

2. (a) For "and" $\bar{w} = \begin{bmatrix} -1.5 \\ 1 \\ 1 \end{bmatrix} \leftarrow w_0$

It's linear separable

x_1	x_2	$\bar{w}x$	$\sigma(\bar{w}x)$	$x_1 \wedge x_2$
0	0	-1.5	0	0
0	1	-0.5	0	0
1	0	-0.5	0	0
1	1	0.5	1	1

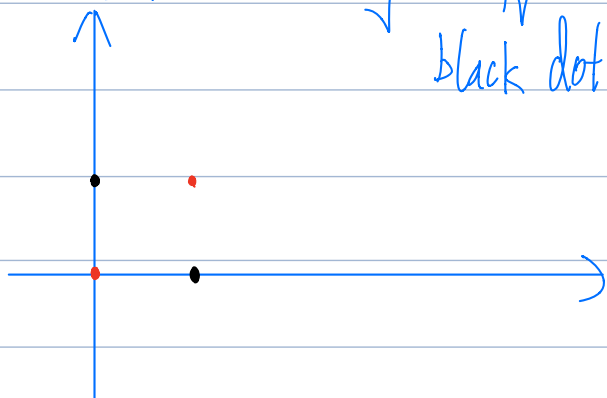
For "or" $\bar{w} = \begin{bmatrix} -0.5 \\ 1 \\ 1 \end{bmatrix} \leftarrow w_0$

It's linear separable

x_1	x_2	$\bar{w}x$	$\sigma(\bar{w}x)$	$x_1 \vee x_2$
0	0	-0.5	0	0
0	1	0.5	1	1
1	0	0.5	1	1
1	1	1.5	1	1

For "exclusive-or", it's not linear separable.

We can see this by the figure shown below, red dot represents class 1
black dot represents class 0



Define $\phi(x_1, x_2) = -x_1 x_2 + 0.4x_1 + 0.4x_2$, $\bar{w} = \begin{bmatrix} -0.1 \\ 1 \\ 1 \end{bmatrix} \leftarrow w_0$

x_1	x_2	$\phi(x_1, x_2)$	$\bar{w}\phi(x)$
0	0	0	-0.1
0	1	0.4	0.3
1	0	0.4	0.3
1	1	-0.2	-0.3

So if we use classifier $\begin{cases} y < 0 \\ y \geq 0 \end{cases}$ $\begin{matrix} \text{class} \\ 0 \\ 1 \end{matrix}$

we get

x_1	x_2	class
0	0	0
0	1	1
1	0	1
1	1	0

which is the same as the result of exclusive-or

For "iff", we can use the same $\phi(x_1, x_2)$ as "exclusive-or",
we define $w = \begin{bmatrix} 0.1 \\ -1 \end{bmatrix}$ same classifier $\begin{cases} y < 0 \\ y \geq 0 \end{cases}$ $\begin{matrix} \text{class} \\ 0 \\ 1 \end{matrix}$

so

x_1	x_2	$\bar{w} \phi(x_1, x_2)$	After classification
0	0	0.1	0
0	1	-0.3	1
1	0	-0.3	1
1	1	0.3	0

which is the same^{as} result of "iff"