\ -8 & -1 & -5 \\ 8 & -4 & 2 \end{vmatrix}$$

$$= i\begin{vmatrix} -1 & -5 \\ -4 & 2 \end{vmatrix} - j\begin{vmatrix} -8 & -5 \\ 8 & 2 \end{vmatrix} + k\begin{vmatrix} -8 & -1 \\ 8 & -4 \end{vmatrix}$$

$= i(-2-20) - j(-16+40) + k(32+8)$

$= -22i - 24j + 40k$

$= -22(x-a) - 24(y-b) + 40(z-c) =$

taking $A(4i+4j+k)$

$= -22(x-4) - 24(y-4) + 40(z-1)$

$= -22x + 88 - 24y + 96 + 40z - 40$

$= -22x - 24y + 40z + 144$

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(b)

At origin Final perpendicular distance

O (0,0,0)

$$D = \frac{|-22(0) - 24(0) + 40(0) + 144|}{\sqrt{(-22)^2 + (24)^2 + (40)^2}}$$

$$D = \frac{144}{2\sqrt{666}}$$

$$D = 2.8$$

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QUESTION 02

$A(7i+4j-k)$
$B(11i+3j)$
$C(2i+6j+3k)$
$D(2i+7j+\lambda k)$

(a)

Show that
$\lambda^2-5\lambda+4=0$

$\lambda^2-5\lambda+4=0$ — (i)
$\lambda^2-\lambda-4\lambda+4=0$
$\lambda(\lambda-1)-4(\lambda-1)=0$
$(\lambda-4)(\lambda-1)=0$

$\lambda=4$	$\lambda=1$
put in (i)	
$16-20+4=0$	$1-5+4=0$
$0=0$	$0=0$

Proved

(b)

ABD

$AB=(11-7,\ 3-4,\ 0+1)=(4,-1,1)$
$BD=(2-11,\ 7-3,\ 1-0)=(-9,4,1)$

$$=\begin{vmatrix} i & j & k \\ 4 & -1 & 1 \\ -9 & 4 & 1 \end{vmatrix}$$

$=i(-1-4)-j(4+9)+k(16-9)$
$=-5i-13j+7k$

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So

$= -5(x-a) - 13(y-b) + 7(z-c) =$

taking A

$= -5(x-7) - 13(y-4) + 7(z+1)$

$= -5x + 35 - 13y + 52 + 7z + 7$

$= -5x - 13y + 7z + 94$

$= -5x - 13y + 7z + 94$

Also

in $r = a + sb + tc$

$-94 = -5a - 13b + 7c$

(c)

when $\lambda = 4$

$AB = (11-7, 3-4, 0+1) = (4, -1, 1)$

$BD = (2-11, 7-3, 4-0) = (-9, 4, 4)$

$$= \begin{vmatrix} i & j & k \\ 4 & -1 & 1 \\ -9 & 4 & 4 \end{vmatrix}$$

~~$= i(4-4) - j(4-4$~~

$= i(-4-4) - j(16+9) + k(16-9)$

$= -8i - 25j + 7k$

So

$= -8(x-a) - 27(y-b) + 7(z-c)$

taking A

$= -8(x-7) - 27(y-4) + 7(z+1)$

$= -8x + 56 - 27y + 108 + 7z + 7$

$= -8x - 27y + 7z + 171$

$= -8x - 27y + 7z + 171.$

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(d)

Acute angle :-

$$\cos\theta = \frac{(-8i - 27j + 7k)(-5i - 13j + 7k)}{\sqrt{64+729+49}\ \sqrt{25+169+49}}$$

$$\cos\theta = \frac{40 + 351 + 49}{\sqrt{842} \cdot \sqrt{243}}$$

$$\theta = \cos^{-1}\left(\frac{}{452.33}\right.$$

$$\theta = 13^\circ\ 24'$$

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QUESTION 03

A $(ti + j)$
B $(-2i - j)$
C $(j + tk)$
D $(-2j + k)$

(b)

$AB = (-2 - t, -1 - 1) = (-2 - t, -2, 0)$
$CD = (-2 - 1, -1 - t) = (0, -3, -1 - t)$

$$= \begin{vmatrix} i & j & k \\ -2-t & -2 & 0 \\ 0 & -3 & -1-t \end{vmatrix}$$

$= i(+2 + 2t) - j(2 + 2t + t + t^2) + k(+6 + 3t)$

$= (2 + 2t)i - j(2 + 3t + t^2) + k(6 + 3t)$

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QUESTION 5(a)

$P(-2,-1), Q(-6,-3)$

find equation of circle

Finding midpoints

$$\left(\frac{-2-6}{2}\right), \left(\frac{-1-3}{2}\right)$$

$$(-4, -2)$$

So $(x+4)^2 + (y+2)^2 = r^2$ —(i)

for r^2 put $P(-2,-1)$ in (x,y) in (i)

$$(-2+4)^2 + (-1+2)^2 = r^2$$

$$(2)^2 + (1)^2 = r^2$$

$$3 = r^2$$

so putting r^2 in eq (i)

$$(x+4)^2 + (y+2)^2 = 3^2$$

(b)

circle passes through $(4,0)$ and $(0,2)$

Find ~~cente~~ radius of circle.

As equation is ~~in~~

$$(x-a)^2 + (y-b)^2 = r^2$$

for $(4,0)$

$$16 + b^2 = r^2 \quad \text{—(i)}$$

for $(0,2)$

$$0 + (2-b)^2 = r^2$$

$$(b-2)^2 = r^2 \quad \text{—(ii)}$$

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~~$-b^2+4-2b = r^2$~~

comparing (i) and (ii)

$(b-2)^2 = 16+b^2$

$\cancel{b^2}+4-4b = 16+\cancel{b^2}$

~~$4(1-b) = 16$~~

~~$-2b =$~~

~~$1-b = 4$~~

~~$b =$~~

$-4(b-1) = 16$

$b-1 = -4$

$b = -3$

by eq (i)

$r^2 = 4^2+(-3)^2$

$r^2 = 16+9$

$r^2 = 25$

$r = 5$

(c)

Find equation of parabola

$y^2 = 100x$

As in parabola

$y^2 = +4ax$

so $+4a = 100$

$a = +25$

so equation of directrix is 25.

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(d)

$x^2 = 24y$

As $x^2 = 4ay$

$4a = 24$

$a = 6$

so equation of axis is 6

(e)

Find major axis length for ellipse

$$\left(\frac{x^2}{25}\right)^2 + \left(\frac{y^2}{16}\right)^2 = 1$$

$$\frac{x^2}{25^2} + \frac{y^2}{16^2} = 1$$

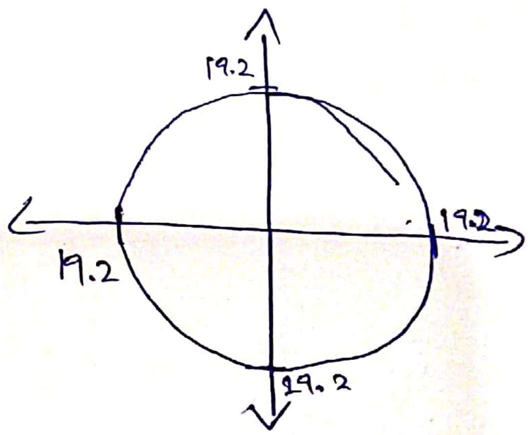
$$c = \sqrt{a^2 - b^2}$$

$$c = \sqrt{25^2 - 16^2}$$

$$= \sqrt{625 - 256}$$

$$= \sqrt{369}$$

$$= \pm 3\sqrt{41} = \pm 19.21$$



(f)

major axis 10

minor axis 8

Find equation of ellipse

As $2a = 10$

$a = 5$

also $2b = 8$

$b = 4$

so $2(x^5) + 2(y^4) = 1$

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