

# Heavy Quark QCD at Finite Temperature and Density Using an Effective Theory

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## **ABSTRACT**

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PART I

# INTRODUCTION





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## INTRODUCTION

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The current state of human knowledge suggests that the majority of the visible matter in the universe is made up of hadrons that are themselves build up from quarks and gluons. We have so far discovered six species, or flavours, of quarks, namely the up, down, strange, charm, top and bottom. These fundamental particles carry three sets of charges: electric charge, flavour charge and colour charge. It is the latter of these that manifest itself through the confinement process that binds the quarks together into inseparable hadrons, and the resulting binding energy is responsible for almost 99 % of the mass of these bound particles. For example the proton weights 938.27 MeV, while its constituents, two up quarks and a single down quark, have a total rest mass of no more than 9.8 (1.9) MeV [Olive et al., 2014].



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## GAUGE THEORIES AND LATTICE GAUGE THEORIES

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- 2.1 CONTINUUM GAUGE THEORIES WITH FERMIONS
- 2.2 SYMMETRY GROUPS AND CONTINUUM SYMMETRIES
- 2.3 LATTICE DISCRETISATION
- 2.4 SYMMETRIES ON THE LATTICE
- 2.5 FERMION DOUBLING AND CHIRAL SYMMETRY
- 2.6 SCALE SETTING AND THE CONTINUUM LIMIT



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## STATISTICAL MECHANICS AND PHASE TRANSITIONS

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- 3.1 THERMAL FIELD THEORY
- 3.2 PHASE TRANSITIONS AND STATISTICAL MECHANICS
- 3.3 THERMAL FIELDS ON THE LATTICE
- 3.4 FINITE DENSITY SIMULATIONS
- 3.5 THE SIGN PROBLEM, NP HARD, EXPONENTIAL CANCELLATIONS
  - 3.5.1 *Reweighting*
  - 3.5.2 *Analytic extrapolation*
  - 3.5.3 *Taylor series*
  - 3.5.4 *Stochastic quantisation*



PART II

# THE EFFECTIVE THEORY





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## THE EFFECTIVE THEORY

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- 4.1 THE CHARACTER EXPANSION
- 4.2 PURE GAUGE EFFECTIVE THEORY
- 4.3 THE HOPPING PARAMETER EXPANSION
- 4.4 THE FULL EFFECTIVE THEORY
- 4.5 NUMERICAL EVALUATION



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## **ANALYTIC EVALUATION OF THE EFFECTIVE THEORY**

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5.1 LINKED CLUSTER EXPANSION

5.2 ANALYTIC RESUMMATION

5.3 LARGE  $N_c$  LIMIT

5.4 YANG LEE ZEROS



PART III

# DISCUSSION AND OUTLOOK



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## DISCUSSION

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**RESEARCH PERSPECTIVES**

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PART IV

# APPENDIX





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## **ANALYTICAL TOOLS FOR $SU(N)$**

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A.1    CALCULATING THE HAAR MEASURE

A.2     $L_n L_m$  INTEGRALS

A.3    FERMION DETERMINANT

A.4     $W_{nm}$  TERMS



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## BIBLIOGRAPHY

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