Optimized Portfolio Weights

1. The one -year adjusted close prices of the stocks in the portfolio were extracted, and their returns were then calculated by using the following formula:

$$returns = \frac{current\ adj.close\ price\ -\ previous\ adj.close\ price}{previous\ adj.close\ price}$$

Stock	MA	DOW	NVDA	AAPL	AMZN	LXRX	UNH
2/13/2023	-1.75%	-0.94%	5.43%	-0.42%	0.16%	0.44%	-0.51%
2/14/2023	-1.43%	-2.22%	-0.90%	1.39%	1.46%	-0.87%	-0.32%
2/15/2023	-0.28%	0.30%	-3.35%	-1.04%	-2.98%	0.00%	-0.79%
2/16/2023	0.16%	0.32%	-2.79%	-0.75%	-0.97%	6.58%	2.41%
2/17/2023	-0.68%	1.05%	-3.43%	-2.67%	-2.70%	-8.64%	-1.56%
2/21/2023	0.67%	-0.18%	0.48%	0.29%	1.28%	6.31%	-0.49%
2/22/2023	-0.05%	0.37%	14.02%	0.33%	0.03%	3.81%	0.57%
2/23/2023	-0.50%	1.43%	-1.60%	-1.80%	-2.42%	-8.98%	-1.50%
2/24/2023	0.59%	0.72%	0.92%	0.82%	0.28%	-2.69%	-0.21%
2/27/2023	1.67%	-0.21%	-1.21%	-0.34%	0.50%	3.69%	-1.53%
2/28/2023	1.50%	-2.07%	-2.23%	-1.42%	-2.19%	-2.22%	-0.15%
3/1/2023	-1.90%	-2.29%	2.71%	0.41%	-0.04%	-0.91%	0.52%
3/2/2023	-0.25%	0.82%	2.47%	3.51%	3.01%	22.94%	0.18%

Fig 1. Sample dataset of the stock returns

2. The annual return of each stock was calculated

$$annual\ return = e^{\bar{x}\times 252} - 1$$
$$\bar{x} = mean\ of\ daily\ returns$$

3. The standard deviation of each stock was calculated

$$standard\ deviation = \sqrt{\frac{\sum_{i=1}^{n}(x_i - \bar{x})^2}{n-1}}$$
$$x_i = returns\ of\ day\ i$$
$$n = number\ of\ days$$

4. The constraints for the optimization model were set

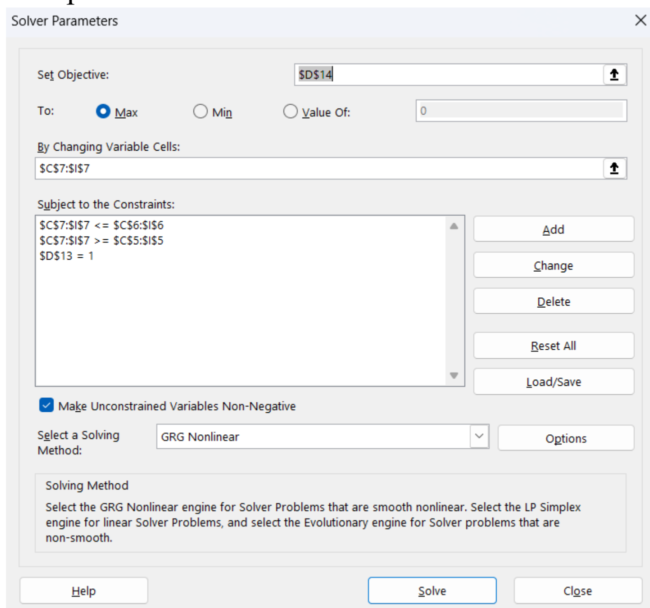

Figure 2. Optimization Model

This model aims to maximize the Sharpe Ratio, subject to three constraints:

- The minimum weight should be greater than or equal to 5%.
- The maximum weight should be less than or equal to 20%.
- The sum of the weights in the portfolio should be equal to 1 or 100%.

5. The expected return of the portfolio was calculated

$$expected\ return\ of\ the\ portfolio = \sum_{j=1}^{n} x_j \times y_j$$

$$x_j = annual\ return\ of\ stock\ n$$

$$y_j = optimal\ weight\ of\ stock\ n$$

6. The standard deviation of the portfolio was calculated

The covariance matrix (cov_m) and weights (w) are given by:

$$cov\_m = \begin{bmatrix} 0.000104324 & 3.54698E-05 & -2.47104E-05 & -6.25134E-06 & -4.31349E-06 & -4.66516E-06 & 6.82228E-07 \\ 3.54698E-05 & 0.000188949 & -3.32626E-07 & 3.34276E-06 & 1.93302E-05 & -1.95802E-06 & 8.70459E-06 \\ -2.47104E-05 & -3.32626E-07 & 0.000841743 & 0.000147598 & 0.000211816 & 1.1272E-05 & -3.3519E-05 \\ -6.25134E-06 & 3.34276E-06 & 0.000147598 & 0.000149499 & 0.000101261 & 8.11931E-05 & 1.26489E-05 \\ -4.31349E-06 & 1.93302E-05 & 0.000211816 & 0.000101261 & 0.000372307 & 0.000101019 & -5.2245E-06 \\ -4.66516E-06 & -1.95802E-06 & 1.1272E-05 & 8.11931E-05 & 0.000101019 & 0.00321983 & 4.47757E-05 \\ 6.82228E-07 & 8.70459E-06 & -3.3519E-05 & 1.26489E-05 & -5.2245E-06 & 4.47757E-05 & 0.000172328 \end{bmatrix}$$

$$w = [0.20 \quad 0.11 \quad 0.20 \quad 0.05 \quad 0.19 \quad 0.05 \quad 0.20]$$

The number of columns in the first matrix should equal the number of rows in the second. Thus, we transpose the weights matrix:

$$cov\_m\ x\ w^T$$

$$= \begin{bmatrix} 0.000104324 & 3.54698\text{E}-05 & -2.47104\text{E}-05 & -6.25134\text{E}-06 & -4.31349\text{E}-06 & -4.66516\text{E}-06 & 6.82228E-07 \\ 3.54698\text{E}-05 & 0.000188949 & -3.32626\text{E}-07 & 3.34276\text{E}-06 & 1.93302\text{E}-05 & -1.95802\text{E}-06 & 8.70459E-06 \\ -2.47104\text{E}-05 & -3.32626\text{E}-07 & 0.000841743 & 0.000147598 & 0.000211816 & 1.1272\text{E}-05 & -3.3519E-05 \\ -6.25134\text{E}-06 & 3.34276\text{E}-06 & 0.000147598 & 0.000149499 & 0.000101261 & 8.11931\text{E}-05 & 1.26489E-05 \\ -4.31349\text{E}-06 & 1.93302\text{E}-05 & 0.000211816 & 0.000101261 & 0.000372307 & 0.000101019 & -5.2245E-06 \\ -4.66516\text{E}-06 & -1.95802\text{E}-06 & 1.1272\text{E}-05 & 8.11931\text{E}-05 & 0.000101019 & 0.00321983 & 4.47757E-05 \\ 6.82228\text{E}-07 & 8.70459\text{E}-06 & -3.3519\text{E}-05 & 1.26489\text{E}-05 & -5.2245\text{E}-06 & 4.47757\text{E}-05 & 0.000172328 \end{bmatrix} \times \begin{bmatrix} 0.20 \\ 0.11 \\ 0.20 \\ 0.05 \\ 0.19 \\ 0.05 \\ 0.20 \end{bmatrix}$$

$$\text{cov\_m} \times \text{w}^{\text{T}} = \begin{bmatrix} 1.85954555 \times 10^{-5} \\ 3.32947178 \times 10^{-5} \\ 2.0485467114 \times 10^{-3} \\ 6.19410106 \times 10^{-5} \\ 1.23434254 \times 10^{-4} \\ 1.943058908 \times 10^{-3} \\ 3.07343255 \times 10^{-4} \end{bmatrix}$$

The annualized variance σ_P^2 of the portfolio is given by:

$$\sigma_{\text{P}}^2 = \text{w} \times \text{cov\_m} \times \text{w}^{\text{T}} \times 252$$

$$\sigma_{\text{P}}^2 = [0.20 \quad 0.11 \quad 0.20 \quad 0.05 \quad 0.19 \quad 0.05 \quad 0.20] \times \begin{bmatrix} 1.85954555 \times 10^{-5} \\ 3.32947178 \times 10^{-5} \\ 2.0485467114 \times 10^{-3} \\ 6.19410106 \times 10^{-5} \\ 1.23434254 \times 10^{-4} \\ 1.943058908 \times 10^{-3} \\ 3.07343255 \times 10^{-4} \end{bmatrix} \times 252$$

$$\sigma_{\text{P}}^2 = \left[\frac{2.2691040679 \times 10^{10}}{2.5 \times 10^{14}}\right] \times 252 = 0.02287266$$

The annualized standard deviation σ_P of the portfolio is:

$$\sigma_P = \sqrt{\sigma_P^2} \times 252 = \sqrt{0.02287266} \approx 0.15$$

7. The risk-free rate was set by utilizing the 10-year U.S Treasury Bond Yield at the time of the analysis

8. The total weight of the optimized portfolio was calculated

$$total\ weight = \sum_{i=1}^{n} optimal\ weight_i$$

9. The Sharpe Ratio of the optimized portfolio was calculated to determine its risk-adjusted return

$$Sharpe\ Ratio = \frac{expected\ retun - risk\ free\ rate}{standard\ deviation}$$

Results

Portfolio Summary	
Expected Return	84.13%
Standard Deviation	15.20%
Risk Free Rate	4.17%
Total Weight	100%
Sharpe Ratio	5.26

Figure 3. Optimized Portfolio Summary

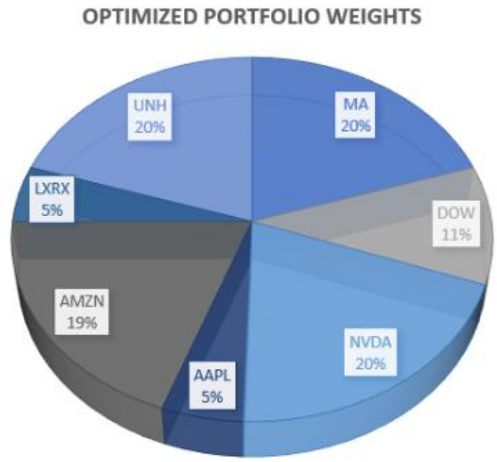

Figure 4. Optimized Portfolio Weights

Efficient Frontier

1. A random 1 x 7 array was generated

2. Random weights of the stocks in the portfolio were assigned by dividing the element in the array of the respective stock by the sum of the elements in the array.

$$\text{Random weight} = \frac{\text{Element of stock n}}{\sum_{i=1}^{n} \text{Element of stock}_i}$$

3. The standard deviation of the randomly weighted portfolio was calculated by using the matrix multiplication process shown in step 6 in the Optimized Portfolio Weights Section.

4. The expected return of the randomly weighted portfolio was calculated by using the sum product multiplication in step 5 in the Optimized Portfolio Weights Section.

5. The risk-free rate was set by utilizing the 10-year U.S Treasury Bond Yield at the time of the analysis

6. The total weight of the randomly weighted portfolio was calculated

$$total\ weight = \sum_{i=1}^{n} \text{random } weight_i$$

7. The Sharpe Ratio of the randomly weighted portfolio was calculated to determine risk-adjusted return

$$Sharpe\ Ratio = \frac{expected\ retun - risk\ free\ rate}{standard\ deviation}$$

8. A 10,000-trial simulation was conducted to determine the standard deviation, returns, and Sharpe Ratio of the randomly weighted stocks in the portfolio.

Efficient Frontier

Trials	St Dev	Return	Sharpe Ratio
1	12.64%	57.39%	4.21
2	21.29%	104.36%	4.71
3	14.88%	78.52%	5.00
4	17.11%	87.20%	4.85
5	20.33%	50.23%	2.27
6	13.33%	67.54%	4.75
7	21.22%	76.05%	3.39
8	21.06%	92.50%	4.19
9	18.47%	93.02%	4.81
10	23.90%	92.41%	3.69

Fig 5. Sample dataset of the simulated standard deviation, expected returns, and Sharpe Ratio

9. An efficient frontier was constructed to compare the risk/return ratios of 10,000 randomly generated portfolios against the optimized portfolio weights.

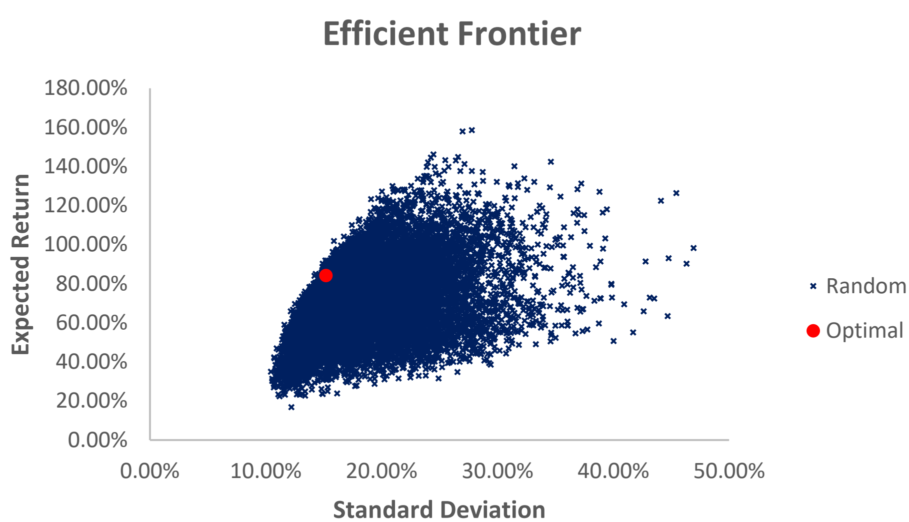

Fig 5. Efficient Frontier