

## Problem 1

$$\xi \sim \text{Poisson}(\lambda)$$

$$\eta \sim \text{Bin}(\xi, p)$$

$$P(\xi = k) = e^{-\lambda} \cdot \frac{\lambda^k}{k!}$$

$$P(\eta = m) = C_{\xi}^m p^m q^{\xi-m}$$

$$P(\eta = m | \xi = n) = C_n^m p^m q^{n-m}$$

$$\begin{aligned} P(\eta = m) &= \sum_{k=m}^{\infty} P(\eta = m | \xi = k) \cdot P(\xi = k) \\ &= e^{-\lambda} \cdot \frac{\lambda^m}{m!} \cdot C_m^m p^m q^0 + e^{-\lambda} \cdot \frac{\lambda^{m+1}}{(m+1)!} \cdot C_{m+1}^m p^m q^1 + e^{-\lambda} \cdot \frac{\lambda^{m+2}}{(m+2)!} \cdot C_{m+2}^m p^m q^2 + \dots + e^{-\lambda} \cdot \frac{\lambda^{m+t}}{(m+t)!} \cdot C_{m+t}^m p^m q^t + \dots \\ &= e^{-\lambda} \cdot \frac{\lambda^m p^m}{(m)!} (1) + e^{-\lambda} \cdot \frac{\lambda^m p^m}{(m)!} (\lambda q) + e^{-\lambda} \cdot \frac{\lambda^m p^m}{(m)!} \left( \frac{\lambda^2 q^2}{2!} \right) + \dots + e^{-\lambda} \cdot \frac{\lambda^m p^m}{(m)!} \left( \frac{\lambda^t q^t}{t!} \right) + \dots \\ &= e^{-\lambda} \cdot \frac{(\lambda p)^m}{m!} \left( 1 + \lambda q + \frac{\lambda^2 q^2}{2!} + \dots + \frac{\lambda^t q^t}{t!} + \dots \right) \\ &= e^{-\lambda} \cdot \frac{(\lambda p)^m}{m!} \cdot e^{\lambda q} \\ &= e^{-\lambda(1-q)} \cdot \frac{(\lambda p)^m}{m!} \\ &= e^{-\lambda p} \cdot \frac{(\lambda p)^m}{m!} \end{aligned}$$

It implies that  $\eta \sim \text{Poisson}(\lambda p)$

## Problem 2

$$t_1 \sim N(30, 100), \quad f_1(x) = \frac{1}{\sqrt{2\pi \cdot 100}} \cdot e^{\left(-\frac{1}{2} \cdot \frac{(x-30)^2}{100}\right)}$$

$$t_2 \sim N(20, 25), \quad f_2(x) = \frac{1}{\sqrt{2\pi \cdot 5}} \cdot e^{\left(-\frac{1}{2} \cdot \frac{(x-20)^2}{25}\right)}$$

$H_1$  - application was checked by a strict reviewer

$H_2$  - application was checked by a kind reviewer

$$P(H_1) = P(H_2) = \frac{1}{2}$$

$$\begin{aligned}
 P(H_2|t=10) &= \frac{P(t=10|H_2) \cdot P(H_2)}{P(t=10|H_2) \cdot P(H_2) + P(t=10|H_1) \cdot P(H_1)} \\
 &= \frac{P(t=10|H_2)}{P(t=10|H_2) + P(t=10|H_1)} \\
 &= \frac{f_2(10) \cdot dx}{f_2(10) \cdot dx + f_1(10) \cdot dx} \\
 &= \frac{f_2(10)}{f_2(10) + f_1(10)} \\
 &= \frac{\frac{1}{\sqrt{2\pi \cdot 5}} \cdot \exp(-\frac{1}{2} \cdot \frac{(10-20)^2}{25})}{\frac{1}{\sqrt{2\pi \cdot 5}} \cdot \exp(-\frac{1}{2} \cdot \frac{(10-20)^2}{25}) + \frac{1}{\sqrt{2\pi \cdot 10}} \cdot \exp(-\frac{1}{2} \cdot \frac{(10-30)^2}{100})} \\
 &= \frac{\frac{1}{\sqrt{2\pi \cdot 5}} \cdot \exp(-2)}{\frac{1}{\sqrt{2\pi \cdot 5}} \cdot \exp(-2) + \frac{1}{\sqrt{2\pi \cdot 10}} \cdot \exp(-2)} \\
 &= \frac{1/5}{1/5 + 1/10} = \frac{2}{3}
 \end{aligned}$$

**Answer:**  $P(H_2|t=10) = \frac{2}{3}$