Competitive Financial Intermediaries in the Market for Student Loans

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Abstract

What are the implications of replacing the federal student loan system with competitive intermediaries that price loans based on the probability of default? I study this question using a quantitative, overlapping generations equilibrium model wherein households mak a costly education decision with access to student loans. To measure the effectiveness of federal student loans, I remove the federal program and allow a competitive intermediary to replace it. The intermediary offers long-term loans where the price depends on the probability of default throughout the entire duration of the loan. I show there are significant losses to welfare and over a 24% decrease in the college educated population with competitive intermediaries. However, the intermediaries facilitate an increase in aggregate production. This increase is largely driven by a more efficient sorting of highly productive individuals to college education.

1. Introduction

The number of workers with a college education and the allocation of individuals to college based on their productivity levels plays a key role in determining the aggregate labor productivity in an economy. This coupled with life-cycle earnings differences (and in turn savings differences) that vary across education levels means that policies affecting the education choices of individuals will significantly impact aggregate output. The market for students loans has become a crucial resource for students seeking an education. Student loans have surpassed credit cards to become the second largest form of consumer credit in the United States (Bricker et al. 2015). There are three unique institutional features that differentiate the market for student loans from other debt markets: borrowers cannot discharge their debts in bankruptcy, all individuals face the same borrowing limits for federal student loans, and all borrowers face the same interest rates.[1] This paper provides an extensive analysis on how a change in the market for students loans to allow debts to be discharged in bankruptcy and financial intermediaries to competitively price debt affect key macroeconomic aggregates such as college completion rates, aggregate output and deadweight loss from bankruptcy.

I develop a quantitative, overlapping generations general equilibrium model to analyze the effects of removing the unique institutional features from the market for student loans. Households face a costly education decision that can be financed with an endowment, work during school and student loan debt. When making an education decision, individuals must weigh the expected increases to lifetime earnings ability against the opportunity cost of attending college and the debt burden of potentially taking on a long-term student loan. Individual workers are subject to idiosyncratic shocks to their productivity levels and incomplete asset markets in the spirit of Huggett (1993) and Aiyagari (1994) which gives rise to a rich distribution of households over labor productivity, asset levels and education.

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[1]The borrowing limit for dependent undergraduate students is set at $31,000 ($23,000 before 2008). The interest rate on student loans is a 2.0 percentage point increase from 10-year treasury bonds.

Individual agents have access to two distinct forms of unsecured debt throughout the life-cycle. Long-term student loan debt is explicitly modeled to capture the institutional features discussed in the previous paragraph. Short-term risky credit is available after schooling is completed, and it can be discharged in bankruptcy a bankruptcy claim that represents a Chapter 7 bankruptcy filing. How households respond to the policies that differentiate these debt markets is key to my analysis.

I use data from the 2007 Survey of Consumer Finances (SCF) and the 1997 National Longitudinal Survey of Youth (NLSY97) to calibrate the model to the US economy in the mid 2000s. The NLSY97 surveys households that were between the ages of 12 and 16 in the year 1996. Most individuals in the survey go through college education in the early 2000s. Previous work by Hubbard et al (1994), Guvenen (2009) and Kim (2016) has shown that earnings risk for workers varies based on education level. I use parameters for earnings risk from Kim (2016) who estimated a wage process from the PSID for workers with and without a college education. I further calibrate the model to match the number of individuals who obtain a college education in 2007 and the fraction of college graduates who have student loan debt. This is done so the model can adequately represent the demand for college education and student debt.

A key aspect of the modeling framework used in this paper is that working households have access to a bankruptcy claim which helps to smooth consumption. When an individual declares bankruptcy, the entire holding of the short-term financial product is discharged. However, the long-term student loan debt payments continue as planned. The punishment for bankruptcy is to be withheld from short-term credit markets for a random number of periods. This punishment is a standard assumption in the literature used to represent the fact that a bankruptcy claim can remain on an individual's credit score for 10 years.[2] When making a bankruptcy decision, individuals with student loan debt must weigh the benefits of smoothing consumption against the cost of being withheld from debt markets in future

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[2]Athreya (2002), Chatterjee et al. (2007) and Livshits et al.(2007) all make similar assumptions.

periods while still making payments.

The main policy experiment run in this paper allows long-term student loan debt to be discharged in the same bankruptcy claim that is prevalent in the market for short-term unsecured consumer credit. More specifically, I assume that a deep pocketed financial intermediary offers a menu of loan prices for the long-term student loans such that zero profits are earned on every individual loan in expectation. This will effectively create endogenous interest rates and borrowing limits for each agent making an education decision. Analyzing this policy allows me to ask the general question: what are the aggregate implications of removing federal policies from the market for student loans and allowing a competitive market system to take its place. I find that a competitive student loan market will lead to a 27% decrease in the college educated population and a 3.4% decrease in aggregate output. However, the average labor productivity of college graduates increases by 17.3% which is driven by a more efficient sorting of highly productive workers to college education. More specifically, there is almost a 13% rise in the college education level of the workers in the top quintile of the idiosyncratic productivity distribution.

A purely competitive market system makes many changes from the current subsidized system. Households respond to changes in bankruptcy policy which allows them to discharge student loan debt, and they respond to the menu of loan prices presented to them by intermediaries. In order to decompose the separate effects of changes in bankruptcy restrictions to changes in the structure of student loan contracts, I solve for an equilibrium where I keep the loan structure constant. In the 'Bankruptcy Only' equilibrium, I allow student loan debt to be discharged in bankruptcy, but students are still able to borrow at subsidized interest rates and borrowing limits.[3] In this market structure, there is over an 18% rise in the number of college educated workers and a 3.0% rise in aggregate output. However, the deadweight loss from bankruptcy is over ten times higher than in the benchmark model.

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[3]This is similar to the policies present in the United States before the 1980s when almost 20% of borrowers declared bankruptcy on student loans

This is driven by a large moral hazard problem. Many moderately productive households take out a student loan and attend college knowing that they can discharge the debt in the first period after school. The moral hazard problem is avoided in a market system with a competitive intermediary.

In addition to quantifying the effects of a competitive student loan market system, this paper makes multiple theoretical contributions to the literature on consumer debt and default. First and foremost, in the policy experiment financial intermediaries offer a menu of loan prices on long-term discount bonds with fixed interest rates, constant debt payments and a terminal date on the loan. To my knowledge, the only previous work to incorporate a competitive pricing system on long-term debt is from Kaplan et al. (2017) and Chatterjee and Eyigungor (2012). However, both of these papers make the assumption that the pricing function of debt can be updated in every period of the loan's existence. My paper adds to this literature by solving for a menu of prices that take into account the entire principal and interest into a single fixed interest rate loan at the origination date. This allows me to quantify what the average price of a competitive fixed interest rate student loan would be, which the model predicts is 10.53%. This paper is also able contribute to the literature by solving for an equilibrium where two distinct financial products can be defaulted on in equilibrium. This allows for the possibility of a household defaulting from a position debt and savings as opposed to just a position of net debt. Although this is not of central importance to answering the current research question, it does provide some useful insights for future work that looks to quantify the effect of default occurring from a realistic position of debt and savings in equilibrium.

1a) Related Literature

This paper is significantly related to a series of literature dating back to Becker (1964) that studies how credit constraints affect human capital accumulation. Work by Keane and Wolpin (2001) uses data largely from the 1980s (NLSY79) to show that family income

played a small role in education decisions. However, using data from the early and mid 2000s (NLSY97), Belley and Lochner (2007) showed that family income is playing an increasingly important role in individuals seeking a higher education. This has led to a recent increase in the study of credit constraints on education attainment from papers such as Lochner and Monge-Naranjo (2011) and Hai and Heckman (2017). The most related work from this literature to the current paper is by Abbott et al. (2018). This paper studies how endogenous parental transfers and education policies feed back into aggregate outcomes of the economy. My work adds to this literature by quantifying the institutional features of the market for student loans by comparing the economy to one with a competitive market based system.

This paper is also related to a series of literature that studies consumer debt and bankruptcy that occur endogenously in equilibrium. Seminal papers in this literature include Chatterjee et al. (2007), Livshits et al. (2007), and Athreya (2002). Recently, Chatterjee and Eyigungor (2012) and Kaplan et al. (2017) have extended this literature to include default pricing functions for long-term debt. The former studies the market for long-term debt in sovereign debt markets, and the latter studies long-term mortgages over the business cycle. I am able to contribute to this literature by extending endogenous default equilibrium models to study the market for student loans.

There is a growing literature that studies student loan debt and default decisions. Notable works in this literature include Ionescu (2009), Chatterjee and Ionescu (2012), and Ionescu and Simpson (2016). Chatterjee and Ionescu (2012) studies how college dropout risk feeds back into default decisions on student loans. However, it is important to note that default in all the papers listed refers to a reorganization of debt with delayed repayment, not something resembling a chapter 7 bankruptcy filing. The most related work from this literature to the current project is a paper by Ionescu (2011) which studies the difference between liquidation and reorganization of student debts. This paper is meant to quantify a change in policy that occurred in the early 1980s. Although this literature is largely focused on reorganization of student loan debts, it is still significantly related to the current paper.

The rest of this paper is organized as follows. Section 2 describes the benchmark model where student loan debt is not dis-chargeable in bankruptcy. Section 3 details the techniques used to choose parameters for the model economy, and outlines how well the model matches important data moments. Section 4 outlines the main policy experiment done in this paper and shows results. Section 5 describes the bankruptcy only model and provides the results to this sensitivity analysis. Finally, section 6 concludes. A description of the computational algorithm and solution method used in this paper is described in the appendix.

2. The Benchmark Model

The model used in this paper contains households, financial intermediaries, a representative firm and a government all operating in equilibrium. The household problem contains five main characteristics: A costly education decision, two distinct forms of debt, a consumer bankruptcy claim, a life-cycle and incomplete asset markets. Below, I will outline the household problem followed by the problems of the representative firm, financial intermediary and government. This section concludes with a description of the equilibrium concept used in this paper.

2.a) Demographics

Every period a cohort of new households, ψ_1 is born. Let ζ_j be the probability of a household surviving from age j to age $j + 1$. Therefore, the cohort of agents that exist in a given generation of the model is equal to $\psi_j = \prod_{t=1}^{j} \zeta_t \psi_1$. I assume that households will retire with certainty at age n_R, and will live to a maximum age of n_j. I also assume there is a continuum of households with mass equal to one, or $\sum_{t=1}^{n_j} \psi_t = 1$.

2.b) Preferences

At age 1 agents maximize their expected discounted lifetime utility from consumption.

$$U = E\Big[\sum_{j=1}^{n_j} \beta^j \Big(\prod_{t=1}^{j} \zeta_t \Big) u(c_j) \Big], \qquad where \quad u(c_j) = \frac{c_j^{1-\sigma}}{1-\sigma}$$

Where $c_j \geq 0$ is consumption at age j. Individual agents discount future utility at rate β, and σ represents the coefficient of relative risk aversion.

2.c) Earnings Process

Households are hired by firms and paid the marginal product of labor for each unit of productivity. An individual's productivity ϵ is composed of three components: an age-specific component, a persistent idiosyncratic shock and a transitory shock. All three of these components are education specific. For notation purposes, let $h \in \{c, l\}$ represent the education levels college and no college respectively. Let $\epsilon_{j,h}$ describe the age-specific component. The persistent component $\epsilon_{p,h}$ follows a persistent AR(1) process with innovation η_h. The persistence and variance of the process are represented by ρ_h and $\sigma_{\eta,h}^2$. I assume the transition probabilities between shocks are defined by $\pi_{mn} = Pr(\epsilon' = \epsilon_n | \epsilon = \epsilon_m)$.

$$log(\epsilon) = \epsilon_{j,h} + \epsilon_{p,h} + \epsilon_{t,h} \quad where \quad log(\epsilon'_{p,h}) = \rho_h log(\epsilon_{p,h}) + \eta'_h \quad \sim \quad N(0, \sigma_{\eta,h}^2)$$

2.d) The Household Problem

The state space of an individual household consists of her labor productivity ϵ, short-term asset level a, age j, student debt level b, education level h and credit status s. Figure 1 outlines the entire timing of household decisions made throughout the life-cycle. Agents are born and immediately make an education decision in period 1. While making an education decision, individuals have an endowment a_0 and have access to students loans, but they do not have access to short-term unsecured consumer credit. If an individual chooses the college path of life, they spend four years in school working at low wages and getting an education. Student loan debt is long term and it has an official end date which occurs in period n_T. Once households are in the working ages of life, they have access to risky short-term credit with endogenous interest rates and borrowing limits set by a financial intermediary.

2.e) The Education Ages

Figure 1: Timing of Household Decisions
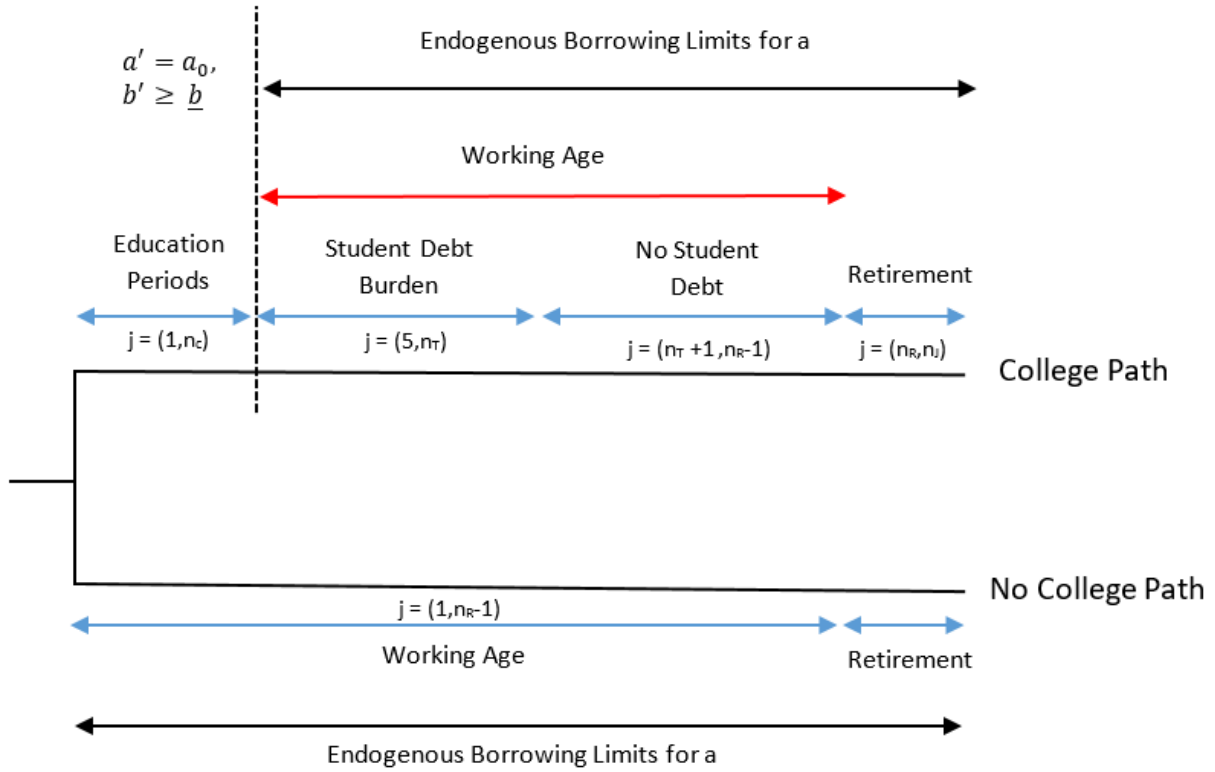
A household is born in period 1 with an endowment a_0, no student loan debt, good credit status s_g and an education opportunity. The endowment is interpreted as parental transfers to education. At age 1 all households make the discrete choice between following the college path through life or the no college path. Equation (1) outlines the education decision where V^c represents attending college and V^l represents following the no college path. I let e represent the subsequent education decision rule.

$$V_1(\epsilon, a_0, b, s_g) = max \left[V_1^c(\epsilon, a_0, b, s_g), V_j^l(\epsilon, a_0, b, s_g) \right] \tag{1}$$

$$\text{where} \quad e_1(\epsilon, a_0, b, s_g) = \begin{cases} 1 & if \quad V_1^c(\epsilon, a_0, b, s_g) \geq V_1^l(\epsilon, a_0, b, s_g) \\ 0 & otherwise \end{cases}$$

While in school, an agent solves the dynamic programming problem outlined in equation (2). Individuals consume and make a decision about how much student loan debt they wish

to receive. The parental transfer is drawn from a distribution Q which is partly estimated from the data and partly calibrated to match model moments. Q is a poisson process where with probability π_R an individual will receive an endowment that allows her to attend college without using student loan debt. With probability $(1 - \pi_R)$ the student will draw an endowment from a normal distribution with mean μ_a and variance σ_a^2 which are estimated from the data on parental transfers. I will use π_R to calibrate the number to individuals who use student loan debt to attend college. The parameter χ represents direct utility from receiving an education. This parameter is the same for all individuals, and it will be calibrated to match the share of workers with a college education.

$$V_1^c(\epsilon, a_0, b, s_g) = \max_{\{c,b'\}} u(c) + \chi + \beta V_2^c(\epsilon, a_0, b', s_g) \tag{2}$$

$$\text{s.t.} \quad c + \phi_c + \frac{q^b b'}{n_C} = a_0 + (1 - \tau)w\epsilon^*$$

$$\text{and } b' \geq \underline{b}, \quad and \quad a_0 \in Q$$

To simplify the computation of the model, I assume that individuals only make a student debt decision in period 1. Then, they receive a fraction of that loan every period they are in school. Individual agents do not experience any uncertainty while in school. Therefore, the future idiosyncratic shock level and the future parental transfer is the same in every period of college as it is in period 1. As in Abbott et al. (2018), I assume that there is no uncertainty while in school to abstract from modeling college dropouts. Since I am mainly worried about aggregate variables, college dropouts should not play a significant role in the results. In periods 2 through 4 there is no uncertainty and no debt decisions, so agents in these periods just consume based on the budget constraint outlined in equation (2). Uncertainty about productivity levels begins in period 5. Therefore, in the last period of college the expected future value function contains uncertainty and is represented as $E\left[V_5^c(\epsilon', 0, b', s_g)|\epsilon\right]$. Note, I assume that all individuals leave college in good credit standing. Any individual who chooses to not go to school, enters the working ages in model period 1.

2.f) The Working Ages

A working age household that enters period j with good credit status s_g has the option to declare bankruptcy on their short-term financial product. Agents solve the discrete choice problem outlined in equation (3) where d is the subsequent bankruptcy decision rule. Households choose between the value of repaying their debt represented by $V_j^{h,r}$ or immediately declaring bankruptcy $V_j^{h,b}$. I assume that debt and bankruptcy are only options for individuals that enter a period with good credit status.

$$V_j^h(\epsilon, a, b, s_g) = \max \left[V_j^{h,r}(\epsilon, a, b, s_g), V_j^{h,b}(\epsilon, a, b, s_g) \right] \tag{3}$$

$$\text{where} \quad d_j^h(\epsilon, a, b, s_g) = \begin{cases} 0 & if \quad V_j^{h,r}(\epsilon, a, b, s_g) \geq V_j^{h,b}(\epsilon, a, b, s_g) \\ 1 & otherwise \end{cases}$$

After making a discrete bankruptcy decision, the household makes a consumption-savings decision. The dynamic programming problem solved by a working age household that repays debts is outlined in equation (4). Given a current a current asset level, a student debt level and an idiosyncratic labor productivity value, the individual decides how much to save for the next period. The financial intermediary offers the households a menu of loan prices, one for each possible choice of a'. The pricing function q^a reflects the households probability of defaulting and will be discussed in more detail in section 2.i. Given that the agent has not defaulted in period j, she will enter period $j+1$ with good credit status.

Student loan debt is modeled to be long-term meaning that it takes more than one model period to repay. It is common in the literature to model this by requiring that the borrower repays a fraction λ of the debt each period. Modeling long-term debt in this fashion implies that there are geometrically decaying loan payments with no terminal date. To avoid this simplification, I assume that the fraction of debt due each period increases as the borrower approaches the terminal date of the loan. Therefore, if a borrower has four more payments to make, the fraction of debt due in the current period is $\frac{1}{4}$. In the terminal date of loan

repayments, $\lambda = 1$ and the borrower finishes making payments on the loan. I assume that debt payments evolve exogenously to avoid having the household make an additional decision during the working periods of life. Essentially, student loan contracts are a long-term discount bond that takes the entire principal and interest into account at the initiation date, and is then repaid in increments until the terminal date.

$$V_j^{h,r}(\epsilon, a, b, s_g) = \max_{\{c,a'\}} \ u(c) + \beta\zeta_j E\left[V_{j+1}^h(\epsilon', a', b', s_g)|\epsilon\right] \tag{4}$$

$$\text{s.t.} \quad c + q_j^h(\epsilon, a', b')a' = \lambda_j b + a + (1-\tau)w\epsilon$$

$$\text{and} \quad b' = [1-\lambda_j]b$$

When a household chooses bankruptcy, the entire holdings of the short-term unsecured consumer credit are discharged. However, the repayments of student loans are not affected. The value function of an individual that declares bankruptcy is expressed in equation (5). No decisions are made in this period to reflect the feature that individuals cannot default and save simultaneously. The agent in the model will enter next period with bad credit s_b. Student loan debt cannot be discharged in bankruptcy, but I assume that a debt payment will never exceed a fraction ξ of the household's earnings. This is to reflect the fact that according to the US Department of Education, if a borrower experiences 'undue hardship' in making debt payments, a fraction of the loan can be forgiven. The undue hardship assumption is rarely used, and I choose a value of ξ so that this almost never occurs in equilibrium.

$$V_j^{h,b}(\epsilon, a, b, s_g) = u(c) + \beta\zeta_j E\left[V_{j+1}^h(\epsilon', 0, b', s_b)|\epsilon\right] \tag{5}$$

$$\text{s.t.} \quad c = \lambda_j b + (1-\tau)\epsilon w$$

$$\text{and} \quad b' = [1-\lambda_j]\,b$$

A household that enters period j with bad credit s_b will solve the dynamic programming problem described in equation (6). The main punishment from bankruptcy is that a household cannot borrow while having bad credit. There is an exogenous probability θ that the

bad credit status will be removed before the next model period. The household continues making student debt payments and can save at the discount price offered by the financial intermediary. The intermediary still offers a menu of loan prices, but since the household cannot borrow, the discount price will always be equal to risk-free price accounting for survival probabilities. This is an equilibrium outcome, not an assumption of the model.

$$V_j^h(\epsilon, a, b, s_b) = \max_{\{c,a'\}} \ u(c) + \beta\zeta_j E\Big[W_{j+1}^h(\epsilon', a', b', s')|\epsilon\Big] \tag{6}$$

$$\text{s.t.} \quad c + q_j^h(\epsilon, a', b')a' = \lambda_j b + a + (1 - \tau)w\epsilon, \quad a' \geq 0$$

$$\text{and} \quad b' = [1 - \lambda_j]\,b$$

$$W_{j+1}^h(\epsilon', a', b', s') = \theta V_{j+1}^h(\epsilon', a', b', s_g) + (1 - \theta)V_{j+1}^h(\epsilon', a', b', s_b)$$

2.g) Retirement

Retired households still have access to the market for short-term credit which includes the option to default. The problems describing a retired household are very similar to those of a household during the working ages. However, retired individuals receive a transfer from the government τ_{ss} in place of earnings. All student loans are repaid before an individual reaches retirement.

2.h) The Firm's Problem

The representative firm hires labor and rents capital in order to produce the consumption good in the model economy. I assume that the firm uses a Cobb-Douglas aggregate production function in which α is the capital share of production, and δ_k is the depreciation rate for capital. The variables K and L are aggregate capital and aggregate labor respectively. Aggregate labor is taking the sum of all individual labor productivity levels of working age households. To simplify notation I let X represent the entire state space of the economy.

$$Y = K^\alpha L^{1-\alpha}$$

$$K = \int_X g(dX)\,\mu_j(dX) \tag{7}$$

$$L = \int_X \epsilon\,\mu_j(dX) \tag{8}$$

2.i) Financial Intermediaries

Financial intermediaries are deep pocketed and own the physical capital in the model economy. The intermediaries offer lending and savings to the households in the form of the financial product a. Let $q_j^h(\epsilon, a', b')$ be the menu of prices for a' such that zero profits are earned in expectation on every transaction. Equation (9) describes the menu of loan prices that will in fact earn zero profits in expectation. The probability of a household defaulting on a' is described by $\delta_{j+1}^h(\epsilon, a', b')$. The function δ_j is taking into account the bankruptcy decision rules for all of the possible realization of ϵ' in period $j+1$. When $a' \geq 0$ there is no incentive to default and q^h will be equal to the risk-free discount price. I assume that intermediaries transform all of the short-term financial product into capital. The aggregate capital described in the firm problem is rented to firms from intermediaries at the risk-free rate r.

$$q_j^h(\epsilon, a', b') = \frac{\zeta_j}{1+r} E\left[1 - \delta_{j+1}^h(\epsilon', a', b')|\epsilon\right] \tag{9}$$

$$\text{where} \quad \delta_{j+1}^h(\epsilon, a', b') = \sum_{n=1}^{N(\epsilon)} \pi_{mn} d_{j+1}^h(\epsilon_n, a', b', s_g)$$

2.j) The Government

The government serves two main purposes in the model economy: execute the fiscal policy affecting households and maintain the market for student loans. The market for student loans is explicitly modeled to represent the key institutional features of the federal loans. Loans are long-term and last for $n_T - n_C$ periods. Equation (10) describes the discount prices

for student loans. It is important to note that the discount price for a long-term loan is not calculated the same way as it is for a short-term loan. The price must account for the discount rate of every payment received on the loan. Let ν represent the two percentage point increase in annual interest rate that is charged on loan contracts. This is one of the features of the market for student loans.

$$q_b = \frac{1}{(n_T - n_C)} \sum_{i=1}^{(n_T - n_C)} \left(\frac{1}{1 + r + \nu} \right)^i \tag{10}$$

The government also handles the taxes and transfers in the model economy. Assume the government charges an income tax rate of τ. The government also supplies transfers to retired households. The aggregate resource constraint of the government is represented by equation (11). In equation (11), G is the aggregate government expenditures that is left over after handling all of the taxes and transfers in the model economy. The third term in equation (11) is the revenue from all student debt payments and the fourth term is the credit that is supplied to households during the education period. I let l represent the student debt decision rule.

$$G = \sum_{j=1}^{n_R - 1} \int_X w\epsilon\tau\mu(dX) - \sum_{j=n_R}^{n_j} \int_X \tau_{ss}\,\mu(dX) + \sum_{j=5}^{n_T} \int_X \lambda_j b\,\mu(dX) - \sum_{j=1}^{n_C} \int_X l(x)\mu(dX) \tag{11}$$

2.k) Recursive Equilibrium

Households are heterogeneous across six possible state variables. Let $x \in X$ be the individual state of a household at a point in time where $X = E \times A \times J \times B \times H \times S$. I define $\Gamma(X)$ to be the Borel algebra for the set of possible asset levels and idiosyncratic shocks, and μ^h to be a probability measure over the subsets of the state space with education h. Therefore, $\mu^h(\Gamma)$ is the fraction of agents whose individual states lie in the set $\Gamma \in \Gamma(X)$.

I assume that all age 1 households are born with good credit status, low education level

and no student loan debt. These age 1 households also have an initial asset level drawn from the distribution Q and initial labor productivity drawn from the invariant distribution of productivity levels. The transition function for asset levels and idiosyncratic shocks is summarized by the function P which implicitly accounts for decision rule g. The Distribution of households is solved for in every period using the transition function P along with decision rules e and d. Equations (12) and (13) describe the distribution of households during education decisions. Equations (14)-(17) describe the distribution during working ages. The distribution during retirement is not formally stated, but resembles the working ages.

Education Ages

$$\mu^c(\Gamma', s_g) = \int_X e(\Gamma, s_g) \, P(X, \Gamma) \, \mu^h(dX, s_g) \tag{12}$$

$$\mu^l(\Gamma', s_g) = \int_X \left[1 - e(\Gamma, s_g)\right] P(X, \Gamma) \, \mu^h(dX, s_g) \tag{13}$$

Working Ages

$$\mu^h(\Gamma', s_g) = \int_X \left[1 - d^h(\Gamma, s_g)\right] P(X, \Gamma) \, \mu^h(dX, s_g) \tag{14}$$

$$\mu^h(\Gamma', s_b) = \int_X d^h(\Gamma, s_g) \, P(X, \Gamma) \, \mu^h(dX, s_g) \tag{15}$$

$$\mu^h(\Gamma', s_g) = \int_X \theta \, P(X, \Gamma) \, \mu^h(dX, s_b) \tag{16}$$

$$\mu^h(\Gamma', s_b) = \int_X (1 - \theta) \, P(X, \Gamma) \, \mu^h(dX, s_b) \tag{17}$$

A *Stationary Recursive Equilibrium* is a set of functions for value V^h, quantities g, c, l, discrete choices d, e, and prices r, w, q_b, q^h and the distribution μ^h such that:

1. V^h solves equations (1)-(6) and subsequent equations for retired households where func-

tions (c, g, l, d, e) are associated decision rules for consumption, asset level a, student debt level b, default and education level respectively.

- After period 1, l describes the deterministic evolution of student debt.
- After period 1, e describes the constant education level of a household.

2. Prices r and w clear markets from the following first-order conditions to the firm problem.

- $r = \alpha K^{\alpha-1} L^{1-\alpha} - \delta_k$
- $w = (1 - \alpha) K^\alpha L^{-\alpha}$

3. Prices q^h are determined by the financial intermediaries problem in equation (9).

4. Price q_b is exogenously set by the government as in equation (10).

5. Aggregate government expenditures is described by equation (11).

6. The endogenous distribution of households solves equations (12-17).

7. Markets Clear in equations (10), (11) and the aggregate resource constraint is balanced.

a. $C = \int_X c(X) \mu(dX)$
b. $C + \delta_k K + G = K^\alpha L^{1-\alpha}$

3. Calibration and Parameterization

The parameters for the model are chosen in a three-step process. First, common parameters are chosen either because they are commonly used in the literature or because they explicitly represent a feature of the economy I want present in the model. Then, parameters for the income process are taken from estimates from the data. As I will discuss in section 2.b, many of theses parameters are estimated in a different research project. Finally, I calibrate the remaining parameters of the model to match key moments seen in the data.

3.a) Common Parameters

Table 1 outlines the parameters that are chosen to be used in the model. The values for the coefficient of relative risk aversion, capital share of production and capital depreciation are all commonly used across the macroeconomic literature. I chose $\theta = 0.10$ so a bad credit

Standard Parameters

Parameter	Description	Value
σ	Risk-Aversion	2.0
δ_k	Capital Depreciation	0.069
α	Capital Share	0.36
θ	Bad Credit Duration	0.10

Government Parameters

Parameter	Description	Value
τ	Income Tax	0.25
ξ_c	Undue Hardship	0.90
ν	Cost of Monitoring Loan	0.02

Demographic Parameters

Parameter	Description	Value
n_J	Final Model Period	64
n_R	Retirement Period	49
n_T	Final Repayment Period	14
n_C	Final Period of College	4
ζ_j	Survival Probabilities	**

Table 1: Parameterization

status will remain with a household for an average of 10 model periods. This represents the fact that a bankruptcy remains on an individual's credit score for 10 years. With regards to parameters affecting the government, the cost of monitoring is taken from the US Department of Education information on student loans. The undue hardship parameter states that an individual never needs to make a student loan payment that exceeds 90% of her current income. As stated in section 2, this is a stylized way of representing a policy. In equilibrium, less than 0.1% of college graduates end up using the undue hardship assumption.

Demographic parameters control how households move through the life-cycle. A model period is assumed to be 1 year. Agents are born at age 18 in model period 1. College lasts for 4 periods, and college graduates enter the workforce at age 23. Student loans are repaid for 10 years. The duration of student loans is chosen to represent the standard repayment option offered by the US Department of Education. Individuals retire at age 67, and after retirement they begin facing survival probabilities that are estimated directly from the 2004 U.S. Life Tables of the National Center for Health Statistics. The final possible year of life

occurs at age 82.

3.b) The Earnings Process

College Graduates

Parameter	Description	Value
ρ_c	Persistence of Shock	0.976*
$\sigma^2_{\eta,c}$	Variance of Persistent Shock	0.060*
$\sigma^2_{t,c}$	Variance of Transitory Shock	0.067*
γ_c	Annual Growth in Earnings	0.029

No College

Parameter	Description	Value
ρ_l	Persistence of Shock	0.980*
$\sigma^2_{\eta,l}$	Variance of Persistent Shock	0.012*
$\sigma^2_{t,l}$	Variance of Transitory Shock	0.090*
γ_l	Annual Growth in Earnings	0.015

Table 2: Parameters of age-specific earnings process

Table 2 outlines all of the parameters for the earnings process that are used in the model.[4] The parameters for the persistence, variance of the persistent shock and variance of the transitory shocks were estimated from the PSID from 1968-2011 in Kim (2016). The annual growth in earnings was estimated directly from the 2007 SCF. The annual growth in earnings was estimated to represent the age-component of earnings for college graduates and non-college graduates from working ages 22-67 in the data. It is commonly known but important to specify that earnings for college graduates grow at a faster rate than they do for their less-educated counterparts.

3.c) Calibration

The remaining nine parameters of the model were calibrated to match a moment of the data. I chose the discount rate of future consumption so the model economy has a risk-free rate of 3.85%. This is the average interest rate on the 10-year treasury bond from 1997-2017. The value for the utility received from education is calibrated to match the share of

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[4]Parameters with an asterisk were estimated in Kim (2016)

Parameter	Value	Target	Description	Model
β	0.992	$r = 3.85\%$	Risk-free Rate	$r = 3.86\%$
χ	2.20	$\mu(h_c) = 0.34$	College Educated Population	$\mu(h_c) = 0.33$
π_R	0.164	$\mu(b) = 0.65$	College Grads with Debt	$\mu(b) = 0.659$
ϕ_c	0.311	$\frac{\phi_c}{\bar{\epsilon}} = 0.17$	Tuition : Median Earnings	$\frac{\phi_c}{\bar{\epsilon}} = 0.17$
τ_{ss}	0.463	$\frac{\tau_{ss}}{\bar{\epsilon}} = 0.25$	Retirement Transfer : Median Earnings	$\frac{\tau_{ss}}{\bar{\epsilon}} = 0.25$
$\underline{b}$	-1.60	$\frac{q_b \underline{b}}{\bar{\epsilon}} = 0.64$	Borrowing Limit : Median Earnings	$\frac{q_b \underline{b}}{\bar{\epsilon}} = 0.64$
μ_a	0.092	$\frac{\mu_a}{\bar{\epsilon}} = 0.05$	Mean Transfers : Median Earnings	$\frac{\mu_a}{\bar{\epsilon}} = 0.05$
σ_a	0.191	$\frac{\sigma_a}{\bar{\epsilon}} = 0.096$	SD Transfers : Median Earnings	$\frac{\sigma_a}{\bar{\epsilon}} = 0.096$
ϵ^*	0.1774	$\frac{\epsilon^*}{\bar{\epsilon}} = 0.0964$	College Earnings : Median Earnings	$\frac{\epsilon^*}{\bar{\epsilon}} = 0.0964$

Table 3: Calibrated Parameters

workers with a college education. According to the National Center for Education Statistics (NCES) the fraction of individuals who obtain a college education in 2007 was 34%. This accounts for high school graduation rates, college enrollment rates and college completion rates. The model slightly under predicts the fraction of educated workers, but this should be okay because the 2007 SCF estimates that about 32% of the working age population has a college education. The poisson arrival rate of high initial endowments is chosen as 16.4% to generate 65% of college graduates holding student loans.

The remaining parameters were all calibrated to match the ratio of a statistic seen in the data to the median earnings. For example, the cost of attending college is estimated as $9,210 in Abbott et al. (2018). The ratio between the cost of college and median earnings is equal to 0.17 in both the model and the data. The borrowing limit on student loans is chosen to be $35,000 which is the estimated value used in Lochner and Monge-Naranjo (2010) for the same time period. The mean transfers, standard deviation of transfers and college earnings were all estimated directly from the NLSY97 for fulltime students age 18-23.

3.d) Model to Data

In this section, I test how well the model meets the data on moments that were not calibration targets. The model predicts that 0.42% of consumers declare bankruptcy in equilibrium. Without publicly available micro-level data on consumer bankruptcies, it is difficult to know how close this is to the actual data. However, there were 467,248 non-

business chapter 7 bankruptcies filed in 2007. If all of these bankruptcies were committed by working age households, then 0.50% of working age households would declare bankruptcy. Since some of these cases must be from older individuals, 0.50% is an over-estimation. This shows that the model has a believable level of consumer bankruptcies.

I also test the validity of the model by comparing the equilibrium results to data on earnings and networth from the 2007 SCF. Specifically, I look at how closely the model matches the distribution of wealth and earnings. Table 4 outlines the distribution of wealth, and it shows what fraction of each quantile listed is represented by college educated households and young households. I chose the ages 22 to 33 to depict young households because this corresponds to the debt repayment periods in the model. For the model to be a valid tool to analyze results on student debt reform, it must have a believable distribution of wealth and earnings amongst college educated and young working households. Results from table 4 shows that the model does a good job representing the distribution of wealth with one exception. The wealth held by the top 1 percent of households is substatially higher in the data. This is not surprising because the inequality literature has shown us that specific modeling techniques need to be used to match the top 1%. This should not significantly affect results because the top 1% largely makes education decisions without regard to student loan policies.

	Gini	Quintiles					Percentiles		
		Q1	Q2	Q3	Q4	Q5	5-10%	1-5%	1%
SCF	0.81	-0.17	1.16	4.48	11.20	83.32	11.19	26.87	33.40
Model	0.68	0.74	3.81	8.69	18.40	68.36	16.29	24.71	9.10
Share of Wealth - College Educated									
SCF		14.43	18.39	24.69	35.01	59.62	76.65	75.55	83.74
Model		18.35	17.48	20.30	29.98	68.19	85.70	92.99	93.07
Share of Wealth - Ages 22-33									
SCF		36.46	24.34	12.91	6.07	2.30	1.84	2.30	3.44
Model		47.90	21.45	10.10	4.59	1.91	1.30	0.35	0.0

Table 4: The Wealth Partition

Table 5 outlines the earnings partition seen in the model and the 2007 SCF. The model

does a more accurate job representing the distribution of earnings that the distribution of wealth. The earnings held by top 1% is is the furthest off, but the remaining values are all within 4 percentage points. The model also does a relatively good job representing the share of earnings held by college graduates and young households. None of these moments were targeted in the calibration technique, which makes the results in tables 4 and 5 a relative success.

	Gini	Quintiles					Percentiles		
		Q1	Q2	Q3	Q4	Q5	5-10%	1-5%	1%
SCF	0.57	1.05	7.19	12.78	20.42	58.57	10.85	15.48	17.02
Model	0.64	4.11	7.41	11.22	18.25	59.01	13.27	19.22	10.95
Share of Earnings - College Educated									
SCF		14.47	14.62	27.14	40.78	62.11	85.12	78.41	88.71
Model		12.71	13.85	16.21	25.37	58.02	71.10	76.12	76.12
Share of Earnings - Ages 22-33									
SCF		18.47	27.24	21.37	17.78	10.52	10.77	3.67	1.64
Model		28.35	22.37	12.64	11.47	11.69	7.74	10.06	0.0

Table 5: The Earnings Partition

4. The Policy Experiment and Results

In this section, I outline the model used to answer the main policy question raised in this paper: What are the implications of allowing student loan debt to be discharged in bankruptcy and the terms of the loan represent the probability of default? Any aspect that is not mentioned in this section remains the same from the benchmark model. An integral part of this model is a long-term debt pricing function that has endogenous default risk in every period of the loan's existence. This pricing function is outlined in section 4.a and could be of independent interest to the literature on consumer debt and default.

4.a) A Financial Intermediary and a Long-term Debt Pricing Function

When student loan debt is considered to be a risky form of debt, the supply of credit is controlled by a deep-pocketed financial intermediary. Specifically, the intermediary will

offer a menu of loan prices such that zero profits are earned in expectation on every loan. The loan prices that earn zero profits are summarized in equation (18) wherein Ω represents the expected cash inflows from every period in the loan's existence. Equation (18) replaces equation (10) in solving for an equilibrium. This new intermediary will earn zero profits on every loan, so the loan repayments are equal to loans issued in equilibrium. Therefore, the market for student loans will clear as long as equation (18) holds.

$$q_b(\epsilon, a', b')b' - E\left[\left(\frac{1 - \delta(\epsilon, a', b')}{1 + r}\right)\Omega(\epsilon, a', b')\right] = 0 \tag{18}$$

The key to solving the above equation is solving for Ω. To solve for this term, I take advantage of the finite duration of the loan. The value for Ω in the final period of the loan is trivially equal to the final payment made. Beginning in period $j = n_T - 1$, the expected cash flows of the loan can be solved for using $\Omega(x_{nT})$ and the probability of defaulting in period n_T. I then recursively solve for this term backwards towards the initiation date of the loan. Solving for Ω in any given period amounts to solving a problem using the expected probability of default in the future period similar to the problem solved in a short-term debt pricing function.

$$\Omega(x_{n_T}) = \lambda_{n_T} b_{n_T}$$

$$\Omega(x_{n_T-1}) = \lambda_{n_T-1} b_{n_T-1} + E\left[\left(\frac{1}{1+r}\right)\{1 - \delta_{n_T}(\hat{x}_{n_T-1})\}\Omega(\hat{x}_{n_T-1})\right]$$

$$...$$

$$\Omega(x_2) = \lambda_2 b_2 + E\left[\left(\frac{1}{1+r}\right)\{1 - \delta_3(\hat{x}_2)\}\Omega(\hat{x}_2)\right]$$

$$q_b(\hat{x}_1)l(x_1) = E\left[\left(\frac{1}{1+r}\right)\{1 - \delta_2(\hat{x}_1)\}\Omega(\hat{x}_1)\right]$$

1. $x_j = \{\epsilon, a, b\}$

2. $\hat{x}_j = \{\epsilon, g(x_j), l(x_j)\}$

The entire algorithm needed to solve for equation (18) is outlined above where g and l are decision rules for savings and debt repayment respectively.. When applying this method

computationally, all of the terms for $\lambda_j b_j$ can be simplified to $\frac{l(x_1)}{(n_T - n_C)}$ where $l(x_1)$ is the student loan decision rule made during the education decision and $(n_T - n_C)$ is the duration of the student loan. This is possible because the modeling of λ terms implies constant loan payments equal to a fraction of the total loan payments (principal and interest). The government budget constraint is still described by equation (11) except it no longer includes supply of student loans or the payments received from these loans.

4.b) The Household with Dis-chargeable Student Debt

Multiple aspects of the household problem will change when student loan debt is allowed to be discharged in bankruptcy. When making an education decision, households will still make a discrete choice for their education path described by equation (1). However, the problem solved by students currently obtaining an education must take into account the menu of loan prices offered by the government. Equation (19) replaces equation (2) to summarize the decision problem of a household that is receiving an education subject to a menu of loan prices. The borrowing limit for student loans is now endogenously determined by solving equation (18).

$$V_j^c(\epsilon, a, b, s_g) = \max_{\{c, b'\}} u(c) + \beta V_{j+1}^h(\epsilon', a', b', s_g) \tag{19}$$

$$\text{s.t.} \quad c + \phi_c + q_b(\epsilon, a', b')\frac{b'}{n_C} = a_0 + (1 - \tau)w\epsilon^*, \quad and \quad a_0 \sim Q$$

The household still makes a discrete default choice described by equation (3). However, the decision problem of a household declaring bankruptcy is now changed to account for student debt being discharged. Equation (20) replaces equation (5) in solving for an equilibrium. Now, a bankruptcy decision simultaneously discharges the holdings of a and b. The household will enter the future period with bad credit status and no asset positions. An individual agent with bad credit status still solves equation (6), except there will be no student debt holdings because it would have been discharged in a previous period.

$$V_j^{h,b}(\epsilon, a, b, s_g) = u(c) + \beta \zeta_j E\left[V_{j+1}(\epsilon', 0, 0, s_b)|\epsilon\right] \tag{20}$$

$$\text{s.t.} \quad c = (1-\tau)w\epsilon$$

The remaining equations describing the household problem remain the same from the benchmark model. The definition of a *Recursive Stationary Equilibrium* is the same as the benchmark model with equations (18)-(20) replacing equations (10), (2) and (5) respectively.

4.c) Results

Model	$\mu(h_2)$	$\mu(d=1)$	DL_b	K	L	$\overline{L}_c$	Y	$\overline{r}_b$
Benchmark	33.1%	0.42%	$6.6e^{-4}$	14.57	2.21	5.26	4.35	5.86%
Experiment	23.9%	0.51%	$1.7e^{-3}$	14.10	2.13	6.17	4.20	10.53%

Table 6: Results

The main results for the policy experiment in this paper are reported in table 5. When student loan debt has endogenous interest rates and borrowing limits accompanied by a bankruptcy option, the percent of households with a college education decreases by 27.8%. Therefore, a non-trivial number of households are either excluded from borrowing or choose not to attend college with the new loan terms. We also see a 3.4% decrease in the aggregate output from the model economy. With regards to bankruptcy, the fraction of working households declaring bankruptcy increases from 0.42% to 0.51%. This is accompanied by the deadweight loss from consumer bankruptcy nearly doubling. The deadweight loss from bankruptcy increases by a large amount because the average student loan that is in bankruptcy is much larger than the average risky credit loan that is in bankruptcy.

Despite the fraction of college graduates decreasing by over 27%, we see that the average college graduate ($\overline{L}_c$) is now 17.3% more productive in the economy with a competitive student loan market. This occurs because a financial intermediary does a more efficient job of sorting the most productive households to college education. Essentially, a competitive
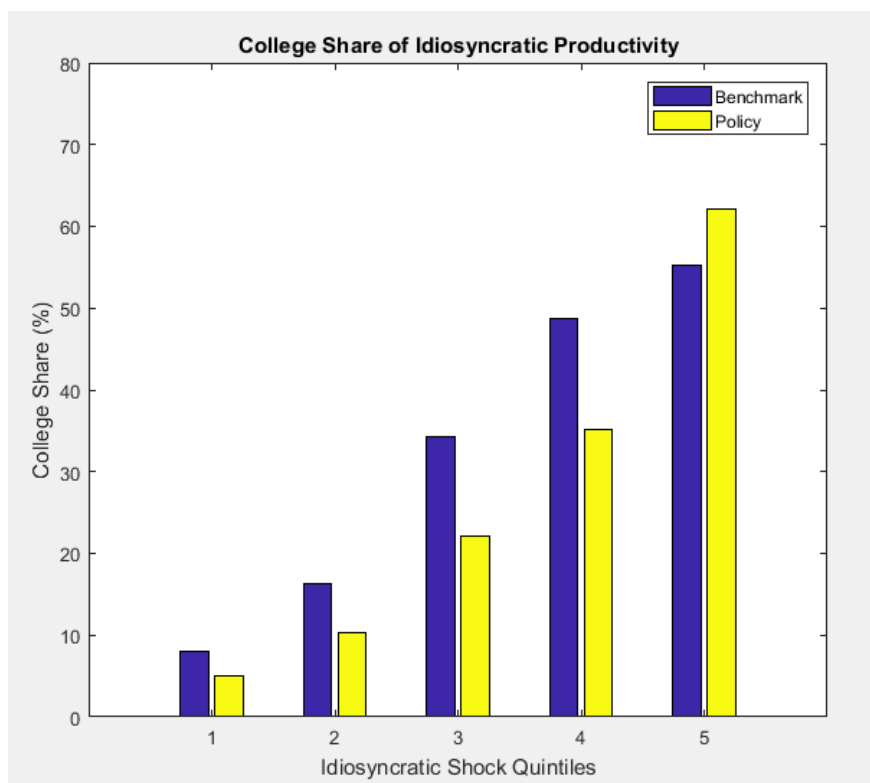
Figure 2:

intermediary will supply merit-based loans where as the current subsidized system provides need-based loans. This fact is represented by figure 2 which displays the fraction of workers in each quintile of the idiosyncratic shock distribution that has a college education. This could also be thought of as which workers would be the most productive if they were college educated. As you can see, there is nearly a 13% rise in the share of college workers in the top quintile. A more efficient sorting of highly productive households is good for the overall economy because more productive households will see larger gains from education which means larger additions to aggregate labor productivity. However, the large decrease in education amongst marginally productive workers offsets this to lead to a net decrease in aggregate output.

One of the strengths of modeling of long-term risky debt with constant loan payments and a fixed interest rate is it allows me to quantify what the price of a fixed rate loan would be in a competitive market place. The model predicts that the average interest rate on a

competitive student loan would be 10.54%. This shows the current system does provide a significant subsidy relative a competitive system in terms of interest rate as well as the credit supplied to borrowers.

5. The Bankruptcy Only Model

Figure 3: The Bankruptcy Only Model

Benchmark Model	Policy Experiment	Bankruptcy Only Model
• Constant q_b • Fixed $\underline{b}$ • b exempt from default	• Endogenous q_b • Endogenous $\underline{b}$ • b is dis-chargeable	• Constant q_b • Fixed $\underline{b}$ • b is dis-chargeable

When we replace the subsidized student loan system with a competitive market based system in section 4, households respond to both the new loan terms and the dis-chargeability of student loan debt. In this section I look to quantify the effects of these two changes separately. To accomplish this, I allow for borrowers to discharge their student loan debt in bankruptcy, but the the government still offers subsidized interest rates and borrowing limits. It is possible that the results from this experiment could go two different ways: more households could go to college because the risk of taking out a student loan has diminished, or fewer individuals could go to college because the credit supplied to households in the working years has contracted.[5]

When households are allowed to declare bankruptcy on student loans offered at subsidized interest rates and borrowing limits, we see the total number of college educated workers increase by 18% and the aggregate output increases by 3.0%. This is an indication that households respond to the change in bankruptcy law by acquiring more human capital. However, the model predicts that the population declaring bankruptcy would double and the deadweight loss associated with consumer bankruptcy would increase ten-fold. If this

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[5]There are still competitive financial intermediaries in the market for short-term risky credit during the working years. These intermediaries will respond to the default-ability of student loans by supplying less credit to households with student loan debt

policy were practically applied, these costs would need to be passed on to the consumer. I refrain from changing taxes in the current equilibrium so I can isolate the specific effect of changing bankruptcy law on individual decisions and the aggregate economy.

Model	$\mu(h_2)$	$\mu(d=1)$	DL_b	K	L	$\overline{L}_c$	Y	$\overline{r_B}$
Benchmark	33.1%	0.42%	$6.6e^{-4}$	14.57	2.21	5.26	4.35	5.86%
Experiment	23.9%	0.51%	$1.7e^{-3}$	14.10	2.13	6.17	4.20	10.53%
Bankruptcy Only	39.1%	0.83%	$6.9e^{-3}$	15.01	2.27	4.95	4.48	5.86%

Table 7: Sensitivity Analysis

Figure 4 outlines the change in the sorting of workers to college education by productivity associated with both policy experiments. Results from the bankruptcy only model indicate that the more efficient sorting of highly productive individuals to education was driven by the change in loan terms, not the option to discharge debts. The bankruptcy only model delivers a large increase in the educated population that is largely driven by the moderately productive working households. The option to discharge debts is particularly appealing to moderately productive households because they can eliminate debt payments while still working at moderately high wages. The results to the bankruptcy only model also show that there is a potentially significant moral hazard problem that occurs when you offer dis-chargeable debt at subsidized loan terms. Borrowers can obtain an education knowing that they can declare bankruptcy before making any substantial payments on loans. This is particularly prevalent in the market for student loans because work by Athreya, Tam and Young (2009) showed that consumer bankruptcy is more attractive for young working households. The moral hazard problem is avoided when we allow competitive intermediaries to supply credit because they can base the terms of the loan based on the probability of default occurring.

6. Conclusion

I studied a quantitative, overlapping generations general equilibrium model in which households make an education decision, are subject to idiosyncratic productivity shocks
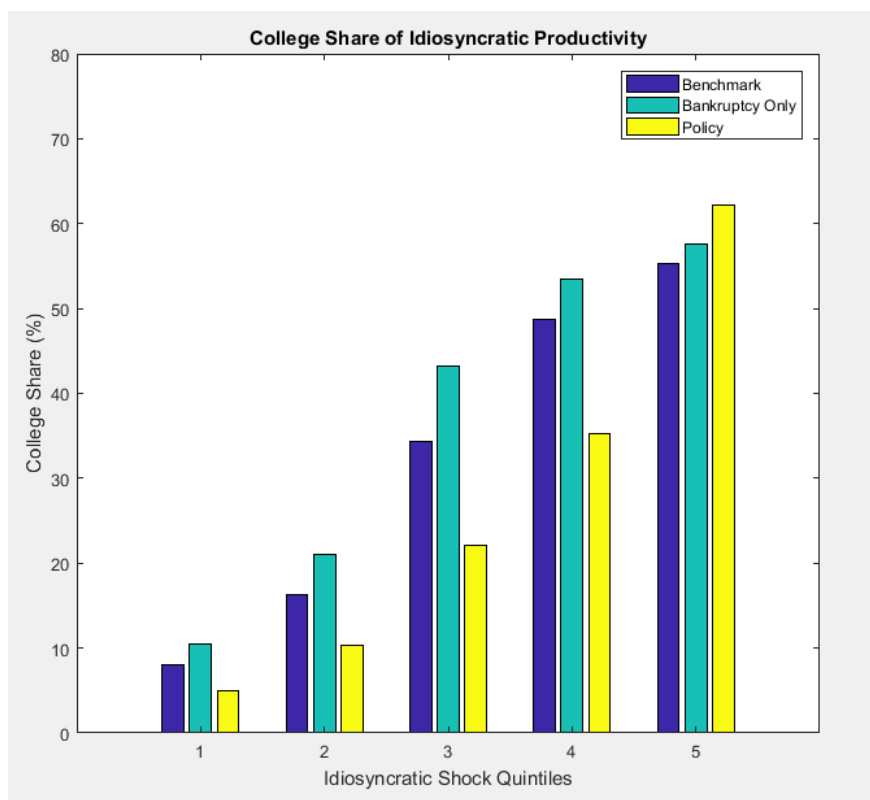
Figure 4:

and have access to two distinct forms of debt. The main contribution made in this paper is quantifying the effects of replacing the subsidized student loan system with a competitive market based system. I find that the market based system would result in a 27% decrease in college completion and a 3.4% decrease in aggregate output. However, competitive financial intermediaries can more efficiently sort the most productive households to college education as seen by a 12.7% increase in the college share workers in the top quintile of idiosyncratic productivity draws. The model also predicts that the average interest rate on a fixed-rate competitive loan would be 10.54%.

This paper makes numerous contributions to the literature. The equilibrium that is solved for includes endogenous education decisions, student loan debt and risky consumer credit after school. This framework is necessary to answer the current research question, but it could be useful for future research projects that study the consumption, debt and bankruptcy patterns of young college graduates. I also am able to solve for an equilibrium

with a long-term fixed interest rate debt pricing function. Financial intermediaries in the policy experiment offer fixed interest rate loans with constant debt payments and a terminal date.

The results to this paper could make a case for or against policies to allow student loans to be discharged in bankruptcy. On one hand, I show that the option to discharge student loans can greatly increase the number of students going to school. Agents in this equilibrium are better able to smooth consumption after graduation which makes them more likely to attend college and take out student loans. However, dis-chargeability of debts creates a severe moral hazard problem with debt decisions. The moral hazard problem can be avoided by allowing competitive financial intermediaries to supply credit, but this leads to a significant decrease in the number of college educated workers and the aggregate output.

6.a) Discussion for Future Work

The results to this paper show that institutional features in the market for student loans can play a significant role in education decisions and aggregate productivity in a model economy. However, it is possible that the results are over-stating the changes that would happen if these policies were ever actually implemented. I make a simplifying assumption that the marginal wages earned by workers is the same regardless of education level. Future work could implement a production function that takes educated workers and their less educated counterparts as substitutes in production. This would create two separate wages for college graduates and non-college graduates. In this equilibrium, when the number of college graduates decreases, their wages would increase which would partially offset the decrease in education level. Therefore, you could think of the results in the current paper as the upper bound for the changes in college education that would occur if these policies were actually implemented.

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Appendix A: Computational Algorithm and Solution Method

A.a) Computational Algorithm

1. Choose a value of $\frac{K}{L}$ to set prices, $\{r, w\}$.

2. Solve for decision rules , $g(x), c(x), l(x), d(x), e(x)$, backwards starting from the last period of life.

   - Solve for $q^h(x)$ in period $j = t$ using decision rules from period $j = t + 1$.
   - Policy Analysis: Solve for $q_b(x)$ in period $j = n_C$ using decision rules from periods $j = (n_c + 1, nT))$.

3. Use decision rules to solve for the distribution of households across states.

4. Use bisection to update the value of $\frac{K}{L}$.

   - Repeat from step 1 until the markets for capital and labor clear.

A.b) Solution Method: The Partial Endogenous Grid Method

In this paper, I develop a new solution method that lends itself well to solving models with consumer default. The Partial Endogenous Grid Method (PEGM) is used to solve household decision rules for consumption, savings and default when there are discontinuous regions of the expected future value function. The solution method outlined in this section adds to the literature that uses first-order conditions to solve for decision rules in the presence of discontinuities.

When solving dynamic programming problems with default decisions, the expected future value function can have points of discontinuity. Whenever the expected future value function has points of discontinuity, methods that use the first-order condition to solve for decision rules can generate multiple internal solutions to a problem. Models with default can be particularly volatile if the shock process is discretized (as I have done in this paper) because then the first-order condition can have discontinuities in both the expected future value function and the pricing function. The PEGM method avoids this problem by only using the first-order condition to solve for decision rules in the continuous region of the first-order condition. The key insight that motivates this method is that in a default model, discontinuities only appear in the far left region of the grid where an individual has debt. Therefore, there is a threshold point where above the threshold the first-order condition is continuous.

$$c = \left\{ \frac{\beta \zeta_j \, E\left[D_2 V_{j+1}(\epsilon_h', a', B', h, s) | \epsilon_h \right]}{D_2 q_j^a(\epsilon_h, a', B', h)a' + q_j^a(\epsilon_h, a', B', h)} \right\}^{\frac{-1}{\sigma}} \tag{21}$$

The first-order condition for equation (4) is contained above in equation (17). The first step to implementing the PEGM method is to identify the points of discontinuity in the

Figure 5: <u>PEGM</u>
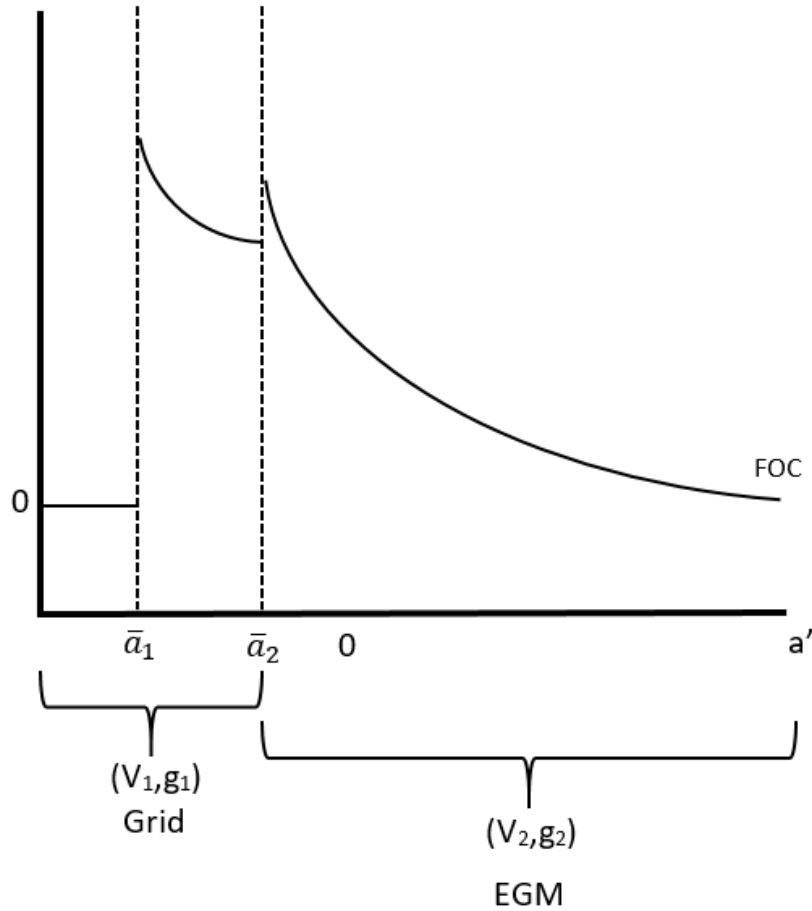
first-order condition. Using figure 3 as a reference, this step is carried out by identifying points $\bar{a}_1$ and $\bar{a}_2$. The second step is to save the largest point of discontinuity or point $\bar{a}_2$ in the figure.

After identifying point $\bar{a}_2$, I segment the first-order condition into two regions. For each point in the state space, I use the first-order condition to find a solution to the problem in the region to the right of $\bar{a}_2$. I then use a grid search method to find a solution to the problem in the region to the left of $\bar{a}_2$.

We now have two solutions to the problem at a given point in the state space. The solution (V_1, g_1) was found using the endogenous grid method and the solution (V_2, g_2) was found using a pure grid-search method. The final step in implementing PEGM, is to discretely choose the maximum of (V_1, V_2) and save the corresponding decision rule. Since the discontinuous region of the grid in any consumer default model is relatively small, this is substantially more accurate than a pure grid-search method. Also, certain versions of the model were solved almost twice as fast in Fortran using PEGM compared to pure grid-search.