



## Path and Function Synthesis of Multi-bar Mechanisms Using Beetle Antennae Search Algorithm

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**Abstract.** Beetle antennae search algorithm (BAS) is based on the searching behavior of longhorn beetles. This newly proposed metaheuristic algorithm is used in the path and function synthesis of multi-bar mechanism. The optimization results are compared with that of other metaheuristic algorithms like genetic algorithm (GA), particle swarm optimization (PSO) and differential evolution (DE) et al. While BAS uses only one group of initial parameters to search the best result, the convergence and efficiency are tested with eight case studies results. Newly added parameter  $q$  increases the possibility to find better results during iterations and deals with a higher number of independent parameters efficiently. Revised BAS exhibits good performance both in path synthesis of four-bar mechanism with or without prescribed timing, while optimized parameters increases from 6 to 34, and extends to path and function combined synthesis of Stephenson III six-bar double dwell mechanism with 14 parameters.

### 1. Introduction

Mechanism synthesis includes function, motion and path generation [1]. The most studied problems are path synthesis of four-bar mechanism, where the coupler can pass a series of desired points with or without prescribed timing [2–20], and function and path combined synthesis of six-bar double dwell mechanism, where the coupler can pass through a series of desired points and the output link can realize the desired angles during the dwell portion [14, 21, 22]. The six-bar dwell mechanism works as an alternative of cam mechanism and to meet certain requirements which are hardly satisfied by four-bar mechanism sometimes [22]. In the optimization of path and function synthesis, the optimization goal is to minimize the combination of the summation square errors of the obtained and desired coupler points and summation square errors of the obtained and desired output angles. The constraints of the Grashof condition and the sequence condition of the crank angle (clockwise or anti-clockwise) are included into objective function by adding penalty factors [8].

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Many studies in the literature focused on applying different algorithms into mechanism synthesis problems [23]. Alizade et al. firstly used penalty factors to include the parameter constraints into the cost function thus simplified the optimization of the four-bar mechanism [2]. Cabrera et al. optimized 3 cases of four-bar mechanism path synthesis problems with genetic algorithm and got accurate and valid results [4]. Laribi et al. applied a combined genetic algorithm-fuzzy logic method (GA-FL) to solve path synthesis of four-bar mechanism, and the case study results showed it was more efficient compared to traditional genetic algorithm [5]. Smaili and Diab applied ant-gradient search method to a hybrid synthesis of four-bar mechanism [6]. Acharyya and Mandal compared the performance of three metaheuristic algorithms, GA, PSO and DE, by solving three examples of four-bar mechanism path synthesis. Results show that DE works better with fast convergence velocity to optimal result and a very low error on target points [8]. Other studies combined two or more metaheuristic methods or newly proposed algorithms were applied to solve path synthesis problems, such as hybrid particle swarm optimization (HPSO) [16], malaga university mechanism synthesis algorithm (MUMSA) [14], and GA-DE hybrid evolutionary algorithm (GA-DE) [9]. Algorithms mimicking human or animal behavior have also been applied to solve path and function synthesis of multi-bar mechanisms, such as the imperialist competitive algorithm (ICA) [24], cuckoo search algorithm [21], and modified krill herd algorithm (MKH) [17] et al. They performed very well in mechanism optimization problems based on the case studies results listed in literature.

The approach presented in this paper to deal with the synthesis of mechanisms using beetle antennae search algorithm. Beetle antennae search algorithm is a newly proposed bio-inspired optimization algorithm which mimics the function of antennae and the random walk mechanism of beetles [25]. With only one group of initial parameter, two main steps of detecting and searching are implemented. This algorithm has been applied to different constraint optimization problems extensively and successfully, and the results show fast convergence velocity to global optimum [26].

The paper is structured as follows: Section 2 describes the position analysis of four-bar mechanism and Stephenson III Six-bar double dwell mechanism; Section 3 defines the goal functions of four-bar and six-bar path synthesis and presents the implement steps of BAS algorithm; Section 4 analyzes the results calculated by the proposed method for eight classical design examples used in path and function synthesis problems; Section 5 discusses the results got by BAS, compares the performance of BAS with other algorithms used in mechanism synthesis and summarizes the conclusions of the paper.

## 2. Position analysis of four-bar mechanism and six-bar mechanism

### 2.1. Four-bar linkage mechanism

The four-bar linkage mechanism is illustrated in Figure 1. Based on closed-loop vector equation, the position of coupler point  $P$  is derived as below:

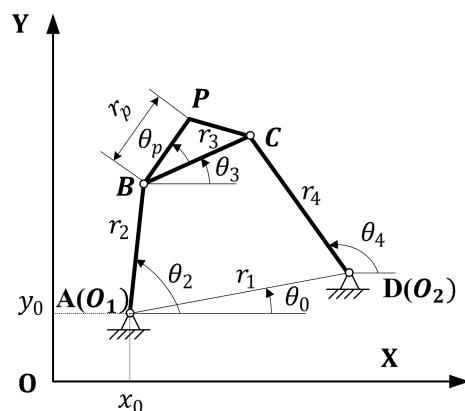


Figure 1: The planar four-bar linkage mechanism.

$$\text{Loop: } r_1 e^{i\theta_0} + r_4 e^{i\theta_4} - r_2 e^{i\theta_2} - r_3 e^{i\theta_3} = 0 \quad (1)$$

$$\begin{cases} r_1 \cos(\theta_0) + r_4 \cos(\theta_4) - r_2 \cos(\theta_2) - r_3 \cos(\theta_3) = 0 \\ r_1 \sin(\theta_0) + r_4 \sin(\theta_4) - r_2 \sin(\theta_2) - r_3 \sin(\theta_3) = 0 \end{cases} \quad (2)$$

$$\theta_3 = 2 \arctan\left(\frac{-A \pm \sqrt{A^2 - 4BC}}{2B}\right) + \theta_0 \quad (3)$$

where

$$\begin{cases} A = \cos(\theta_2 - \theta_0) - K_1 + K_2 \cos(\theta_2 - \theta_0) + K_3 \\ B = -2 \sin(\theta_2 - \theta_0) \\ C = K_1 + (K_2 - 1) \cos(\theta_2 - \theta_0) + K_3 \\ K_1 = r_1/r_2, K_2 = r_1/r_3, K_3 = (r_4^2 - r_1^2 - r_2^2 - r_3^2)/(2r_2r_3) \end{cases} \quad (4)$$

So we get the positions of  $P$  as below:

$$\begin{cases} x_p = x_0 + r_2 \cos(\theta_2) + r_p \cos(\theta_3 + \theta_p) \\ y_p = y_0 + r_2 \sin(\theta_2) + r_p \sin(\theta_3 + \theta_p) \end{cases} \quad (5)$$

## 2.2. Stephenson III six-bar linkage mechanism

The six-bar linkage mechanism is illustrated in Figure 2. Based on closed-loop vector equation, the position of coupler point  $P$  and the angle of output link  $\theta_6$  are derived as below:

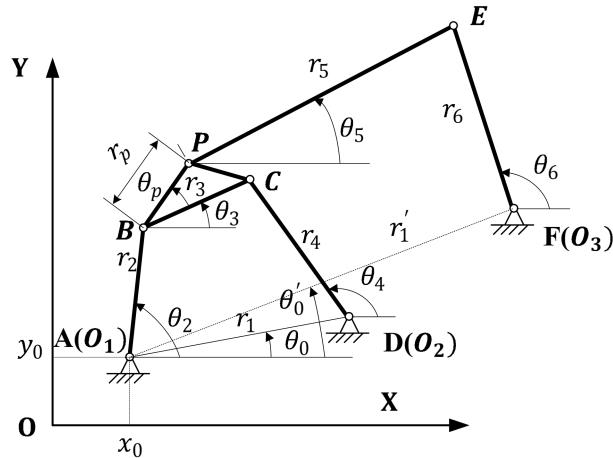


Figure 2: The planar six-bar linkage mechanism.

$$\begin{cases} \text{Loop1: } r_1 e^{i\theta_0} + r_4 e^{i\theta_4} - r_2 e^{i\theta_2} - r_3 e^{i\theta_3} = 0 \\ \text{Loop2: } r'_1 e^{i\theta'_0} + r_6 e^{i\theta_6} - r_2 e^{i\theta_2} - r_p e^{i(\theta_3 + \theta_p)} - r_5 e^{i\theta_5} = 0 \end{cases} \quad (6)$$

Loop1:

$$\begin{cases} \alpha = r_2 \cos(\theta_2) - r_1 \cos(\theta_0), \beta = r_2 \sin(\theta_2) - r_1 \sin(\theta_0) \\ \gamma = (r_4^2 + \alpha^2 + \beta^2 - r_3^2)/(2r_4), \lambda = \text{atan2}(\alpha, \beta) \\ \theta_4 = \text{atan2}(\cos(\lambda)\gamma/\beta, [1 - (\cos(\lambda)\gamma/\beta)^2]^{1/2}) - \lambda \\ \theta_3 = \text{atan2}(r_4 \sin(\theta_4) - \beta, r_4 \cos(\theta_4) - \alpha) \end{cases} \quad (7)$$

Loop2:

$$\begin{cases} \alpha_1 = r_2 \cos(\theta_2) + r_p \cos(\theta_3 + \theta_p) - r_6 \cos(\theta_6) \\ \beta_1 = r_2 \sin(\theta_2) + r_p \sin(\theta_3 + \theta_p) - r_6 \sin(\theta_6) \\ \gamma_1 = (r_6^2 + \alpha_1^2 + \beta_1^2 - r_5^2)/(2r_6), \lambda_1 = \text{atan2}(\alpha_1, \beta_1) \\ \theta_6 = \text{atan2}(\cos(\lambda_1)\gamma_1/\beta_1, [1 - (\cos(\lambda_1)\gamma_1/\beta_1)^2]^{1/2}) - \lambda_1 \\ \theta_5 = \text{atan2}(r_6 \sin(\theta_6) - \beta_1, r_6 \cos(\theta_6) - \alpha_1) \end{cases} \quad (8)$$

So we get the positions as below:

$$\begin{cases} x_A = x_0, y_A = y_0 \\ x_D = x_0 + r_1 \cos(\theta_0), y_D = y_0 + r_1 \sin(\theta_0) \\ x_F = x_0 + r'_1 \cos(\theta'_0), y_F = y_0 + r'_1 \sin(\theta'_0) \\ x_P = x_0 + r_2 \cos(\theta_2) + r_p \cos(\theta_3 + \theta_p) \\ y_P = y_0 + r_2 \sin(\theta_2) + r_p \sin(\theta_3 + \theta_p) \end{cases} \quad (9)$$

### 3. Optimization implementation

#### 3.1. Goal function of four-bar mechanism optimization

Equation (10) computes the position error between a set of target points indicated by the designer that should be met by coupler point  $P$  of a four-bar mechanism and the set of positions of the coupler of the designed four-bar mechanism (see Figure 1). In (10),  $N$  is the number of required target points,  $(P_{Xd}^i, P_{Yd}^i)$  and  $(P_X^i, P_Y^i)$  are the coordinates of the desired and generated precision points respectively. The coordinates of the generated precision points are calculated using (5). Also, three constraints have been used in the optimization problem: the Grashof criterion, the sequence of the crank angle (clockwise or anti-clockwise) and the range of the design variables. To define the complete optimization problem, the first two constraints were included by adding penalty functions.

$$\min \left\{ \sum_{i=1}^N [(P_{Xd}^i - P_X^i)^2 + (P_{Yd}^i - P_Y^i)^2] + M_1 h_1(X) + M_2 h_2(X) \right\} \quad (10)$$

where  $x_i \in [l_{min}^i, l_{max}^i], \forall x_i \in X, X = [r_1, r_2, r_3, r_4, r_p, \theta_p, \theta_0, x_0, y_0, \theta_1^1, \dots, \theta_2^N]$ .

$$\begin{aligned} h_1(X) &= \begin{cases} 1, & \text{the Grashof condition false} \\ 0, & \text{the Grashof condition true} \end{cases} \\ h_2(X) &= \begin{cases} 1, & \text{the sequence condition of the crank angle false} \\ 0, & \text{the sequence condition of the crank angle true} \end{cases} \end{aligned} \quad (11)$$

where  $h_1(X)$  and  $h_2(X)$  evaluate the Grashof condition and the sequence condition of the crank angle respectively,  $M_1$  and  $M_2$  are the penalty factors for two penalty functions and  $X$  denotes the design variables.

#### 3.2. Goal function of Stephenson III six-bar mechanism optimization

Equation (12) computes the position error and output angle errors between a set of target values indicated by the designer and the set of values of the coupler of the designed six-bar mechanism (see Figure 2). (12) consists of two different parts and it is used to optimize a six-bar dwell mechanism that will pass through the precision points of coupler point  $P$ , while satisfying the coordinated requirement between input and output angles in the dwell portion with desired accuracy level (see Figure 2).

$$\min \left\{ \sum_{i=1}^N [(P_{Xd}^i - P_X^i)^2 + (P_{Yd}^i - P_Y^i)^2] + \sum_{i=1}^M (\theta_{6d}^i - \theta_6^i)^2 + M_1 h_1(X) + M_2 h_2(X) + M_3 h_3(X) \right\} \quad (12)$$

where  $x_i \in [l_{min}^i, l_{max}^i], \forall x_i \in X, X = [r_1, r_2, r_3, r_4, r_5, r_6, r_p, \theta_p, \theta_0, x_0, y_0, \theta_2^1, \dots, \theta_2^N]$

Therefore, the objective function is consisted of two parts. The first part which is formulated in the same way as (10). The second part defines the error of the output angle at the dwell period and it can be formulated as:  $\sum_{i=1}^M (\theta_{6d}^i - \theta_6^i)^2$ , where  $M$  is the required number of target angles during dwell period,  $\theta_{6d}^i$  and  $\theta_6^i$  are the desired and generated output angles respectively. The coordinates of point P and the output angle  $\theta_6$  are calculated using position analysis in Section 2.2.

Four constraints have been used in this optimization problem: the satisfaction of the Grashof criterion, the sequence of the crank angle, the range of the design variables and the non-violation of the transmission angle (the transmission angle is defined as an acute angle between the coupler and output links). The first three constraints are the same as in the first objective problem defined in Section 3.1. The last constraint is verified at each target point. The goal is to keep the minimum transmission angle of the mechanism larger than the desired value when the designed mechanism passes through those target points. To define the complete optimization problem, the first two and the last constraints were included into goal function by adding penalty functions.

$$\begin{aligned} h_1(X) &= \begin{cases} 1, & \text{the Grashof condition false} \\ 0, & \text{the Grashof condition true} \end{cases} \\ h_2(X) &= \begin{cases} 1, & \text{the sequence condition of the crank angle false} \\ 0, & \text{the sequence condition of the crank angle true} \end{cases} \\ h_3(X) &= \begin{cases} 1, & \text{non-violation of transmission angle false} \\ 0, & \text{non-violation of transmission angle true} \end{cases} \end{aligned} \quad (13)$$

where  $h_1(X)$ ,  $h_2(X)$  and  $h_3(X)$  evaluate the Grashof condition, the sequence condition of the crank angle (clockwise or anti-clockwise) and non-violation of transmission angle (more than  $20^\circ$ ) respectively,  $M_1, M_2$  and  $M_3$  are the penalty factors for those functions and  $X$  denotes the design variables.

### 3.3. Optimization algorithm: Beetle Antennae Search Algorithm

Beetle antennae search algorithm (BAS) is a newly proposed metaheuristic algorithm inspired by the searching behavior of longhorn beetles [25, 26]. It imitates the function of antennae and the random walk mechanism of beetles in nature, and then two main steps of detecting and searching are implemented. Unlike other swarm intelligence algorithms and evolutionary algorithms like GA, DE and PSO, BAS uses only one initial particle to search the best value instead of a group of particles. The main formula of the natural inspired BAS consist of two aspects: searching behavior and detecting behavior.

The position of the beetle is denoted as a vector  $\mathbf{x}_t$  at  $t^{th}$  iteration ( $t = 1, 2, \dots, T^{max}$ ) and the fitness function at position  $\mathbf{x}$  is denoted as  $f(\mathbf{x})$  and minimum value of  $f(\mathbf{x})$  corresponds to the optimization goal.  $T^{max}$  is defined as maximum iteration times.

The searching behavior is used to explore the next antennae positions by introducing a serial of normalized random unit vectors  $\mathbf{b}_i, i = 1, 2, \dots, q$ . Newly added parameter  $q$  serves to explore design space by increasing the number of antennae pairs. Parameter  $q$  increases the possibility to find better results during iterations and deals with a higher number of independent parameters efficiently. The searching behaviors of both right-hand and left-hand sides of  $q$  antennae pairs respectively are presented as below:

$$\mathbf{x}_{ri} = \mathbf{x}_t + d^t \mathbf{b}_i, \mathbf{x}_{li} = \mathbf{x}_t - d^t \mathbf{b}_i, i = 1, 2, \dots, q \quad (14)$$

$$\mathbf{x}_{ti} = \mathbf{x}_{t-1} - \delta^t \text{sign}(f(\mathbf{x}_{ri}) - f(\mathbf{x}_{li})), i = 1, 2, \dots, q \quad (15)$$

where  $\mathbf{x}_{ri}, \mathbf{x}_{li}, i = 1, 2, \dots, q$  denote the positions lying in the searching area of right-hand side and left-hand side of  $q$  groups of antennae.  $q$  is newly proposed in this paper to adapt the dimension increasing of variable  $x$ .  $f(\mathbf{x}_{ri})$  and  $f(\mathbf{x}_{li}), i = 1, 2, \dots, q$  are fitness functions of  $\mathbf{x}_{ri}, \mathbf{x}_{li}, i = 1, 2, \dots, q$  individually.  $\text{sign}(\bullet)$  represents a sign function.  $d^t$  and  $\delta^t$  are antennae length and step length individually, both account for the convergence speed and follow a decreasing function of iteration number  $t$  both. The initialization of  $d$  and  $\delta$  should be

adapted to the searching space. Using equation (15), each antenna goes to left or right by a step size with smaller fitness function.

In cost function, we normalize  $\mathbf{x}_{ti}$ ,  $i = 1, 2, \dots, q$  based on the side constraints, evaluate the values of cost function  $f(\mathbf{x}_{ti})$ ,  $i = 1, 2, \dots, q$ , refresh the value of  $\tilde{\mathbf{x}}_{best}, f_{best}$ , and get the normalized  $\mathbf{x}_{best}$  with  $\min(f_{ti}), i = 1, 2, \dots, q$  and corresponding  $\mathbf{x}_t$  during each iteration.  $f_{best}, \tilde{\mathbf{x}}_{best}$  and  $\mathbf{x}_{best}$  are defined as the minimum goal function and corresponding variable  $\mathbf{x}$  and normalized variable.

To tune the parameters, we decrease the searching length  $d$  and step length  $\delta$  as follow:

$$d^t = c_1 d^{t-1}, \delta^t = c_2 d^t \quad (16)$$

where  $c_1$  and  $c_2$  are constant, and used to decrease the value of the searching length  $d$  and step length  $\delta$  during each iteration. Based on trajectory optimization problems, suggested parameters are listed as below, while it is adjustable based on the optimization problems.

$$d_0 = 0.10, \delta_0 = 0.05, c_1 = 0.9998, c_2 = 0.5, q = 40, T_{max} = 50000 \quad (17)$$

Table 1: Flow chart of the variable BAS algorithm

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**Algorithm:** Variable BAS algorithm for multi-dimensional constrained optimization

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**Input:** Initialize variable  $\mathbf{x}^0$  in standard normalization form so as to satisfy optimization constraints and set the parameters  $d_0, \delta_0, c_1, c_2, q, T_{max}$ .

**Output:**  $\mathbf{x}_{best}, f_{best}$

While  $t < T_{max}$  or stopping criterion is not satisfied do

- 1) Search in design space with normalized random unit vector in left and right directions for all antennae according to (14);
- 2) Update the state variable  $\mathbf{x}_{ti}, i = 1, 2, \dots, q$  according to equation 15;
- 3) Normalize  $\mathbf{x}_{ti}, i = 1, 2, \dots, q$  and get the cost function  $\min(f_{ti}), i = 1, 2, \dots, q$ ;
- 4) If  $\min(f_{ti}), i = 1, 2, \dots, q$  satisfies optimum condition then refresh the value  $\tilde{\mathbf{x}}_{best}, f_{best}$  and get the normalized  $\mathbf{x}_{best}$  with  $\min(f_{ti}), i = 1, 2, \dots, q$  and corresponding  $\mathbf{x}_t$ ;
- 5) Decrease the searching length  $d$  and step length  $\delta$  according to (16);

**Return:**  $\tilde{\mathbf{x}}_{best}, \mathbf{x}_{best}, f_{best}$ .

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## 4. Results

Using the goal function and algorithm described in Section 3, eight different and classic path or function synthesis problems are solved with BAS. Optimization results obtained by BAS for the four-bar mechanism path synthesis problem (solved with or without prescribed timing) and the Stephenson III six-bar double dwell mechanism path and function synthesis problem (with prescribed timing) are compared with the results in literature.

### 4.1. Case 1: Path generation without prescribed timing

The first case of this section is a four-bar path synthesis problem with six points aligned in a vertical straight line without prescribed timing. The meaning of letters of designed variables is the same as in Section 2. The final error is computed by (10) and the problem is defined as:

Design Variables:

$$X = [r_1, r_2, r_3, r_4, r_p, \theta_p, \theta_0, x_0, y_0, \theta_2^1, \dots, \theta_2^6]$$

Target Points:

$$\{C_d^i\} = \{(20, 20), (20, 25), (20, 30), (20, 40), (20, 45)\}$$

Limits of design variables:

$$r_1, r_2, r_3, r_4, r_p \in [0, 60]; x_0, y_0 \in [-60, 60]; \theta_0, \theta_2^1, \dots, \theta_2^6, \theta_p \in [0, 2\pi]$$

Parameters of the BAS algorithm:

$$d_0 = 0.10, \delta_0 = 0.05, c_1 = 0.9998, c_2 = 0.5, q = 40, T_{max} = 50000$$

Table 2 shows that BAS found the best design overall corresponding to an error of  $1.241e - 5$  on target positions. The comparison of coupler paths of different algorithms and the best mechanism designed by the BAS algorithm are shown in Figure 3. In Figure 3(b), blue line is the trajectory of coupler during whole rotation of crank, black points are target points and red points are obtained points by BAS. The joints with hexagon background are fixed joints and the joints without hexagon background are rotating joints. In the rest part, the meanings of signals in best mechanism figures of other cases are the same. All the simulations of best mechanisms obtained by BAS are given in supplementary materials.

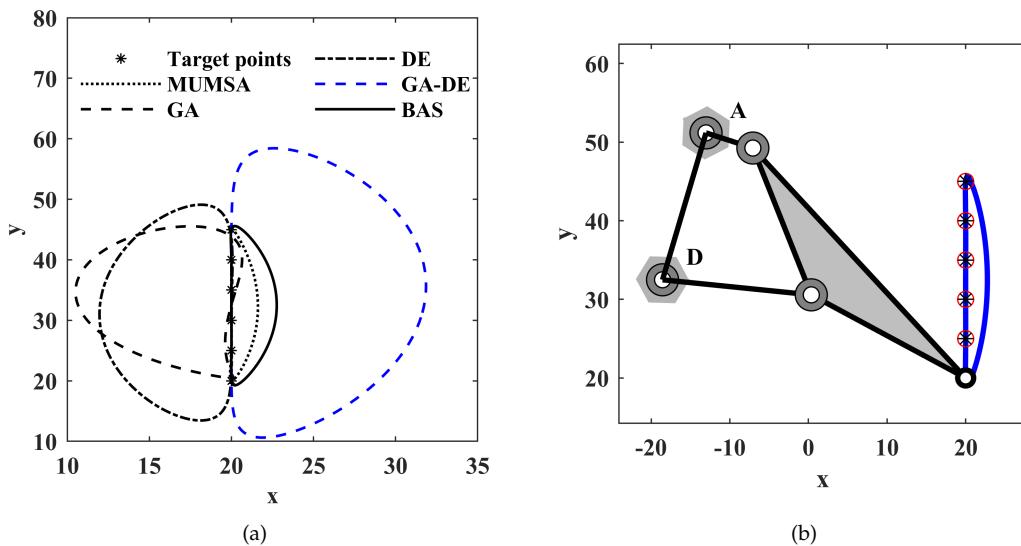


Figure 3: (a) Coupler paths of best mechanisms obtained from listed algorithms in Case 1. (b) The optimized mechanism in Case 1 (BAS).

#### 4.2. Case 2: Path generation with prescribed timing

The second test problem considered in this study regards a four-bar path synthesis problem with five non-aligned points with prescribed timing. The four-bar mechanism has its crank fixed in the origin of the coordinate system and the fixed link is parallel to the X-axis, which means  $\theta_0 = x_0 = y_0 = 0$ . The final error is also computed by (10) and the problem is defined as:

Design Variables:

$$X = [r_1, r_2, r_3, r_4, r_p, \theta_p]$$

Table 2: Optimization results obtained for Case 1

	MUMSA[14]	GA[8]	DE[8]	GA-DE[9]	BAS
$r_1$	31.788264	28.771330	35.020740	13.251600	19.492081
$r_2$	8.204647	5.000000	6.404196	5.940780	6.264472
$r_3$	24.932131	35.365480	31.607220	58.311800	20.100163
$r_4$	31.385926	59.136810	50.599490	53.720700	19.021916
$r_p$	37.108246	14.850370	46.461261	61.301156	39.853805
$\theta_p$	0.398977	1.570796	1.106544	-1.300260	0.367966
$\theta_0$	4.015959	5.287474	0.000000	0.196076	4.424562
$x_0$	-6.366519	29.913290	60.000000	-35.362100	-13.030072
$y_0$	56.836760	32.602280	18.077910	36.770400	51.179667
$\theta_2^1$	1.366547	6.283185	6.283185	1.660150	5.969982
$\theta_2^2$	2.330773	0.318205	0.264935	2.046840	0.455500
$\theta_2^3$	2.871039	0.638520	0.500377	2.428110	1.020271
$\theta_2^4$	3.394591	0.979950	0.735321	2.809010	1.555070
$\theta_2^5$	3.970960	1.412732	0.996529	3.190090	2.111741
$\theta_2^6$	4.963490	2.076254	1.333549	3.573790	2.883925
Error	0.002057	1.101697	0.122738	0.000017	0.000012

Target Points:

$$\{C_d^i\} = \{(3, 3), (2.759, 3.363), (2.372, 3.663), (1.890, 3.862), (1.355, 3.943)\}$$

$$[\theta_2^1, \theta_2^2, \theta_2^3, \theta_2^4, \theta_2^5] = [\pi/6, \pi/4, \pi/3, 5\pi/12, \pi/2]$$

Limits of design variables:

$$r_1, r_2, r_3, r_4, r_p \in [0, 5]; \theta_p \in [0, 2\pi]$$

Parameters of the BAS algorithm:

$$d_0 = 0.10, \delta_0 = 0.05, c_1 = 0.9998, c_2 = 0.5, q = 40, T_{max} = 10000$$

Table 3 shows BAS found a better solution than the other referenced algorithms achieving a final error of  $7.467e - 7$  on target positions. The comparison of coupler paths of different algorithms and the best mechanism designed by the BAS algorithm are shown in Figure 4.

Table 3: Optimization results obtained for Case 2

	PSO[16]	HPSO[16]	GA[24]	MUMSA[14]	BAS
$r_1$	3.057620	3.787720	3.063042	3.773269	3.713526
$r_2$	1.861840	1.998420	1.995962	2.000004	1.997874
$r_3$	3.845910	4.133130	3.305823	4.116971	4.046953
$r_4$	2.970630	2.745130	2.524706	2.746157	2.719239
$r_p$	2.496827	2.369722	2.372148	2.368437	2.370297
$\theta_p$	0.724356	0.784148	0.804418	0.783157	0.786537
Error	0.000586	0.000986	0.000002	0.000002	0.000001

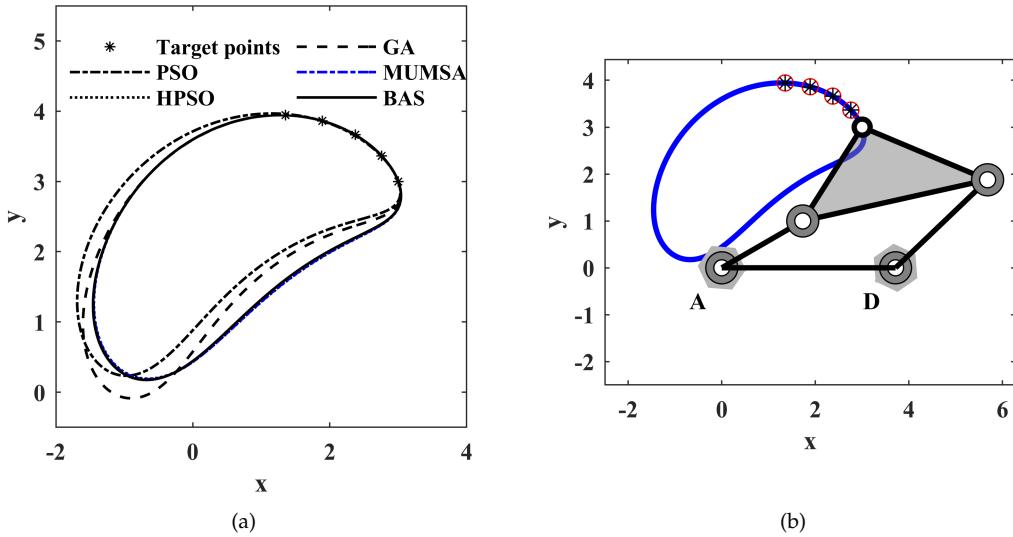


Figure 4: (a) Coupler paths of best mechanisms obtained from listed algorithms in Case 2. (b) The optimized mechanism in Case 2 (BAS).

#### 4.3. Case 3: Path generation with prescribed timing

In case 3, the coupler point of the four-bar mechanism has to pass a close-loop path generation with prescribed timing. The error function is described by (10). The problem is described as below:

Design Variables:

$$X = [r_1, r_2, r_3, r_4, r_p, \theta_p, \theta_0, x_0, y_0, \theta_2^1]$$

Target Points:

$$\{C_d^i\} = \left\{ \begin{array}{ll} (0.5, 1.1), & (0.4, 1.1), \\ (0.02, 0.6), & (0, 0.5), \\ (0.2, 0.3), & (0.3, 0.4), \end{array} \begin{array}{ll} (0.3, 1.1), & (0.2, 1.0), \\ (0.0, 0.4), & (0.03, 0.3), \\ (0.4, 0.5), & (0.5, 0.7), \end{array} \begin{array}{ll} (0.1, 0.9), & (0.05, 0.75), \\ (0.1, 0.25), & (0.15, 0.2), \\ (0.6, 0.9), & (0.6, 1.0). \end{array} \right\}$$

Limits of design variables:

$$r_1, r_2, r_3, r_4, r_p \in [0, 5]; x_0, y_0 \in [-5, 5]; \theta_0, \theta_2^1, \theta_p \in [0, 2\pi]$$

Parameters of the BAS algorithm:

$$d_0 = 0.10, \delta_0 = 0.05, c_1 = 0.9998, c_2 = 0.5, q = 8, T_{max} = 50000$$

It can be seen from Table 4 that BAS found a better solution than the other referenced algorithms. The final error,  $9.029e - 3$ , is comparable with the result obtained by MUMSA (error= 0.00911) [14]. The comparison of coupler paths of different algorithms and the best mechanism designed by the BAS algorithm are shown in Figure 5.

#### 4.4. Case 4: Path generation with prescribed timing

The fourth test case considered in this study is a problem of path generation with prescribed timing. The six coupler optimized points consist of a semi-archer arc and the problem (error is defined by (10)) is:

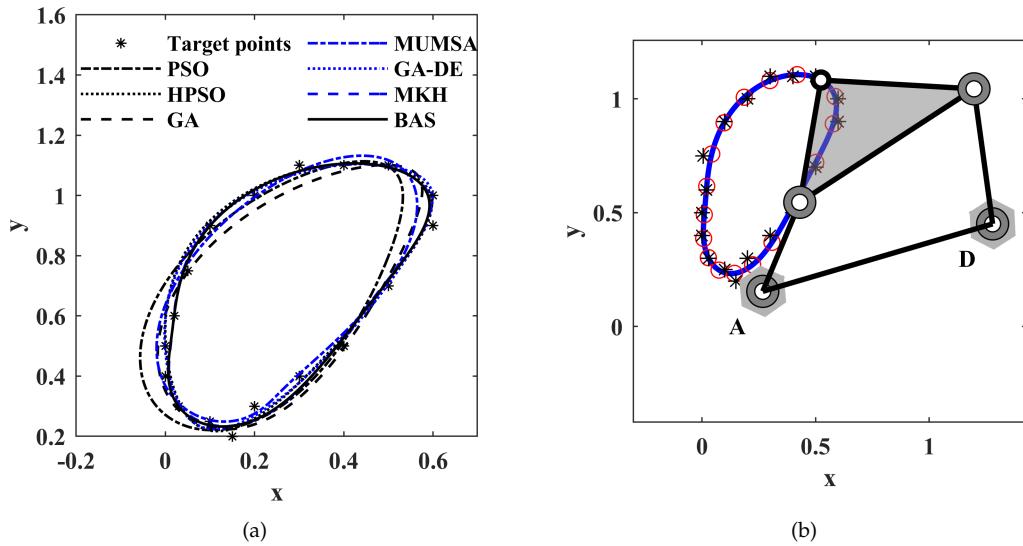


Figure 5: (a) Coupler paths of best mechanisms obtained from listed algorithms in Case 3. (b) The optimized mechanism in Case 3 (BAS).

Table 4: Optimization results obtained for Case 3

	PSO[16]	HPSO[16]	GA[8]	MUMSA[14]	GA-DE[9]	MKH[17]	BAS
$r_1$	2.926100	2.850000	3.057878	4.453772	47.437900	1.004290	1.054180
$r_2$	0.487700	0.370000	0.237803	0.297057	0.324770	0.421800	0.423871
$r_3$	2.909900	2.904800	4.828954	3.913095	0.472857	0.878210	0.914564
$r_4$	2.150300	0.500000	2.056465	0.849372	47.309300	0.580130	0.598871
$r_p$	1.493947	1.973700	2.003475	2.651983	0.341251	0.523400	0.545027
$\theta_p$	-0.332546	1.027396	1.177913	2.464734	-1.215383	0.814773	0.822747
$\theta_0$	0.719000	0.760000	1.002168	2.738736	3.320290	0.292940	0.285040
$x_0$	-0.384600	0.940000	1.776808	-1.309243	0.526988	0.268860	0.267700
$y_0$	-0.675200	-1.171200	-0.641991	2.806964	0.723930	0.177150	0.154427
$\theta_2^1$	0.215900	0.513400	0.226186	4.853543	3.512330	0.885950	1.176411
Error	0.049200	0.011100	0.033700	0.019600	0.010861	0.009110	0.009029

Design Variables:

$$X = [r_1, r_2, r_3, r_4, r_p, \theta_p, \theta_0, x_0, y_0]$$

Target Points:

$$\{C_d^i\} = \{(0, 0), (1.9098, 5.8779), (6.60989, 5.106), (13.09, 9.5106), (18.09, 5.8779), (20, 0)\}$$

$$[\theta_2^1, \theta_2^2, \theta_2^3, \theta_2^4, \theta_2^5] = [\pi/6, \pi/3, \pi/2, 2\pi/3, 5\pi/6, \pi]$$

Limits of design variables:

$$r_1, r_2, r_3, r_4, r_p \in [0, 50]; x_0, y_0 \in [-50, 50]; \theta_0, \theta_p \in [0, 2\pi]$$

Parameters of the BAS algorithm:

$$d_0 = 0.10, \delta_0 = 0.05, c_1 = 0.9997, c_2 = 0.5, q = 40, T_{max} = 30000$$

Table 5 shows that BAS found the best design overall corresponding to an error of 0.786 on target positions. This solution is significantly better than those available in literature. The comparison of coupler paths of different algorithms and the best mechanism designed by the BAS algorithm are shown in Figure 6.

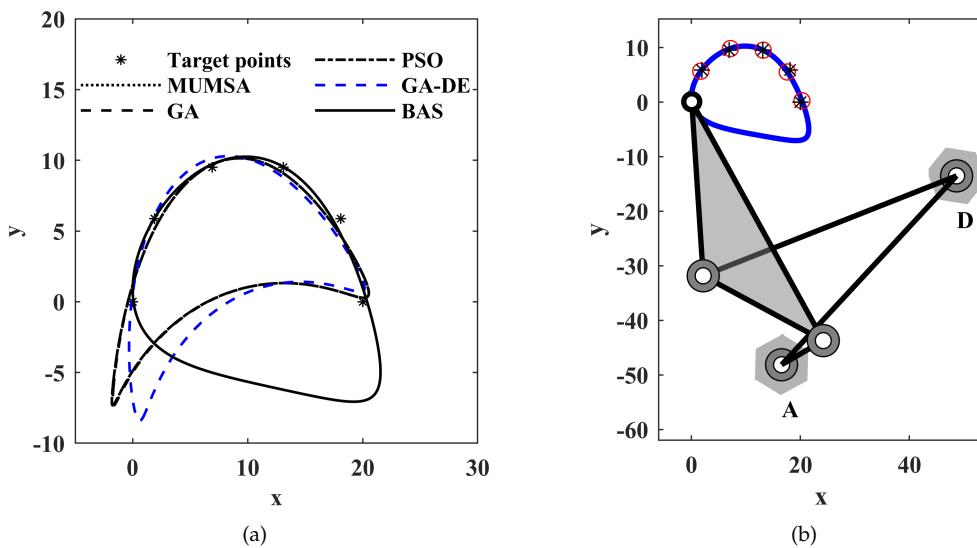


Figure 6: (a) Coupler paths of best mechanisms obtained from listed algorithms in Case 4. (b) The optimized mechanism in Case 4 (BAS).

Table 5: Optimization results obtained for Case 4

	MUMSA[14]	GA[8]	PSO[8]	GA-DE[9]	BAS
$r_1$	50.000000	50.000000	50.000000	50.000000	47.234502
$r_2$	5.000000	5.000000	5.000000	5.000000	8.847399
$r_3$	7.031047	6.970090	7.031020	5.905343	25.047471
$r_4$	48.134183	48.199300	48.134200	50.000000	50.000000
$r_p$	21.353356	21.219120	21.353282	18.819312	50.000000
$\theta_p$	0.651729	0.638006	0.651724	0.000000	5.710719
$\theta_0$	0.042825	0.050845	0.042829	0.463633	0.822595
$x_0$	12.197494	12.237700	12.197500	14.373772	16.553117
$y_0$	-15.998203	-15.833200	-15.998100	-12.444295	-48.147379
Error	2.580350	2.582860	2.580360	2.349649	0.786368

#### 4.5. Case 5: Path generation without prescribed timing

This test case regards an elliptical path generation synthesis problem without prescribed timing in which the trajectory is defined by 10 points. The problem (error is defined by (10)) is:

Design Variables:

$$X = [r_1, r_2, r_3, r_4, r_p, \theta_p, \theta_0, x_0, y_0, \theta_2^1, \dots, \theta_2^{10}]$$

Target Points:

$$\{C_d^i\} = \left\{ \begin{array}{lllll} (20,10), & (17.66,15.142), & (11.736,17.878), & (5,16.928), & (0.60307,12.736), \\ (0.60307,7.2638), & (5,3.0718), & (11.736,2.1215), & (17.66,4.8577), & (20,0). \end{array} \right\}$$

Limits of design variables:

$$r_1, r_2, r_3, r_4, r_p \in [0, 80]; x_0, y_0 \in [-80, 80]; \theta_0, \theta_2^1, \dots, \theta_2^{10}, \theta_p \in [0, 2\pi]$$

Parameters of the BAS algorithm:

$$d_0 = 0.10, \delta_0 = 0.05, c_1 = 0.9998, c_2 = 0.5, q = 40, T_{max} = 40000$$

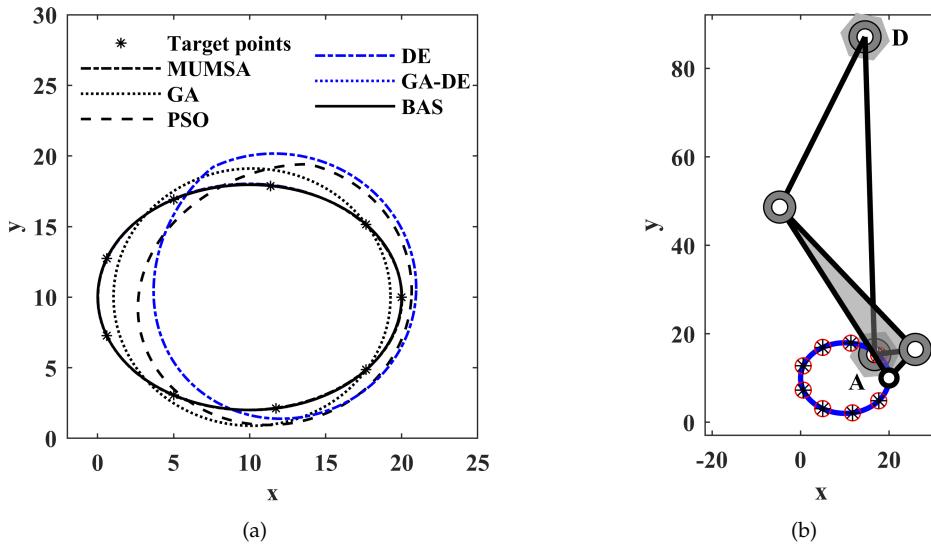


Figure 7: (a) Coupler paths of best mechanisms obtained from listed algorithms in Case 5. (b) The optimized mechanism in Case 5 (BAS).

Table 6 shows that BAS found the best design overall corresponding to an error of  $4.252e - 4$  on target positions. The comparison of coupler paths of different algorithms and the best mechanism designed by the BAS algorithm are shown in Figure 7.

#### 4.6. Case 6: Path generation and function synthesis with prescribed timing

This test case regards a path and function combined synthesis problem with prescribed timing in which the coupler of six-bar mechanism must pass through a set of precision points and its output link has to maintain an accuracy angle in the dwell portion (Figure 2). The final error is computed by (12) and the problem is defined as below:

Design Variables:

$$X = [r_1, r_2, r_3, r_4, r_5, r_6, r_p, r'_1, \theta_0, \theta'_0, \theta_2^1, x_0, y_0, \theta_2^1]$$

Table 6: Optimization results obtained for Case 5

	MUMSA[14]	GA[8]	PSO[8]	DE[8]	GA-DE[9]	BAS
$r_1$	79.516068	79.981513	52.535162	54.360893	80.000000	71.868123
$r_2$	9.723973	9.109993	8.687886	8.683351	8.420320	9.261862
$r_3$	45.842524	72.936511	36.155078	34.318634	51.342600	44.454296
$r_4$	51.438480	80.000000	80.000000	79.996171	42.453200	43.053351
$r_p$	8.728939	0.000000	1.481055	1.465250	10.653040	8.782072
$\theta_p$	-0.345226	0.000000	1.570796	1.570669	2.646545	1.636258
$\theta_0$	5.596945	0.026149	1.403504	2.129650	4.281770	1.601166
$x_0$	2.021109	10.155966	11.002124	10.954397	5.533720	16.754036
$y_0$	13.216588	10.000000	11.095585	11.074534	0.477183	15.298668
$\theta_2^1$	0.637687	6.283185	6.282619	6.283185	2.093500	0.125842
$\theta_2^2$	1.325533	0.600745	0.615302	0.616731	2.812910	0.816721
$\theta_2^3$	2.008034	1.372812	1.305421	1.310254	3.516050	1.535133
$\theta_2^4$	2.695566	2.210575	2.188053	2.193570	4.206380	2.181149
$\theta_2^5$	3.384579	2.862639	2.913049	2.917170	4.890510	2.875384
$\theta_2^6$	4.082938	3.420547	3.499313	3.490746	5.573980	3.572807
$\theta_2^7$	4.798455	4.072611	4.125586	4.132017	6.264580	4.285406
$\theta_2^8$	5.511706	4.910373	4.919977	4.922075	0.676198	5.001621
$\theta_2^9$	6.212792	5.682440	5.685021	5.695372	1.383070	5.713342
$\theta_2^{10}$	0.637187	6.283185	6.282323	6.282970	2.093480	0.125829
Error	0.004700	2.281273	1.971004	1.952326	0.000602	0.000425

Target Points:

$$\{C_d^i\} = \left\{ \begin{array}{lll} (-0.5424, 2.3708), & (0.2202, 2.9871), & (0.9761, 3.4633), \\ (1.0618, 3.6380), & (0.8835, 3.7226), & (0.5629, 3.7156), \\ (0.1744, 3.6128), & (-0.2338, 3.4206), & (-0.6315, 3.1536), \\ (-1.0, 2.8284), & (-1.3251, 2.4600), & (-1.5922, 2.0622), \\ (-1.7844, 1.6539), & (-1.8872, 1.2654), & (-1.8942, 0.9448), \\ (-1.8096, 0.7665), & (-1.6349, 0.8522), & (-1.1587, 1.6081). \end{array} \right\}$$

$$\{\delta_2^i\} = \left\{ \begin{array}{llllllll} 0^\circ, & 15^\circ, & 40^\circ, & 60^\circ, & 80^\circ, & 100^\circ, & 120^\circ, & 140^\circ, & 160^\circ, \\ 180^\circ, & 200^\circ, & 220^\circ, & 240^\circ, & 260^\circ, & 280^\circ, & 300^\circ, & 320^\circ, & 345^\circ. \end{array} \right\}$$

where  $\theta_2^i = \theta_2^1 + \delta_2^i, i = 1, \dots, 18$

Input-output angle correlation during dwell period:

$$\theta_2^i + \{160^\circ, 180^\circ, 200^\circ, 220^\circ\} \rightarrow \theta_6^i = 210^\circ$$

$$\theta_2^i + \{345^\circ, 0^\circ, 15^\circ\} \rightarrow \theta_6^i = 225^\circ$$

Parameters of the BAS algorithm:

$$d_0 = 0.05, \delta_0 = 0.025, c_1 = 0.9999, c_2 = 0.5, q = 10, T_{max} = 50000$$

Table 7 shows that BAS found the best design overall corresponding to final error of  $1.952e - 4$  on target positions and angles. The comparison of coupler paths and output angles of different algorithms and the best mechanism designed by the BAS algorithm are shown in Figure 8. In Table 7, DE(B), MUMSA and BAS considered the direct synthesis of six-bar mechanism while DE(A) first considered the four-bar mechanism and later considered the output angle, DE(C) only considered ten coupler points of dwell portion and ignored the other eight coupler points. The minimum transmission angle constraint used in different optimization methods maintains at the same value,  $20^\circ$ .

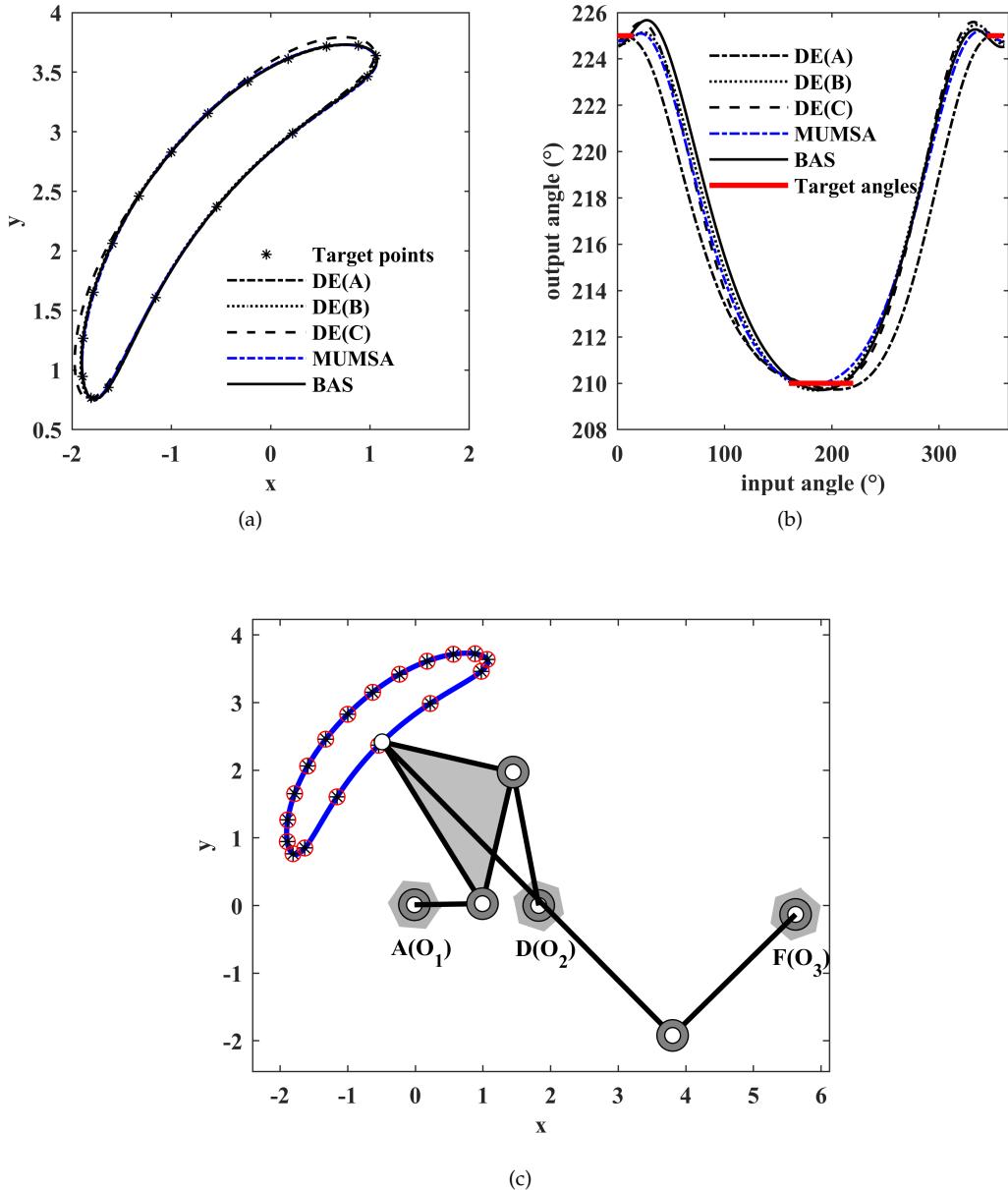


Figure 8: (a) Coupler paths of best mechanisms obtained from listed algorithms in Case 6. (b) Output angle in Case 6. (c) The optimized mechanism in Case 6 (BAS).

#### 4.7. Case 7: Path generation without prescribed timing

This test case regards an "8" shape path generation synthesis problem without prescribed timing in which the trajectory is defined by 12 points. The problem (error is defined by (10)) is:

Design Variables:

$$X = [r_1, r_2, r_3, r_4, r_p, \theta_0, \theta_0, x_0, y_0, \theta_2^1, \dots, \theta_2^{12}]$$

Table 7: Optimization results obtained for Case 6

	DE[22]A	DE[22]B	DE[22]C	MUMSA[14]	BAS
$r_1$	1.814500	1.806500	2.092600	1.713529	1.838058
$r_2$	0.991100	0.982600	1.146400	0.926020	1.006098
$r_3$	1.999500	2.017700	1.989000	1.991373	1.997801
$r_4$	2.031500	2.000900	1.972700	1.848672	2.008472
$r_5$	4.367400	5.776900	6.663300	5.354980	6.106587
$r_6$	2.492400	2.529600	2.551700	2.549790	2.551492
$r_p$	2.817400	2.871100	2.717800	2.975936	2.815041
$\theta_p$	0.777600	0.783712	0.845246	0.831651	0.783870
$\theta_0$	6.269879	0.011582	6.242261	0.067677	6.277428
$r'_1$	4.415800	5.281700	6.290700	4.873740	5.638282
$\theta'_0$	0.235595	0.017031	6.181183	0.096703	6.257864
$x_0$	0.011500	0.041500	-0.272900	0.175257	-0.013849
$y_0$	0.015700	-0.037700	0.093100	-0.118703	0.011592
$\theta_2^1$	0.000520	0.003648	-0.040620	6.222361	6.281768
Evaluations	53310	93405	93405	93405	50000
Error	0.000251	0.005653	0.035375	0.001400	0.000195

Target Points:

$$\{C_d^i\} = \left\{ \begin{array}{l} (4.15, 2.21), (4.50, 2.18), (4.53, 1.83), (4.13, 1.68), (3.67, 1.58), (2.96, 1.33), \\ (2.67, 1.06), (2.63, 0.82), (2.92, 0.81), (3.23, 1.07), (3.49, 1.45), (3.76, 1.87). \end{array} \right\}$$

Limits of design variables:

$$r_1 \in [0, 5]; r_2, r_3, r_4 \in [0, 10]; r_p \in [0, 14]; x_0, y_0 \in [-5, 5]; \theta_0, \theta_2^1, \dots, \theta_2^{12}, \theta_p \in [0, 2\pi]$$

Parameters of the BAS algorithm:

$$d_0 = 0.05, \delta_0 = 0.025, c_1 = 0.9995, c_2 = 0.5, q = 40, T_{max} = 20000$$

Table 8 shows that BAS found the best design overall corresponding to an error of  $9.942e - 5$  on target positions. Only  $\theta_2^1$  is listed in Table 8 for simplicity, and all the angle parameters of BAS are given in supplementary materials. The comparison of coupler paths of different algorithms and the best mechanism designed by the BAS algorithm are shown in Figure 9.

#### 4.8. Case 8: Path generation without prescribed timing

This test case regards a leaf shape path generation synthesis problem without prescribed timing in which the trajectory is defined by 25 points [27]. The problem (error is defined by equation 10) is:

Design Variables:

$$X = [r_1, r_2, r_3, r_4, r_p, \theta_p, \theta_0, x_0, y_0, \theta_2^1, \dots, \theta_2^{25}]$$

Target Points:

$$\{C_d^i\} = \left\{ \begin{array}{l} (7.03, 5.99), (6.95, 5.45), (6.77, 5.03), (6.4, 4.6), (5.91, 4.03), \\ (5.43, 3.56), (4.93, 2.94), (4.67, 2.6), (4.38, 2.2), (4.04, 1.67), \\ (3.76, 1.22), (3.76, 1.97), (3.76, 2.78), (3.76, 3.56), (3.76, 4.34), \\ (3.76, 4.91), (3.76, 5.47), (3.8, 5.98), (4.07, 6.4), (4.53, 6.75), \\ (5.07, 6.85), (5.05, 6.84), (5.89, 6.83), (6.41, 6.8), (6.92, 6.58). \end{array} \right\}$$

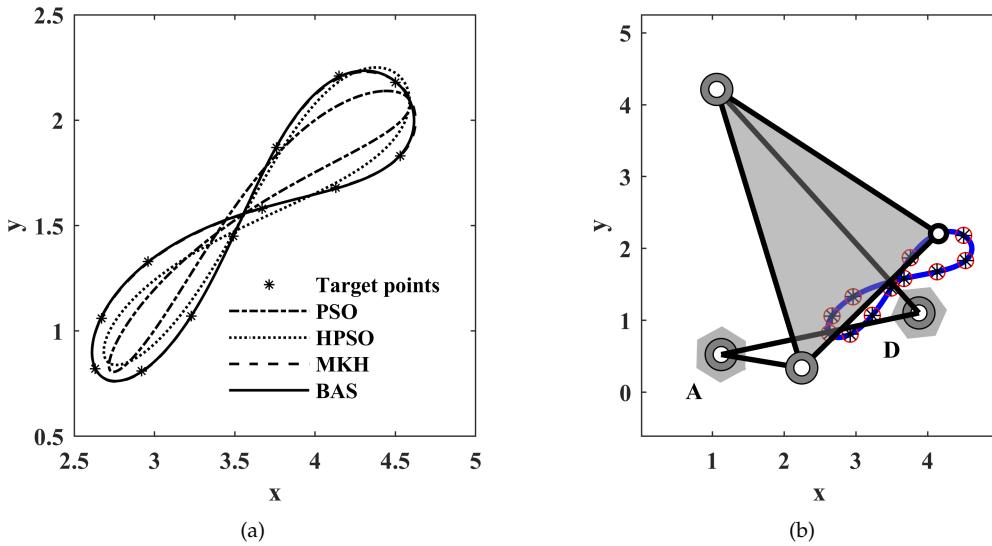


Figure 9: (a) Coupler paths of best mechanisms obtained from listed algorithms in Case 7. (b) The optimized mechanism in Case 7 (BAS).

Table 8: Optimization results obtained for Case 7

	PSO[16]	HPSO[16]	MKH[17]	BAS
$r_1$	4.550300	4.535900	2.876450	2.816277
$r_2$	1.101300	1.113300	1.146440	1.138263
$r_3$	3.955800	14.738100	4.407730	4.050992
$r_4$	3.933000	16.801700	4.757130	4.195776
$r_p$	3.941572	3.941866	2.666575	2.668280
$\theta_p$	-0.539171	-1.521104	-1.136351	5.191079
$\theta_0$	0.000000	0.000000	0.165020	0.207060
$x_0$	0.000000	0.000000	1.142650	1.122700
$y_0$	0.000000	0.000000	0.482370	0.524666
$\theta_2^1$	-0.201400	-0.181600	-0.313990	6.119106
Error	0.171600	0.096400	0.000160	0.000099

Limits of design variables:

$$r_1, r_2, r_3, r_4, r_p \in [0, 5]; x_0, y_0 \in [-5, 5]; \theta_0, \theta_2^1, \dots, \theta_2^2, \theta_p \in [0, 2\pi]$$

Parameters of the BAS algorithm:

$$d_0 = 0.10, \delta_0 = 0.05, c_1 = 0.9998, c_2 = 0.5, q = 40, T_{max} = 40000$$

Table 9 shows that BAS found the second best design: the final error of  $3.978e - 2$  on target positions is slightly larger than the optimum value of 0.03916 obtained by the MKH algorithm [17]. Only  $\theta_2^1$  is listed in Table 9 for simplicity, and all the angle parameters of BAS are given in supplementary materials. The comparison of coupler paths of different algorithms and the best mechanism designed by the BAS algorithm

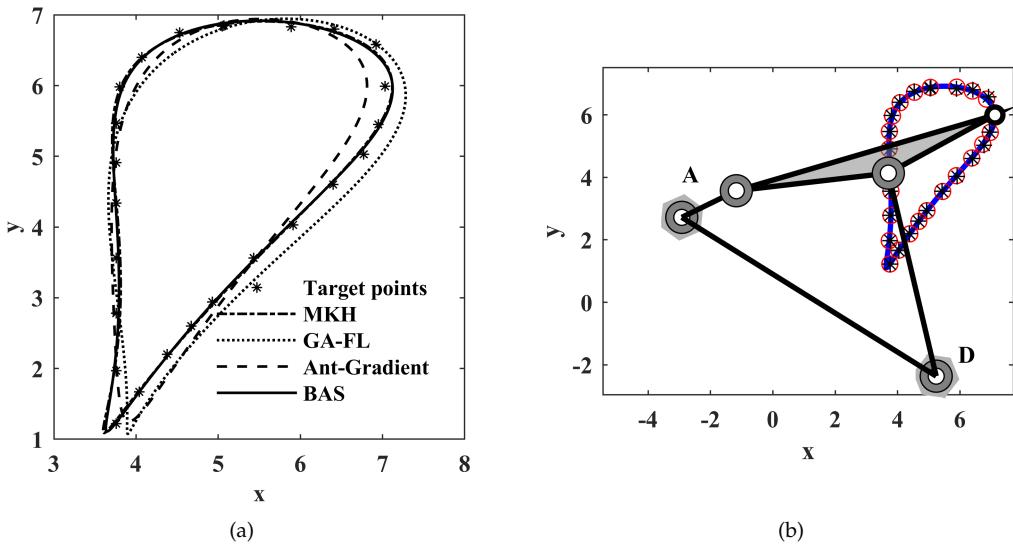


Figure 10: (a) Coupler paths of best mechanisms obtained from listed algorithms in Case 8. (b) The optimized mechanism in Case 8 (BAS).

Table 9: Optimization results obtained for Case 8

	MKH[17]	GA-FL[5]	Ant-Gradient[6]	BAS
$r_1$	9.994320	9.000000	13.080000	9.613219
$r_2$	1.930270	3.010000	1.890000	1.945984
$r_3$	4.572420	8.800000	8.410000	4.908082
$r_4$	7.366740	8.800000	6.750000	6.676828
$r_p$	8.042540	11.099500	14.450000	8.640923
$\theta_p$	0.187430	-0.681000	0.195000	0.169408
$\theta_0$	-0.617630	0.489000	-0.381500	5.726073
$x_0$	-2.313010	-2.400000	-8.770000	-2.921463
$y_0$	2.861890	-4.000000	1.200000	2.719026
$\theta_2^1$	*	*	*	0.452979
Error	0.039160	0.902200	0.550400	0.039778

are shown in Figure 10. The coupler trajectories of the best mechanisms obtained by MKH and BAS are very similar in Figure 10(a).

## 5. Discussion and conclusions

This study applied the BAS algorithm to path synthesis of four-bar mechanisms with (test problems 2, 3 and 4) or without prescribed timing (test problems 1, 5, 7 and 8), as well as to function and path combined synthesis of a Stephenson III six-bar dwell mechanism (test problem 6). These are classical benchmark problems usually selected in literatures to test different methodologies. Optimization constraints were included in the cost function using a penalty function approach. The final error on target point positions and output angles evaluated for the optimum designs of BAS was always lower than for the other algorithms except for Case 8 (see Tables 2 through 9). Since target trajectories set for problems 1, 5, 7 and 8 must fit an

increasing number of control points, the number of optimization variables increased from 15 (Case 1) to 34 (Case 8). Remarkably, the BAS algorithm could always work well requiring only with small adjustments of algorithm parameters. The number of trajectory control points was 6 for problem 2 but raised to 10 for problems 3 and 4. Remarkably, in problem 4, the error obtained by BAS algorithm was almost three times smaller than for the other algorithms. In test problem 6, the error function combined errors on positions of control points and the angle of output link in dwell portion. BAS was again much more accurate than other algorithms such as DE and MUMSA. The most important advantage of BAS algorithm is that it needs only one group of initial parameters but not a population like other metaheuristic algorithms such as DE, GA, PSO, MKH and MUMSA. This simplifies the optimization process as it may not be easy to generate good initial designs. Furthermore, BAS is inherently much faster than population-based algorithms because it works with only one particle. The results obtained in the analyzed mechanism design problems prove the validity of the developed algorithm. To the best of our knowledge, this is first work that extends beetle antennae search (BAS) algorithms to mechanical design. BAS is applied to path synthesis of four-bar mechanism and extended to path and function combined synthesis of six-bar dwell mechanism. BAS achieves better convergence and efficiency compared to other metaheuristic algorithms and opens a new avenue for path and function synthesis problem.

## References

- [1] George Randall Wandling Sr. Synthesis of mechanisms for function, path, and motion generation using invariant characterization, storage and search methods. 2000.
- [2] RI Alizade, IG Novruzbekov, and GN Sandor. Optimization of four-bar function generating mechanisms using penalty functions with inequality and equality constraints. *Mechanism and Machine Theory*, 10(4):327–336, 1975.
- [3] Arun K Natesan. Kinematic analysis and synthesis of four-bar mechanisms for straight line coupler curves. 1994.
- [4] JA Cabrera, A Simon, and M Prado. Optimal synthesis of mechanisms with genetic algorithms. *Mechanism and Machine theory*, 37(10):1165–1177, 2002.
- [5] MA Laribi, A Mlika, L Romdhane, and S Zeghloul. A combined genetic algorithm–fuzzy logic method (ga–fl) in mechanisms synthesis. *Mechanism and machine theory*, 39(7):717–735, 2004.
- [6] Ahmad Smaili and Nadim Diab. Optimum synthesis of hybrid-task mechanisms using ant-gradient search method. *Mechanism and Machine Theory*, 42(1):115–130, 2007.
- [7] N Nariman-Zadeh, M Felezi, A Jamali, and M Ganji. Pareto optimal synthesis of four-bar mechanisms for path generation. *Mechanism and Machine Theory*, 44(1):180–191, 2009.
- [8] SK Acharyya and M Mandal. Performance of eas for four-bar linkage synthesis. *Mechanism and Machine Theory*, 44(9):1784–1794, 2009.
- [9] Wen-Yi Lin. A ga-de hybrid evolutionary algorithm for path synthesis of four-bar linkage. *Mechanism and Machine Theory*, 45(8):1096–1107, 2010.
- [10] Mahmoud Sharifiati and M Norouzi. Optimal synthesis of function generator of four-bar linkages based on distribution of precision points. *Meccanica*, 46(5):1007–1021, 2011.
- [11] E Martinez, C Romero, MV Carbonell, and M Flórez. On the geometry and design of four bar linkage mechanisms. In *4th International Conference on Education and New learning Technologies EDULEARN12 Proceedings*, pages 6275–6279, 2012.
- [12] Mahya Sharifiati. *Geometric Design of Planar Four-Bar Mechanisms*. McGill University (Canada), 2012.
- [13] Jun Wu, QJ Ge, Feng Gao, and WZ Guo. On the extension of a fourier descriptor based method for planar four-bar linkage synthesis for generation of open and closed paths. *Journal of Mechanisms and Robotics*, 3(3):031002, 2011.
- [14] JA Cabrera, A Ortiz, F Nadal, and JJ Castillo. An evolutionary algorithm for path synthesis of mechanisms. *Mechanism and Machine Theory*, 46(2):127–141, 2011.
- [15] Yuxuan Tong, David H Myszka, and Andrew P Murray. Four-bar linkage synthesis for a combination of motion and path-point generation. In *ASME 2013 International Design Engineering Technical Conferences and Computers and Information in Engineering Conference*, pages V06AT07A054–V06AT07A054. American Society of Mechanical Engineers, 2013.
- [16] Chun-Te Lee and Chun-Che Lee. On a hybrid particle swarm optimization method and its application in mechanism design. *Proceedings of the Institution of Mechanical Engineers, Part C: Journal of Mechanical Engineering Science*, 228(15):2844–2857, 2014.
- [17] Radovan R Bulatović, Goran Miodragović, and Marina S Bošković. Modified krill herd (mkh) algorithm and its application in dimensional synthesis of a four-bar linkage. *Mechanism and Machine Theory*, 95:1–21, 2016.
- [18] Jianwei Sun, Lu Chen, and Jinkui Chu. Motion generation of spherical four-bar mechanism using harmonic characteristic parameters. *Mechanism and Machine Theory*, 95:76–92, 2016.
- [19] A Shinde, S Kulkarni, and S Shete. Dimensional synthesis of mechanism using genetic algorithm. *International Journal of Current Engineering and Technology*, 2017.
- [20] Sahand Hadizadeh Kafash and Ali Nahvi. Optimal synthesis of four-bar path generator linkages using circular proximity function. *Mechanism and Machine Theory*, 115:18–34, 2017.
- [21] Radovan R Bulatović, Stevan R Djordjević, and Vladimir S Djordjević. Cuckoo search algorithm: a metaheuristic approach to solving the problem of optimum synthesis of a six-bar double dwell linkage. *Mechanism and Machine Theory*, 61:1–13, 2013.

- [22] PS Shiakolas, D Koladiya, and J Kebrle. On the optimum synthesis of six-bar linkages using differential evolution and the geometric centroid of precision positions technique. *Mechanism and Machine Theory*, 40(3):319–335, 2005.
- [23] Xinxing Tong, Wenjie Ge, Yonghong Zhang, and Zhenfei Zhao. Topology design and analysis of compliant mechanisms with composite laminated plates. *Journal of Mechanical Science and Technology*, 33(2):613–620, 2019.
- [24] Saeed Ebrahimi and Pedram Payvandy. Efficient constrained synthesis of path generating four-bar mechanisms based on the heuristic optimization algorithms. *Mechanism and Machine Theory*, 85:189–204, 2015.
- [25] Xiangyuan Jiang and Shuai Li. Bas: beetle antennae search algorithm for optimization problems. *arXiv preprint arXiv:1710.10724*, 2017.
- [26] Xiangyuan Jiang and Shuai Li. Beetle antennae search without parameter tuning (bas-wpt) for multi-objective optimization. *arXiv preprint arXiv:1711.02395*, 2017.
- [27] John R McGARVA. Rapid search and selection of path generating mechanisms from a library. *Mechanism and machine theory*, 29(2):223–235, 1994.
- [28] A Ortiz, JA Cabrera, F Nadal, and A Bonilla. Dimensional synthesis of mechanisms using differential evolution with auto-adaptive control parameters. *Mechanism and Machine Theory*, 64:210–229, 2013.