Problem 1

$\xi \sim Poisson(\lambda)$
$\eta \sim Bin(\xi, p)$
$P(\xi = k) = e^{-\lambda} \cdot \frac{\lambda^k}{k!}$
$P(\eta = m) = C_{\xi}^{m} p^m q^{\xi - m}$
$P(\eta = m | \xi = n) = C_n^m p^m q^{n-m}$

$$\begin{aligned}
P(\eta = m) &= \sum_{k=m}^{\infty} P(\eta = m|\xi = k) \cdot P(\xi = k) \\
&= e^{-\lambda} \cdot \frac{\lambda^m}{m!} \cdot C_m^m p^m q^0 + e^{-\lambda} \cdot \frac{\lambda^{m+1}}{(m+1)!} \cdot C_{m+1}^m p^m q^1 + e^{-\lambda} \cdot \frac{\lambda^{m+2}}{(m+2)!} \cdot C_{m+2}^m p^m q^2 + ... + e^{-\lambda} \cdot \frac{\lambda^{m+t}}{(m+t)!} \cdot C_{m+t}^m p^m q^t + ... \\
&= e^{-\lambda} \cdot \frac{\lambda^m p^m}{(m)!}(1) + e^{-\lambda} \cdot \frac{\lambda^m p^m}{(m)!}(\lambda q) + e^{-\lambda} \cdot \frac{\lambda^m p^m}{(m)!}(\frac{\lambda^2 q^2}{2!}) + ... + e^{-\lambda} \cdot \frac{\lambda^m p^m}{(m)!}(\frac{\lambda^t q^t}{t!}) + ... \\
&= e^{-\lambda} \cdot \frac{(\lambda p)^m}{m!} \left(1 + \lambda q + \frac{\lambda^2 q^2}{2!} + ... + \frac{\lambda^t q^t}{t!} + ...\right) \\
&= e^{-\lambda} \cdot \frac{(\lambda p)^m}{m!} \cdot e^{\lambda q} \\
&= e^{-\lambda(1-q)} \cdot \frac{(\lambda p)^m}{m!} \\
&= e^{-\lambda p} \cdot \frac{(\lambda p)^m}{m!}
\end{aligned}$$

It implies that $\eta \sim Poisson(\lambda p)$

Problem 2

$t_1 \sim N(30, 100),\ f_1(x) = \frac{1}{\sqrt{2\pi} \cdot 10} \cdot e^{\left(-\frac{1}{2} \cdot \frac{(x-30)^2}{100}\right)}$
$t_2 \sim N(20, 25),\ f_2(x) = \frac{1}{\sqrt{2\pi} \cdot 5} \cdot e^{\left(-\frac{1}{2} \cdot \frac{(x-20)^2}{25}\right)}$
H_1 - application was checked by a strict reviewer
H_2 - application was checked by a kind reviewer

$P(H_1) = P(H_2) = \frac{1}{2}$

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\begin{aligned}
P(H_2|t=10) &= \frac{P(t=10|H_2)\cdot P(H_2)}{P(t=10|H_2)\cdot P(H_2)+P(t=10|H_1)\cdot P(H_1)}\\
&= \frac{P(t=10|H_2)}{P(t=10|H_2)+P(t=10|H_1)}\\
&= \frac{f_2(10)\cdot dx}{f_2(10)\cdot dx+f_1(10)\cdot dx}\\
&= \frac{f_2(10)}{f_2(10)+f_1(10)}\\
&= \frac{\frac{1}{\sqrt{2\pi}\cdot 5}\cdot exp(-\frac{1}{2}\cdot\frac{(10-20)^2}{25})}{\frac{1}{\sqrt{2\pi}\cdot 5}\cdot exp(-\frac{1}{2}\cdot\frac{(10-20)^2}{25})+\frac{1}{\sqrt{2\pi}\cdot 10}\cdot exp(-\frac{1}{2}\cdot\frac{(10-30)^2}{100})}\\
&= \frac{\frac{1}{\sqrt{2\pi}\cdot 5}\cdot exp(-2)}{\frac{1}{\sqrt{2\pi}\cdot 5}\cdot exp(-2)+\frac{1}{\sqrt{2\pi}\cdot 10}\cdot exp(-2)}\\
&= \frac{1/5}{1/5+1/10}=\frac{2}{3}
\end{aligned}
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Answer: $P(H_2|t=10)=\frac{2}{3}$