ASSIGNMENT 02

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QUESTION 01

Evaluate Limit.

a) $\lim_{(x,y)\to(2,1)} \frac{x^2-2xy}{x^2-4y^2}$

$= \lim_{(x,y)\to(2,1)} \frac{x(x-2y)}{x^2-(2y)^2}$

$= \lim_{(x,y)\to(2,1)} \frac{x\cancel{(x-2y)}}{\cancel{(x-2y)}(x+2y)}$

$= \lim_{(x,y\to 2,1)} \frac{x}{x+2y}$

$= \frac{2}{2+2(1)}$

$= \frac{2}{4}$

$= \frac{1}{2}$

b) $\lim_{(x,y)\to(0,0)} \cancel{\frac{x^2-2xy}{x^2-4y^2}} \frac{x-4y}{6y+7x}$

By ~~using~~ L-Hopital rule

$= \cancel{\lim_{x,y\to 0,0}} \frac{1}{7}$ B

put $x=0$

$= \frac{0-4y}{6y}$

$= \frac{-2}{3}$

put $y=0$

$= \frac{x}{7x}$

$= \frac{1}{7}$

$= \left(-\frac{2}{3}, \frac{1}{7}\right)$

c) $\lim_{(x,y)\to(0,0)} \frac{x^2-y^6}{xy^3}$

$x=0$

$= \frac{-y^6}{0\cdot(y^3)}$

$= -\infty$

$y=0$

$= \frac{x^2}{x(0)}$

$= \infty$

$= (-\infty, \infty)$

d) $\lim_{(x,y,z)\to(-1,0,4)} \frac{x^3-z e^{2y}}{6x+2y-3z}$

$= \frac{(-1)^3-(4)\,e^{2(0)}}{6(-1)+2(0)-3(4)}$

$= \frac{-1-4(1)}{-6-12}$

$= \frac{-5}{-18}$

$= \frac{5}{18}$

QUESTION 02

Determin $D_{\vec{u}}f$

a) $f(x,y) = \cos\left(\frac{x}{y}\right)$ in the direction of $\vec{v} = (3,-4)$

$\vec{v} = (3,-4)$

for $\frac{\partial}{\partial x} = \frac{\partial}{\partial x} \cos(x/y)$

Solving

$$\frac{\partial}{\partial x} = \frac{-\sin\frac{x}{y}}{y}$$

for $\frac{\partial}{\partial y} = \frac{\partial}{\partial y} \cos(x/y)$

$$\frac{\partial}{\partial y} = \frac{x\sin\left(\frac{x}{y}\right)}{y^2}$$

So

$$f_x = -\frac{\left(\sin\frac{x}{y}\right)}{y}$$

and $f_y = \frac{x\left(\sin\left(\frac{x}{y}\right)\right)}{y^2}$

Now for

$$D_uf(x,y) = -\frac{\left(\sin\frac{x}{y}\right)}{y}(3) + \frac{x\left(\sin\frac{x}{y}\right)}{y^2}(-4)$$

$$= -3\frac{\left(\sin\frac{x}{y}\right)}{y} - 4\frac{\sin\frac{x}{y}}{y^2}$$

(h)

(b)

$f(x,y,z) = x^2y^3 - 4xz$ in direction of $\vec{v} = (-1,2,0)$

$\vec{v} = (-1, 2, 0)$

finding $f_x = \frac{\partial}{\partial x}(x^2y^3 - 4xz)$

$= 2xy^3 - 4z$

For $f_y = \frac{\partial}{\partial y}(x^2y^3 - 4xz)$

$= 3y^2x^2.$

For $f_z = \frac{\partial}{\partial z}(x^2y^3 - 4xz)$

$= -4x$

Now for $D_{\vec{u}}f$

$D_{\vec{u}}f = (2xy^3 - 4z)(-1) + (3x^2y^2)(2) - (4x)(0)$

$= -2xy^3 + 4z + 6x^2y^2$

$D_{\vec{u}}f = -2xy^3 + 4z + 6x^2y^2$

QUESTION 03

Find directional derivative of $f(x,y,z) = 4x - y^2 e^{3xz}$ at $(3,-1,0)$ in the direction of $\vec{V} = (-1,4,2)$

Solution:-

$f(x,y,z) = 4x - y^2 e^{3xz}$

$\frac{\partial}{\partial x} = 4 - zy^2 e^{3xz}$

at $(3,-1,0)$

$\frac{\partial}{\partial x} = 4 - 0$

$= 4$

$\frac{\partial}{\partial y} = -2ye^{3xz}$

at $(3,-1,0)$

$= -2(-1)e^{3(0)}$

$= 2$

$\frac{\partial}{\partial z} = -3y^2 x e^{3xz}$

at $(3,-1,0)$

$= -3(-1)^2(3)$

$= -9$

$f(x,y,z) = \langle 4, 2, -9 \rangle$

$\vec{V} = (-1,4,2)$

$= \sqrt{(-1)^2 + (4)^2 + (2)^2}$

$= \sqrt{21}$

$\hat{u} = \frac{-i}{\sqrt{21}} + \frac{4}{\sqrt{21}} j + \frac{2}{\sqrt{21}} k$

$$D_u f(x,y) = \langle 4, 2, -9 \rangle \langle -\frac{i}{\sqrt{21}} + \frac{4}{\sqrt{21}} j + \frac{2}{\sqrt{21}} k$$

$$= -\frac{4}{\sqrt{21}} i + \frac{8}{\sqrt{21}} j - \frac{18}{\sqrt{21}} k$$

$$D_u f(x,y,z) = \frac{-4}{\sqrt{21}} \hat{i} + \frac{8}{\sqrt{21}} \hat{j} + -\frac{18}{\sqrt{21}} \hat{k}$$

QUESTION 04

(a)

$$f(x,y) = \sqrt{x^2+y^2} \text{ at } (-2,3)$$

Solution:-

$$\frac{\delta}{\delta x} = \frac{1}{2\sqrt{x^2+y^2}} 2x$$

at $(-2,3)$

$$= \frac{1}{2\sqrt{4+9}} \cdot 2(-2)$$

$$= \frac{-2}{\sqrt{13}}$$

$$\frac{\delta}{\delta y} = \frac{1}{2\sqrt{x^2+y^2}} 2y$$

at $(-2,3)$

$$= \frac{1}{\sqrt{2^2+3^2}} (3)$$

$$= \frac{1}{\sqrt{13}} 3$$

$$= \frac{3}{\sqrt{13}}$$

$$\hat{u} = \frac{-2}{\sqrt{13}} i + \frac{3}{\sqrt{13}} j$$

(b)

$f(x, y, z) = e^{2x}\cos(y-2z)$ at $(4, -2, 0)$

Solution:-

$\frac{\partial}{\partial x} = 2e^{2x}\cos(y-2z)$

at $(4, -2, 0)$

$= 2e^{8}\cos(-2-0)$

$= 2e^{8}\cos(-2)$

$\frac{\partial}{\partial y} = \frac{\partial}{\partial y}\left[e^{2x}\cos(y-2z)\right]$

$= -e^{2x}\sin(y-2z)$

at $(4, -2, 0)$

$= -e^{8}\left[\sin(-2-0)\right]$

$= -e^{8}\sin(-2)$

$\frac{\partial}{\partial z} = \frac{\partial}{\partial z} e^{2}\cos(y-2z)$

$= 2e^{2x}\sin(y-2z$

at $(4, -2, 0)$

$= 2e^{8}\sin(-2-0)$

$= 2e^{8}\sin(-2)$

~~$f(x, y, z)$~~

$f(\hat{x}, y, z) = \langle 2e^{8}(0.999), -e^{8}\cancel{(-0.999)}^{(-0.034)}, 2e^{8}(-0.034)$

$\hat{u} = 2e^{8}(0.999)\hat{i} + e^{8}(0.034)\hat{j} - 2e^{8}(0.034)\hat{k}$

QUESTION 05

Compute div $\vec{F}$ and Curl $\vec{F}$

(a)

$$\vec{F} = x^2 y \hat{i} - (z^3 - 3x)\hat{j} + 4y^2 \hat{k}$$

As $\nabla = \frac{\partial}{\partial x}\hat{i} + \frac{\partial}{\partial y}\hat{j} + \frac{\partial}{\partial z}\hat{k}$

$\nabla . F$ for divergence

$$\nabla . F = \frac{\partial}{\partial x}(x^2 y) - \frac{\partial}{\partial y}(z^3 - 3x) + \frac{\partial}{\partial z}(4y^2)$$

$$= 2xy - 0 + 0$$

$$\nabla . F = 2xy$$

Now for curl $\nabla \times F$

$$\nabla \times F = \begin{vmatrix} i & j & k \\ \partial/\partial x & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ x^2 y & -\{z^3 + 3x\} & 4y^2 \end{vmatrix}$$

$$= \hat{i}\left[\left(\frac{\partial}{\partial y}(4y^2) + \frac{\partial}{\partial z}(-z^3 + 3x)\right] - \hat{j}\left[\left(\frac{\partial}{\partial x}(4y^2) - \frac{\partial}{\partial z}(x^2 y)\right]\right.\right.$$

$$+ \hat{k}\left[\frac{\partial}{\partial x}(-z^3 + 3x) - \frac{\partial}{\partial y}(x^2 y)\right)$$

$$= (8y - 3z^2)\hat{i} - \hat{j}(0 - 0) + \hat{k}(3 - x^2)$$

$$\nabla \times F = (8y - 3z^2)\hat{i} + (3 - x^2)\hat{k}$$

(b)

$$\vec{F} = (8x+2z^2)\hat{i} + \frac{x^3y^2}{z}\hat{j} - (z-7x)\hat{k}$$

As $\nabla = \frac{\partial}{\partial x}\hat{i} + \frac{\partial}{\partial y}\hat{j} + \frac{\partial}{\partial z}\hat{K}$

For Div $\vec{F}$

$$\nabla \cdot F = (8x+2z^2)\frac{\partial}{\partial x} + \left(\frac{x^3y^2}{z}\right)\frac{\partial}{\partial y} + (-z+7x)\frac{\partial}{\partial z}$$

$$= 8 + \frac{2x^3y}{z} + -1$$

$$\nabla \cdot F = \frac{2x^3y}{z} + 7$$

For Curl $\vec{F}$

$$\nabla \times F = \begin{vmatrix} i & j & K \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ (8x+2z^2) & \frac{x^3y^2}{z} & -z+7x \end{vmatrix}$$

$$= i\left[\left(\frac{\partial}{\partial y}\right)(-z+7x) - \left(\frac{\partial}{\partial z}\right)\left(\frac{x^3y^2}{z}\right)\right] - j\left[\frac{\partial}{\partial x}(-z+7x) - \frac{\partial}{\partial z}(8x+2z^2)\right]$$

$$+ K\left[\frac{\partial}{\partial x}\left(\frac{x^3y^2}{z}\right) - \frac{\partial}{\partial y}(8x+2z^2)\right]$$

$$= \left(\frac{-2x^3y^2}{z^2}\right)i - (7-4z)j + \left(\frac{3x^2y^2}{z} - 0\right)K$$

$$\nabla \times F = \left(\frac{-2x^3y^2}{z^2}\right)\hat{i} - (7-4z)\hat{j} + \frac{3x^2y^2}{z}\hat{K}$$

QUESTION 06
(a)

$$F = \left(4y^2 + \frac{3x^2y}{z^2}\right)\hat{i} + \left(8xy + \frac{x^3}{z^2}\right)\hat{j} + \left(11 - \frac{2x^3y}{z^3}\right)\hat{k}$$

The vector field is conservative if and only if

$$\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}, \quad \frac{\partial N}{\partial z} = \frac{\partial P}{\partial y}, \quad \frac{\partial M}{\partial z} = \frac{\partial P}{\partial x}$$

As

$$F = \underbrace{\left(4y^2 + \frac{3x^2y}{z^2}\right)}_{M}\hat{i} + \underbrace{\left(8xy + \frac{x^3}{z^2}\right)}_{N}\hat{j} + \underbrace{\left(11 - \frac{2x^3y}{z^3}\right)}_{P}\hat{K}$$

$$\frac{\partial M}{\partial y} = 8y + \frac{3x^2}{z}$$

$$\frac{\partial N}{\partial x} = 8y - \frac{3x^2}{z^2}$$

$$, \frac{\partial N}{\partial z} = (x^3 z^{-3}) - 2$$

$$= \left(\frac{x^3}{z^3}\right) - 2$$

$$, \frac{\partial P}{\partial y} = -\frac{2x^3}{z^3}$$

and $\dfrac{\partial M}{\partial z} = -6 \dfrac{x^2y}{z^3}$

and $\dfrac{\partial P}{\partial x} = -\dfrac{6x^2y}{z^3}$

Hence

$$\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}, \quad \frac{\partial N}{\partial z} = \frac{\partial P}{\partial y}, \quad \frac{\partial M}{\partial z} = \frac{\partial P}{\partial x}$$

So vector field is conservative.

(6)

$F = 6x\hat{i} + (2x - y^2)\hat{j} + (6z - x^3)\hat{k}$

The vector field is conservative if and only if

$$\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}, \quad \frac{\partial N}{\partial z} = \frac{\partial P}{\partial y}, \quad \frac{\partial M}{\partial z} = \frac{\partial P}{\partial x}$$

$$F = \underbrace{6x}_{M}\hat{i} + \underbrace{(2x - y^2)}_{N}\hat{j} + \underbrace{(6z - x^3)}_{P}\hat{k}$$

$\frac{\partial M}{\partial y} = 0$	$\frac{\partial N}{\partial x} = 2$
, $\frac{\partial N}{\partial z} = 0$	$\frac{\partial P}{\partial y} = 0$
and $\frac{\partial M}{\partial z} = 0$	$\frac{\partial P}{\partial x} = -3x^2$

So $\frac{\partial M}{\partial y} \neq \frac{\partial N}{\partial x}$, $\frac{\partial N}{\partial z} = \frac{\partial P}{\partial y}$, $\frac{\partial M}{\partial z} \neq \frac{\partial P}{\partial x}$.

So the vector field is not conservative

QUESTION 07

(a)

Find $\frac{dz}{dt}$

$z = \frac{x^2 - w}{y^4}$, $x = t^3 + 7$, $y = \cos(2t)$, $w = 4t$

$$\frac{dz}{dt} = \frac{\partial z}{\partial x} \cdot \frac{dx}{dt} + \frac{\partial z}{\partial y} \cdot \frac{dy}{dt} \quad \text{--- (i)}$$

So

$$\frac{dx}{dt} = \frac{d}{dt}(t^3 + 7)$$

$$= 3t^2$$

$$\frac{\partial z}{\partial x} = \frac{1}{y^4}(2x)$$

$$\frac{dy}{dt} = \frac{d}{dt}(\cos(2t))$$

$$= -\sin 2t \ (2)$$

$$\frac{\partial z}{\partial y} = \frac{\partial}{\partial y}\left(\frac{x^2 - 4t}{y^4}\right)$$

$$= \frac{-5}{y^5}(x^2 - 4t)$$

putting values in eq (i):

$$\frac{dz}{dt} = \frac{2x}{y^4} \cdot 3t^2 + \left(\frac{-5}{y^5}(x^2 - 4t)\right) \cdot (-2\sin 2t)$$

$$\frac{dz}{dt} = \frac{2}{y^4}\left(3xt^2 + \frac{5(x^2 - 4t)\sin 2t}{y}\right)$$

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(b)

Find $\frac{dz}{du}$, when

$z = x^2y^4 - 2y$, $y = \sin x^2$

put y value in z.

$z = x^2(\sin x^2)^4 - 2\sin x^2$

taking differentiation with respect to x

$$\frac{dz}{du} = \frac{d}{du}\left[x^2(\sin x^2)^4 - 2\sin x^2\right]$$

$$= 2u\left[(\sin(x^2))^4\right] + x^2\left[4(\sin x^2)\right]^3 \cdot \cos(x^2) \cdot 2x$$

$$-4u\cos(x^2$$

$$= 2u\sin^4(x^2) + 8x^3\cos x^2\sin^3 x^2 - 4u\cos(x^2)$$

(c)

compute $\frac{dy}{du}$ for the following equation

$x^2y^4 - 3 = \sin xy$

differentiating w.r.t x on b/s

$$\frac{d}{du}(x^2y^4 - 3) = \frac{d}{du}\sin xy$$

$$\frac{dy}{du}x^2 3y^3 + y^4 2u = \cos xy \cdot \left(y + \frac{dy}{du} \cdot x\right)$$

$$\frac{dy}{du}(3x^2y^3) - \cos xy\,\frac{dy}{du} \cdot x = \cos xy \cdot y - y^4 2u$$

$$\frac{dy}{du}\left(3x^2y^3 - \cos(xy)x\right) = \cos xy \cdot y - y^4 2x$$

$$\frac{dy}{dx} = \frac{\cos xy \cdot y - 2y^4 x}{3x^2y^3 - \cos xy \cdot x}$$

$$\frac{dy}{dx} = \frac{y}{x}\left(\frac{\cos xy - 2y^3 x}{3xy^3 - \cos xy}\right)$$

(d)

Use chain rule to determine $\frac{\partial w}{\partial t}, \frac{\partial w}{\partial s}$

where

$w = \sqrt{x^2+y^2} + \frac{6y}{z}$, $x = \sin(P)$

$y = 1+3t-4s$, $z = \frac{t^3}{s^2}$, $p = 1-2t$

For $\frac{\partial w}{\partial t}$

$w = \sqrt{x^2+y^2} + \frac{6y}{z}$ — (i)

As $x = \sin(P)$

and $P = 1-2t$

$x = \sin(1-2t)$

also $y = 1+3t-4s$

puting in (i)

$$w = \sqrt{(\sin(1-2t))^2 + (1+3t-4s)^2} + \frac{6(1+3t-4s)}{t^3/s^2}$$

$$\frac{\partial w}{\partial t} = \frac{1}{2\sqrt{(\sin(1-2t))^2 + (1+3t-4s)^2}} - \cos^2(-1-2t)^2 \cdot 2 + 2(1+3t-4s)3 + \frac{3(-4)}{t^4 s^2}$$

$$= \frac{1}{2\sqrt{(\sin(1-2t))^2 + (1+3t-4s}} - \cos^2(-1-2t)^2 \cdot 2 + 6 + 18t - 24s + \frac{-12}{t^4 s}$$