Question 1

$$k(x,z) = e^{-x^T x/2\sigma^2} \sum_{i=0}^{\infty} \frac{1}{i!}\left(\frac{x^T x'}{\sigma^2}\right)^i e^{-(x')^T x'/2\sigma^2}$$ by Taylor expansion

$$= \left[\sum_{i=0}^{\infty} \frac{1}{\sigma^{2i}\, i!} \sum_{j_1=0}^{i} \sum_{j_i=0}^{j} (x_{j_1} \cdots x_{j_i})(x'_{j_1} \cdots x'_{j_i})\right] e^{-x^T x/2\sigma^2} e^{-(x')^T x'/2\sigma^2}$$

$$= \sum_{i=0}^{\infty} \left[\frac{(x_{j_1} \cdots x_{j_i})}{\sigma^i \sqrt{i!}} e^{\left(-\frac{x^T x}{2\sigma^2}\right)}\right]\left[\frac{(x'_{j_1} \cdots x'_{j_i})}{\sigma^i \sqrt{i!}} e^{\left(-\frac{x'^T x'}{2\sigma^2}\right)}\right]$$

let each element in $\phi(x)$ be $\frac{(x_{j_1} \cdots x_{j_i})}{\sigma^i \sqrt{i!}} e^{\left(-\frac{x^T x}{2\sigma^2}\right)}$ $\forall\ i \in [0, \infty)$

$$k(x,z) = \phi(x)^T \phi(x)$$

so Guassian kernel can be expressed as a inner product of an infinite-dimensional feature space.