ASSIGNMENT 01

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QUESTION 01

Position Vectors:

$4\hat{i}-4\hat{j}+\hat{k}$, $-4\hat{i}+3\hat{j}-4\hat{k}$ and $4\hat{i}-\hat{j}-2\hat{k}$

a) Find equation of plane

As we have

$A(4,-4,1)$
$B(-4,3,-4)$
$C(4,-1,-2$

$$\vec{AB} = \vec{OB} - \vec{OA}$$

$$= \begin{bmatrix} -4 \\ 3 \\ -4 \end{bmatrix} - \begin{bmatrix} 4 \\ -4 \\ 1 \end{bmatrix}$$

$$= \begin{bmatrix} -8 \\ 7 \\ 5 \end{bmatrix}$$

$$= -8\hat{i} + 7\hat{j} - 5\hat{k}$$

$$\vec{AC} = \vec{OC} - \vec{OA}$$

$$= \begin{bmatrix} 4 \\ -1 \\ -2 \end{bmatrix} - \begin{bmatrix} 4 \\ -4 \\ 1 \end{bmatrix} = \begin{bmatrix} 0 \\ 3 \\ -3 \end{bmatrix}$$

$$= 3\hat{j} - 3\hat{k}$$

$$n = \vec{AB} \times \vec{AC}$$

$$= \begin{vmatrix} i & j & k \\ -8 & 7 & -5 \\ 0 & 3 & -3 \end{vmatrix}$$

$$= \hat{i} + 4\hat{j} + 4\hat{k}$$

$$d = a.n = (4\hat{i} - 4\hat{j} + \hat{k}).(\hat{i} + 4\hat{j} + 4\hat{k})$$

$$= 4 - 16 + 4 = -8$$

Equation of Plane:-

$r.n = d$

$r.(\hat{i}+4\hat{j}+4\hat{k}) = -8$

$\therefore (x\hat{i}+y\hat{j}+z\hat{k}).(\hat{i}+4\hat{j}+4\hat{k}) = -8$

$x+4y+4z = -8$

b) Find perpendicular distance from O to the plane. ABC.

prep distance : $\frac{d}{|n|} = \frac{8}{33}$

$= 1.39$

c) point D has position vector $2\hat{i}+3\hat{j}-3\hat{k}$
Find coordinates of the ~~plane~~ plane ABC

line OD : $r = a + \lambda b$

at some value of λ

$$\begin{pmatrix} 2\lambda' \\ 3\lambda' \\ -3\lambda' \end{pmatrix} \cdot \begin{pmatrix} 1 \\ 4 \\ 4 \end{pmatrix} = 8$$

$2\lambda' + 12\lambda' - 12\lambda' = -8$

$2\lambda' = -8$

$\lambda' = -4$

$$r = \begin{pmatrix} 2(-4) \\ 3(-4) \\ -3(-4) \end{pmatrix}$$

$r = (-8, -12, 12)$

QUESTION 02

$A\,(7i+4j-k)$
$B\,(11i+3j)$
$C\,(2i+6j+3K)$
$D\,(2i+7j+\lambda K)$

(a)

Show that
$\lambda^2-5\lambda+4=0$

$\lambda^2-5\lambda+4=0 \quad -\ (i)$
$\lambda^2-\lambda-4\lambda+4=0$
$\lambda(\lambda-1)-4(\lambda-1)=0$
$(\lambda-4)(\lambda-1)=0$

$\lambda=4$	$\lambda=1$
put in (i)	
$16-20+4=0$	$1-5+4=0$
$0=0$	$0=0$

Proved

(b)

ABD

$AB=(11-7,\ 3-4,\ 0+1)=(4,-1,1)$
$BD=(2-11,\ 7-3,\ 1-0)=(-9,4,1)$

$$=\begin{vmatrix} i & j & k \\ 4 & -1 & 1 \\ -9 & 4 & 1 \end{vmatrix}$$

$=i(-1-4)-j(4+9)+k(16-9)$
$=-5i-13j+7K$

So

$= 5(x-a) - 13(y-b) + 7(z-c) =$

taking A

$= -5(x-7) - 13(y-4) + 7(z+1)$

$= -5x + 35 - 13y + 52 + 7z + 7$

$= -5x - 13y + 7z + 94$

$= -5x - 13y + 7z + 94$

Also

in $r = a + sb + tc$

$-94 = -5a - 13b + 7c$

(c)

when $\lambda = 4$

$AB = (11-7, 3-4, 0+1) = (4, -1, 1)$

$BD = (2-11, 7-3, 4-0) = (-9, 4, 4)$

$$= \begin{vmatrix} i & j & k \\ 4 & -1 & 1 \\ -9 & 4 & 4 \end{vmatrix}$$

~~$= i(4-4) - j(4-4$~~

$= i(-4-4) - j(16+9) + k(16-9)$

$= -8i - 27j + 7k$

So

$= -8(x-a) - 27(y-b) + 7(z-c)$

taking A

$= -8(x-7) - 27(y-4) + 7(z+1)$

$= -8x + 56 - 27y + 108 + 7z + 7$

$= -8x - 27y + 7z + 171$

$= -8x - 27y + 7z + 171.$

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(d)

Acute angle :-

$$\cos\theta = \frac{(-8i-27j+7k)(-5i-13j+7k)}{\sqrt{64+729+49}\ \sqrt{25+169+49}}$$

$$\cos\theta = \frac{40+351+49}{\sqrt{842}\cdot\sqrt{243}}$$

$$\theta = \cos^{-1}\left(\frac{}{452.33}\right.$$

$$\theta = 13^{\circ}\,24'$$

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QUESTION 03

Let t be a positive constant. The line L_1 passes through.

a) Find value of t.

$L_1 = t\hat{i} + \hat{j} \qquad -2\hat{i} - \hat{j}$

$L_2 = g\hat{i} + t\hat{j} \qquad -2\hat{j} + \hat{k}$

The shortest between L_1 and L_2 is $\sqrt{21}$

$r_1 = OA + \lambda AB$

$r_2 = OA + \lambda AB$

$$L_1 = r_1 = \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix} + \lambda \begin{pmatrix} -2 \\ -10 \\ 0 \end{pmatrix}$$

$$L_2 = r_2 = \begin{pmatrix} 0 \\ 1 \\ t \end{pmatrix} + \lambda \begin{bmatrix} 0 \\ -2 \\ 1 \end{bmatrix}$$

$$D = \frac{(b_1 \times b_2) \cdot (a_2 - a_1)}{(b_1 \times b_2)}$$

$$b_1 \times b_2 = \begin{vmatrix} i & j & K \\ -2 & -10 & 0 \\ 0 & -2 & 1 \end{vmatrix}$$

$$= -\hat{i} + 2\hat{j} + 4\hat{k}$$

$$|b_1 \times b_2| = \sqrt{(-1)^2 + (2)^2 + (4)^2}$$

$$= \sqrt{1 + 4 + 16}$$

$$= \sqrt{21}$$

$$(a_2-a_1) = \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix} - \begin{bmatrix} t \\ 1 \\ 0 \end{bmatrix}$$

$$= -t\hat{i} + t\hat{k}$$

$$D = \frac{(-\hat{i}+2\hat{j}+4\hat{k}) \cdot (-t\hat{i}+t\hat{k})}{\sqrt{4}}$$

$$\sqrt{21} = \frac{t+4t}{\sqrt{21}}$$

$$t+4t = 21$$

$$5t = 21$$

$$t = \frac{21}{5}$$

Part - b

$$\vec{r}_1 = \frac{21}{5}\hat{i} + \hat{j} + \lambda(2\hat{i} - \hat{j})$$

$$\vec{r}_2 = i - \frac{21}{5}\hat{k} + \mu(-2i + \hat{k})$$

$$\vec{r} = \frac{-21}{5}\hat{i} + \hat{j} + \lambda(-2\hat{i} - \hat{j}) + \mu(-2\hat{j} + \hat{k})$$

Part - C

$$\pi_2 = 5x - 6y + 5 = 20$$

$$\lambda_2 = \frac{x-0}{0}, \ \lambda_2 = \frac{y-1}{2}$$

$$\lambda_2 = \frac{2-4.2}{1}$$

From L_2 direction vector is

$$= \{0, -2, 1\}$$

From π_2 normal vector is

$$= (5, -6, 7)$$

$$\cos\theta = \frac{a_1 b_1 + a_2 b_2 + a_3 b_3}{|a| \cdot |b|}$$

$$(a \cdot b) = \begin{bmatrix} 0 \\ 1 \\ -21/5 \end{bmatrix} \begin{bmatrix} 5 \\ -6 \\ 7 \end{bmatrix}$$

$$= -6 + 29.4 = 23.4$$

$$|a| = \sqrt{1^2 + \left(\frac{21}{5}\right)^2}$$

$$|a| = 4.3$$

$$|b| = \sqrt{(5)^2 + (6)^2 + (7)^2}$$

$$|b| = 10.49$$

$$\theta = \cos^{-1}\left(\frac{23.4}{4.3 \times 10.49}\right)$$

$$\theta = 59.34^\circ$$

d

$$\pi_1 = \begin{bmatrix} -21/5 \\ 1 \\ 0 \end{bmatrix}, \quad \pi_2 = \begin{bmatrix} 5 \\ -6 \\ 7 \end{bmatrix}$$

$$\pi_1 \cdot \pi_2 = \begin{bmatrix} -21/5 \\ 1 \\ 0 \end{bmatrix} \cdot \begin{bmatrix} 5 \\ -6 \\ 7 \end{bmatrix}$$

$$= -21 - 6 = -27$$

$$|a| = \sqrt{17.64 + 1} = 4.3$$

$$|b| = \sqrt{5^2 + (-6)^2 + 7^2} = 10.49$$

$$\theta = 126.78$$

acute angle $= 180^\circ - 126.78^\circ$

$$\theta = 53.23^\circ$$

QUESTION 5(a)

$P(-2,-1), Q(-6,-3)$

find equation of circle

Finding midpoints

$\left(\frac{-2-6}{2}\right), \left(\frac{-1-3}{2}\right)$

$(-4, -2)$

So $(x+4)^2+(y+2)^2 = r^2$ — (i)

for r^2 put $P(-2,-1)$ in (x,y) in (i)

$(-2+4)^2 + (-1+2)^2 = r^2$

$(2)^2 + (1)^2 = r^2$

$5 = r^2$

so putting r^2 in eq (i)

$(x+4)^2+(y+2)^2 = 5$

(b)

circle passes through $(4,0)$ and $(0,2)$

Find ~~cente~~ radius of circle.

As equation is in

$(x-a)^2+(y-b)^2 = r^2$

for $(4,0)$

$16 + b^2 = r^2$ — (i)

for $(0,2)$

$0+(2-b)^2 = r^2$

$(b-2)^2 = r^2$ — (ii)

$\cancel{-b^2+4-2b=r^2}$

comparing (i) and (ii)

$(b-2)^2 = 16+b^2$

$\cancel{b^2}+4-4b = 16+\cancel{b^2}$

$\cancel{4(1-b) = 16}$

$\cancel{-2b}$

$\cancel{1-b = 4}$

$\cancel{b = 2}$

$-4(b-1) = 16$

$b-1 = -4$

$b = -3$

by eq (i)

$r^2 = 4^2+(-3)^2$

$r^2 = 16+9$

$r^2 = 25$

$r = 5$

(c)

Find equation of parabola

$y^2 = 100x$

As in parabola

$y^2 = +4ax$

so $+4a = 100$

$a = +25$

so equation of directrix is $\cancel{25}$ $x = -a$

$x = -25$

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(d)

$x^2 = 24y$

v/s $x^2 = 4ay$

$4a = 24$

$a = 6$

~~so equation of axis is 6~~

$x = -a$

$x = -6$

(e)

Find major axis length for ellipse

$$\left(\frac{x}{25}\right)^2 + \left(\frac{y}{16}\right)^2 = 1$$

$$\frac{x^2}{25^2} + \frac{y^2}{16^2} = 1$$

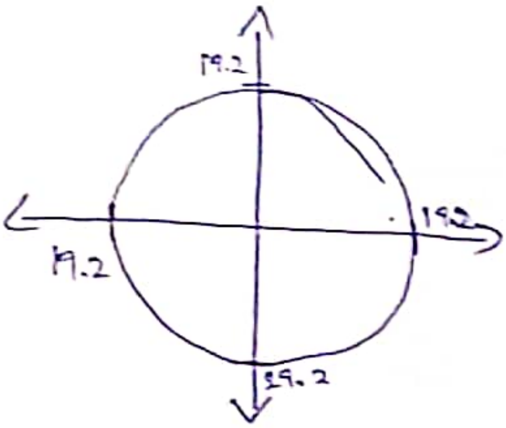
$c = \sqrt{a^2 - b^2}$

$c = \sqrt{25^2 - 16^2}$

$= \sqrt{625 - 256}$

$= \sqrt{369}$

$= \pm 3\sqrt{41} = \pm 19.21$



(f)

major axis 10

minor axis 8

Find equation of ellipse

v/s $2a = 10$

$a = 5$

also $2b = 8$

$b = 4$

so $\cancel{2(x^5) + 2(y^4)} = 1$

$$\left(\frac{x}{5}\right)^2 + \left(\frac{y}{4}\right)^2 = 1$$