

3/11/17

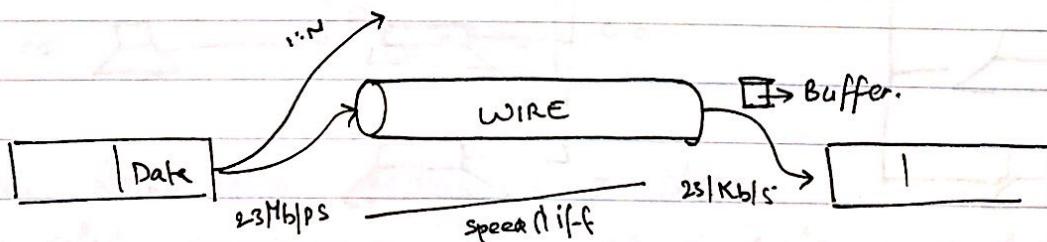
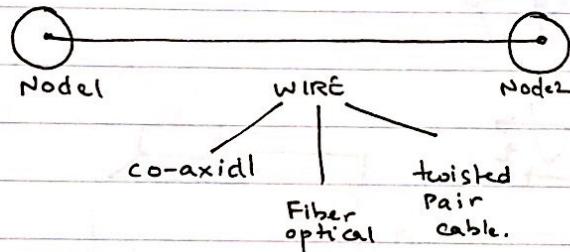
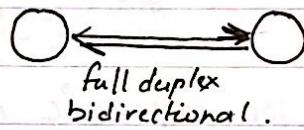
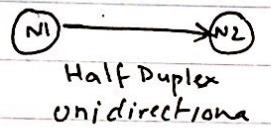
## Computer Networks

Book : FRosen / Tannenbaum  
PPTs  
Theory

## Topologies

3/11/17

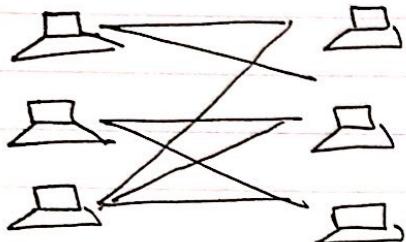
Mesh, Star, Hybrid, Ring



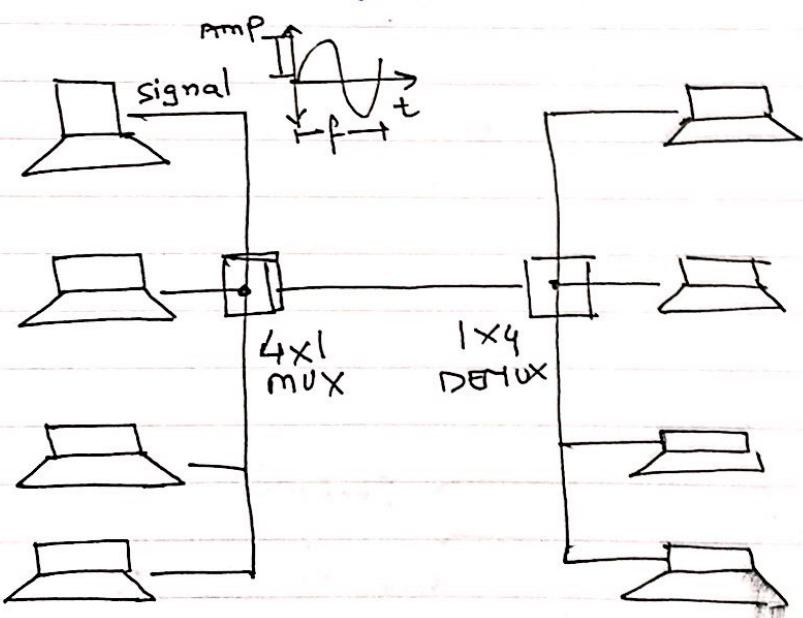
- Physical layer: data + checksum + error
- Datalink layer: congestion control
- Network layer: confirmation of receiver. // port-to port
- Transport layer: end-to end transfer "App to App".
- Session layer: create a session b/w "App" [ports / sockets]
- Presentation layer: encode/decode; security
- Application layer: user level program

16/11/18

## 1> No-multiplexing



- 1> No facility for dynamic needs
- 2> High cable lengths



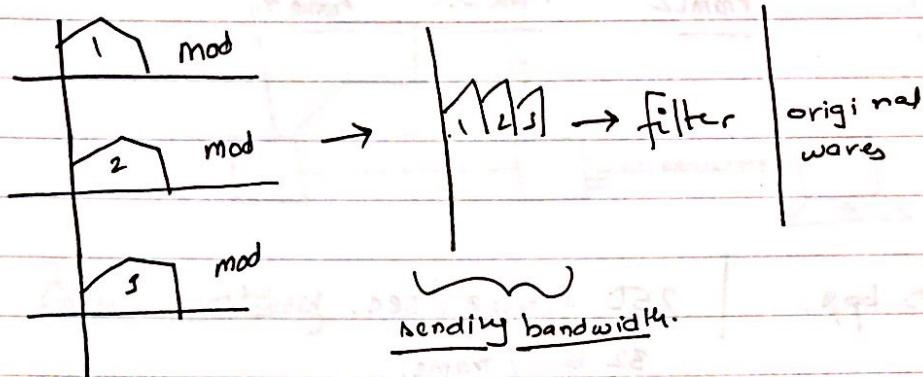
## 2> Multiplexing-

To avoid collision

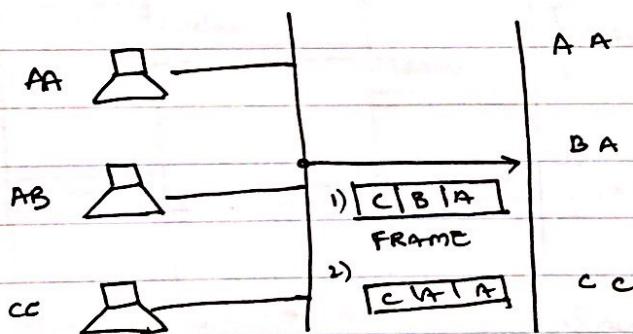
- keep some traffic control logic
- change the frequency of each source by fft & add them up.

1) Frequency Domain Multiplexing.

2) Time domain Multiplexing.

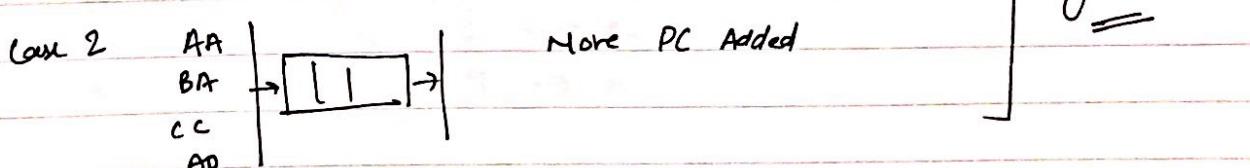
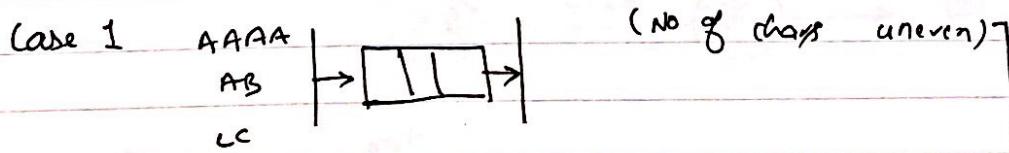


## Time Domain Multiplexing



like Round Robin Scheduling of processes

Also called Synchronous TDM. No of devices = no of frame slots.

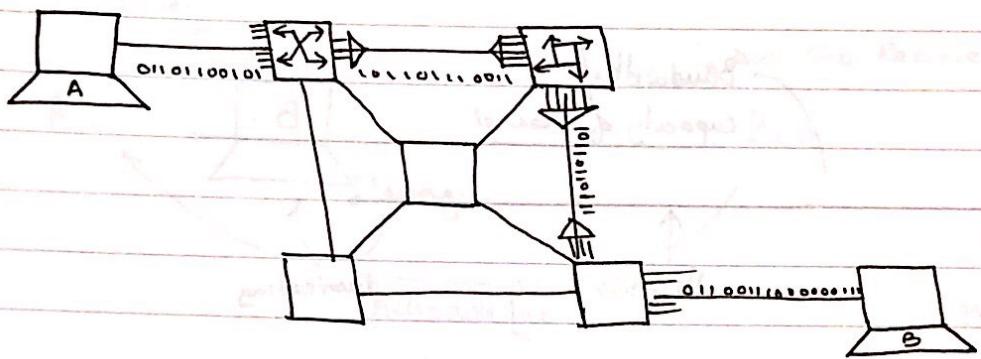


Case 1 →  $\boxed{A|A}$ ,  $\boxed{B|G}$ ,  $\boxed{A|A}$ ,  $\boxed{B_2|G}$

Frame 1. Frame 2. Frame 3. Frame 4.

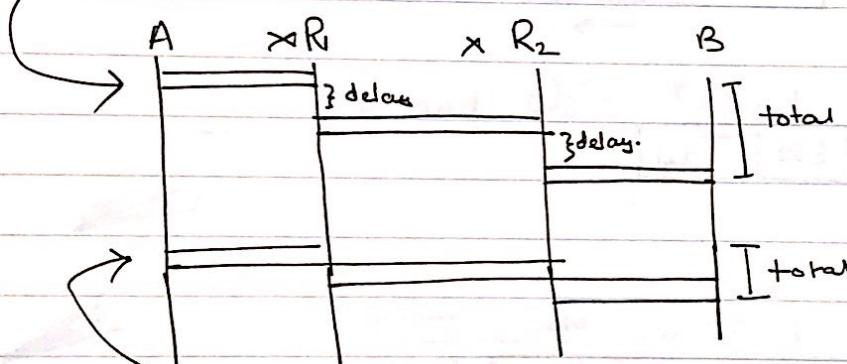
Example

 8250 bps. | 250 frames/sec.  
52 bits/frame.

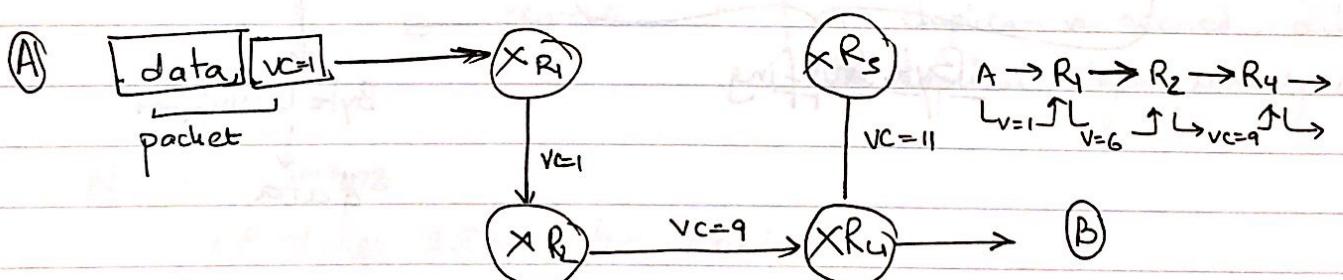


### Circuit Switching

### Packet Switching

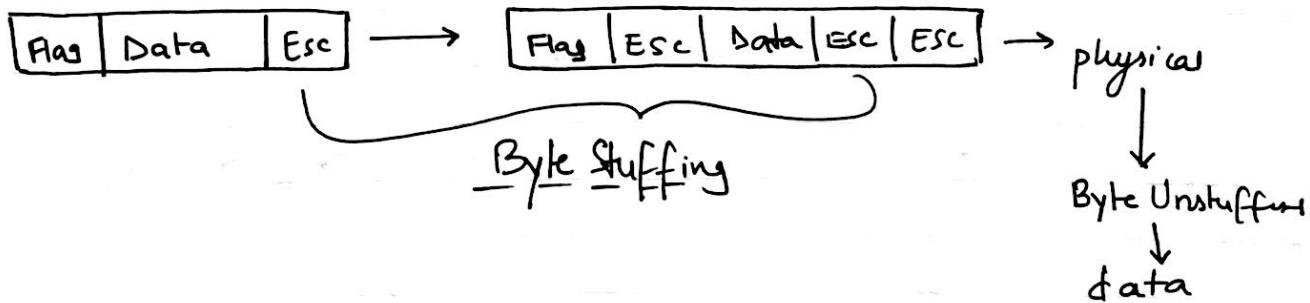
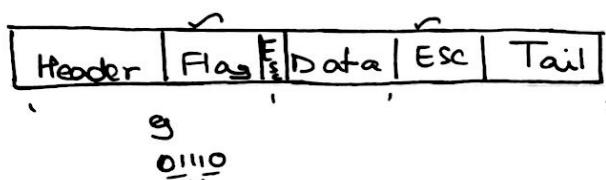
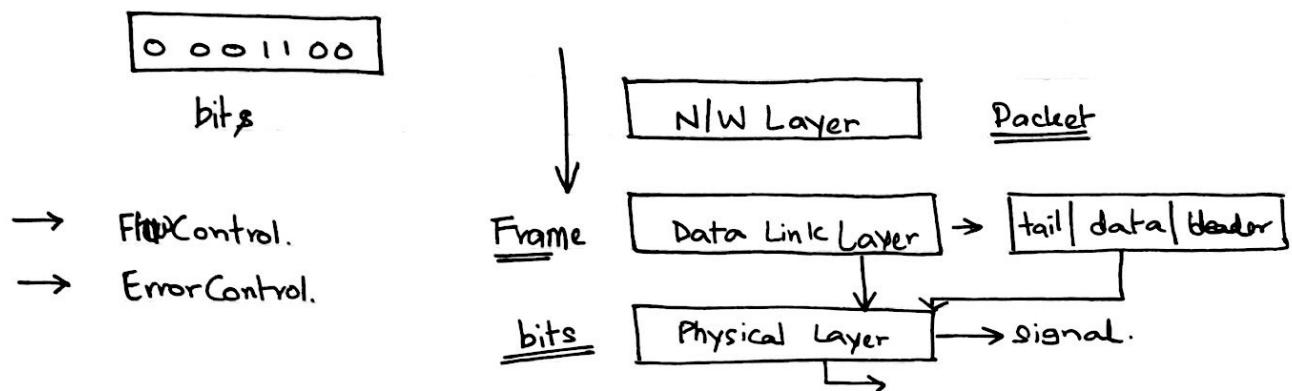
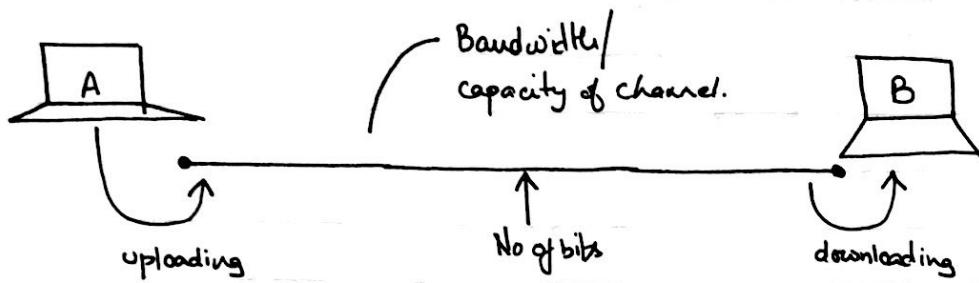


→ Virtual Circuit → mimics circuit switch in packet switching.



$P^{in}$	$VC$	$P^{out}$	$VC$
1	1	5	6
5	6	7	9

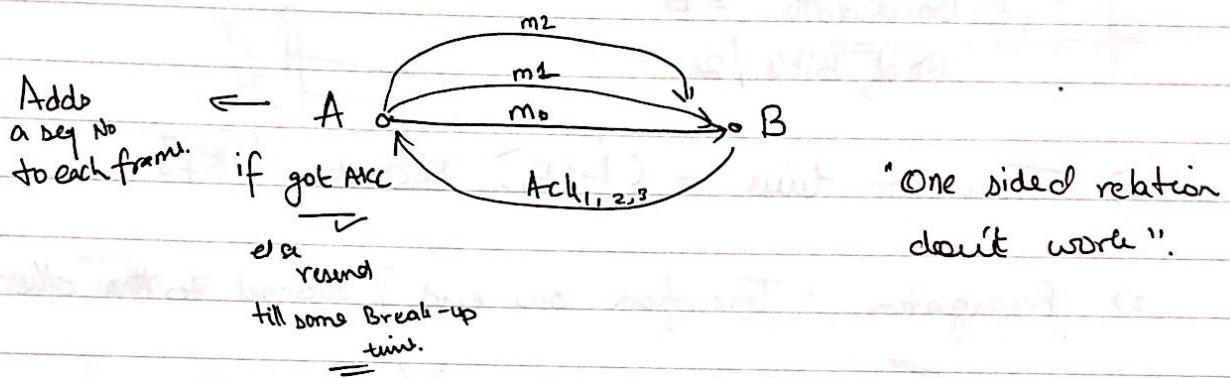
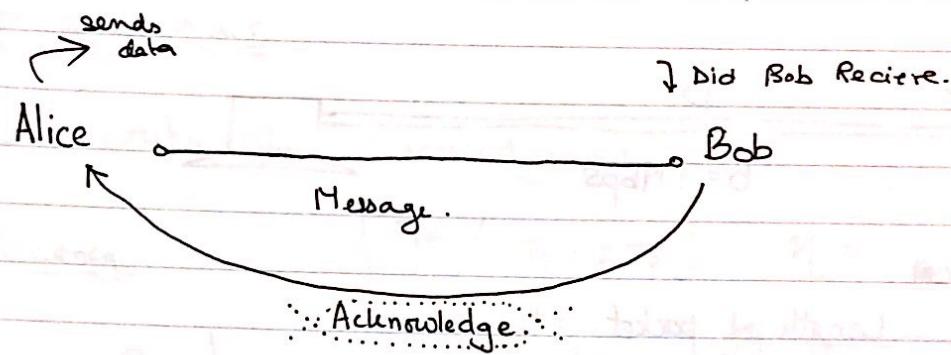
$\rightarrow R_1 \rightarrow R_2$



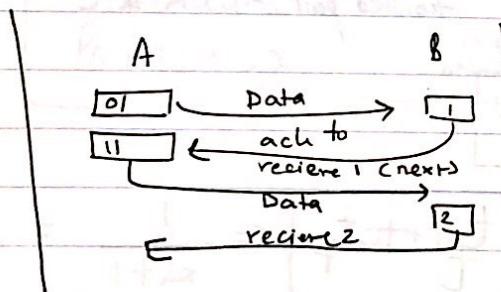
## Bit Stuffing

flag : 0111110

data : 01101111/01000  
↑  
stuffed bit.



### " Stop and Wait Protocol "



if ack is lost then also Alice resends that package.  
if duplicate is received, delete the duplicate before sending ack.

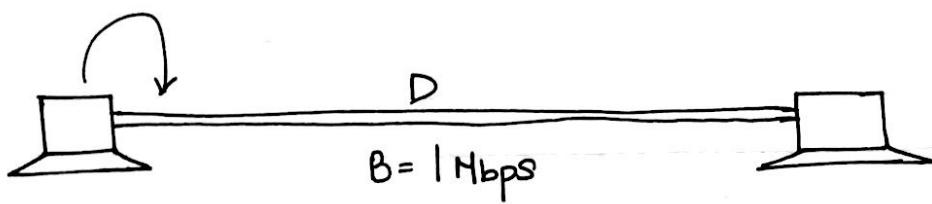
$$B(\omega) = 1 \text{ Mbps}$$

1 bit takes 200ms for round trip.

Frame = 1000 bits.

Channel Util :  $\frac{\text{FrameSize}}{1000} \times 100\%$

$\frac{200000 \text{ (bits)}}{1000}$



### Simpliest Protocol

Length of packet = L

Bandwidth = B

No of Bit's / Sec =

1) Transmission time =  $(L/B)$  seconds  $[t_f]$

2) Propagation : Time from one end of channel to the other.

$$\overline{t_p} = (\text{Length of channel}) / (\text{Speed of prop}) //$$

Speed of prop :  $\text{ofc} = c$   
 twisted pair = ' $k$ ' \* c  $[k \in [0, 1]]$

$$t_p = D/V$$

3) Total Time taken :  $t_f + t_p$

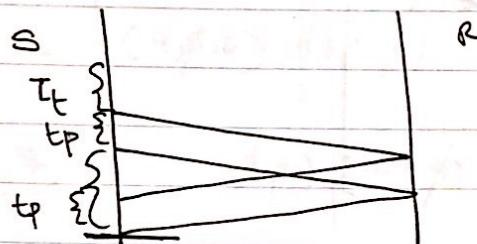
4) Efficiency :  $\frac{\text{useful time}}{\text{Total time}}$

$$= \frac{t_f}{t_f + t_p}$$

Stop and Wait.

Sender waits for acknowledgement.

Efficiency :  $\left[ \frac{T_t}{T_t + 2T_p} \right] = \eta = \boxed{\frac{1}{1+2a}}$



$$a = T_p / T_t$$

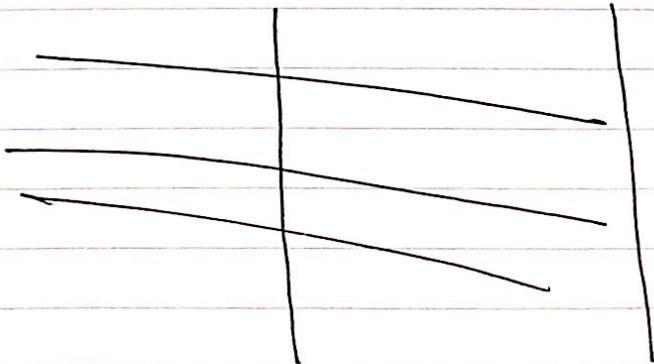
$$\text{Total} = T_t + 2T_p$$

# The eff ( $\eta$ ) is only 6% find best params for 50% A

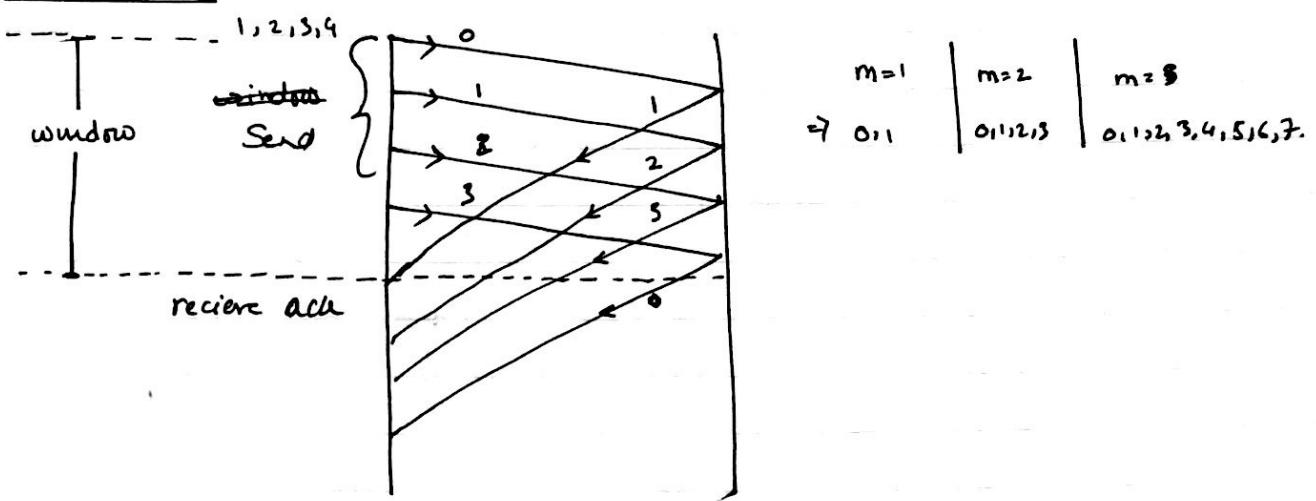


Bandwidth : B  
Package size :  $\frac{2DB}{V}$

$$\frac{1}{2} = \frac{1}{1+2a} \Rightarrow 2a+1 = 2 \quad a = 1/2 \Rightarrow T_p / T_t = 1/2$$
$$\Rightarrow L = 2dB/V$$



## GoBackN



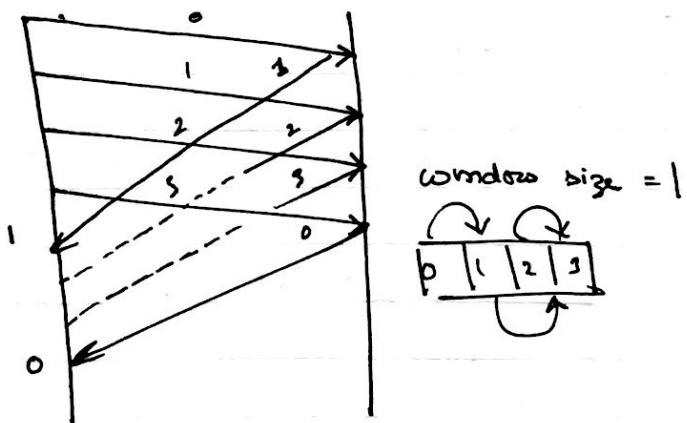
Problem : What if packet is lost?

→ Resend the complete window.

To improve efficiency.

window size =  $2^m$

0	1	2	3
0	1	2	3
0	1	2	3
...			



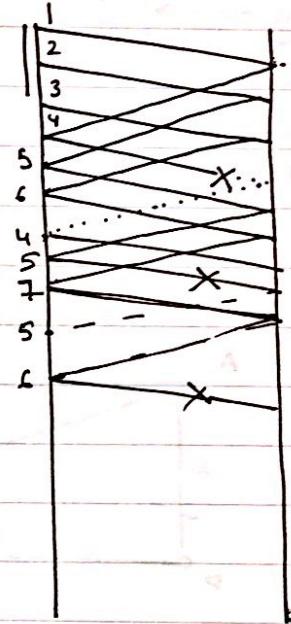
31/11/18

$(N) \rightarrow (N+1)$

111 | + 111

Sender is trying to send 10 packets @ window size = 3.  
Every 4<sup>th</sup> frame is lost.

$(1, 2, 3)$	$(4, 5, 6)$	$(7, 8, 9)$	$(10, 11, 12)$
$(7, 8, 9)$	$(10, 11, 12)$	$(1, 2, 3)$	$(4, 5, 6)$
$(10)$	$(9, 10)$		

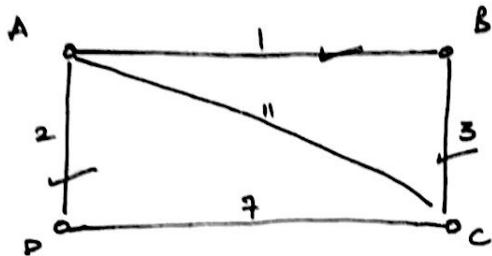
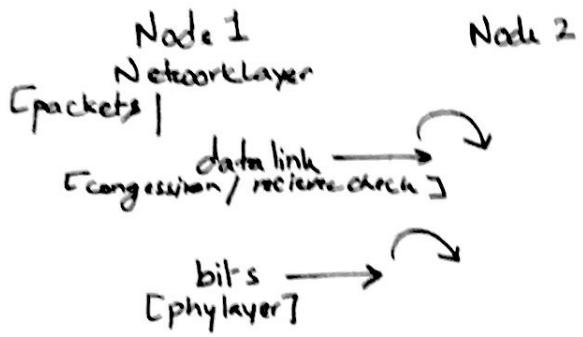


Every 5<sup>th</sup> Packet is lost

$(1, 2, 3)$	$(4, 5, 6)$	$(7, 8, 9)$	$(10, 11, 12)$
$(7, 8, 9)$	$(10, 11, 12)$	$(1, 2, 3)$	$(4, 5, 6)$

10/2/18

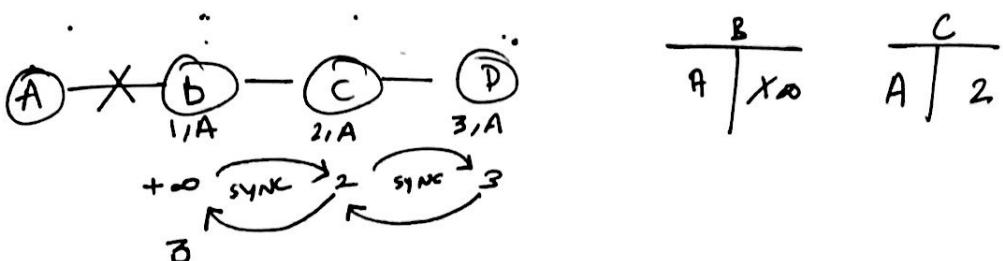
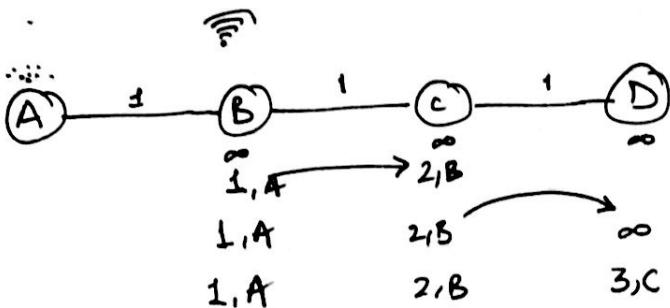
## Network Layer

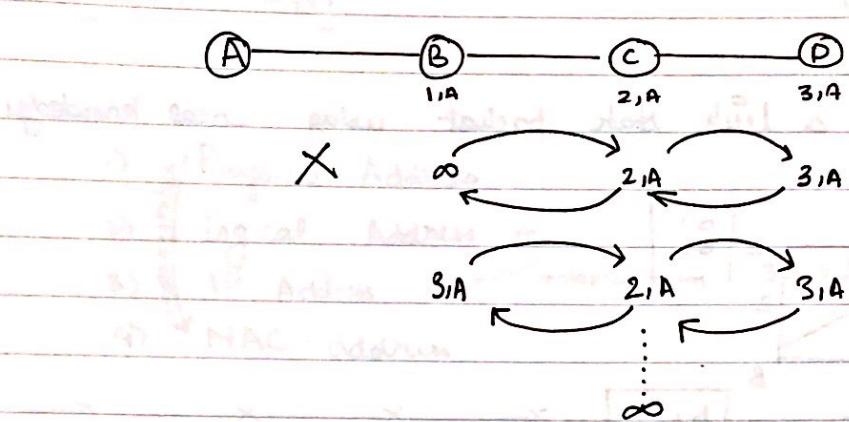


[AC and DC will be unused.]

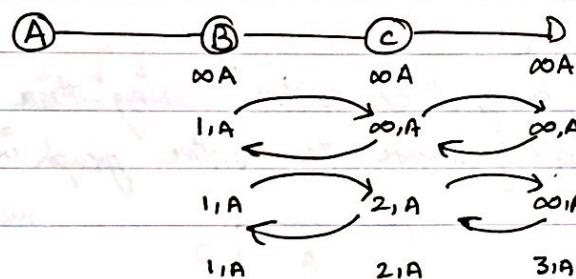
- Distance Vector Routing - (DVR) Algorithm

- # Greedy Approach #

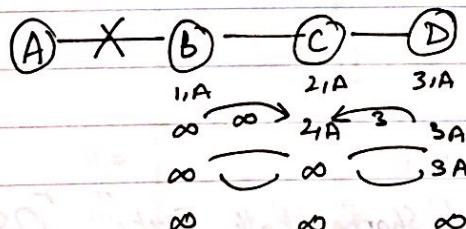




count to infinity problem



Solution to counting to infinity problem



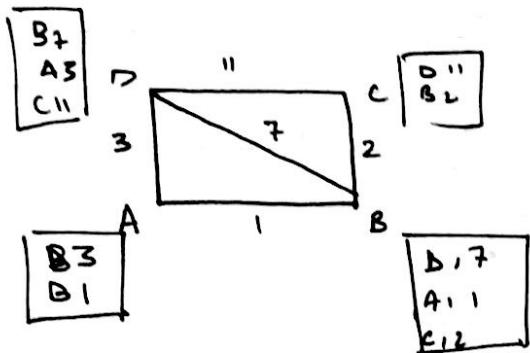
A	B	C
dest Net ID	next hop IPAdd	wt matrix

D cannot reach A

if a node is dependent on some other node then the former node sends the dist vect as infinity to the later node.

## Link State Routing (LSR)

1. Every router will prepare a link state packet using local knowledge



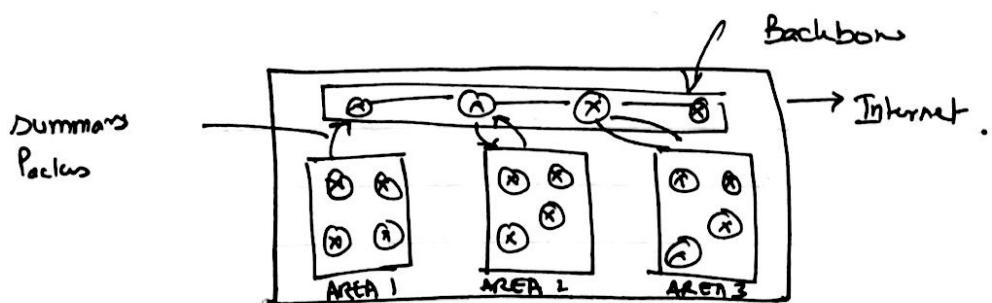
2. All the packets are flooded to all the routers

3. Every router will get LSP from every other router, using these link state packets, every router will generate the entire graph in its local memory

4. Every Router will do single source shortest path algo to find the final routing table.

- # count to infinity not possible.
- # high congestion.
- # duplicates.
- # high bit errors.

LSR is implemented using "Open Shortest Path First." OSPF.

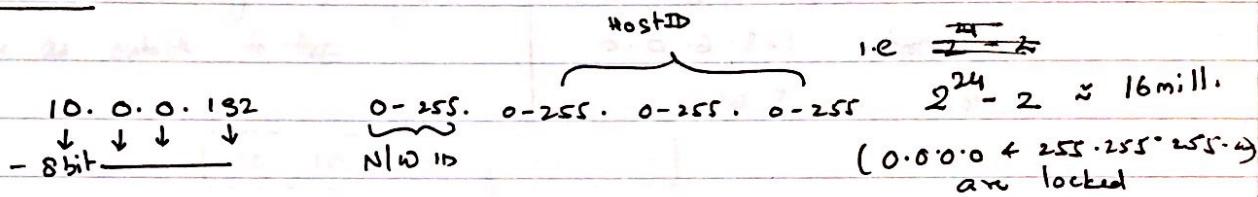


## Network Layer

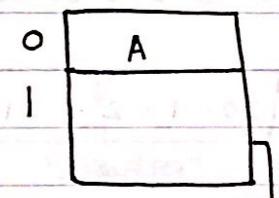
- 1) Physical Address
- 2) Logical Address
- 3) IP Address implementation 32 bit.
- 4) MAC - address



## IP Address



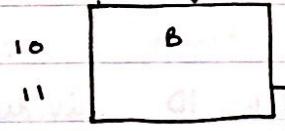
Classes



0.0.0.0 to 127.0.0.0

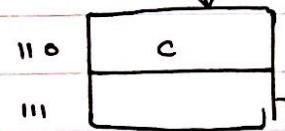
0 | 000 0000

$2^4 - 2$  Host  
 $126$  N/W ID



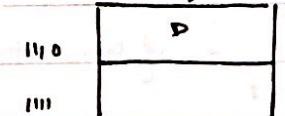
128.0.0.0 to 191.0.0.0  
N/W ID

$2^{16} - 2$  Host  
 $2^{14} - 2$  N/W



192.0.0.0 N/W to 223.0.0.0  
110 | 00000 . 0. 0. 0] Host

$2^8 - 2$  Host  
 $2^{21} - 2$  N/W



224.0.0.0 to 255.0.0.0

Choice of class depends on no. of networks & connected computers.

## Classless [ CIDR ]

$125 \cdot 0 \cdot 0 \cdot 16 | 26$  → total no of networks  
 $\underline{24} + 2$

$2^{\underline{26}-2} \Rightarrow 62$  Local IDs  
 $\Rightarrow \boxed{125 \cdot 0 \cdot 0 \cdot 00} \boxed{010000}$   
 Network Host

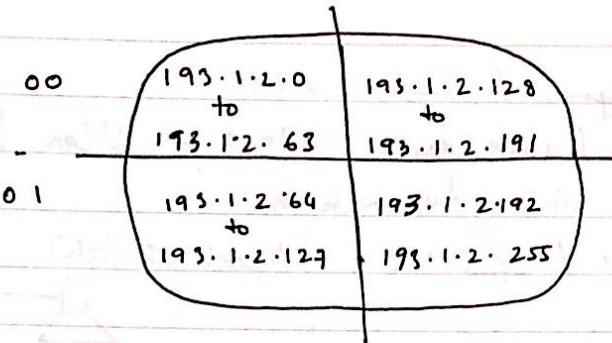
eg  $128 \cdot 6 \cdot 3 \cdot 64 | 16$   
 Network  $128 \cdot 6 \cdot 0 \cdot 0$   
 Host  $\dots 3 \cdot 64$

eg  $20 \cdot 1 \cdot 2 \cdot 100$   
 $\boxed{20} \dots \boxed{1} \dots \boxed{2} \quad \boxed{00100010}$   
 $20 \cdot 1 \cdot 2 \cdot 011 \quad \underline{00100}$   
 Network Host  
 $20 \cdot 1 \cdot 2 \cdot 96$  to  $20 \cdot 1 \cdot 2 \cdot 127$

Q Given 192.1.2.0 as the network ID divided into 4 parts

$192 \cdot 1 \cdot 2 \cdot 0 | 24$

$192 \cdot 1 \cdot 2 \cdot 00000000]_{127} \quad ]$  make 1 bit a part  
 $10000000]_{128} \quad ]$  of the network ID  
 $\dots \dots \dots$   
~~.....~~

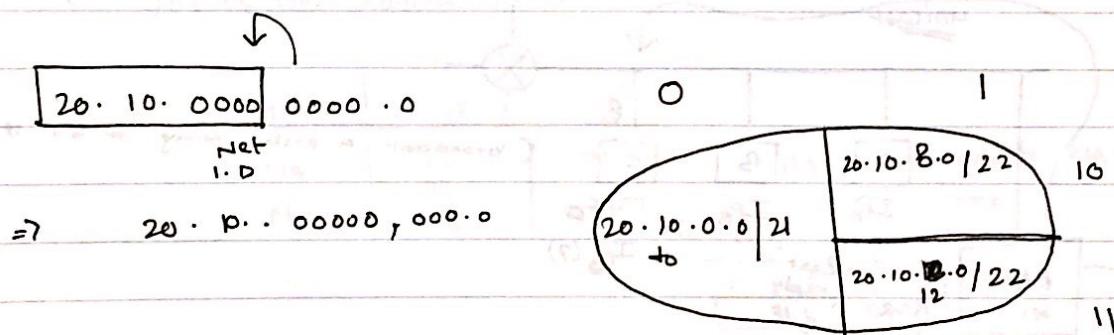


Q. ISP has block of  $20 \cdot 10 \cdot 0 \cdot 0$  to  $20 \cdot 10 \cdot 15 \cdot 255$  and wants to give half of its IP to Company 1) quarter to other and keep the rest with itself propose as suitable to the

$20 \cdot 10 \cdot 0 \cdot 0$	$1100 \cdot 11$
-------------------------------	-----------------

Network      Host

$20 \cdot 10 \cdot 0 \cdot 0$  to  $20 \cdot 10 \cdot 15 \cdot 255$



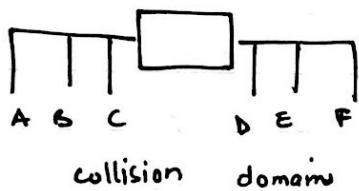
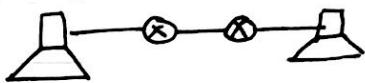
1) Repeater

2) Hub → Physical layer / Passive device

3) switch → PL + DL [active device] allows for V-Lan. [within network]

4) Router → Active device between two networks

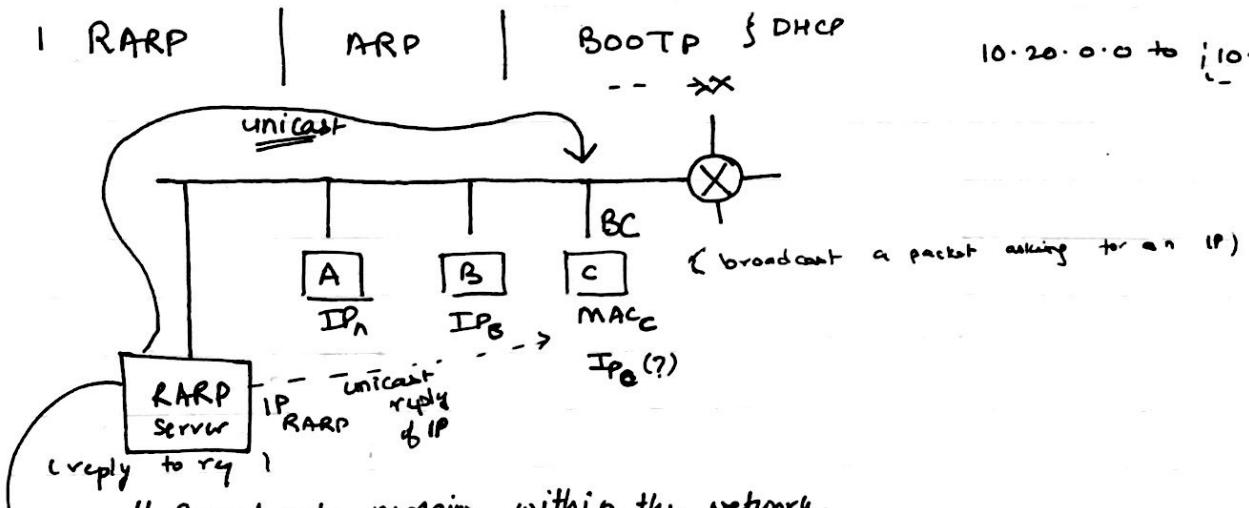
5) Bridge → Active device / only PL+DL [multiport switch]



3 layer switch ≈ Router.

1 RARP | ARP | BOOTP & DHCP

10.20.0.0 to {10.20.255.255}

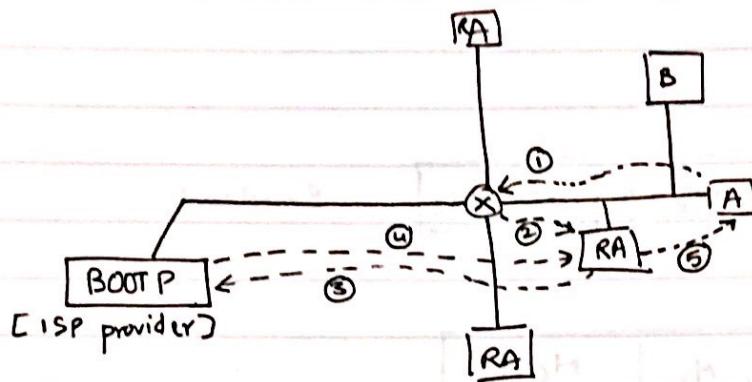


M <sub>A</sub>	IP <sub>A</sub>
M <sub>B</sub>	IP <sub>B</sub>
M <sub>C</sub>	IP <sub>C</sub>

static static

RARP → waste IPs

BOOTP



- ① send Broadcast for IP
- ② send to RA
- ③ send Req for A
- ④ Reply back to RA.
- ⑤ Reply to A it's IP

IP are still static |

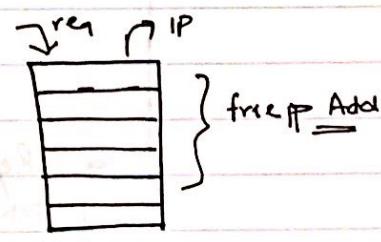
今日  
発誕年月  
birth life day

AX止

## DHCP [ Dynamic Host Control Protocol ]

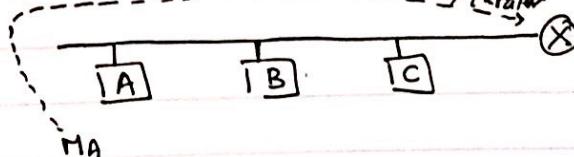
MAC	IP	Time
MA <sub>A</sub>	IP <sub>A</sub>	time 1
MA <sub>B</sub>	IP <sub>B</sub>	time 2

IPA is assigned to MA  
only for time 1



## ARP Address Resolution Protocol

req the MAC add of Host C or A



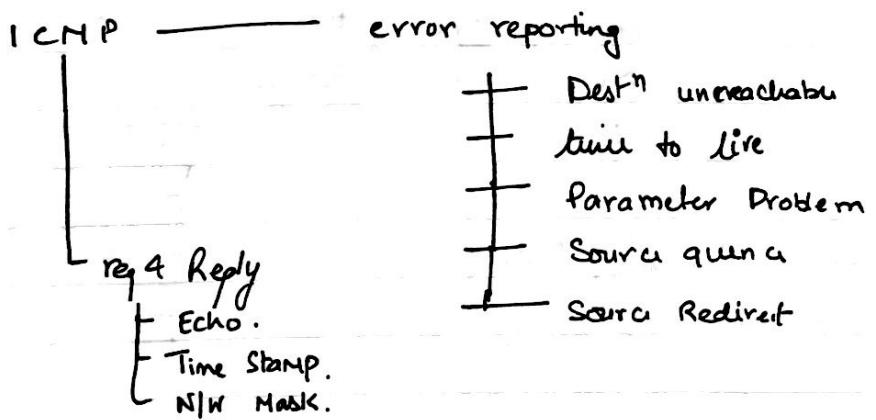


ARP req | IPs Ms = ? | Mc Mls  
Broadcast

ARP reply | IPs Ms | Mc Ms  
at dest

## ICMP

- N/W layer.
- feedback messages.



21/3/18

## Transport Layer

- End-To-End Connectivity

- Packet Duplication

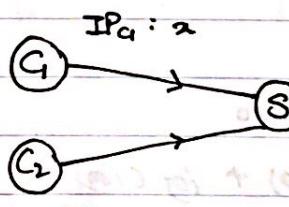
-

TCP

UDP

Source Port (16)	Destination Port (16)
Sequence No (32)	
ACK. NO. (32)	
TCP header C4 S N G A K D H R T F Checksum (16) Urgent Pointer	Adv Window Size (16)
Options (32)	
Data (32)	

**Socket:** Combination of IP address + Port i.e. 48 bit  
Random number



Seq No:  $(2^{32}+1)$  bytes → whenever this is crossed, repeats  
If the sent seq is  $4G+1$  then a wrap around happens  
and the time is called a wraparound time after which  
the seq repeats

$BW = 15/s$  what is wrt.

$$WAT = 2^{32} B / 1 b/s = 2^{32} \text{ sec} \approx 142 \text{ years.}$$

$$\text{Let BW} = 1 \text{ Mbs} = \frac{2^{32}}{2^{16}} = 2^8 \text{ sec}$$

Life Time Max time the packet can be alive in the network before being absorbed by the receiver.  $\approx 3 \text{ min.}$

$$WAT > LT$$

when  $B/W = 2^{32}/2^{16} = 2^2 = 48$  [

To increase the WAT we add bits from options and add to the start.

Eg  $LT = 180 \text{ s}$   
 $B/W = 2^{16}$

what is min no of bits in seq to prevent wrap around.

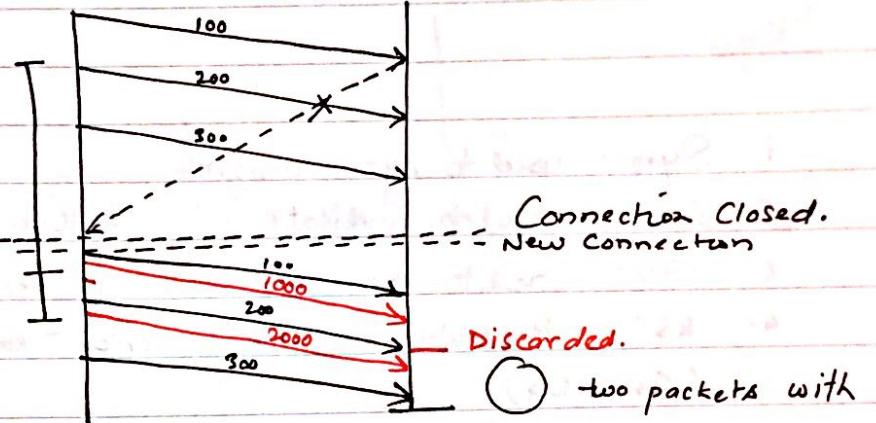
$$\frac{2^x}{2^{16}} = 180$$

$$2^x = 2^{16} \cdot 180$$

$$x \log(2) = 16 \log(2) + \log(180)$$

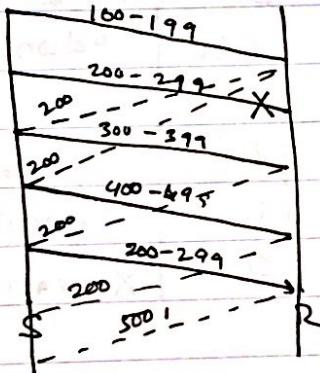
$$\Rightarrow x = 38$$

6 more bits.



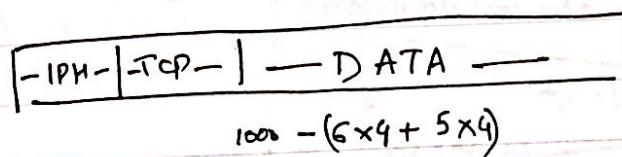
To avoid this, the sequence is started with random number.

Acknowledgement Contains the seq no of next expected byte.



Header Length (4 bits)

if the total len field in ~~IP~~ datagram is 1000 & header len is also 5, find data size in packet.



## Flag

1. Syn : used to synchronize
2. Ack : used to indicate the ACK no field in the header is valid
3. FIN : used to terminate the connection
4. RST : Reset; The connection has to be terminated before est (reset flag)
5. PSH : TCP buffers the data in order to increase efficiency  
# PSH flag pushes the data directly into the application.
6. URG : (Urgent) : data in segment is urgent & has to be given the highest priority.

SYN	ACK	Action
1	0	Request for Data
1	1	Reply back
0	1	Only ACK
0	0	X Invalid X

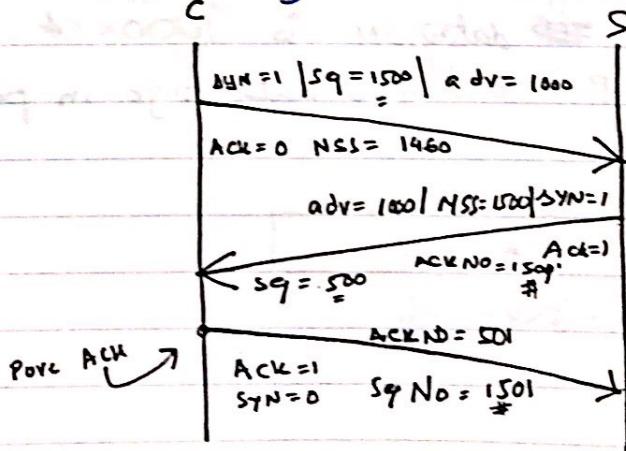
## Maximum Segment Size

The Max Datagram size.

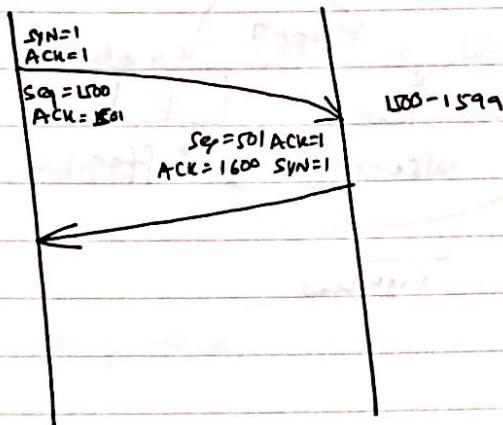
Data → any size

(App N)

Initial Connection

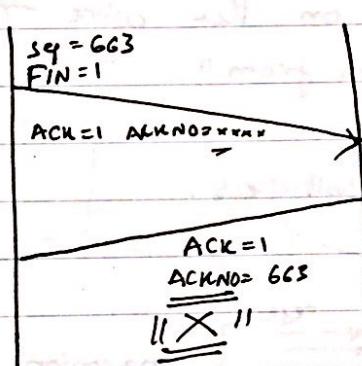


## Data Transfer



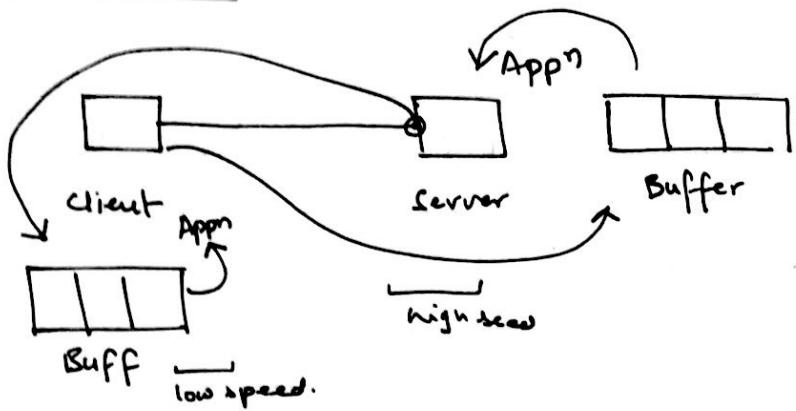
Three-way Handshaking

## Termination



28/3/18

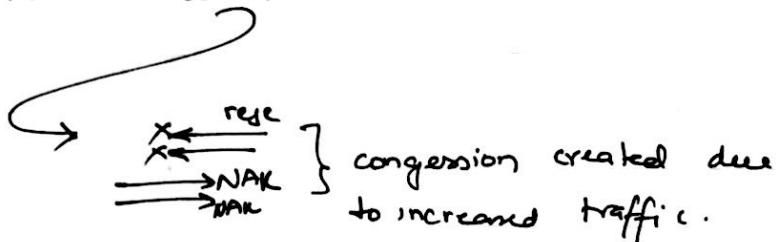
## Congestion Control



// Buffer size is limited.

Rate @ which data is put on the wire  $\rightarrow$  uploading.  
" " " " " taken from " "  $\rightarrow$  downloading.

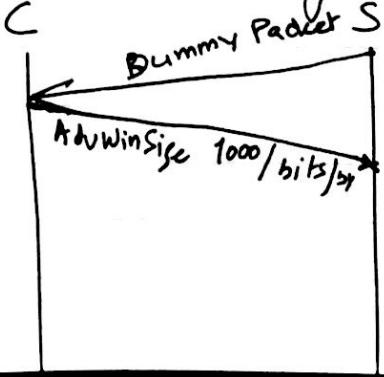
# The buffer can be a bottleneck.



To avoid this sync the speed of uploader and download.

Advanced Window Size This is the window size & then based on this the sender decides upload logic. (16 bits i.e.  $2^{16} = 65464$ )

To extend it to more we take bits from option. i.e. 32 bits,

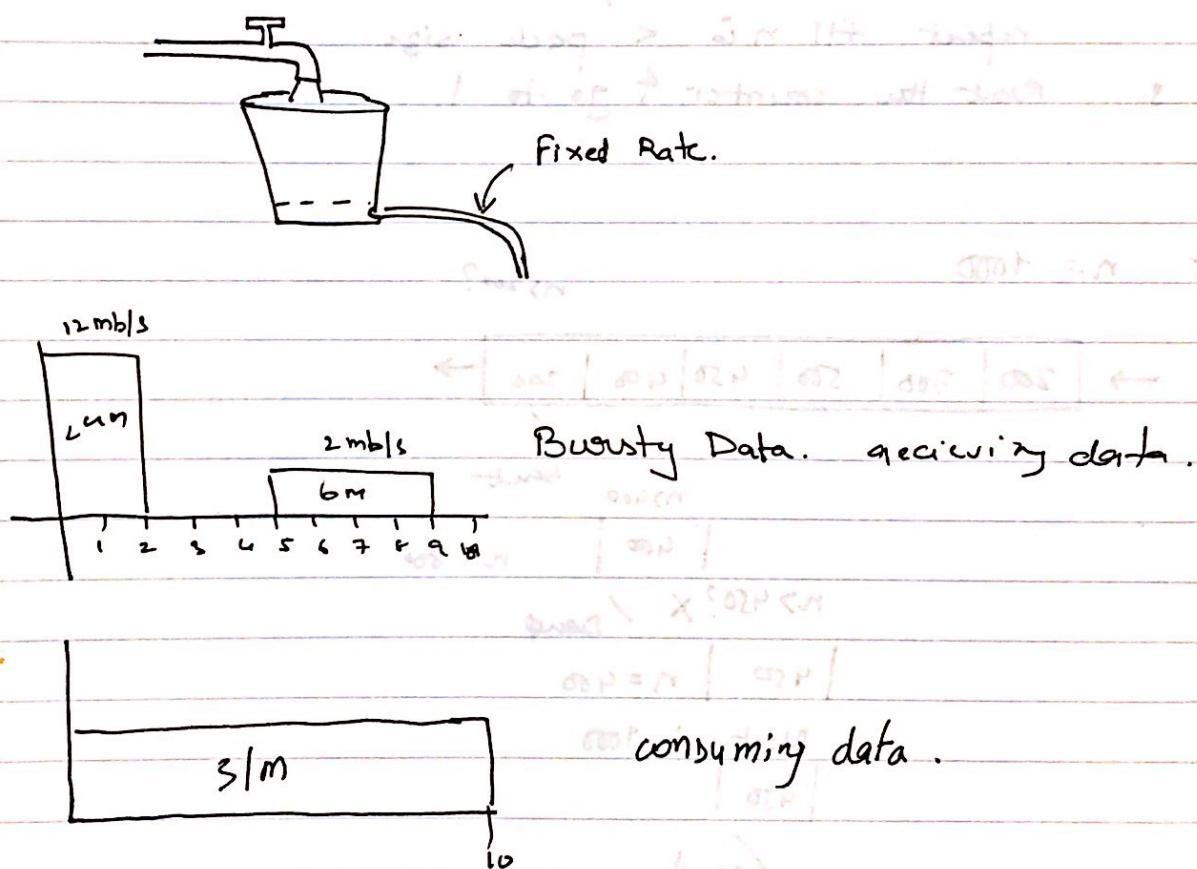


If the  $\times$  has advertised 1GB should it send it directly?

→ The sender should not send 1GB because the net may not be able to hold this data. ∴ sender will use congestion window to keep the track of capacity of net.

### Congestion Control protocol.

#### @ Recent Leaky Bucket Protocol.

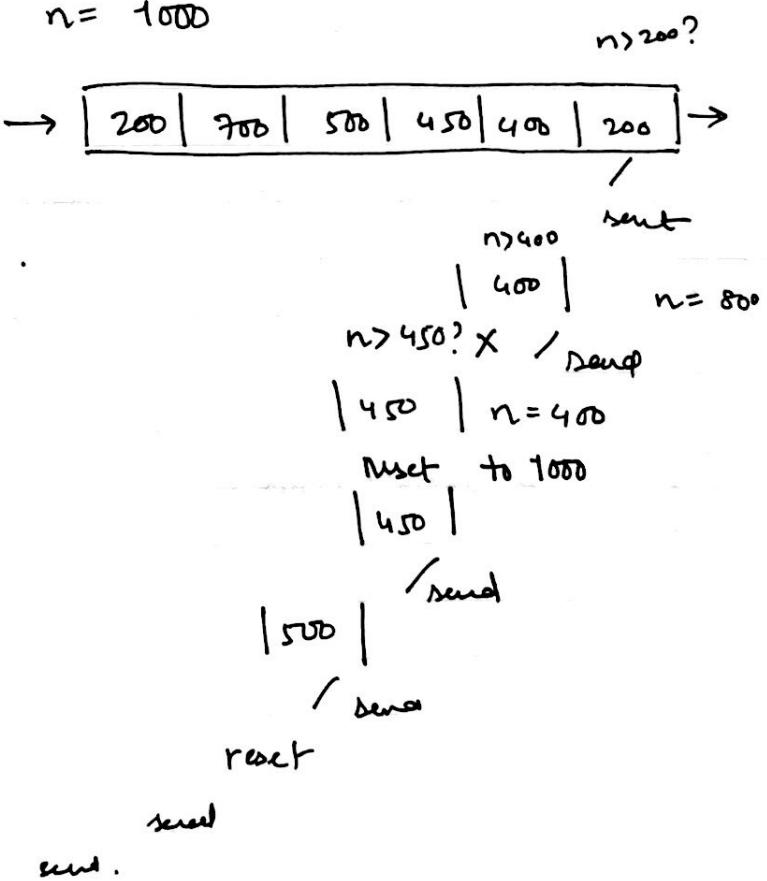


- A leaky bucket can be implemented using FIFO
- A FIFO holds the packets if the traffic is of fixed sized packets the process removes a fixed no of packets at each tick of the clock, if the traffic is of variable len, the fixed output burst must be based on the no of device/bits

Algo

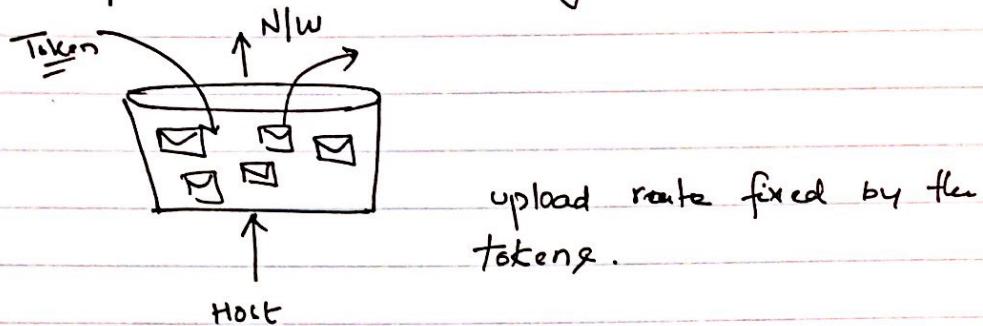
1. init counter = 6 at tick of clock
2. if  $n >$  size of pack :
  - send packet & inc the counter by pack size.
  - repeat
  - repeat till  $n$  is  $<$  pack size
3. Reset the counter & go to 1.

let  $n = 1000$



## Token Bucket Protocol

To remove a pack add equal value of tokens.



28/0  
14/00  
2024/0

10/4/18

## Transport layer

TCP | UDP → High Transmission Speed

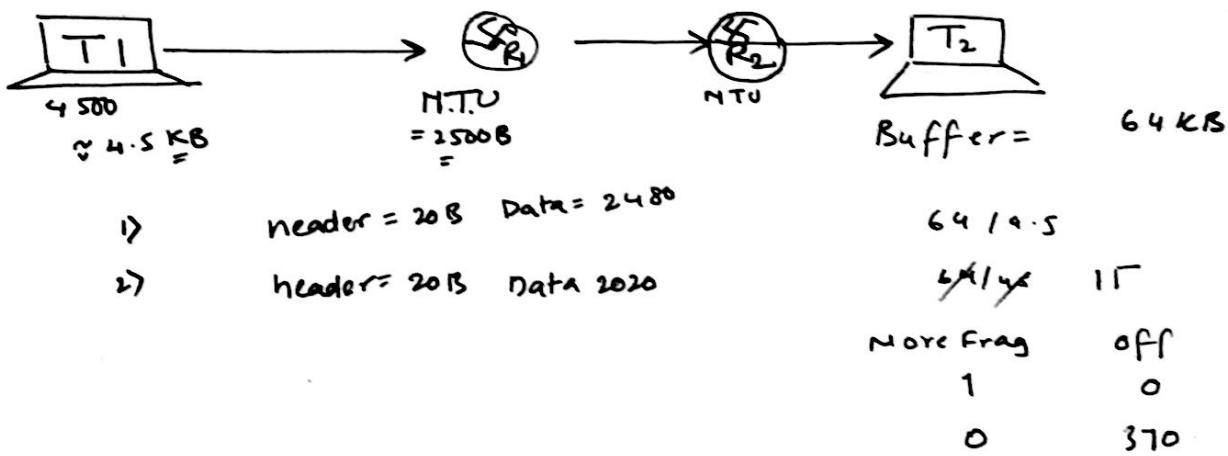
connectionless | connection based

Youtube but normal video streaming uses UDP.

→ SMS → UDP

→ Amazon . NK → TCP.

## Congestion Control



Case 2

Source MTU	Router R <sub>1</sub> MTU	Router R <sub>2</sub> MTU	Destination Buffer
4500B	2500B	1500	64 KB / buffer
h=20 D=2480	h=20 d= 1480	1 0	
h=20 D=2020	h=20 d= 1000	1 1480/15	
h=20	h=20 d= 1480	1 26180/15	
	h=20 d= 540	0 3960/15	

11/4/13

## Session Layer

E      Security. | Privacy.

E

unauthorized  
No modification  
available  
etc



E

## CIA Properties

- Attacks

EVE (Adversary)

Alice ————— M —————> Bob

- Message not received.
- Message received is altered.
- M is overhead.
- corrupted.
- delayed.
- fabrication of new msg.
- Stealing ID of sender
- His broadcasted.
- Replay Message.

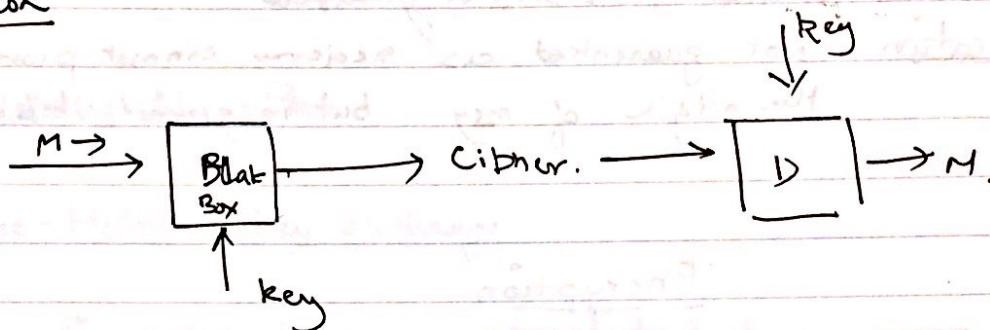
あいえお  
かきくけこ  
さしすせそ

## Attacks

Passive

Active

## Encryption



1) Cyphertext attack.

$$c \rightarrow M$$

$$m, c \in A^*$$

Algo

$$c = (m+k) \% 26$$

} if  $k=3$  Caesar cipher

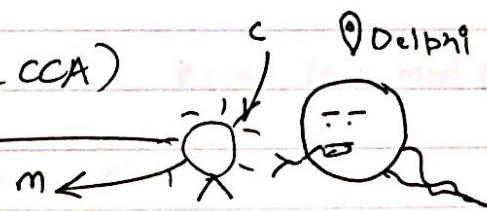
2) Plain Text Attack

$$P_1, P_2, \dots, P_n$$

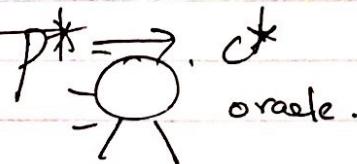
$$P^* \rightarrow C^*$$

3) Chosen Cipher Attack (CCA)

Choose from set of ciphers



4) Chosen Plain Text Attack



$$m \longrightarrow \text{encrypt} \longrightarrow c = E(m, k) \rightarrow D(c, k)$$

→ Encryption

→ achieves confidentiality.

(only when safe) → Integrity. ~~Integrity is still not guaranteed.~~ ✓  
from attacks

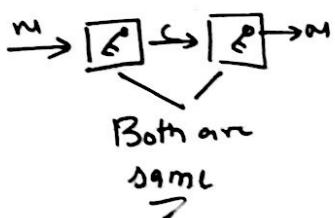
→ Availability is still not guaranteed.

→ Authentication: Not guaranteed as receiver cannot prove the origin of msg. but in general it is.

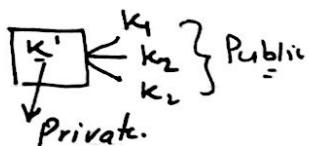
### Encryption

#### Symmetric

- Locks the msg with key



#### Asymmetric (Public Key)



- sharing the keys is hard
- requires a lot of keys.

Problem with symmetric

- $\sim C_2$  pairs
- Sharing the key. (?)

User  $\xrightarrow{z}$  gmail / Amazon.

### Key Sharing - Algorithm

### Diffie-Hellman Key Exchange

$\mathbb{Z}_7 = \{1, 2, \dots, 5, 6\} \text{ if } 7 \text{ is a group.}$   
 $\uparrow$   
prime

extended euclidean alg / PLT  
Fermat's little theorem.

$\mathbb{Z}_p^*$  is a group  $G \subset (G, p, g)$   $p = \text{prime no.}$   
 $\xrightarrow{\text{generator}}$

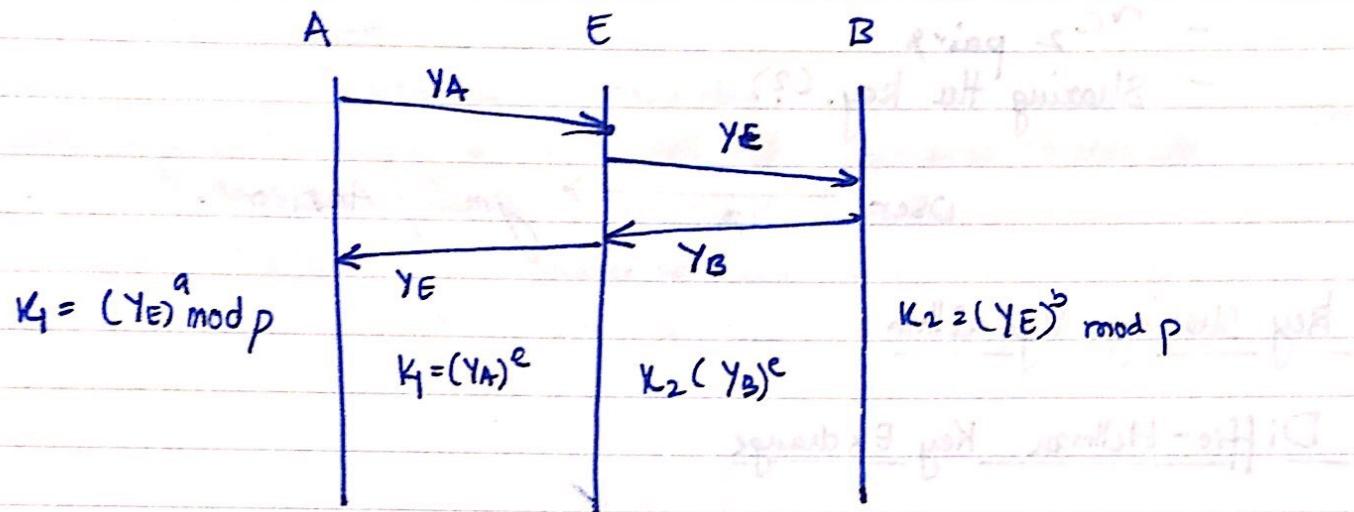
Alice.  
Random 'a'  
 $y_A = g^a \bmod p$

Eve  
 $y_A \rightarrow$  Bob.  
 $'b'$   $y_B = g^b \bmod p$

Given  $y_A, y, p$  we cannot  
find 'a':  
 $K_1 = (y_B)^a \bmod p$

$$K_2 = (y_A)^b \bmod p.$$
$$K_1 = K_2$$
$$g^{ab} \bmod p$$
$$y_A \cdot y_B = g^{(a+b)} \bmod p$$

Drawback  $\rightarrow$  No authentication



No authentication in the middle attack / Mit.

Solution: Add authentication.

Everyone has a digital signature.

Digital Signatures.

$$\begin{aligned} & (Sk, Pu) \\ & (Pu, Pubk) \end{aligned} \quad \left. \begin{array}{l} \\ \end{array} \right\} \text{Public Key Cryptography.}$$

$m = D(E(m, Sk), pk).$

$m = D(E(m, pk), pk).$

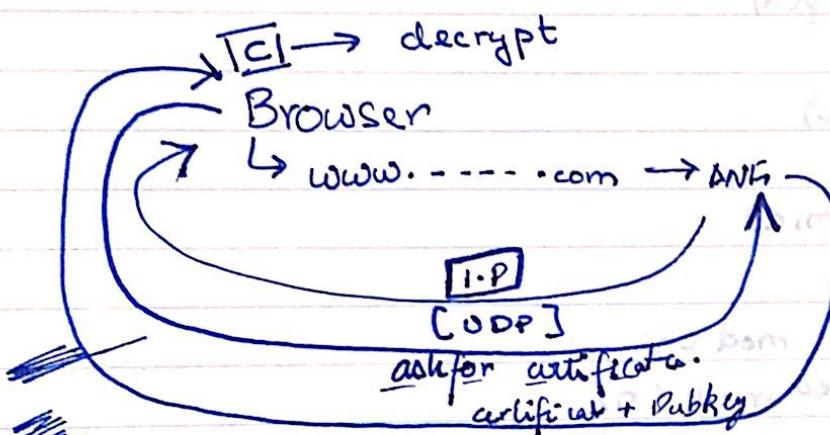
→ receiver is authentic

$A \rightarrow Y_A$  encrypt with  $S_k$ .  
 $B \rightarrow Y_B$  encrypt with  $S_k$ .

Certificate authority.

SSL

Secure Sockets Layer



Browser → Machine go → Server.

DES, AES, IDEA

↳ symmetric protocol.

$\begin{cases} 128 \text{ bit} \\ 256 \text{ bit} \end{cases}$

alt to DHKS is MAC digest.

RSA

[Rivest & Shamir & Adleman]

$$\phi(n) = (p-1)(q-1)$$

Toiler's

$$d \times e \bmod \phi(n) = 1$$

$$e = d^{-1} \bmod \phi(n)$$

$$c = m^d \bmod n$$

$$c = E(m, d)$$

$$D(c, e) = c^e \bmod n \\ = m \bmod n$$

$$ed \equiv 1 \\ = m$$

- Fermat's little theorem
- Extended Euclidean
- Euler's theorem

19/4/18

## 27 Elgamal Encryption Key

$$G: \mathbb{Z}_p^* \ni g$$

Public  $(G, g, p)$

$$\mathbb{Z}_p = \{1, 2, 3, \dots, 10\}$$

↳ generates the group.

$$c = (y^x m \bmod p)$$

$$c = (c \cdot k) \bmod (p).$$

~~g<sup>x</sup>~~

$$(y^x)^{-1} m [g^x]^{-1} \bmod p.$$

$$p = 107$$

$$g = 2$$

$$\text{Rkey} = 67$$

$$b = 45$$

$$\text{Aliu} \xleftarrow{B=66} \text{Bob}$$

$$b = 45$$

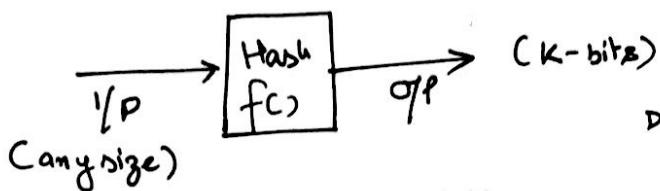
$$\begin{array}{|c|c|} \hline \text{Pubkey} = 97 & c = 28 \\ \hline \end{array} \quad \text{Prkey}_{\text{Bob}} =$$

Find vulnerability in Elgamal Enc  
10 marks

25/4/18

## Hashing -

1>



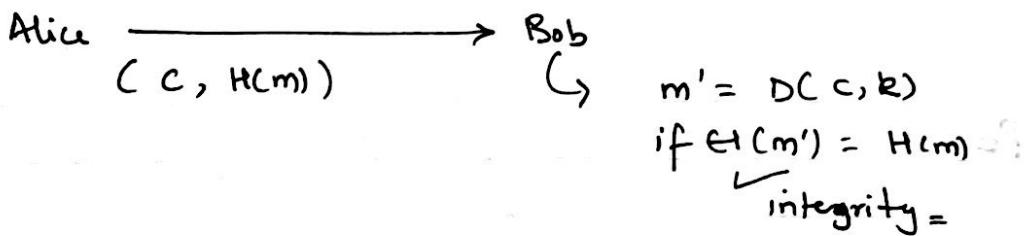
A                      B  
 $C = g^m \cdot (y_A)^{x_B}$

$$\begin{aligned} D(c, a) &= c \cdot (y_B)^{a_1} \bmod p \\ &= g^m (y_A^b) (y_B^{a_1})^{-1} \bmod p \\ &= g^m y \bmod p \end{aligned}$$

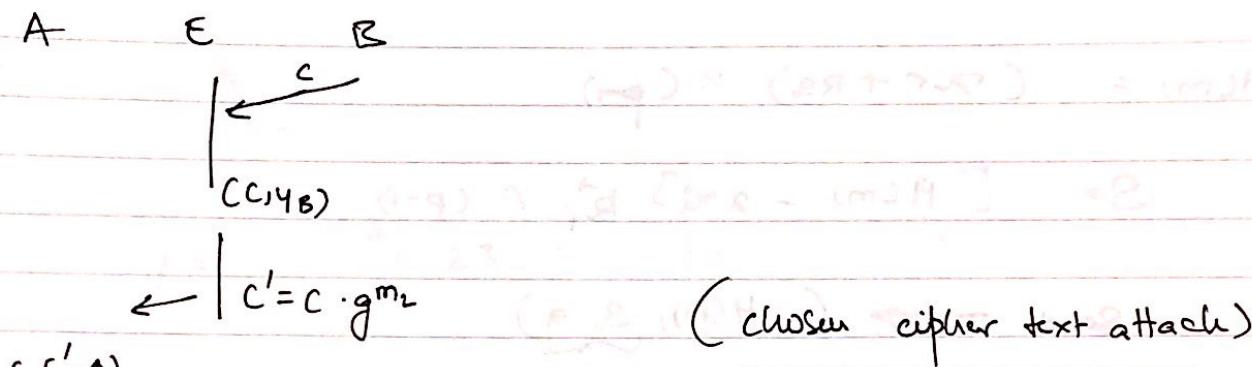
- 2> Irreversible function.  $f_{\text{inv}}$  exists  $f_{\text{inv}}^2$  does not.
- 3> even if input differs by 1 bit, output differs by many.
- 4) Least collisions

Security : C            I            A

acts like checksum.



if hash of m is added to the data, then we can overcome the chosen cipher text attack.



$$\begin{aligned}
 & (c', r_A) \\
 &= c' (y_B)^{r_A} \cdot p \\
 &= g^m \cdot y_B^{r_A} \cdot g^{m_2} \cdot y_B^{r_A} \cdot p \\
 &= g^{(m+m_2)} \cdot r \cdot p \\
 &= g^{m'} \cdot r \cdot p \quad // \text{message modified. (Integrity Violated)}
 \end{aligned}$$

## Digital Signature Scheme

Sender

$$m \downarrow \quad C = E(m, k) \rightarrow$$

$$H(m) \xrightarrow{\text{Digital Signature } (H(m))}$$

Receiver

$$D(C, k) = m'$$

$$\begin{aligned} & \text{verif } MD \\ &= H(m) \end{aligned}$$

if  $(H(m) = H(m'))$   
 $\rightarrow \underline{\text{verified}}$

Egamil DSS  
 $(G, P, g)$

A

$a \in \mathbb{Z}_p$

$$y = g^a \cdot r \cdot p \quad (\text{pub})$$

$$a = g^k \cdot r \cdot p \quad \text{for some } k \in \{0, 1\}$$

$$H(m) = (x_r + r_s) \cdot (p-1)$$

$$S = [H(m) - x_r] \cdot k_i^{-1} \cdot (p-1)$$

Send  $\rightarrow (H(m), \underbrace{S, a}_{\text{signature}})$

$$\underline{g^{H(m)}} = g^a \cdot g^s$$

Pf:

$$\begin{aligned} g^{a+s} &= (g^a)^r \cdot (g^s)^e \pmod{p-1} \\ &= \underline{g^{(xr+rs)} \pmod{p-1}} \\ &= \underline{g^{H(m)}} \end{aligned}$$

$$\begin{array}{llll} p = 23 & x = 3 & m = 7 & H(m) = 7 \\ g = 5 & G = \mathbb{Z}_{23} \end{array}$$

$$\text{Let } k = 1$$

$$q = g^k \cdot p = 5^1 \cdot 23 = 5$$

$$q = 5$$

$$H = 3 \times 5 + 11 \times 1$$

$$S \neq [7 = 3 \cdot 5] \cdot k_i^{-1} \pmod{p-1}$$

$$\begin{aligned} 7 &- 15 = 1 \\ &\equiv 8 \pmod{23} \end{aligned}$$

$$2 \quad S = 4$$

$$Y = 5^3 \cdot 23 = 10$$

$$y^{r_s} = 10^5 \cdot 5^{14} \cdot 1 \cdot 22 \quad | \quad g^{H(m)} = 5^7$$

$$= 10 \cancel{5^7} \cdot \cancel{23} \cdot 22 \quad | \quad (17) \quad (17)$$

matched

$$k = 9$$

$$S = (7 - 3 \times 5^9 \cdot 23) - 5 \cdot 22$$

$$= 20$$

$$\Rightarrow (H(m), 20, 11).$$

### Euler's Theorem.

$M, N$  s.t. both are coprime ( $\text{GCD}(M, N) = 1$ )

$$N = a^p b^q c^s$$

$$\phi(N) = \left[ (1 - \frac{1}{a}) (1 - \frac{1}{b}) (1 - \frac{1}{c}) \right] \times N$$

euclid's totient  $\Rightarrow [N^{\phi(N)} \bmod N = 1]$

$$32^{32} \mod 9$$

$$a = 3^2$$

$$\begin{aligned}\phi(n) &= (1 - 1/3) \times 9 \\ &= 2/3 \times 3 = 2\end{aligned}$$

$$32^6 \mod 9 = 1$$

$$32^{32} \% 6$$

=

$$\begin{aligned}6 &= 2' 3' \\ &= (1 - 1/2)(1 - 1/3) 6 \\ &= 2\end{aligned}$$

$$\underline{32^2} \mod 6 = 4 \quad 21$$

$$\Rightarrow 1$$

$$32^2 \Rightarrow$$

$$32 \% 9 = 5$$