

Heavy Quark QCD at Finite Temperature and Density Using an Effective Theory

Jonas Rylund Glesaaen

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ABSTRACT

DEUTCHE ZUSAMMENFASSUNG

PART I

INTRODUCTION

INTRODUCTION

The current state of human knowledge suggests that the majority of the visible matter in the universe is made up of hadrons that are themselves built up from the more fundamental *quarks*. We have so far discovered six species, or flavours, of quarks, namely the up, down, strange, charm, top and bottom. These fundamental particles carry three sets of charges: electric charge, flavour charge and colour charge. It is the latter of these that manifest itself through the confinement process that binds the quarks together into inseparable hadrons, and the resulting binding energy is responsible for almost 99 % of the mass of these bound particles. For example the proton has a mass of 938.27 MeV, while its constituents, two up quarks and a single down quark, have a total rest mass of no more than 9.8 (1.9) MeV [Olive et al., 2014]. It is therefore of great importance to understand the dynamics governing interactions between particles carrying colour charges and the force's mediators, the *gluons*.

Through the various stages of discoveries within the world of particle physics, the theories we use to describe nature has evolved. The current reigning model of the universe is called the Standard Model of particle physics which categorises the known world consisting of 6 leptons, 18 quarks, 13 mediators, and their anti-particles into symmetry groups. This theory describes three of the four established fundamental forces as a quantum theory of fields, and is the most successful theory to date, predicting experimental values with astonishing accuracy.

GAUGE THEORIES AND LATTICE GAUGE THEORIES

In this chapter we will formally introduce the quantum field theory describing fermions invariant under local group transformations and introduce the Yang Mills Lagrangian. This is covered in SECTION 2.1.

- 2.1 CONTINUUM GAUGE THEORIES WITH FERMIONS
- 2.2 SYMMETRY GROUPS AND CONTINUUM SYMMETRIES
- 2.3 LATTICE DISCRETISATION
- 2.4 SYMMETRIES ON THE LATTICE
- 2.5 FERMION DOUBLING AND CHIRAL SYMMETRY
- 2.6 SCALE SETTING AND THE CONTINUUM LIMIT

STATISTICAL MECHANICS AND PHASE TRANSITIONS

- 3.1 THERMAL FIELD THEORY
- 3.2 PHASE TRANSITIONS AND STATISTICAL MECHANICS
- 3.3 THERMAL FIELDS ON THE LATTICE
- 3.4 FINITE DENSITY SIMULATIONS
- 3.5 THE SIGN PROBLEM, NP HARD, EXPONENTIAL CANCELLATIONS
 - 3.5.1 *Reweighting*
 - 3.5.2 *Analytic extrapolation*
 - 3.5.3 *Taylor series*
 - 3.5.4 *Stochastic quantisation*

PART II

THE EFFECTIVE THEORY

THE EFFECTIVE THEORY

- 4.1 THE CHARACTER EXPANSION
- 4.2 PURE GAUGE EFFECTIVE THEORY
- 4.3 THE HOPPING PARAMETER EXPANSION
- 4.4 THE FULL EFFECTIVE THEORY
- 4.5 NUMERICAL EVALUATION

ANALYTIC EVALUATION OF THE EFFECTIVE THEORY

5.1 LINKED CLUSTER EXPANSION

5.2 ANALYTIC RESUMMATION

5.3 LARGE N_c LIMIT

5.4 YANG LEE ZEROS

PART III

DISCUSSION AND OUTLOOK

DISCUSSION

RESEARCH PERSPECTIVES

PART IV

APPENDIX



ANALYTICAL TOOLS FOR SU(N)

A.1 CALCULATING THE HAAR MEASURE

A.2 $L_n L_m$ INTEGRALS

A.3 FERMION DETERMINANT

A.4 W_{nm} TERMS

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