MECH 474

Linear Single Inverted Pendulum

Preliminary report

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# INTRODUCTION

## 1.1 Objective

The objective of the Mechatronic project is to understand how to efficiently stabilize a static system becoming unstable when inverting a simple pendulum from rest. The mathematical system will be integrated in the control system that will first invert a pendulum upwards by moving horizontally a cart with a pendulum. A DC-motor, installed on the cart, represents the force to the pendulum driving on a pinion as observed on the mathematical model on page 9 .

The inverted pendulum is a classical and fundamental Control Systems model. The challenge is to balance a pendulum vertically upwards from rest, using a mobile cart that can move horizontally in two directions left or right.

The inverted pendulum is the best model to learn as control system engineers because it is the prototype for many types of real life systems such as motion control systems, altitude control of aircrafts or satellites, landing gears, etc. The different systems that the inverted pendulum exemplifies can be categorized into three different criterias:

1. The types of bases that actuates the pendulum. It can either have a linear motion or a rotary on a horizontal plane. This is the Rotary Inverted System by Quanser systems[1], as shown in Figure 1.1.1 and Figure 1.1.2 [1].
2. The number of links of the mechanism; each link represents a degree of freedom of the system. The more links a system has the more challenging the solution will be, as seen on Figure 1.1.3
3. There are two types of mass distribution along the pendulum rod. The links can be a homogenous rod and have the mass at the center of gravity or the second option is to have a massless rod and have a mass at the end of the rod.

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| Figure 1.1.1: Linear base with pendulum |

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| Figure 1.1.2: Rotary base with pendulum |

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| Figure 1.1.3: Number of degrees of freedom equates the number of links |

## 1.2 Contributions

The Project consisted of understanding how the lab equipment functions, understanding the scope of the problem, deriving a mathematical model for our system, performing tests, understanding the results and writing a report on our progress. While every member of our team dedicated time to understanding the equipment and the scope of the project, the division of each individual’s contributions are outlined below.

Christopher- Derivation of mathematical models, Preliminary coding and testing, Report

Irwin- Derivation of Mathematical model, testing and Report

Safrine- Preliminary testing and Report

## 1.3 Literature Review

Nowadays, The study of control systems is essential in order to design any dynamic system. By means of Control systems, Mechatronics focuses on governing the dynamic mechanical system with the electrical system to make sure that they perform in an ordained manner, accurately, efficiently and fast no matter the external environment.

This project is a complete analysis of a physical double inverted pendulum. The first step to understand the system is to derive the mathematical model of the entire system by utilizing the classical field theory such as Lagrange theory and Newton’s law of motion in a formal framework. The poles will then be determined for the double inverted pendulum and the stability of the system will be determined. This project will be completed using MATLAB and SIMULINK. The method we will be using to complete the task will be the LQR method, Linear Quadratic Regulator, to enhance the poles position with the combination of a Kalman filter for a complete LQG system, Linear Quadratic Gaussian system. The system can only be finalized when the control matrices and the observation matrices have been found. The challenge will be to attain the desired outputs for a double inverted pendulum.

## 1.4 Description of the Linear inverted Pendulum

The system, shown in Figure 1.1.4, used for this project is made by QUANSER, “a global leader in the design and manufacture of refined products”[1] for engineering lab equipment. The system specifications can be examined in Appendix I on page 15 .

THe Quanser Double Inverted Pendulum has the following components:

|  |  |
| --- | --- |
| ID No. | Description |
| 1 | IP02 |
| 2 | Stainless Steel Shaft |
| 3 | Rack |
| 4 | Cart Position Pinion |
| 5 | Cart Motor Pinion |
| 6 | Cart Motor Pinion Shaft |
| 7 | Pendulum Axis |
| 8 | IP02 Cart Encoder |
| 9 | IP02 Pendulum Encoder |
| 10 | IP02 Cart Encoder Connector |
| 11 | IP02 Pendulum Encoder Connector |

|  |  |
| --- | --- |
| 12 | Motor Connector |
| 13 | DC Motor |
| 14 | Planetary Gearbox |
| 15 | Linear Bearing |
| 16 | Pendulum Socket |
| 17 | IP02 Weight |
| 21 | S1 and S2 Connector |
| 22 | Rack End PLate |
| 23 | Rack Set SCrew |
| 24 | Track Discontinuity |

Table 1.1: Components nomenclature of the Quanser Double Inverted Pendulum [1]

Hardware and Set-up and Specifications:

* A rack (3), as shown in figure 2.1, is mounted on a test bench[1].
* On the rack, there is an IP02 (1) Cart that can travel on the rack(3) guided by a stainless steel rod (2) via linear bearing(15), as seen in Figure 1.1.5[3]. The pendulum is attached in the front of the IP02 cart which has a quadrature incremental encoder attached to the shaft where the pendulum is secured.

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| Figure 1.1.4: Quanser Double inverted Pendulum [1] |

* A DC motor simulates the force on the system through a driver gear(4) travels on the rack(3). The position of the cart(1) can be monitored by the encoder(8) next to the DC motor on the cart(1). The planetary gearbox(14) attached to the DC Motor(13) reduces the angular velocity and the horizontal velocity of the cart(1) and produces a higher torque at the pinion.

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| Figure 1.1.5: Quanser System, Linear servo base unit with inverted pendulum [3] |

On the IP02 Cart, the massless Pendulum is attached to the socket(16), as seen on the next figure. THe shaft connected to the socket (16) is attached to the encoder (9) that keeps track of the pendulum with respect to its initial position.

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| --- |
| Figure 1.1.6: IP02 Cart Isometric view [3] |

The DC Motor (13) used from Quanser is a Micro Mo Coreless(2338S006) with a high-efficiency and low inductance motor. It is driven by The Quanser Power Module. To protect the system, there are two Rack end plates (22) to hold the IP02 Cart on the rack. There is also a track discontinuity (24) as seen on Figure 1.1.5.

The I/O Board

The Quanser Board, as shown in the following figure, is responsible for the communication between the MATLAB/SIMULINK software and the hardware. It gives power to the DC Motor, reads the values from the encoders and transfers the data to the software. We will be using the Q4 Quanser board module with all the necessary inputs and outputs for our project. The Q4 specifications consists of the following characteristics[3]:

1. 4 x 14-bit analog inputs
2. 4x 12-bit D/A voltage outputs
3. 4 quadrature encoder inputs
4. 16 programmable digital I/O channels
5. Simultaneous sampling of both analog, digital and encoder sections
6. 2x 32-bit dedicated counter/timers, including watchdog functionality
7. 4x 24-bit reconfigurable encoder counter/timers
8. 2x on-board PWM outputs
9. 32-bit, 33 MHz PCI bus interface
10. Supports Quanser real-time control software QuarC

|  |
| --- |
| Figure1.1.7: Q4 Quanser Input/Output Board [3] |

The Power Module

The power module from Quanser, as seen on Figure 1.1.8, for this project is a UPM2405 with the following characteristics [3]:

1. A regulated dual output DC power supply set
2. A linear Power Operational Amplifier which can supply a +12v, Gnd and -12V.

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| Figure 1.1.8 : UPM2405 Quanser Power Module [3] |

# MODELLING OF A SINGLE INVERTED PENDULUM ON A CART

## 2.1 MODEL OF THE SYSTEM

The goal of this Project is to balance a pendulum to an upward position from a system that is inherently at rest by moving a cart in a horizontal manner, moving it right and left. The Model can be seen on figure 2.1.1.

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| Figure 2.1.1: Free body diagram of the Linear Single Inverted Pendulum |

### Energy Equations

Kinetic Energy

Where:

𝑇𝑐 is the kinetic energy of the cart

is the mass of the cart

is the position of the cart

𝑇𝑝 is the kinetic energy of the pendulum

𝑚𝑝 is the mass of the pendulum

is the distance on the rod from the pivot to the center of gravity

and are the positions of the pendulum’s centroid

is the angular position of the pendulum

The total kinetic energy is

Which yields

Potential Energy

Where: is the potential energy of the system

is the mass of the pendulum

is the gravitational acceleration

is the distance on the rod from the pivot to the center of gravity

is the position of the pendulum’s centroid

Lagrangian

The Euler-Lagrange equation is another method to obtain the equation of motion of a mechanical system. With the use of this method, we can get the same equation as with Newton’s second law. The interest of using the Euler-Lagrange equation is that you can obtain the equations of motion of a complicated system using the energy of the system. It is easier to compute the energy of a system compared to a force analysis. The Lagrangian of a system can be expressed by 𝐿 = 𝑇 − V

Therefore

  =





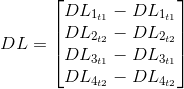




Neglecting friction,



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The value for the motor torque can be defined as the following, where:

R is the resistance of the motor = 2.6 Ohm;

Kt is the motor torque constant = .00767 N\*m/A;

Km is the back-emf constant = .00767 V\*s/rad;

Kg is the gear ratio of the motor = 3.71;

rp is the radius of the motor pinion = .00635 m;

d(q1)/dt is the instantaneous velocity of the cart; and

V is the instantaneous voltage input sent to the motor

As the encoder has 4096 counts per revolution, the values for pendulum angle (q2) can be calculated by multiplying by pi/2048 and the values for cart displacement (q1) in meters can be calculated by multiplying by 0.00002275.

## 2.2 PARAMETER IDENTIFICATION

Using the following matlab code, it will be possible to find the parameters (theta):

dL1 = 0.5\*(dq1.^2);

dL2 = dq1.\*dq2.\*cos(q2);

dL3 = 0.5\*(dq2.^2);

dL4 = 1 - cos(q2);

tau = (Kt/R)\*(V - dq1\*Kg\*Km/rp);

SQ3V750mHz\_taudq1 = tau.\*dq1;

SQ3V750mHz\_Fdq1 = SQ3V750mHz\_taudq1.\*(1/rp)

for i=1:10:(length(DL1)-1),

DL(i,1)=(DL1(i+1)-DL1(i));

DL(i,2)=(DL2(i+1)-DL2(i));

DL(i,3)=(DL3(i+1)-DL3(i));

DL(i,4)=(DL4(i+1)-DL4(i));

Itq(i,1)=trapz(t(1,1),(SQ3V750mHz\_Fdq1(1:i+1,1)));

end

theta=lsqnonneg(DL,Itq)

zero=DL\*theta-Itq

The output for theta is [ 1.3594 ]  
 [ 0.0889 ]  
 [ 0. ]  
 [ 0.5014 ]

By using 10 values to average out the noise:

for i=1:10:(length(DL1)-10),

DL(i,1)=(DL1(i+10)-DL1(i));

DL(i,2)=(DL2(i+10)-DL2(i));

DL(i,3)=(DL3(i+10)-DL3(i));

DL(i,4)=(DL4(i+10)-DL4(i));

Itq(i,1)=trapz(t(1,1),(SQ3V750mHz\_Fdq1(1:i+10,1)));

end

theta=lsqnonneg(DL,Itq)

zero=DL\*theta-Itq

The new output for theta is [ 1.3740 ]  
 [ 0.0889 ]  
 [ 0. ]  
 [ 0.499 ]

All parameters should have positive non-zero values, which means the solution is incorrect, this could be due to the fact that theta2 and theta4 are dependent on each other through the constant g for gravity, however, the zero values acquired are for theta1 and theta3.

Using an incremental method for summing the integral at each integer, the trapz() function can be modified to use :

for i=1:10:(length(DL1)-10),

DL(i,1)=(DL1(i+10)-DL1(1));

DL(i,2)=(DL2(i+10)-DL2(1));

DL(i,3)=(DL3(i+10)-DL3(1));

DL(i,4)=(DL4(i+10)-DL4(1));

Itq(i,1)=trapz(t(1,1:i+10),(SQ3V750mHz\_Fdq1(1:i+10,1)));

end

theta=lsqnonneg(DL,Itq)

The new output for theta is [ 2.1088 ]  
 [ 0.01666 ]  
 [ 0.0058 ]  
 [ 0.4329 ]

## 

## Parameter Verification

When remultiplying the DL matrix with the theta values, it would be expected to get a result equivalent to the Itq matrix. However when comparing both values, there is a margin of error varying from 0-30%, which is quite substantial.

for i=1:(length(DL1)-10),

zero(i)=DL(i,1)\*theta(1)+DL(i,2)\*theta(2)+DL(i,3)\*theta(3)+DL(i,4)\*theta(4)-Itq(i);

end

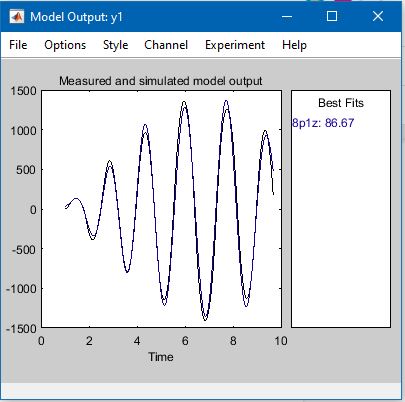
Since the values had a great deal of error, an attempt to use the built-in matlab function ident() with different inputs to determine a transfer function for the system from voltage input to pendulum angle output.  
  


Figure 2.2.1 Matlab Ident Transfer Function with 8 poles and 1 zero

As it is possible to determine visually the output from the transfer function simulation is very similar to the measured experimental output. The percentage precision is much better than the earlier parameter estimation, but will require some refining and testing in order to model and control the inverted pendulum vertically stable.

# DISCUSSION

## Source of Errors during preliminary design

1. The mathematical model is designed assuming friction is negligible. While this might work in an ideal scenario, experimentally, the translational motion of the cart may be affected by friction substantially enough to hinder us when using the energy equation comparing the changes in work put into the system to the changes in kinetic and potential energy of the system
2. The calculation for torque(tau) is derived from the motor system dynamics and was translated in order to find linear translation force to describe the work input energy into the system. It is assumed that the coefficients for the calculation are accurate, however it is possible that the resistance of the motor may vary with normal wear.
3. The code that was able to yield 4 potential parameters uses an integral of summation of differentials instead of simply an integral of differentials. In other words, the timestep between each iteration when calculating values for Itq increases by 0.002s and uses DL1(i+1)-DL1(1) instead of having a constant timestep of 0.002s and using DL1(i+1)-DL1(i).

# CONCLUSION

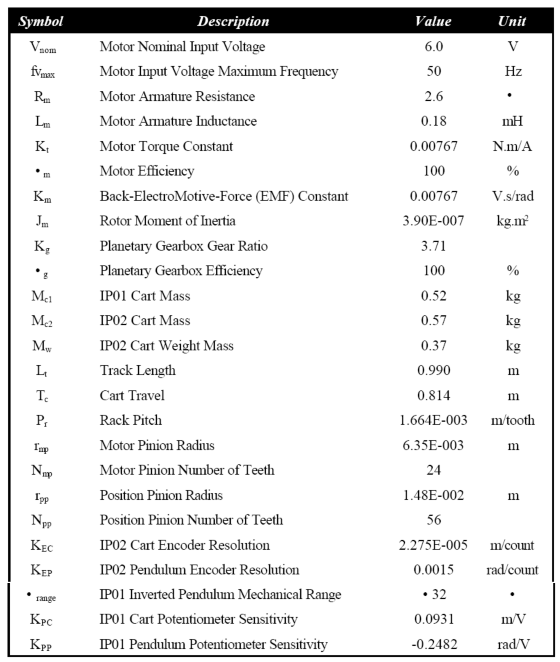
In conclusion, many sources of error have been identified thus far. Our next steps are to continue working on the code and testing while learning from our calculation errors in order to optimize our system and finally attain the goal of the project. We will also try to model the system using simply the dynamic equations of motion as they may be less sensitive to friction losses than the energy equation method.

### 

### REFERENCE

1. <https://www.quanser.com/products/>
2. <https://www.semanticscholar.org/paper/Swing-up-and-Stabilizing-Control-of-Classical-and-Sarnovsk%C3%BD/762a0baaa96492225b5131e2773d15906d776450>
3. C. Su, T.Wen, G. Huard, MECH 474 Project Manual, Montreal, Quebec: Concordia University Department of Mechanical and Industrial Engineering , 2015

### APPENDIX I : IP02 SYSTEM SPECIFICATIONS [3]



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