Flapping-wing Aerial Vehicle Project Report*

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2. Design of the flapping-wing vehicle
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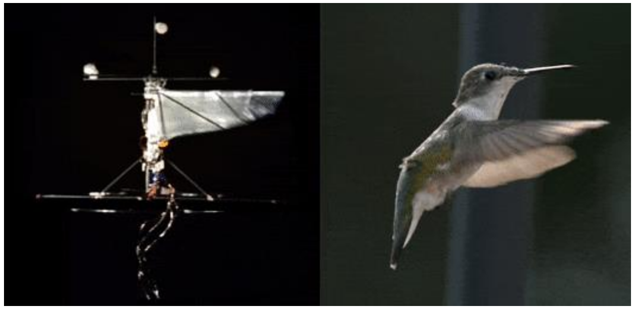
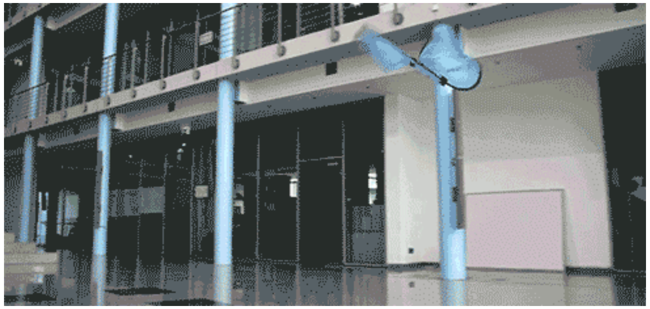
Introduction of flapping-wing vehicle



Hummingbird robot

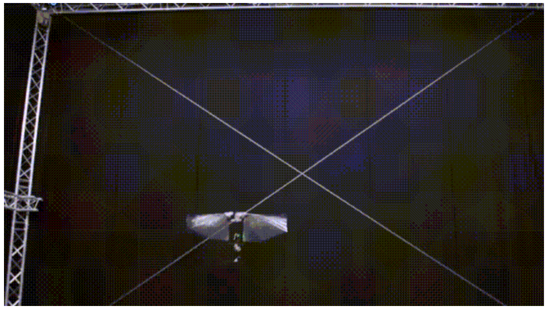
FESTO Smartbird

Bat bot

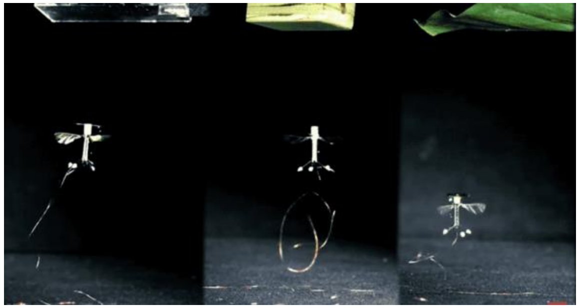


FESTO emotion butterfly

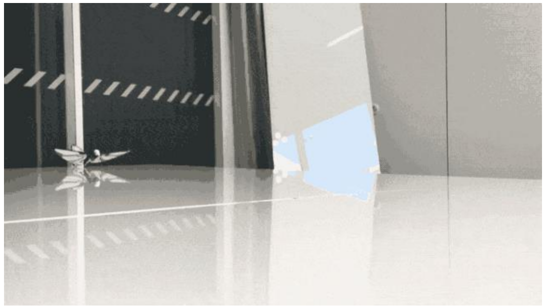
Dragonfly robot



Delfly nimble



Robobee



Metafly

FESTO bionic swift

Introduction of flapping-wing vehicle

Rotor craft

Advantages

- Hovering flight
- Simple structure
- High mobility

Disadvantages

- Low duration
- Aerodynamic efficiency
- Flight noise

Fixed wing craft

Advantages

- High duration
- Aerodynamic efficiency
- Loading capacity

Disadvantages

- Can’t hover
- Low mobility
- Flight noise

Flapping-wing vehicle

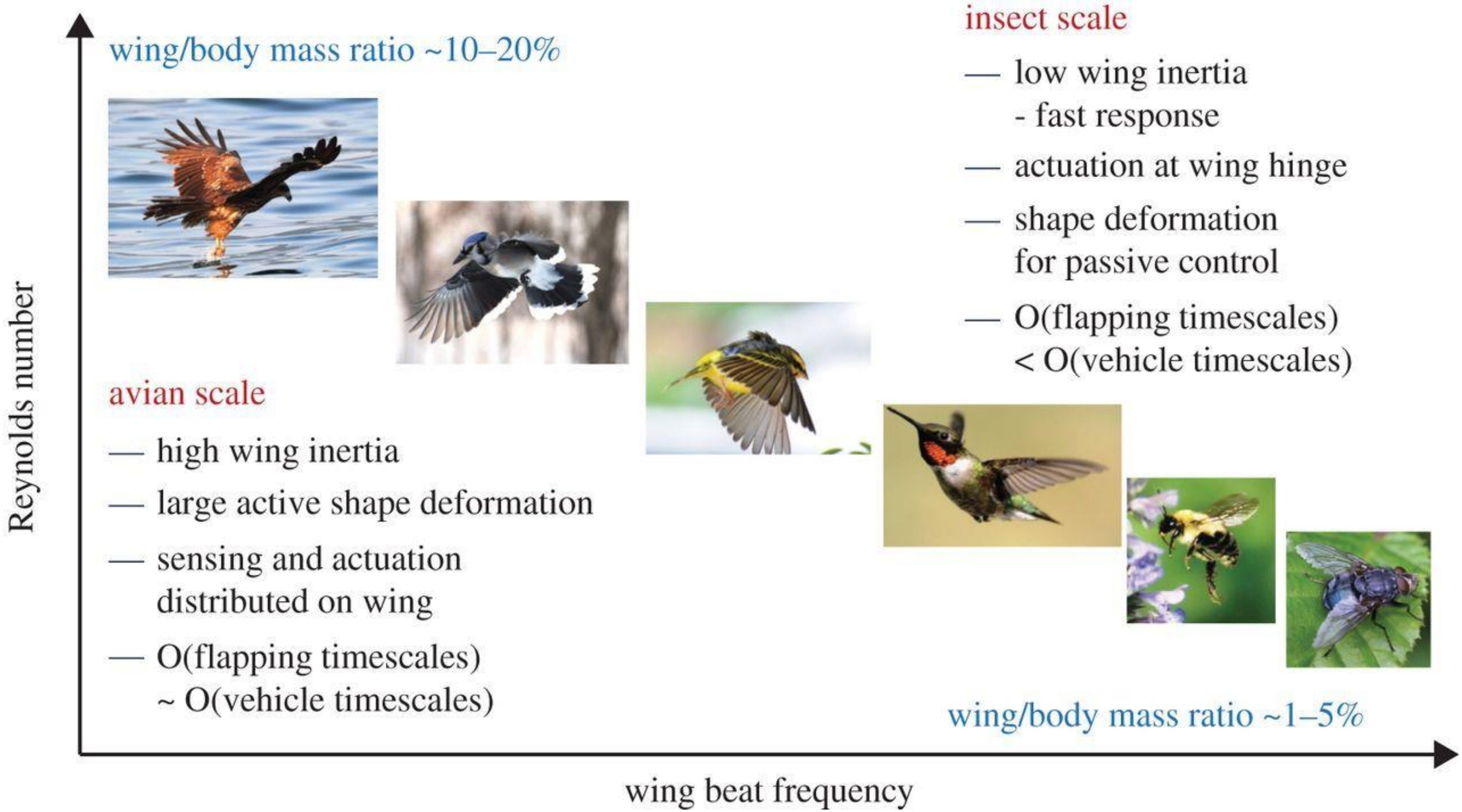
Advantages

- Aerodynamic efficiency
- Low flight noise
- Bionic capacity (conceal)

Disadvantages

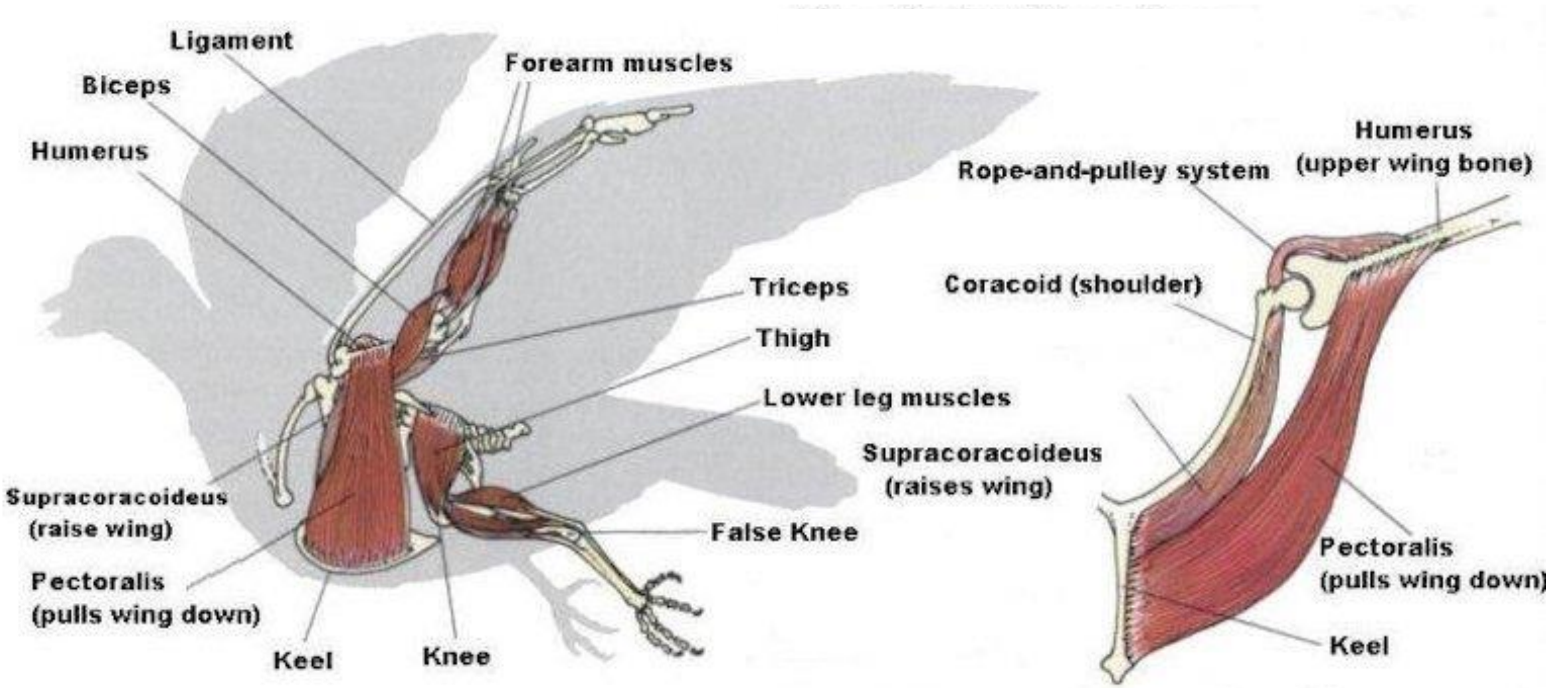
- Complex structure
- Loading capacity
- Vibration

Design of the flapping-wing vehicle



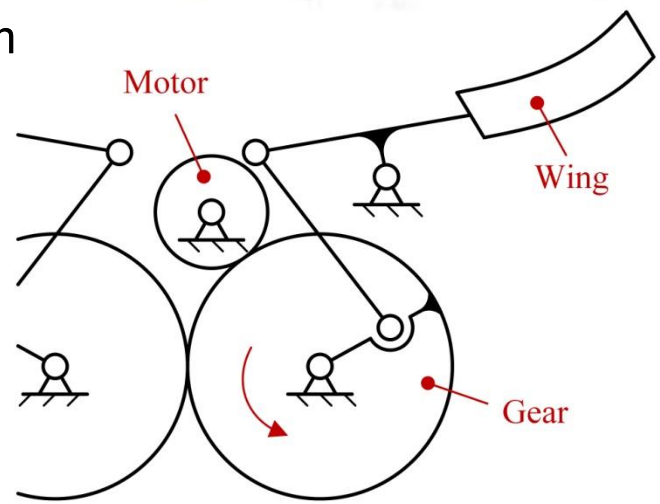
Characteristics of biological flapping flight based on the Reynolds number, flapping frequency and wing/body mass ratio. (Shyy W. et al., 2016)

Design of the flapping-wing vehicle



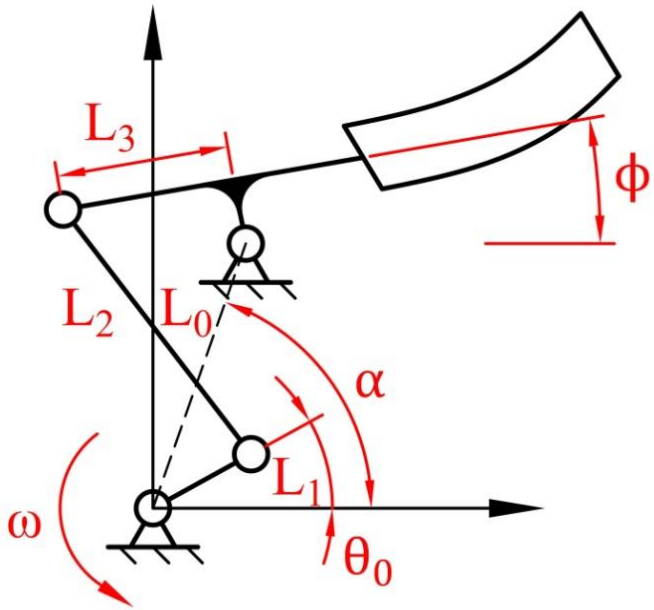
Bird's muscle system

Seagull flight animation



Flapping-wing mechanism

Design of the flapping-wing vehicle



Parameters of four-bar linkage

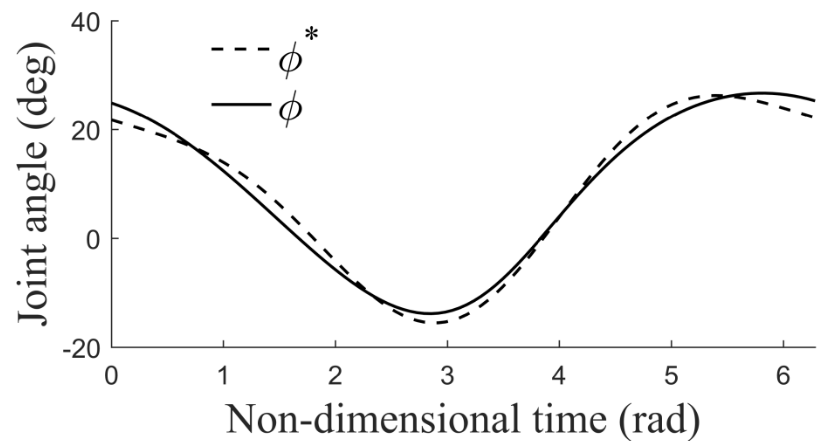
Parameters	Value
θ_0	296.16°
α	38.56°
L_0	47.86 mm
L_1	13.25 mm
L_2	42.12 mm
L_3	40 mm

Target flapping wing motion (Liu T., et al, 2006):

$$\phi^*(t) = 8.4654 - 8.5368 \cdot sin(\omega t) + 17.8798 \cdot cos(\omega t) + 1.0898 \cdot sin(2\omega t) - 4.5880 \cdot cos(2\omega t)$$

Optimization problem:

$$\begin{cases} \min f(X) = \sum_{i=1}^{100} [\phi(i) - \phi^*(i)]^2 \\ X \in [\theta_0, \alpha, L_0, L_1, L_2]^T \\ \text{S.T.}\ L_1 = \min\{L_0, L_1, L_2, L_3\} \\ \qquad \max\{L_0, L_1, L_2, L_3\} + L_1 < \dfrac{\text{sum}\{L_0, L_1, L_2, L_3\}}{2} \end{cases}$$

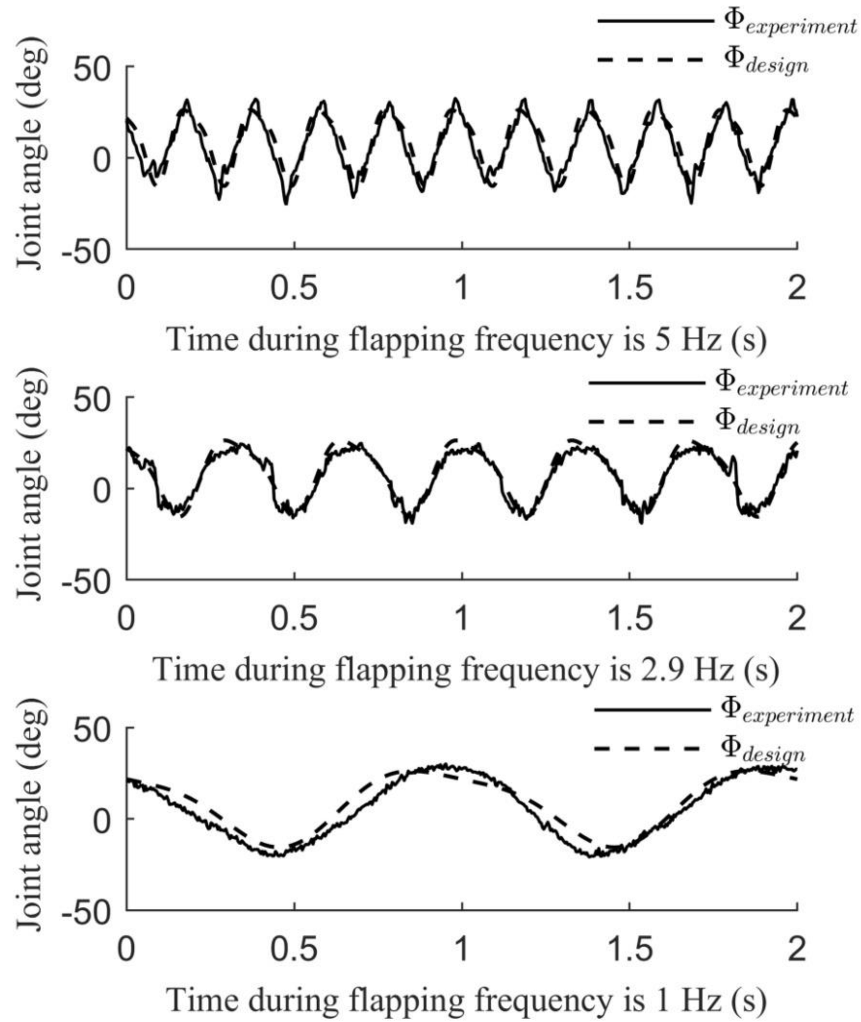


Shoulder joint angle during one beat

Design of the flapping-wing vehicle

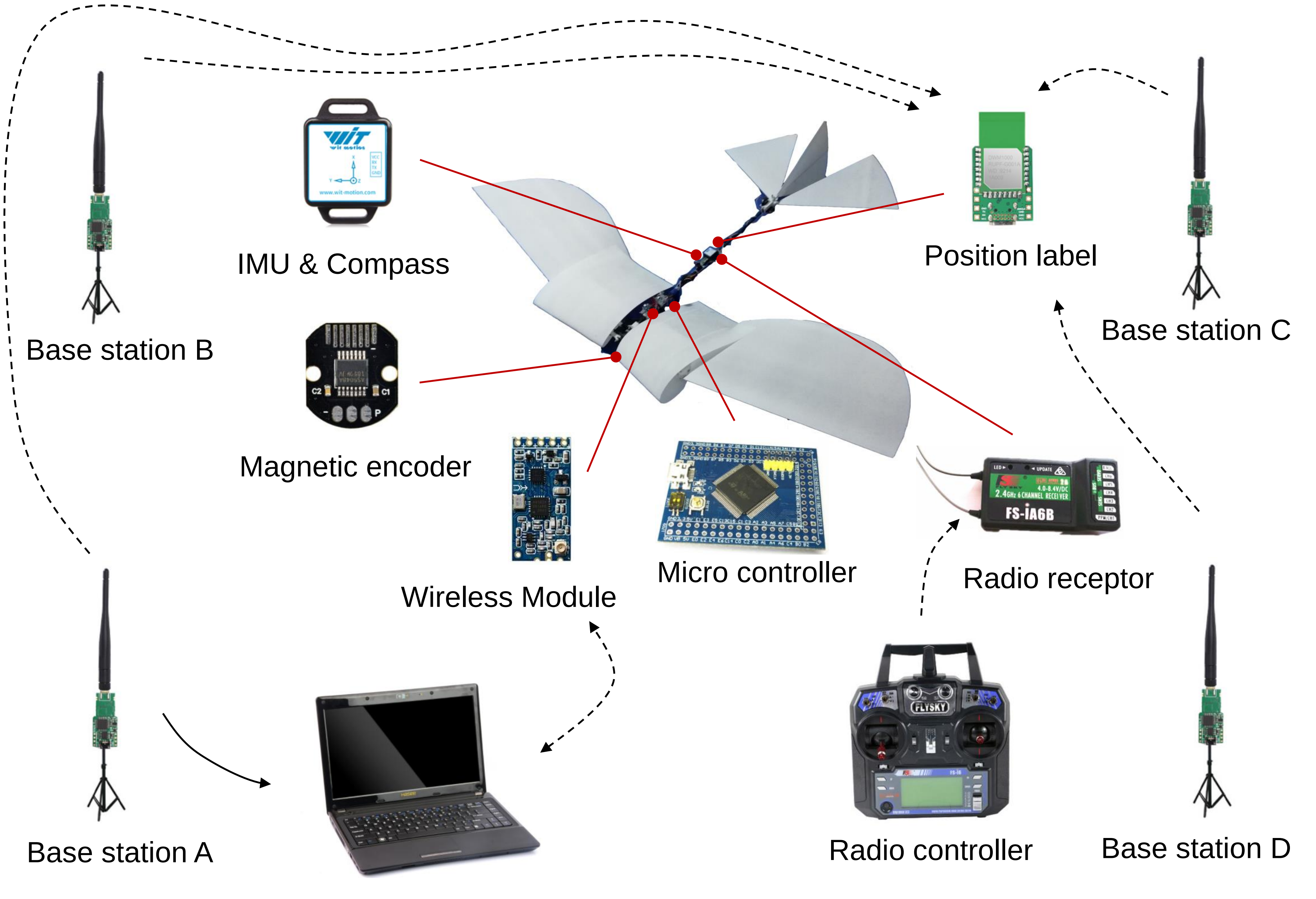
Wing motion at 2.9 Hz (speed X 0.02)

Wing motion at 2.9 Hz (speed X 0.02)

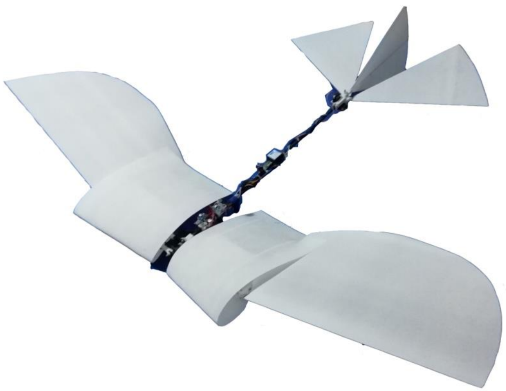


Shoulder joint angle variation

Design of the flapping-wing vehicle



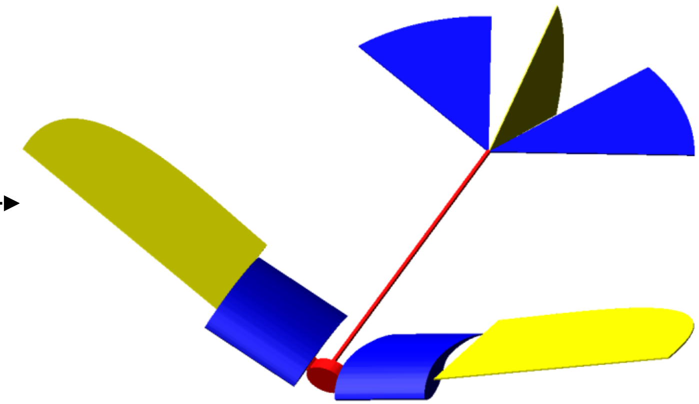
Mathematical modeling



Physical prototype

Modeling process

1. Dynamic modeling
2. Simulation & visualization



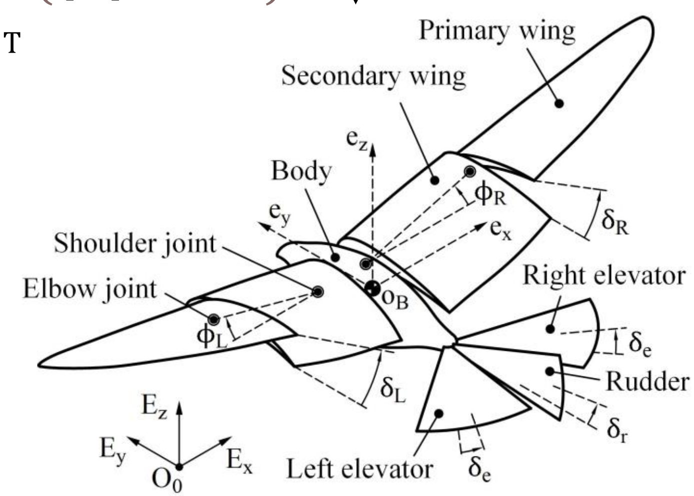
Virtual prototype

$$M(q)\ddot{q} + C(q,\dot{q})q + G(q) = Q(q,\dot{q},\delta,\dot{\delta},u)$$

$$q = [p_x, p_y, p_z, q_x, q_y, q_z, \phi_R, \phi_L]^{\mathrm{T}}$$

Assumptions

- Neglect wing inertial variation
- Ignore mass of tail
- Neglect body's aerodynamic
- Ignore tail's rotational velocity
- Flapping motion follow design

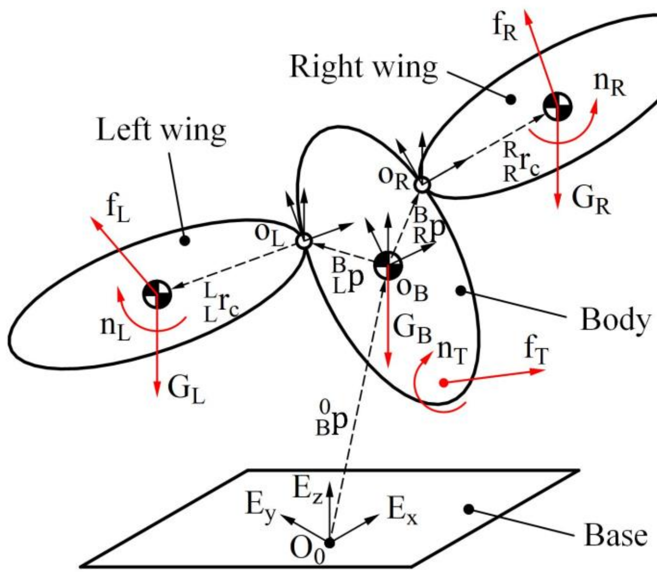


Vehicle dynamic model

$$\dot{x} = f(x,t,u)$$

1. CAD modeling
2. Data interface

Mathematical modeling



Simplified rigid body model

Generalized coordinates

Gravity vector

$$M(q)\ddot{q} + C(q,\dot{q})q + G(q) = Q(q,\dot{q},\delta,\dot{\delta},u)$$

Inertial matrix

Generalized force vector

Coriolis and centrifugal force matrix

Newtonian Euler recursive dynamic modeling

Step 1 (forward kinematics):

Translation

$$\begin{cases} {}^{i+1}_{i+1}\omega = {}^{i+1}_{i}R \cdot {}^{i}_{i}\omega \\ {}^{i+1}_{i+1}\dot{\omega} = {}^{i+1}_{i}R \cdot {}^{i}_{i}\dot{\omega} \\ {}^{i+1}_{i+1}v = \dot{q}_{i+1} \cdot {}^{i}_{i}k + {}^{i+1}_{i}R \cdot ({}^{i}_{i}\omega \times {}^{i}_{i+1}p + {}^{i}_{i}v) \\ {}^{i+1}_{i+1}\dot{v} = \ddot{q}_{i+1} \cdot {}^{i}_{i}k + 2({}^{i+1}_{i+1}\omega \times \dot{q}_{i+1} \cdot {}^{i}_{i}k) \\ \qquad + {}^{i+1}_{i}R \cdot [{}^{i}_{i}\dot{\omega} \times {}^{i}_{i+1}p + {}^{i}_{i}\omega \times ({}^{i}_{i}\omega \times {}^{i}_{i+1}p) + {}^{i}_{i}\dot{v}] \end{cases}$$

Rotation

$$\begin{cases} {}^{i+1}_{i+1}\omega = {}^{i+1}_{i}R \cdot {}^{i}_{i}\omega + \dot{q}_{i+1} \cdot {}^{i}_{i}k \\ {}^{i+1}_{i+1}\dot{\omega} = {}^{i+1}_{i}R \cdot {}^{i}_{i}\dot{\omega} + \ddot{q}_{i+1} \cdot {}^{i}_{i}k + {}^{i+1}_{i}R \cdot [{}^{i}_{i}\omega \times (\dot{q}_{i+1} \cdot {}^{i}_{i}k)] \\ {}^{i+1}_{i+1}v = {}^{i+1}_{i}R \cdot ({}^{i}_{i}\omega \times {}^{i}_{i+1}p + {}^{i}_{i}v) \\ {}^{i+1}_{i+1}\dot{v} = {}^{i+1}_{i}R \cdot [{}^{i}_{i}\dot{\omega} \times {}^{i}_{i+1}p + {}^{i}_{i}\omega \times ({}^{i}_{i}\omega \times {}^{i}_{i+1}p) + {}^{i}_{i}\dot{v}] \end{cases}$$

$${}^{i}_{i}\dot{v}_c = {}^{i}_{i}\dot{v} + {}^{i}_{i}\dot{\omega} \times {}^{i}_{i}r_c + {}^{i}_{i}\omega \times ({}^{i}_{i}\omega \times {}^{i}_{i}r_c)$$

$${}^{i}_{i}f_c = m_i \cdot {}^{i}_{i}\dot{v}_c$$

$${}^{i}_{i}n_c = {}^{i}_{i}I_c \cdot {}^{i}_{i}\dot{\omega} + {}^{i}_{i}\omega \times ({}^{i}_{i}I_c \cdot {}^{i}_{i}\omega)$$

Step 2 (backward dynamics):

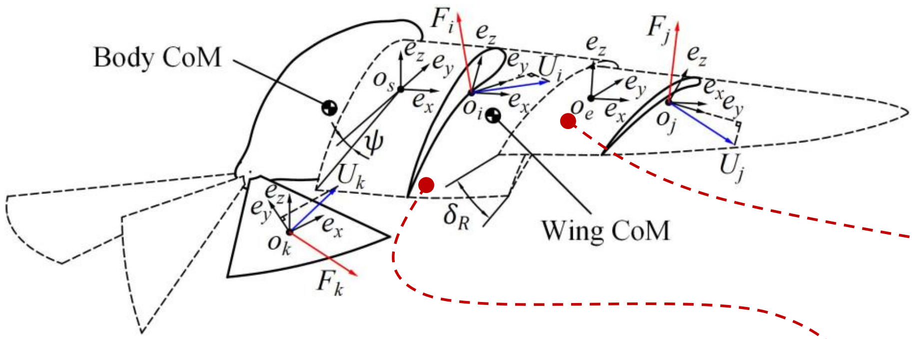
$${}^{i}_{i}f = {}^{i}_{i}f_c + {}^{i}_{i}f_{ext} + {}^{i}_{i+1}R \cdot {}^{i+1}_{i+1}f$$

$${}^{i}_{i}n = {}^{i}_{i}n_c + {}^{i}_{i}n_{ext} + {}^{i}_{i+1}R \cdot {}^{i+1}_{i+1}n$$
$$\qquad + {}^{i}_{i}r_c \times ({}^{i}_{i}f_c + {}^{i}_{i}f_{ext}) + {}^{i}_{i+1}p \times ({}^{i}_{i+1}R \cdot {}^{i+1}_{i+1}f)$$

$$\tau_i = {}^{i}_{i}n \cdot {}^{i}_{i}k$$

$$f_i = {}^{i}_{i}f \cdot {}^{i}_{i}k$$

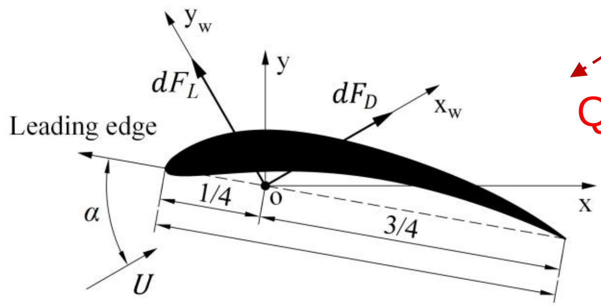
Mathematical modeling



Aerodynamic model

Linear torsion model:

$$\begin{cases} J\ddot{\delta} + C\dot{\delta} + K\delta = \tau \\ \tau = [1\ 0\ 0]\cdot[\sum {}^{e}_{e}r_j \times ({}^{e}_{j}R \cdot \begin{bmatrix} 0 \\ -dF_D(t,r)_j \\ dF_L(t,r)_j \end{bmatrix}) \\ \qquad + {}^{e}_{e}r_c \times ({}^{O}_{e}R^{-1} \cdot \begin{bmatrix} 0 \\ 0 \\ -m_p g \end{bmatrix})] \end{cases}$$



Aerodynamic forces on airfoil

Quasi-steady aerodynamic model

$$\begin{cases} dF_L(t,r) = \frac{1}{2}C_L(\alpha(t))\rho c(r)U^2(t,r)dr \\ \qquad + \frac{1}{2}C_{rot}\rho c^2(r)U(t,r)\dot{\alpha}(t)dr \\ dF_D(t,r) = \frac{1}{2}C_D(\alpha(t))\rho c(r)U^2(t,r)dr \\ C_{rot} = 2\pi((3/4) - \hat{x}_0) \end{cases}$$

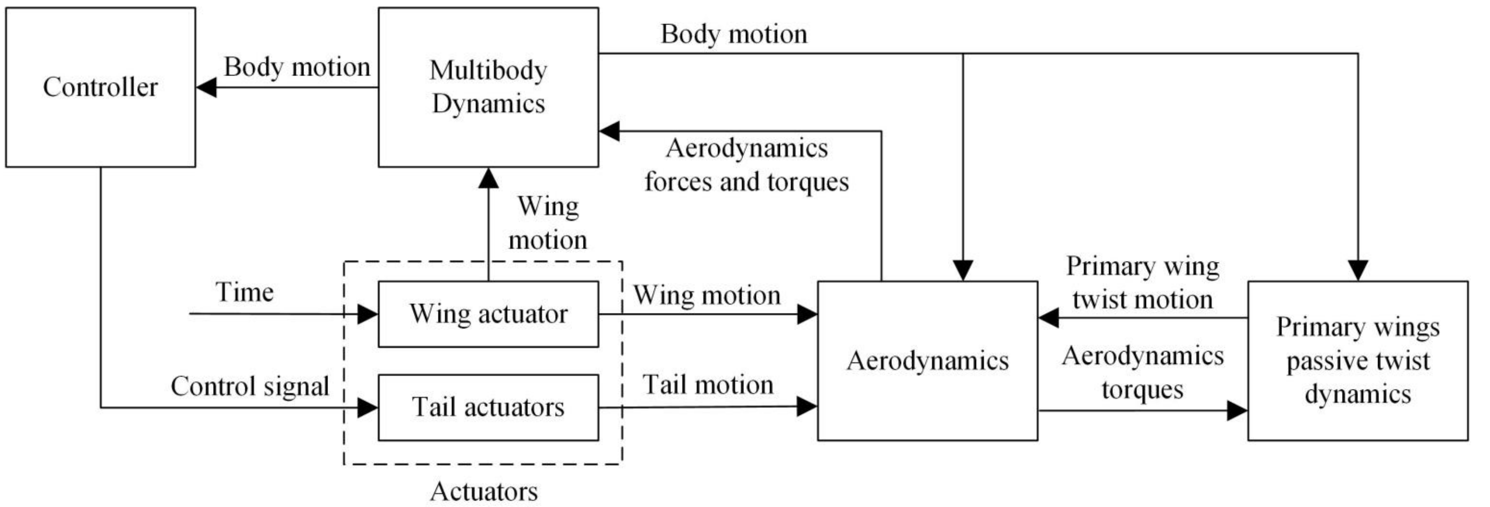
From Dickinson (Dickinson M. H., et al, 1999)

$$C_L = 0.225 + 1.58sin(2.13\alpha - 7.20)$$
$$C_D = 1.92 - 1.55cos(2.04\alpha - 9.82)$$

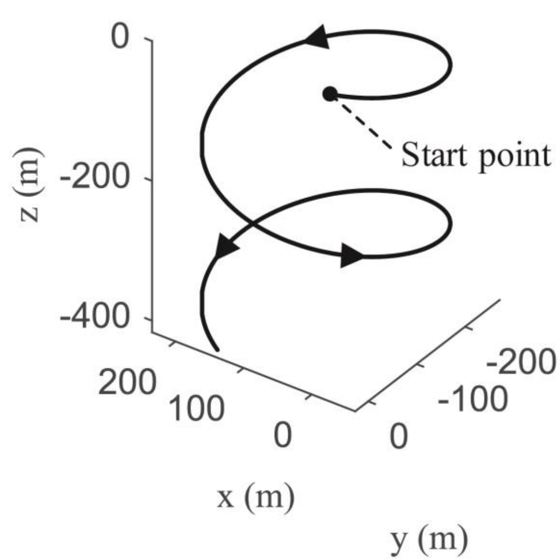
$$\begin{aligned} C_L =& 1.125sin(1.04\alpha + 1.325) \\ &+ 1.126sin(2.056\alpha + 0.06919) \\ C_D =& 5.096sin(0.02086\alpha + 0.2175) \\ &+ 1.087sin(1.945\alpha - 1.505) \end{aligned}$$

From CFD simulation

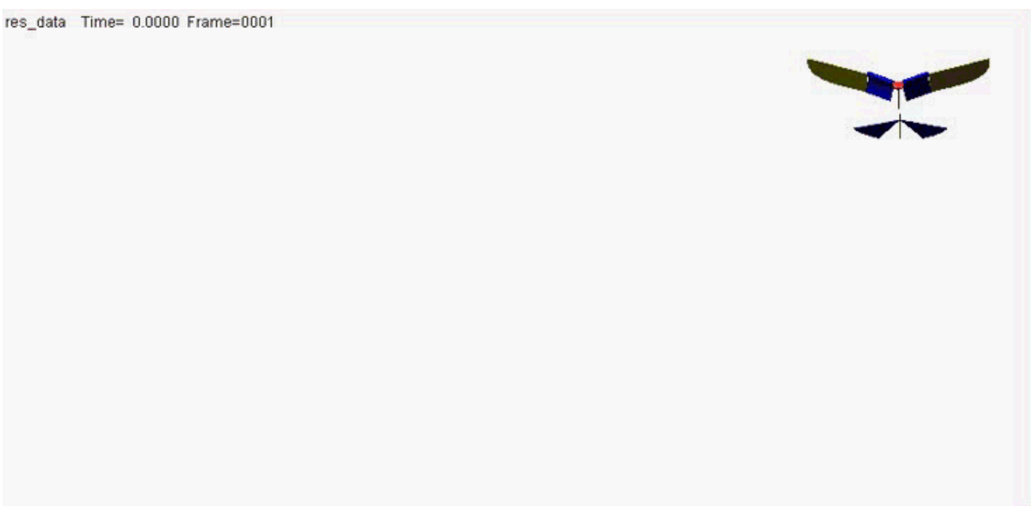
Mathematical modeling



Simulation structure of virtual prototype



Uncontrolled flight trajectory



Flight with simple PID attitude controllers

Nonlinear flight controller design

$$M(q)\ddot{q} + C(q,\dot{q})q + G(q) = Q\left(q,\dot{q},\delta,\dot{\delta},u\right)$$

Approximate decoupling

Attitude dynamic function (robust control)

$$\left\{\begin{aligned}
&\begin{bmatrix} m_{44} & m_{45} & m_{46} & m_{47} & m_{48} \\ m_{54} & m_{55} & m_{56} & m_{57} & m_{58} \\ m_{64} & m_{65} & m_{66} & m_{67} & m_{68} \\ m_{74} & m_{75} & m_{76} & m_{77} & m_{78} \\ m_{84} & m_{85} & m_{86} & m_{87} & m_{88} \end{bmatrix} \begin{bmatrix} \ddot{q}_x \\ \ddot{q}_y \\ \ddot{q}_z \\ \ddot{\phi}_R \\ \ddot{\phi}_L \end{bmatrix} + \begin{bmatrix} c_4 \\ c_5 \\ c_6 \\ c_7 \\ c_8 \end{bmatrix} \\
&= \begin{bmatrix} T_n \cdot \left[{}^{B}_{R}R \cdot \left({}^{R}_{R}n_w + {}^{\mathrm{R}}_{\mathrm{R}}\mathrm{r_c} \times {}^{R}_{R}f_w\right) + {}^{B}_{R}p \times \left({}^{B}_{R}R \cdot {}^{R}_{R}f_w\right) + {}^{B}_{L}R \cdot \left({}^{L}_{L}n_w + {}^{\mathrm{L}}_{\mathrm{L}}\mathrm{r_c} \times {}^{L}_{L}f_w\right) + {}^{B}_{L}p \times \left({}^{B}_{L}R \cdot {}^{L}_{L}f_w\right) + {}^{B}_{B}n_T\right] \\ [0 \quad 1 \quad 0] \cdot \left({}^{R}_{R}n_w + {}^{\mathrm{R}}_{\mathrm{R}}\mathrm{r_c} \times {}^{R}_{R}f_w\right) + n_R \\ [0 \quad 1 \quad 0] \cdot \left({}^{L}_{L}n_w + {}^{\mathrm{L}}_{\mathrm{L}}\mathrm{r_c} \times {}^{L}_{L}f_w\right) + n_L \end{bmatrix} \qquad ① \\
&\begin{bmatrix} m_{11} & 0 & 0 \\ 0 & m_{22} & 0 \\ 0 & 0 & m_{33} \end{bmatrix} \begin{bmatrix} \ddot{p}_x \\ \ddot{p}_y \\ \ddot{p}_z \end{bmatrix} + \begin{bmatrix} c_1 \\ c_2 \\ c_3 \end{bmatrix} = {}^{O}_{B}R \cdot \left({}^{B}_{R}R \cdot {}^{R}_{R}f_w + {}^{B}_{L}R \cdot {}^{L}_{L}f_w + {}^{B}_{B}f_T\right) \qquad ②
\end{aligned}\right.$$

Position dynamic function (tracking control)

$$\left\{\begin{aligned}
&\mathrm{m}_{44}\ddot{q}_x + \mathrm{m}_{45}\ddot{q}_y + \mathrm{m}_{46}\ddot{q}_z + \mathrm{m}_{47}\ddot{\phi}_R + \mathrm{m}_{48}\ddot{\phi}_L + c_4 = f_{w1}(q,\dot{q}) + f_{t1}(q,\dot{q},\delta_r,\delta_e) \\
&\mathrm{m}_{54}\ddot{q}_x + \mathrm{m}_{55}\ddot{q}_y + \mathrm{m}_{56}\ddot{q}_z + \mathrm{m}_{57}\ddot{\phi}_R + \mathrm{m}_{58}\ddot{\phi}_L + c_5 = f_{w2}(q,\dot{q}) + f_{t2}(q,\dot{q},\delta_r,\delta_e) \\
&\mathrm{m}_{64}\ddot{q}_x + \mathrm{m}_{65}\ddot{q}_y + \mathrm{m}_{66}\ddot{q}_z + \mathrm{m}_{67}\ddot{\phi}_R + \mathrm{m}_{68}\ddot{\phi}_L + c_6 = f_{w3}(q,\dot{q}) + f_{t3}(q,\dot{q},\delta_r,\delta_e)
\end{aligned}\right. \qquad ③$$

Where, $\mathrm{m}_{44} \gg \mathrm{m}_{45}, \mathrm{m}_{44} \gg \mathrm{m}_{46}, \mathrm{m}_{55} \gg \mathrm{m}_{54}, \mathrm{m}_{55} \gg \mathrm{m}_{56}, \mathrm{m}_{66} \gg \mathrm{m}_{64}, \mathrm{m}_{66} \gg \mathrm{m}_{65}$

Nonlinear attitude controller

$$\begin{cases} \ddot{q}_x = \dfrac{1}{m_{44}}\left[f_{w1}(q,\dot{q},\delta_R,\delta_L,\dot{\delta}_R,\dot{\delta}_L) - c_4 - m_{47}\ddot{\phi}_R - m_{48}\ddot{\phi}_L\right] + \Delta_1 + \dfrac{f_{t1}(q,\dot{q},0,0)}{m_{44}} + h_{r1}(q,\dot{q})\delta_r + h_{e1}(q,\dot{q})\delta_e + \Delta_{t1} \\ \ddot{q}_y = \dfrac{1}{m_{55}}\left[f_{w2}(q,\dot{q},\delta_R,\delta_L,\dot{\delta}_R,\dot{\delta}_L) - c_5 - m_{57}\ddot{\phi}_R - m_{58}\ddot{\phi}_L\right] + \Delta_2 + \dfrac{f_{t2}(q,\dot{q},0,0)}{m_{55}} + h_{r2}(q,\dot{q})\delta_r + h_{e2}(q,\dot{q})\delta_e + \Delta_{t2} \\ \ddot{q}_z = \dfrac{1}{m_{66}}\left[f_{w3}(q,\dot{q},\delta_R,\delta_L,\dot{\delta}_R,\dot{\delta}_L) - c_6 - m_{67}\ddot{\phi}_R - m_{68}\ddot{\phi}_L\right] + \Delta_3 + \dfrac{f_{t3}(q,\dot{q},0,0)}{m_{66}} + h_{r3}(q,\dot{q})\delta_r + h_{e3}(q,\dot{q})\delta_e + \Delta_{t3} \end{cases}$$

Affine form

④

Where,
$$\begin{cases} h_{ri}(q,\dot{q}) = \dfrac{\dfrac{f_{t1}(q,\dot{q},h,0)}{m_{i+3\,i+3}} - \dfrac{f_{t1}(q,\dot{q},0,0)}{m_{i+3\,i+3}}}{h} \\ h_{ei}(q,\dot{q}) = \dfrac{\dfrac{f_{t1}(q,\dot{q},0,h)}{m_{i+3\,i+3}} - \dfrac{f_{t1}(q,\dot{q},0,0)}{m_{i+3\,i+3}}}{h} \end{cases}$$

$$h_{r1}(q,\dot{q}) \approx 0, h_{r2}(q,\dot{q}) \approx 0, h_{e2}(q,\dot{q}) \approx 0, h_{e3}(q,\dot{q}) \approx 0$$

$$\begin{cases} \ddot{q}_x = \dfrac{1}{m_{44}}\left[f_{w1}(q,\dot{q},\delta_R,\delta_L,\dot{\delta}_R,\dot{\delta}_L) - c_4 - m_{47}\ddot{\phi}_R - m_{48}\ddot{\phi}_L\right] + \dfrac{f_{t1}(q,\dot{q},0,0)}{m_{44}} + h_{e1}(q,\dot{q})\delta_e + (\Delta_1 + \Delta_{t1}) \\ \ddot{q}_z = \dfrac{1}{m_{66}}\left[f_{w3}(q,\dot{q},\delta_R,\delta_L,\dot{\delta}_R,\dot{\delta}_L) - c_6 - m_{67}\ddot{\phi}_R - m_{68}\ddot{\phi}_L\right] + \dfrac{f_{t3}(q,\dot{q},0,0)}{m_{66}} + h_{r3}(q,\dot{q})\delta_r + (\Delta_3 + \Delta_{t3}) \end{cases}$$

⑤

$$\begin{cases} \dot{e}_x = -\dot{q}_x \\ \ddot{e}_x = f_x(q,\dot{q},\delta_R,\delta_L,\dot{\delta}_R,\dot{\delta}_L,\ddot{\phi}_R,\ddot{\phi}_L) + g_x(q,\dot{q})\delta_e + \Delta_x \\ \dot{e}_z = -\dot{q}_z \\ \ddot{e}_z = f_z(q,\dot{q},\delta_R,\delta_L,\dot{\delta}_R,\dot{\delta}_L,\ddot{\phi}_R,\ddot{\phi}_L) + g_z(q,\dot{q})\delta_r + \Delta_z \end{cases}$$

⑥

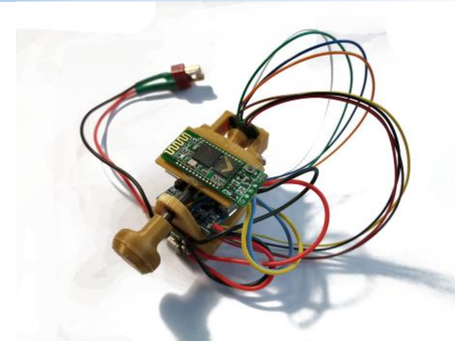
Suitable for sliding mode method

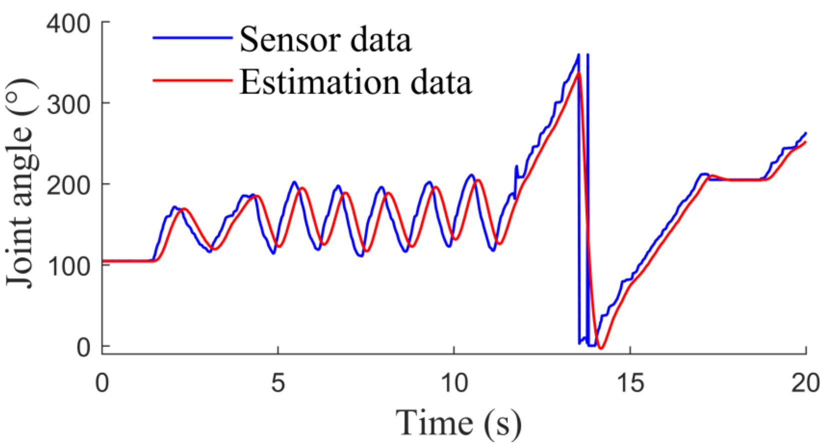
Where, $e_x = q_x^{target} - q_x$ and $e_z = q_z^{target} - q_z$

Flapping motion estimation

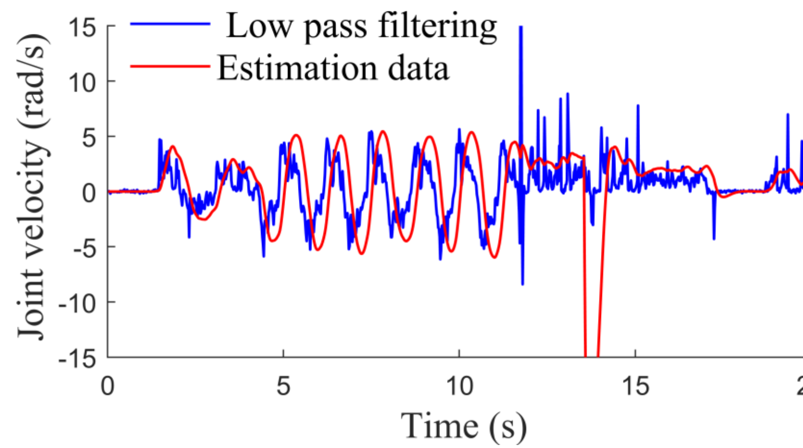
Tracking Differentiator:

$$\begin{cases} \phi(k+1) = \phi(k) + h \cdot \dot{\phi}(k) \\ \dot{\phi}(k+1) = \dot{\phi}(k) + h \cdot \ddot{\phi}(k) \\ \ddot{\phi}(k+1) = \ddot{\phi}(k) + h \cdot R^3 \left[u(k) - \phi(k) - \dfrac{\dot{\phi}(k)}{R} - \dfrac{\ddot{\phi}(k)}{R^2} \right] \end{cases}$$





Joint angle measurement



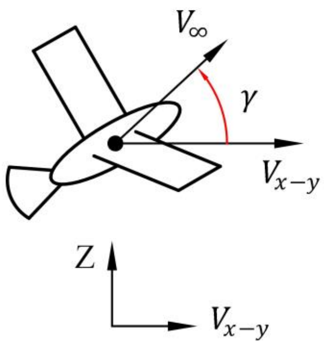
Joint velocity estimation

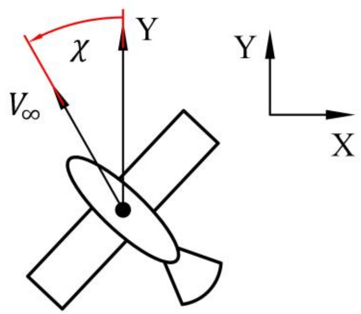
Secondary wing deformation:

$$J\ddot{\delta} + C\dot{\delta} + K\delta = \tau \quad \xrightarrow[J \ll K,\ C \ll K]{\text{Simplified model}} \quad K\delta = \tau(\delta) \quad \text{(Algebraic equation)}$$

Path tracking controller

① $$\begin{bmatrix} m_{11} & 0 & 0 \\ 0 & m_{22} & 0 \\ 0 & 0 & m_{33} \end{bmatrix} \begin{bmatrix} \ddot{p}_x \\ \ddot{p}_y \\ \ddot{p}_z \end{bmatrix} + \begin{bmatrix} c_1 \\ c_2 \\ c_3 \end{bmatrix} = {}^{O}_{B}R \cdot \left({}^{B}_{R}R \cdot {}^{R}_{R}f_w + {}^{B}_{L}R \cdot {}^{L}_{L}f_w + {}^{B}_{B}f_T \right)$$





Coordinate transformation

② $$\begin{cases} \begin{bmatrix} \dot{p}_x \\ \dot{p}_y \\ \dot{p}_z \end{bmatrix} = \begin{bmatrix} -V_\infty \cdot \cos\gamma \cdot \sin\chi \\ V_\infty \cdot \cos\gamma \cdot \cos\chi \\ V_\infty \cdot \sin\gamma \end{bmatrix} \\ \begin{bmatrix} \dot{V}_\infty \\ \dot{\gamma} \\ \dot{\chi} \end{bmatrix} = \begin{bmatrix} \sqrt{\ddot{p}_x^2 + \ddot{p}_y^2 + \ddot{p}_z^2} \\ \dfrac{\ddot{p}_z \cdot \cos\gamma}{V_\infty} - \dfrac{\ddot{p}_x \cdot \dot{p}_z \cdot \dot{p}_x + \ddot{p}_y \cdot \dot{p}_z \cdot \dot{p}_y}{V_\infty^3 \cdot \cos\gamma} \\ \dfrac{\ddot{p}_y \cdot \dot{p}_x - \ddot{p}_x \cdot \dot{p}_y}{\dot{p}_x^2 + \dot{p}_y^2} \end{bmatrix} \end{cases}$$

Average dynamic function

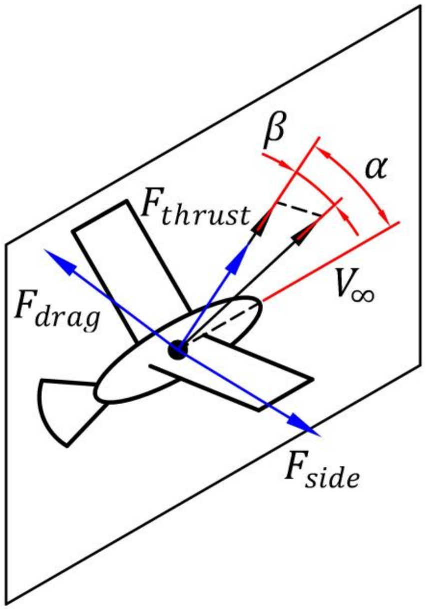
③ $$\begin{bmatrix} \dot{\bar{V}}_\infty \\ \dot{\bar{\gamma}} \\ \dot{\bar{\chi}} \end{bmatrix} = f\left(\bar{V}_\infty, \bar{\gamma}, \bar{\chi}, \underline{\bar{q}_x, \bar{q}_y, \bar{q}_z}\right)$$

Virtual control input, $\bar{q}_y \approx 0$

New problems

- Built three-dimension function f
- Design path tracking strategy

Path tracking controller

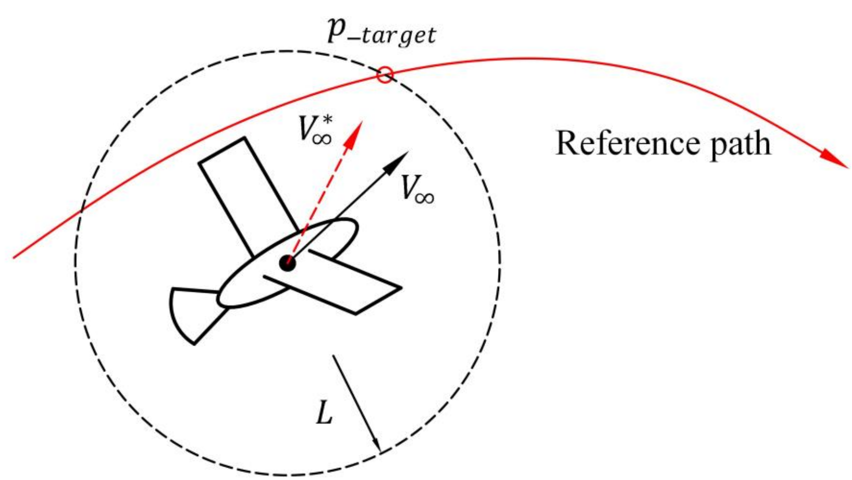


$$
\begin{cases}
F_{thrust} = \frac{1}{2}\rho \cdot C_{thrust}(\alpha, f) \cdot V_\infty^2 \\
F_{drag} = \frac{1}{2}\rho \cdot C_{drag}(\alpha, f) \cdot V_\infty^2 \\
F_{side} = \frac{1}{2}\rho \cdot C_{side}(\beta) \cdot V_\infty^2
\end{cases}
$$

Where, C_{thrust}, C_{drag} and C_{side} need to be identified.
And, $\alpha \approx \gamma - q_x$, $\beta \approx q_z - \chi$

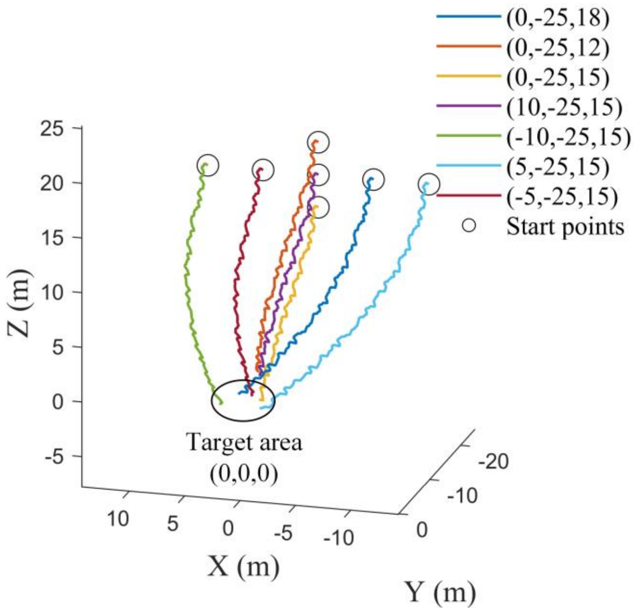
Pure-pursuit or MPC ?

$$V_\infty^* \xrightarrow{\text{Geometric method}} \gamma^*, \chi^* \xrightarrow{\text{Optimization method}} q_x^*, q_z^* \xrightarrow{\text{Sliding mode method}} \delta_r, \delta_e$$

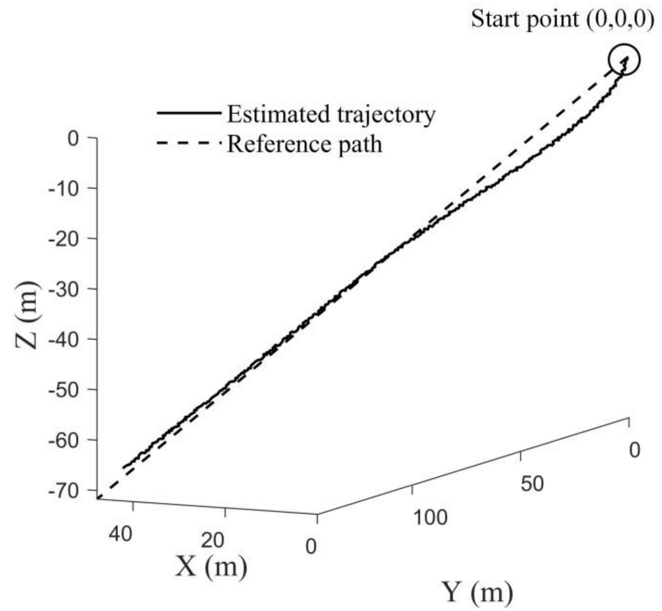


Virtual tracking experiments

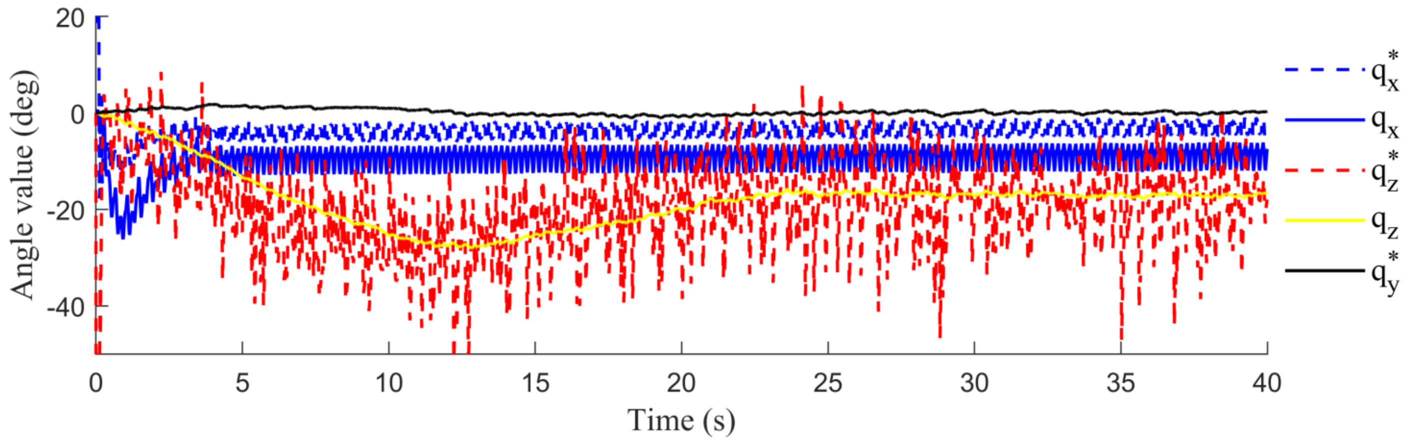
Experiment I (fixed point tracking)



Experiment II (path tracking)



Initial condition: flapping frequency 5 Hz, velocity 2 m/s along Y axis



Attitude data at experiment II

Conclusions

- Background of flapping-wing vehicle is simply introduced.
- Design process of the vehicle prototype is listed.
- Process of Mathematical modeling is derived.
- Nonlinear path tracking controller is designed.
- Two tracking experiments is demonstrated.