
AI

Artificial Intelligence

8.2.7.
Constraint Satisfaction (Ch. 6)

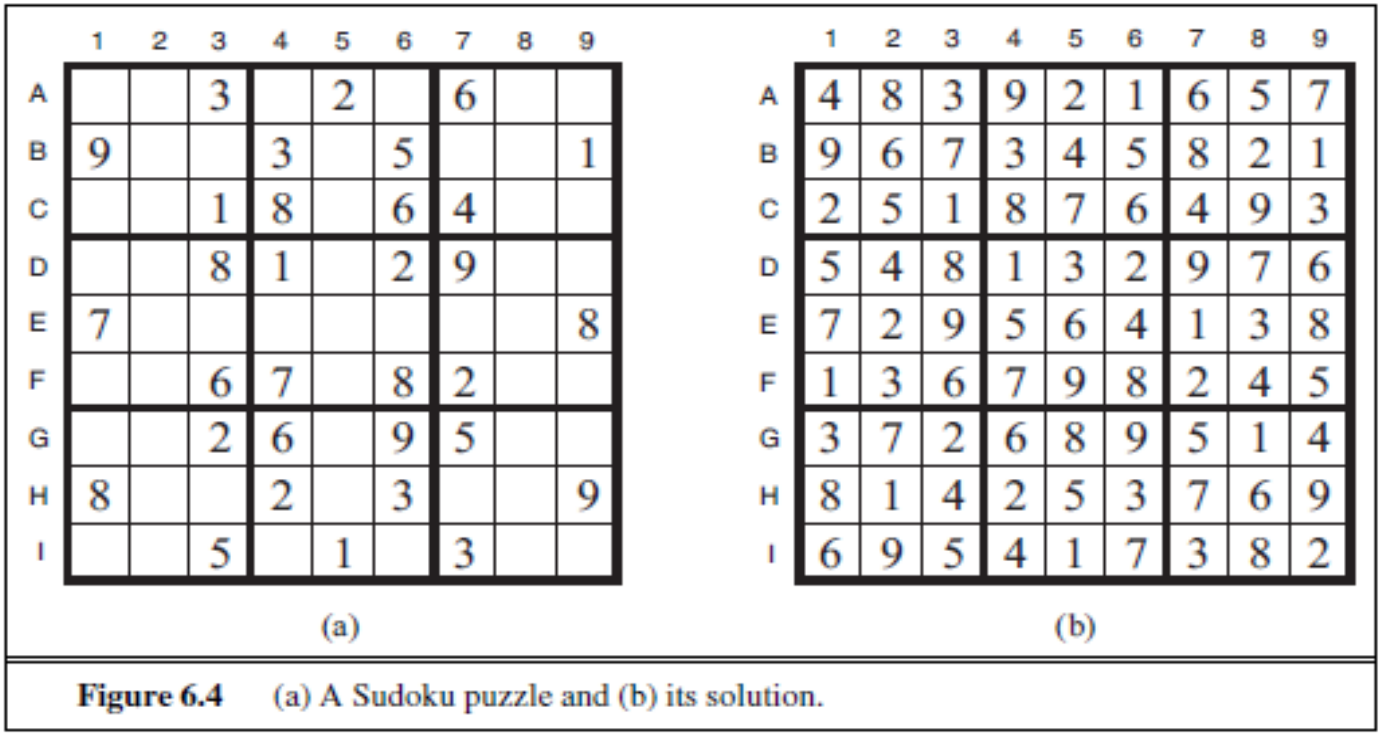
Outline

- Definition
- Inference
- Backtracking search
- Local search
- The structure of problems
- Examples: sudoku, crosswords

Sudoku

	1	2	3	4	5	6	7	8	9
A			3		2		6		
B	9			3		5			1
C			1	8		6	4		
D			8	1		2	9		
E	7								8
F			6	7		8	2		
G			2	6		9	5		
H	8			2		3			9
I			5		1		3		

Sudoku



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F			6	7		8	2		
G			2	6		9	5		
H	8			2		3			9
I			5		1		3		

(a)

	1	2	3	4	5	6	7	8	9
A	4	8	3	9	2	1	6	5	7
B	9	6	7	3	4	5	8	2	1
C	2	5	1	8	7	6	4	9	3
D	5	4	8	1	3	2	9	7	6
E	7	2	9	5	6	4	1	3	8
F	1	3	6	7	9	8	2	4	5
G	3	7	2	6	8	9	5	1	4
H	8	1	4	2	5	3	7	6	9
I	6	9	5	4	1	7	3	8	2

(b)

Figure 6.4 (a) A Sudoku puzzle and (b) its solution.

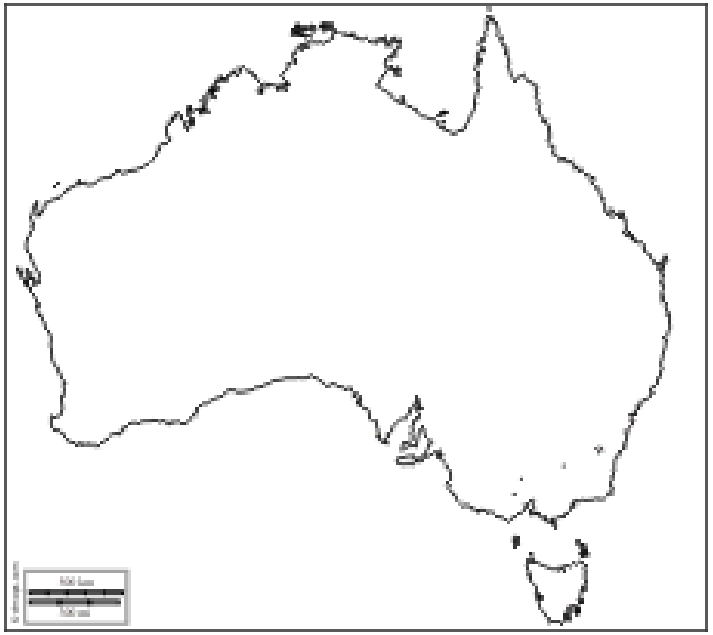
Sudoku Links

- http://norvig.com/sudoku.html
- http://pythonsudoku.sourceforge.net/
- https://pypi.python.org/pypi/sudoku-solver
- https://medium.com/towards-data-science/peter-norvigs-sudoku-solver-25779bb349ce
- http://www.sudokudragon.com/sudoku.htm
- http://www.websudoku.com/

Constraint satisfaction problems (CSPs)

- Standard search problem:
  - state is a "black box" – any data structure that supports successor function, heuristic function, and goal test
- CSP:
  - state is defined by variables X_i with values from domain D_i
  - goal test is a set of constraints specifying allowable combinations of values for subsets of variables
- Simple example of a formal representation language
- Allows useful general-purpose algorithms with more power than standard search algorithms

Another foreign trip



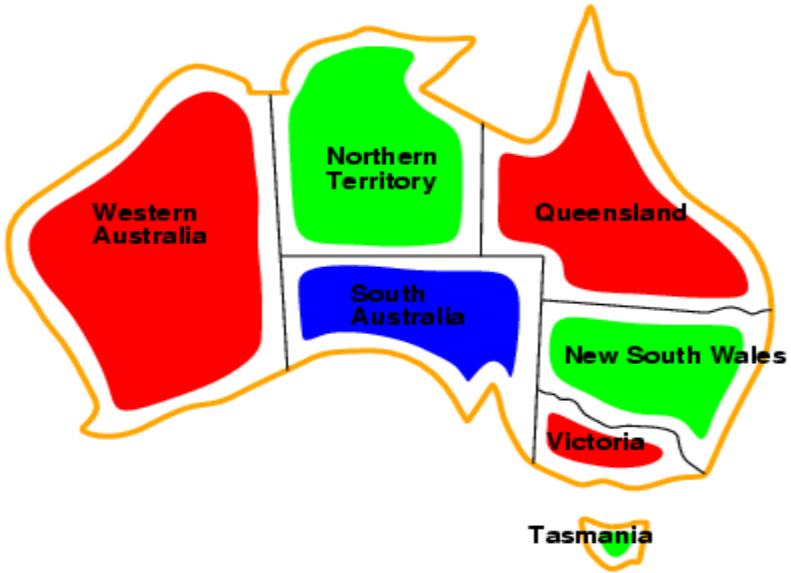

DOG
HEAD
CAT'S
HEAD
CAT FOOD

https://imgur.com/7I6Lnk7

Example: Map Coloring

- Variables *WA, NT, Q, NSW, V, SA, T*
- Domains D_i = {red,green,blue}
- Constraints: adjacent regions must have different colors
- e.g., WA ≠ NT, or (WA,NT) in {(red,green),(red,blue),(green,red), (green,blue),(blue,red),(blue,green)}

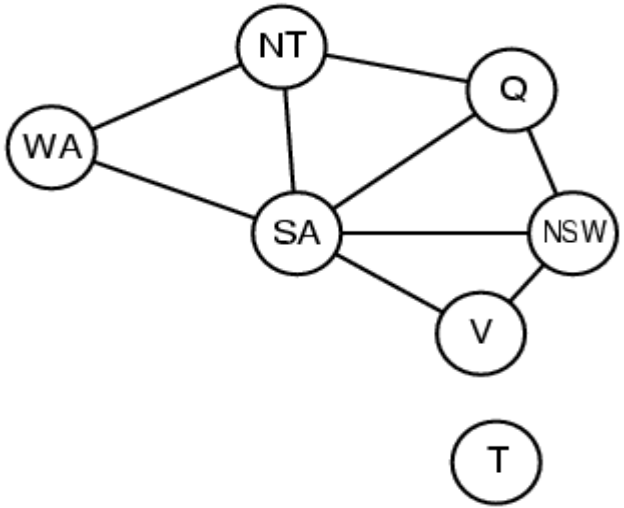
Example: Map Coloring



- Solutions are complete and consistent assignments, e.g., WA = red, NT = green,Q = red,NSW = green,V = red,SA = blue,T = green

Constraint graph

- Binary CSP: each constraint relates two variables
- Constraint graph: nodes are variables, arcs are constraints



Varieties of CSPs

- Discrete variables
  - finite domains:
    - n variables, domain size $d \rightarrow O(d^n)$ complete assignments
    - e.g., Boolean CSPs, incl.~Boolean satisfiability (NP-complete)
  - infinite domains:
    - integers, strings, etc.
    - e.g., job scheduling, variables are start/end days for each job
    - need a constraint language, e.g., $StartJob_1 + 5 \leq StartJob_3$
- Continuous variables
  - e.g., start/end times for Hubble Space Telescope observations
  - linear constraints solvable in polynomial time by linear programming

Varieties of constraints

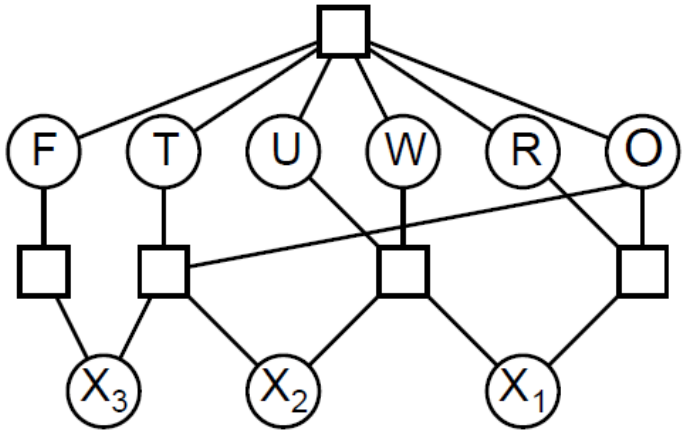
- Unary constraints involve a single variable,
  - e.g., SA ≠ green
- Binary constraints involve pairs of variables,
  - e.g., SA ≠ WA
- Higher-order constraints involve 3 or more variables,
  - e.g., cryptarithmetic column constraints

Puzzle

```
  T W O
+ T W O
-------
F O U R
```

Example: Cryptarithmetic

$$\begin{array}{r} \,\mathrm{T\ W\ O} \\ +\ \mathrm{T\ W\ O} \\ \hline \mathrm{F\ O\ U\ R} \end{array}$$



Variables: $F\ T\ U\ W\ R\ O\ X_1\ X_2\ X_3$

Domains: $\{0, 1, 2, 3, 4, 5, 6, 7, 8, 9\}$

Constraints

alldiff(F, T, U, W, R, O)

$O + O = R + 10 \cdot X_1$, etc.

Solution

- 734+734=1468

Real-world CSPs

- Assignment problems
  - e.g., who teaches what class
- Timetabling problems
  - e.g., which class is offered when and where?
- Transportation scheduling
- Factory scheduling
- Circuit layout

- Notice that many real-world problems involve real-valued variables

State-based models

- State:
  - Keeps track of the relevant information so far, that is needed to find an optimal solution
- Search problem
- Min-cost path
- Tree-based algorithms
  - Backtracking
- Graph-based algorithms
  - Dynamic Programming
  - Uniform Cost Search
  - A*

Standard search formulation (incremental)

Let's start with the straightforward approach, then fix it

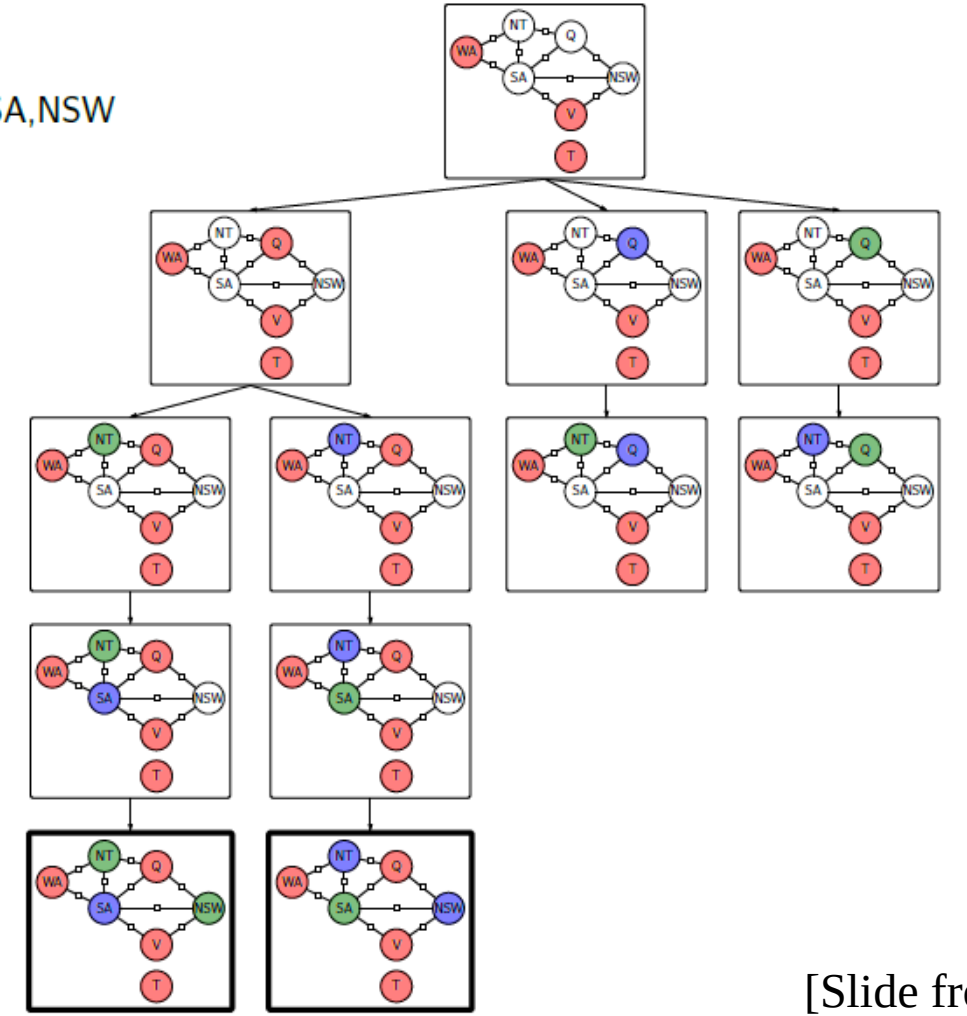
States are defined by the values assigned so far

- Initial state: the empty assignment { }
- Successor function: assign a value to an unassigned variable that does not conflict with current assignment
  → fail if no legal assignments
- Goal test: the current assignment is complete

1. This is the same for all CSPs
2. Every solution appears at depth *n* with *n* variables
   → use depth-first search
3. Path is irrelevant, so can also use complete-state formulation
4. b = (n - ℓ)d at depth ℓ, hence n! · d^n leaves

Search

WA,V,T,Q,NT,SA,NSW



[Slide from Percy Liang]

Backtracking search

- Variable assignments are commutative, i.e.,

  [WA = red then NT = green] same as [NT = green then WA = red]
- Only need to consider assignments to a single variable at each node

  → b = d and there are d^n leaves
- Depth-first search for CSPs with single-variable assignments is called backtracking search
- Backtracking search is the basic uninformed algorithm for CSPs
- Can solve *n*-queens for $n \approx 25$

Backtracking search

```
function BACKTRACKING-SEARCH( csp) returns a solution, or failure
    return RECURSIVE-BACKTRACKING({}, csp)

function RECURSIVE-BACKTRACKING( assignment, csp) returns a solution, or
failure
    if assignment is complete then return assignment
    var ← SELECT-UNASSIGNED-VARIABLE( Variables[csp], assignment, csp)
    for each value in ORDER-DOMAIN-VALUES(var, assignment, csp) do
        if value is consistent with assignment according to Constraints[csp] then
            add { var = value } to assignment
            result ← RECURSIVE-BACKTRACKING(assignment, csp)
            if result ≠ failue then return result
            remove { var = value } from assignment
    return failure
```

Backtracking example

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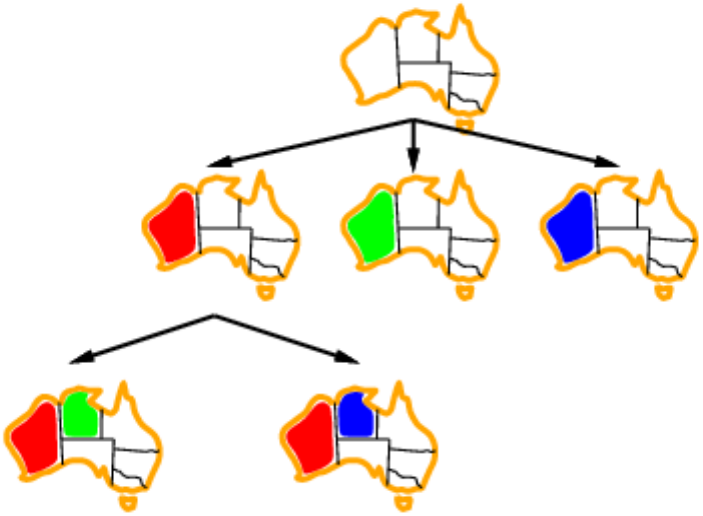

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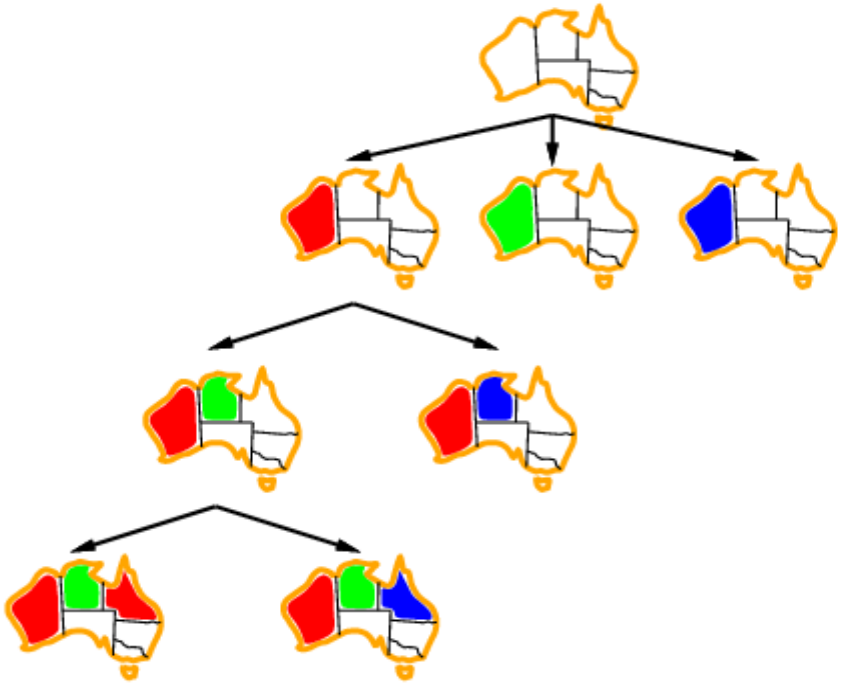


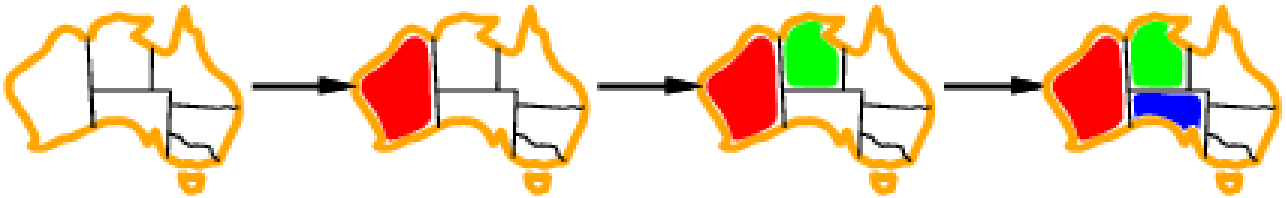
Improving backtracking efficiency

- General-purpose methods can give huge gains in speed:
  - Which variable should be assigned next?
  - In what order should its values be tried?
  - Can we detect inevitable failure early?

Most constrained variable

- Most constrained variable:

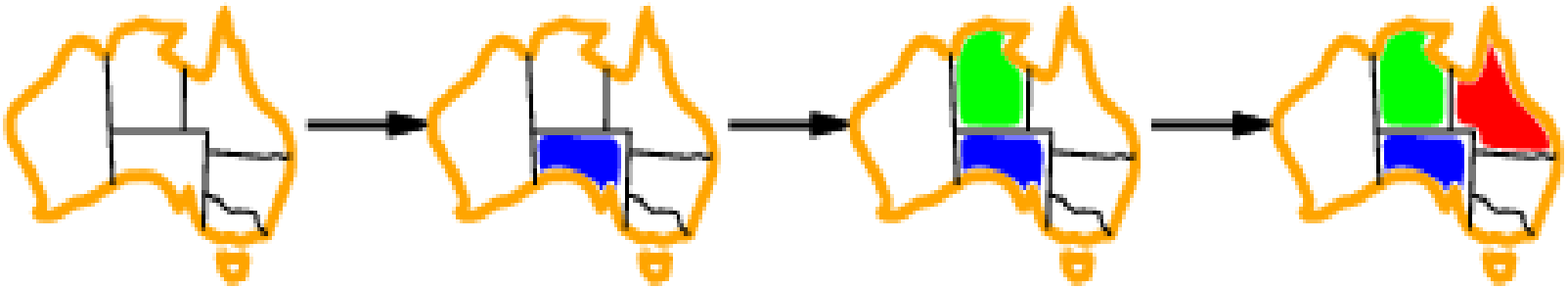
  choose the variable with the fewest legal values



- a.k.a. minimum remaining values (MRV) heuristic

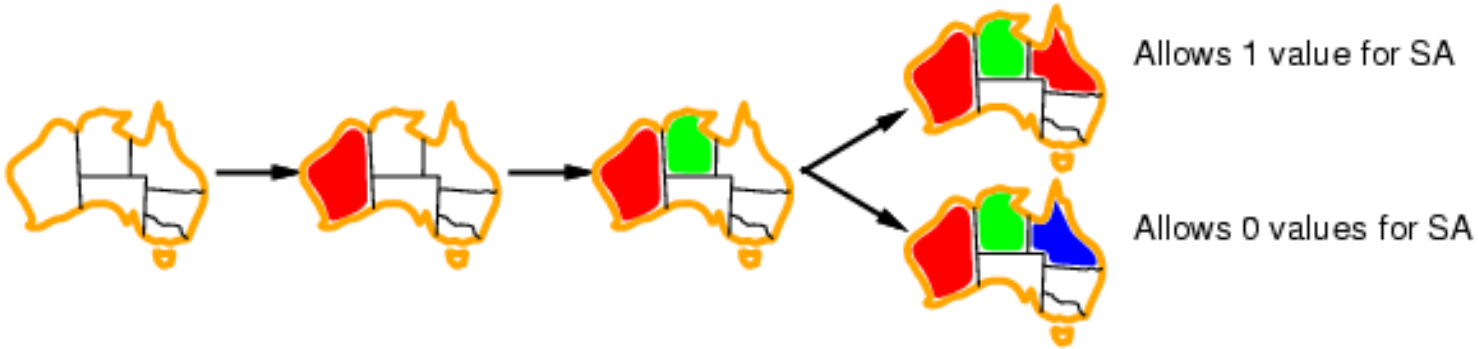
Most constraining variable

- Tie-breaker among most constrained variables
- Most constraining variable:
  - choose the variable with the most constraints on remaining variables



Least constraining value

- Given a variable, choose the least constraining value:
  - the one that rules out the fewest values in the remaining variables



- Combining these heuristics makes 1000 queens feasible

Forward checking

- Idea:
  - Keep track of remaining legal values for unassigned variables
  - Terminate search when any variable has no legal values

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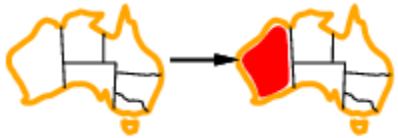

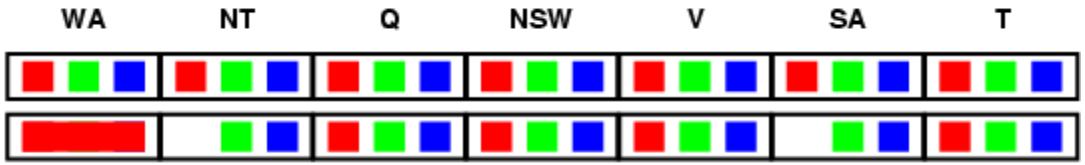

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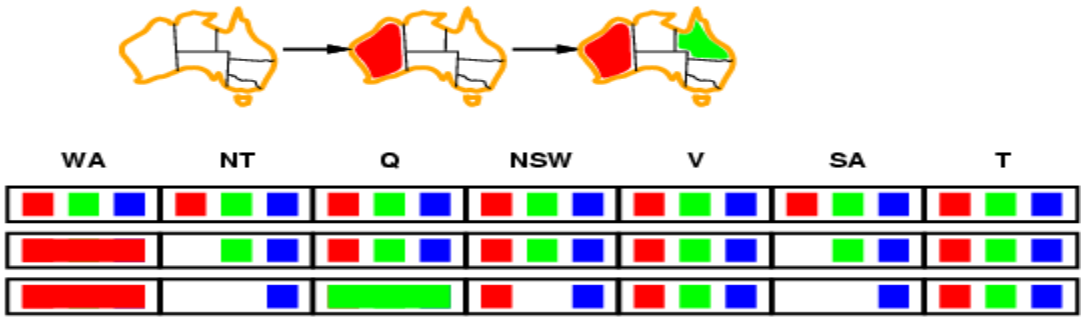

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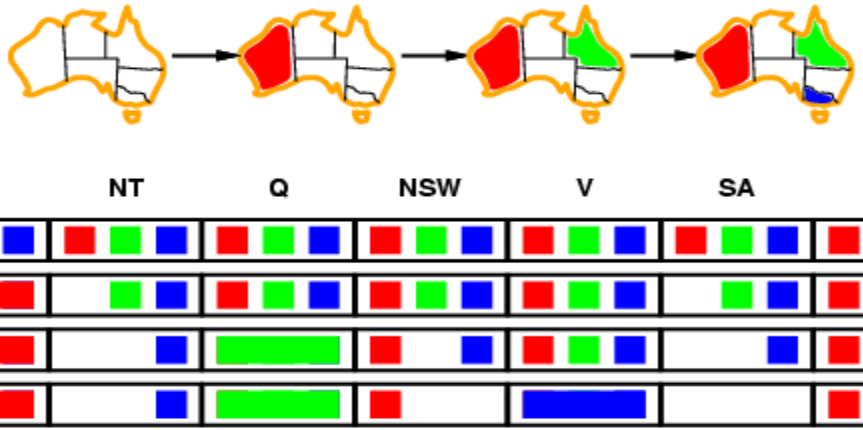


Constraint propagation

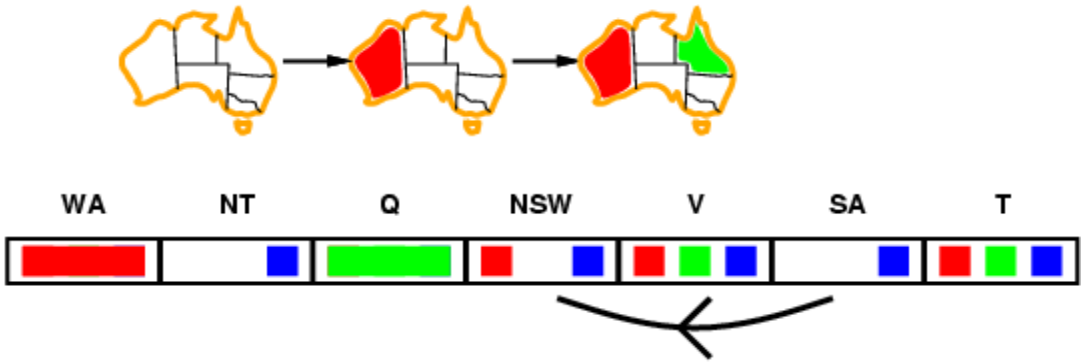
- Forward checking propagates information from assigned to unassigned variables, but doesn't provide early detection for all failures:

- NT and SA cannot both be blue!
- Constraint propagation repeatedly enforces constraints locally

Arc consistency

- Simplest form of propagation makes each arc consistent
- $X \rightarrow Y$ is consistent iff

  for every value x of X there is some allowed y



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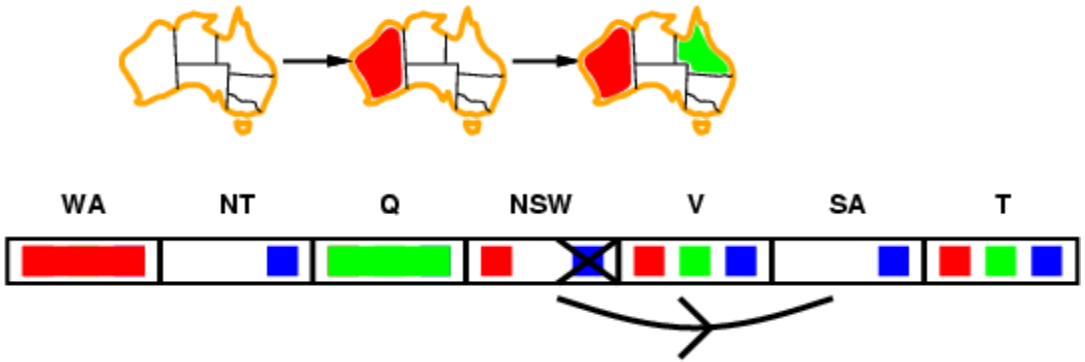

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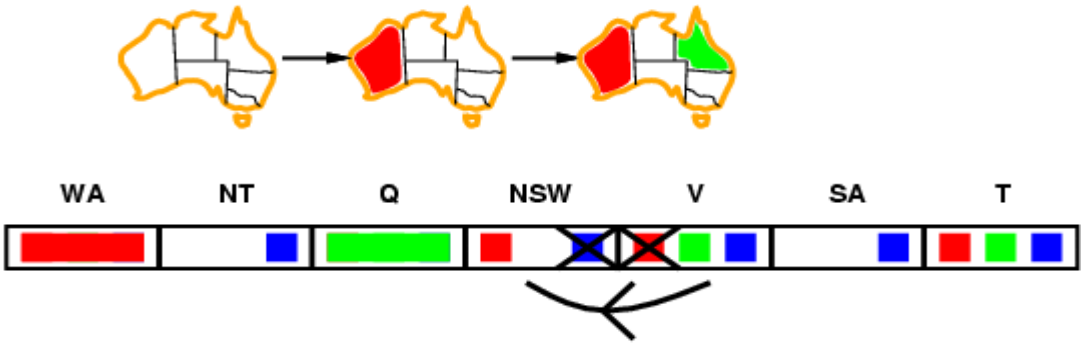

- If X loses a value, neighbors of X need to be rechecked

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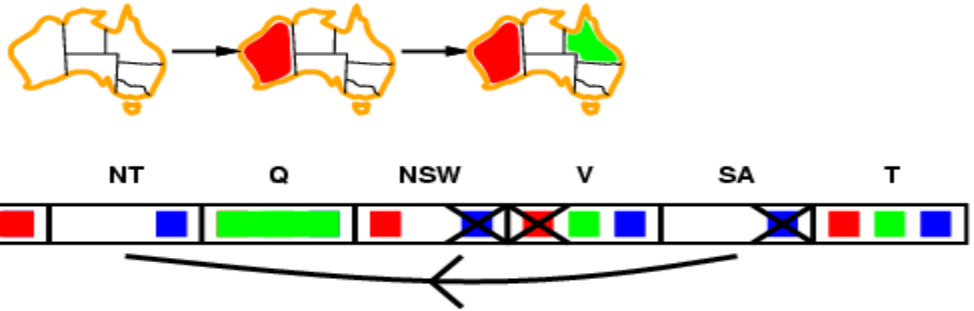


- If X loses a value, neighbors of X need to be rechecked
- Arc consistency detects failure earlier than forward checking
- Can be run as a preprocessor or after each assignment

Arc consistency algorithm AC-3

```
function AC-3( csp) returns the CSP, possibly with reduced domains
   inputs: csp, a binary CSP with variables {X_1, X_2, ..., X_n}
   local variables: queue, a queue of arcs, initially all the arcs in csp

   while queue is not empty do
      (X_i, X_j) ← REMOVE-FIRST(queue)
      if RM-INCONSISTENT-VALUES(X_i, X_j) then
         for each X_k in NEIGHBORS[X_i] do
            add (X_k, X_i) to queue

---

function RM-INCONSISTENT-VALUES( X_i, X_j) returns true iff remove a value
   removed ← false
   for each x in DOMAIN[X_i] do
      if no value y in DOMAIN[X_j] allows (x,y) to satisfy constraint(X_i, X_j)
         then delete x from DOMAIN[X_i]; removed ← true
   return removed
```

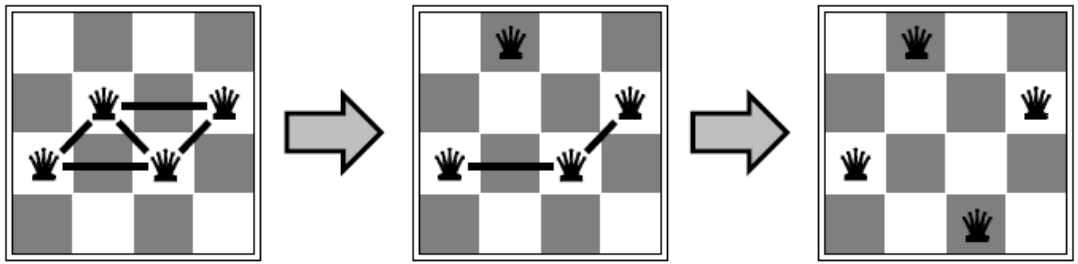
- Time complexity: $O(n^2d^3)$

Local search for CSPs

- Hill-climbing, simulated annealing typically work with "complete" states, i.e., all variables assigned
- To apply to CSPs:
  - allow states with unsatisfied constraints
  - operators reassign variable values
- Variable selection: randomly select any conflicted variable
- Value selection by min-conflicts heuristic:
  - choose value that violates the fewest constraints
  - i.e., hill-climb with *h(n)* = total number of violated constraints

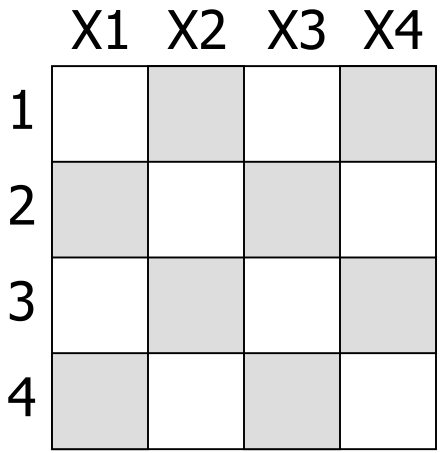
Example: 4-Queens

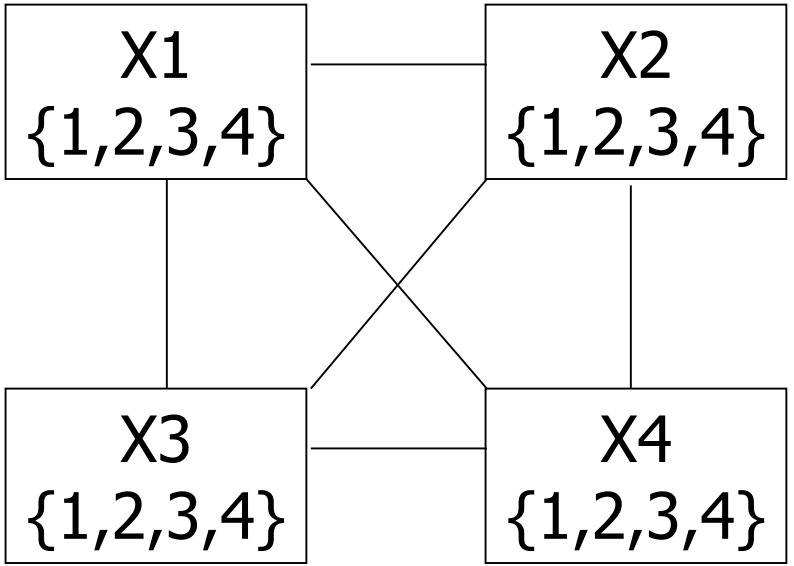
- States: 4 queens in 4 columns (4^4 = 256 states)
- Actions: move queen in column
- Goal test: no attacks
- Evaluation: *h(n)* = number of attacks



- Given random initial state, can solve *n*-queens in almost constant time for arbitrary *n* with high probability (e.g., *n* = 10,000,000)

Example: 4-Queens Problem





(Slides from Bonnie Dorr)

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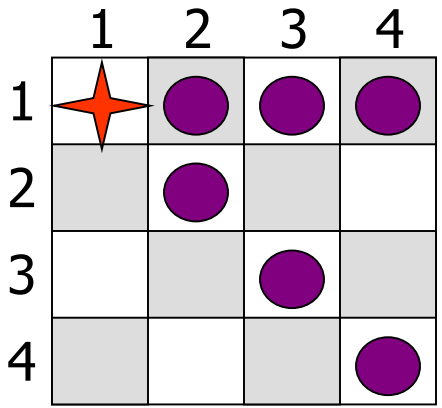

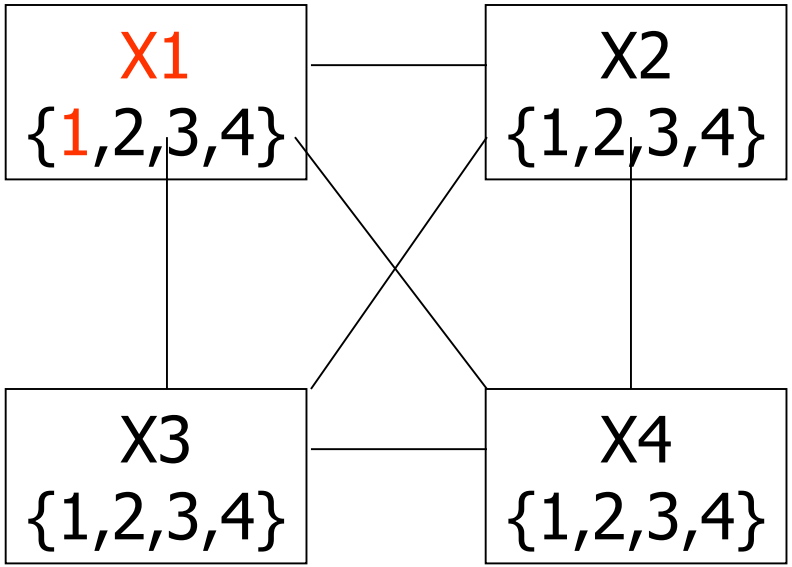

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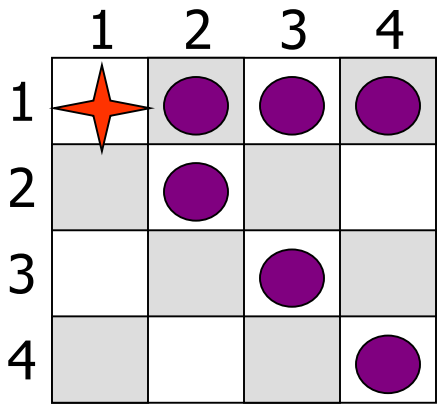

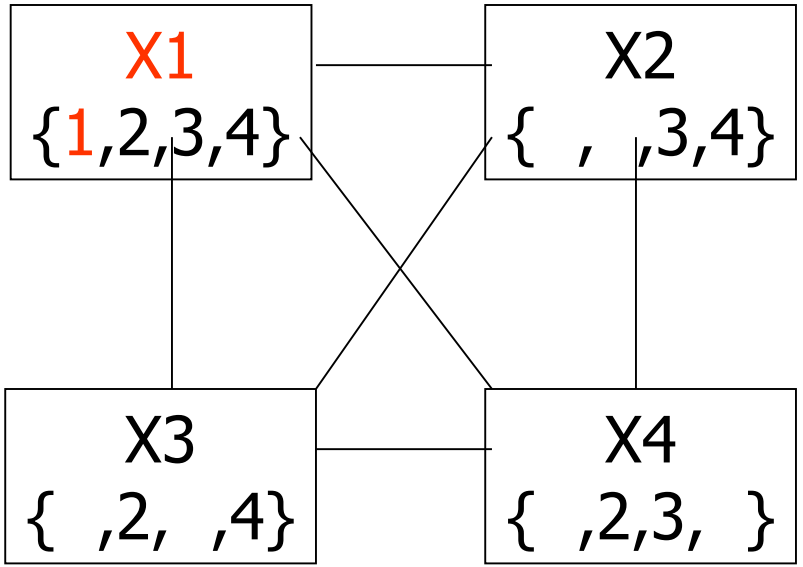

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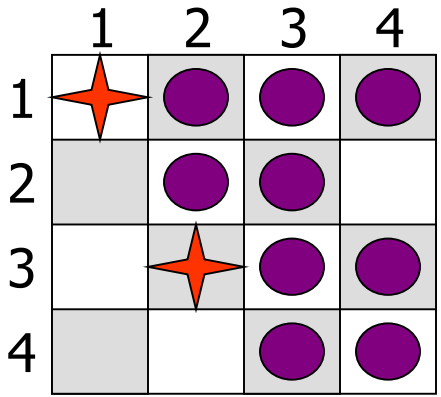

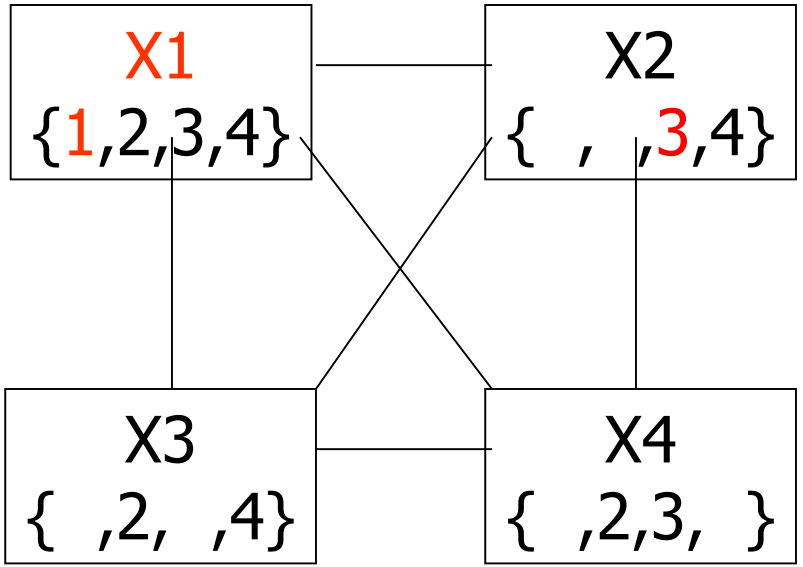

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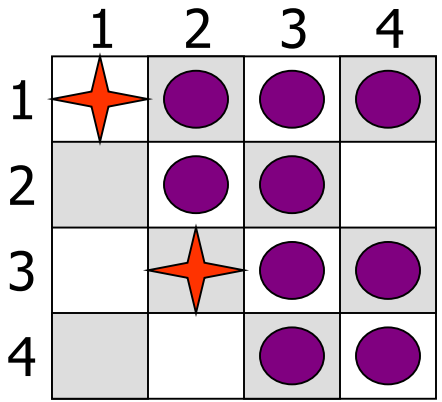
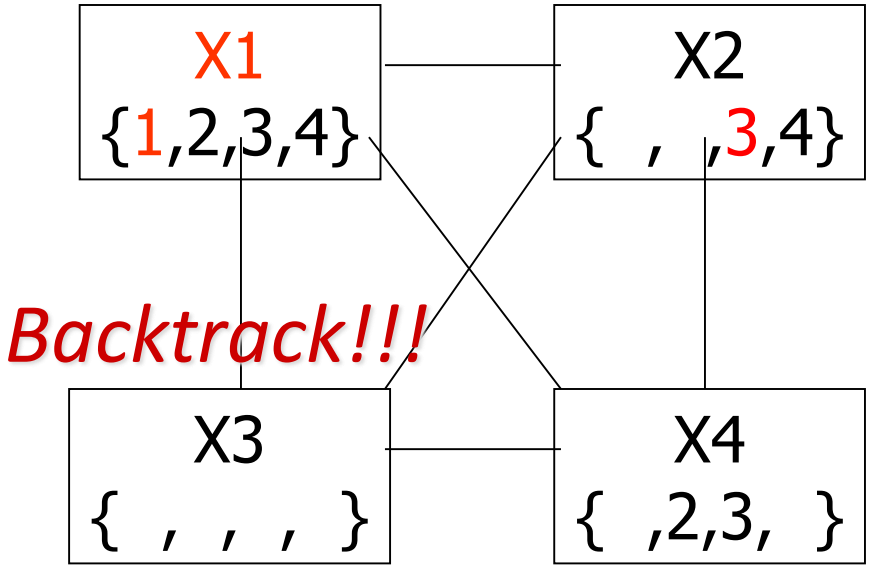
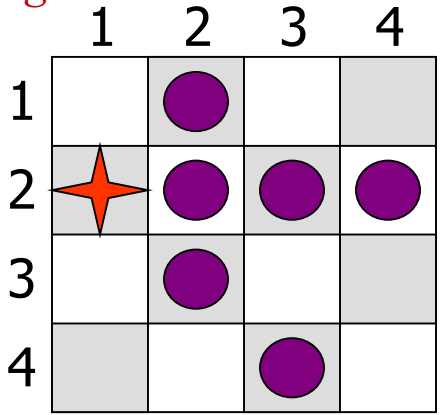
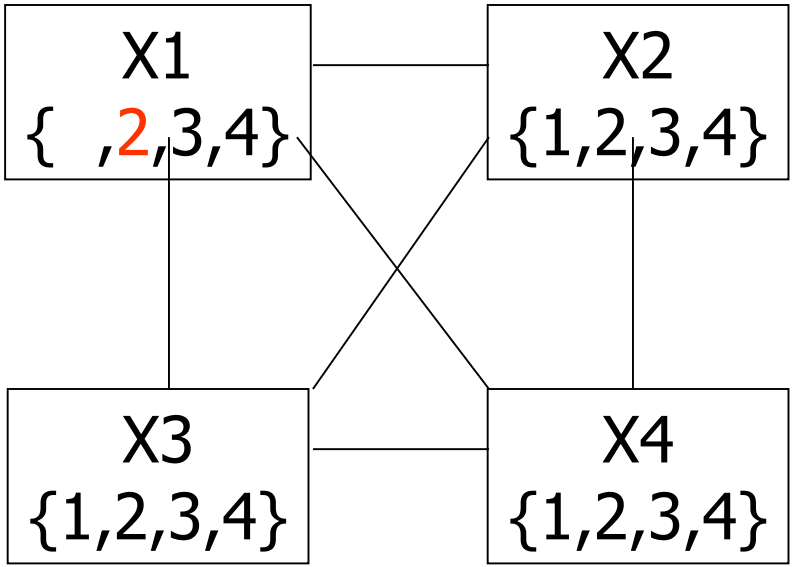




Example: 4-Queens Problem

Picking up a little later after two steps of backtracking….





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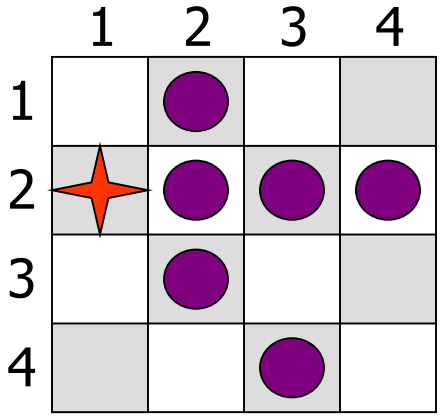

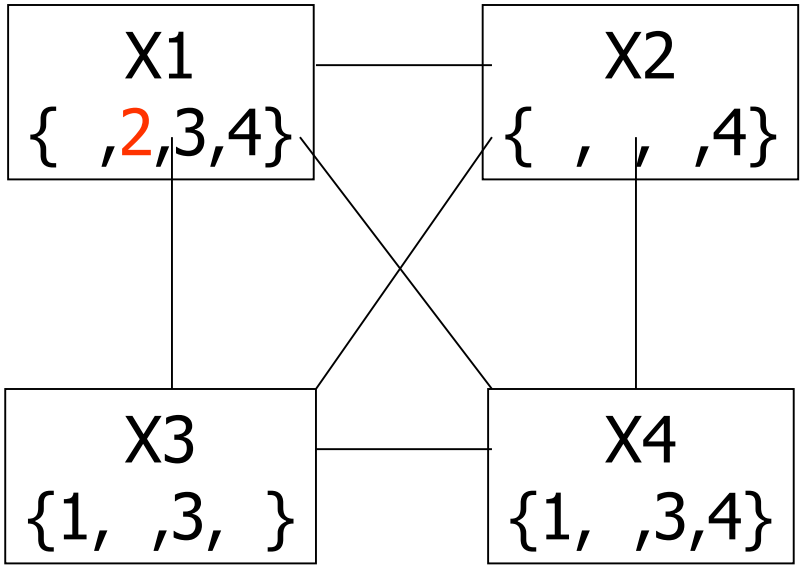

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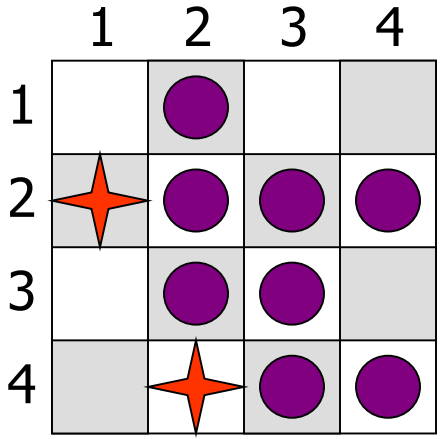
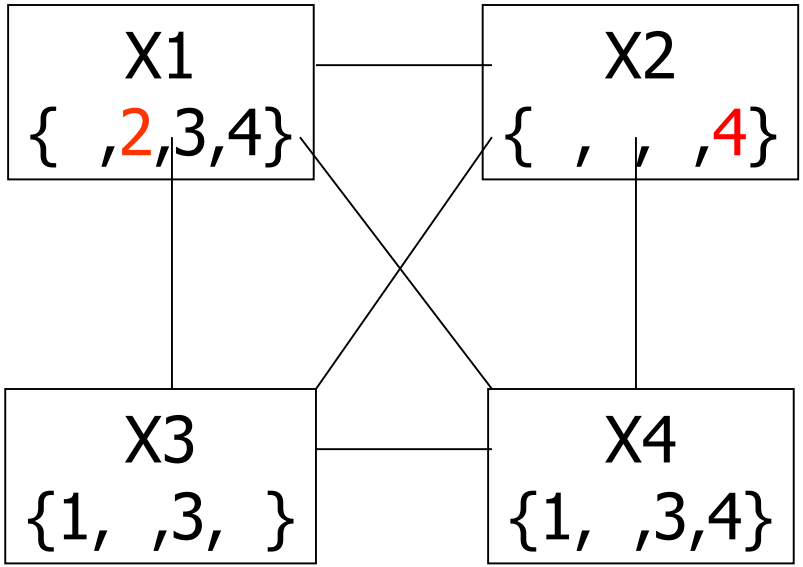
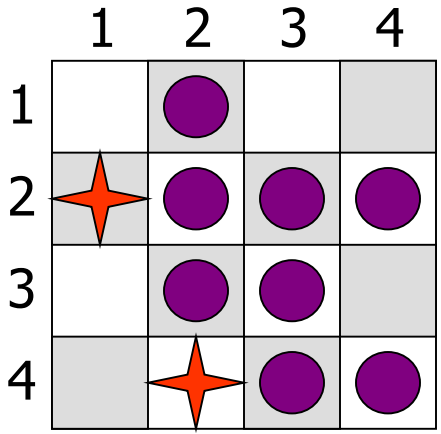
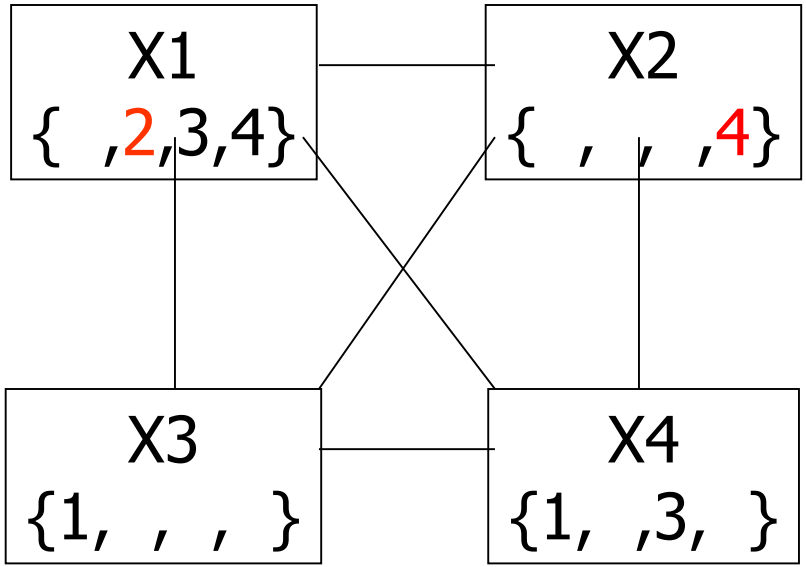


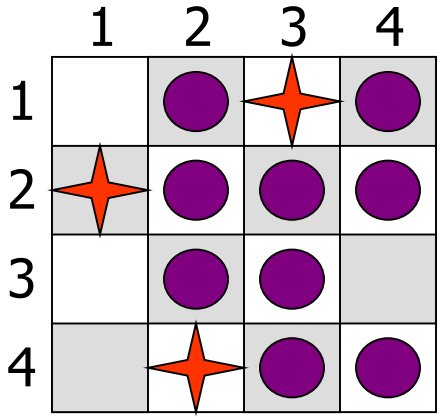


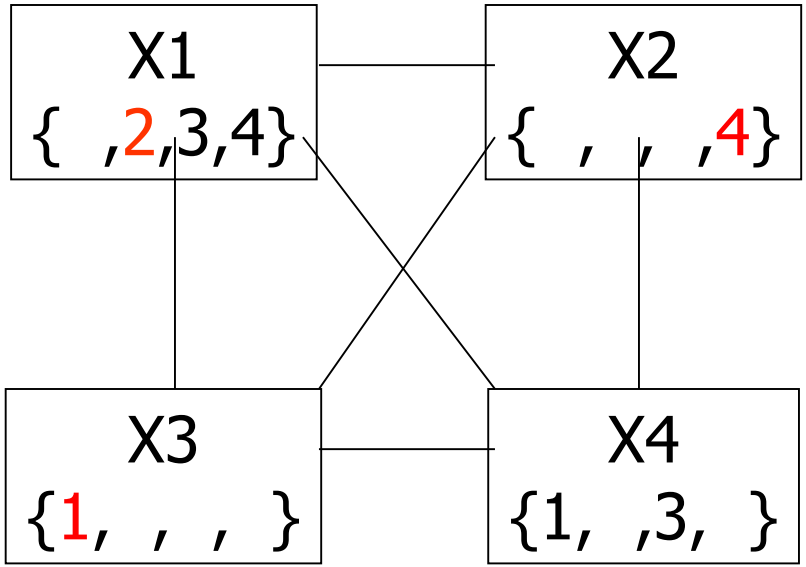
Example: 4-Queens Problem



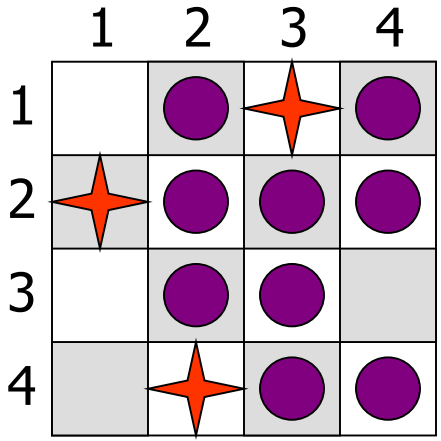


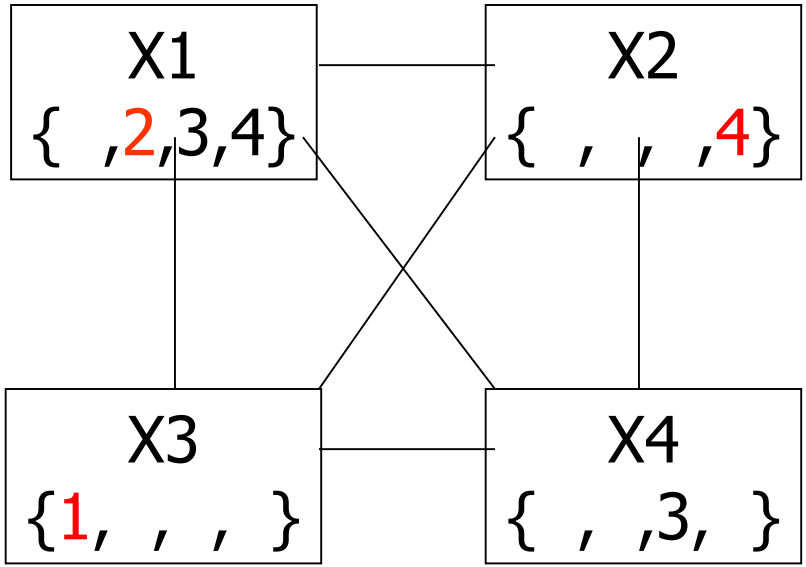
Example: 4-Queens Problem



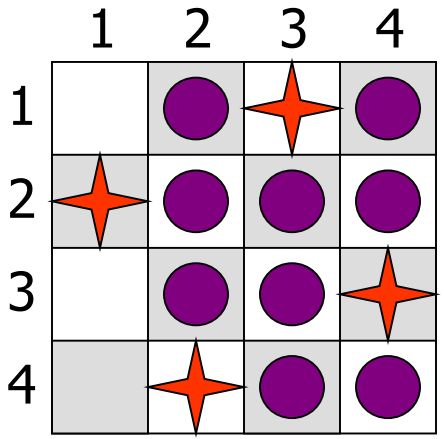


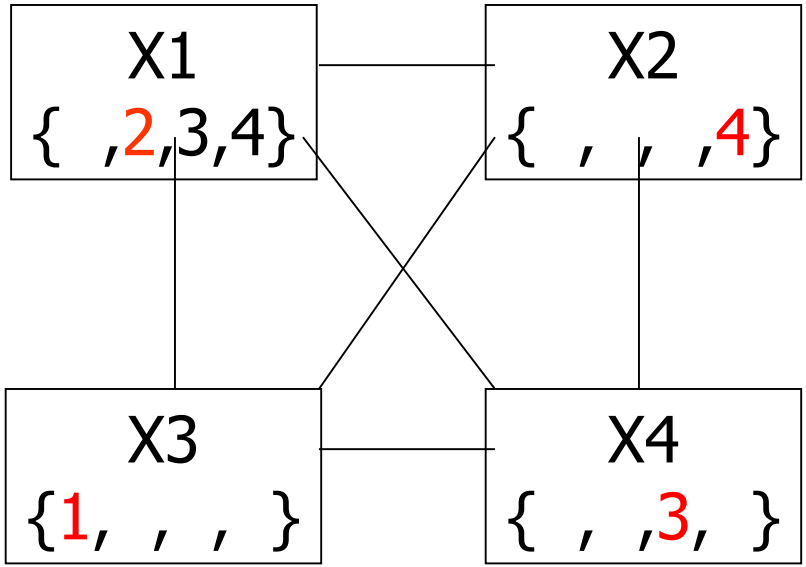
Example: 4-Queens Problem





Example: 4-Queens Problem



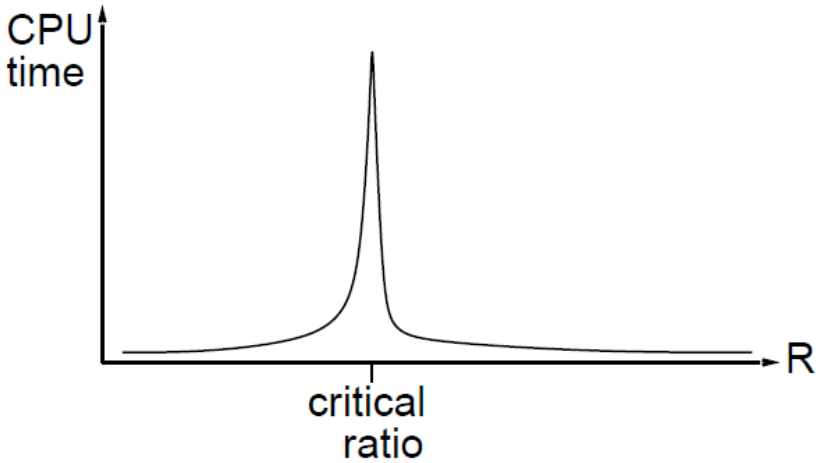


Performance of min-conflicts

Given random initial state, can solve n-queens in almost constant time for arbitrary n with high probability (e.g., $n = 10{,}000{,}000$)

The same appears to be true for any randomly-generated CSP
except in a narrow range of the ratio

$$R = \frac{\text{number of constraints}}{\text{number of variables}}$$



Simple csp example

```
vars=['Alex','Bob'] # We will define our vars to be undergrads, Alex and Bob
domains={'Alex':[1,2,3,4],'Bob':[1,2,3,4]} # Alex and Bob are both in year 1...4
neighbors={'Alex':['Bob'],'Bob':['Alex']} # Specify that Alex is affected by Bob & vice versa
def constraints(A, a, B, b): # Constraint: Alex is 3 years ahead of Bob
    # Fail if Alex isn't 3 years ahead of Bob
    if (A=='Alex' and B=='Bob' and a - b <> 3):
        return False
    # Fail if Bob isn't 3 years behind Alex
    # (nb: we cover all possible input with logical consistency)
    if (B=='Alex' and A =='Bob' and b - a <> 3):
        return False
    return True
csp=CSP(vars, domains, neighbors, constraints) # Get a CSP object that defines this problem
assignments=min_conflicts(csp) # Solve the CSP with the min conflicts algorithm
print assignments # Should observe that Bob: 1 Alex: 4
```

Example by Andrew Mercer-Taylor

Crossword

Edited by Will Shortz

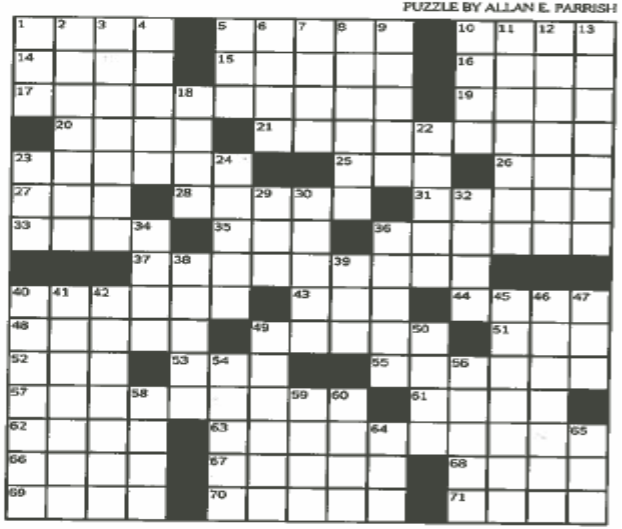
PUZZLE BY ALLAN E. PARRISH

ACROSS

1 Pyramid city close to Cairo
5 Mocking remarks
10 "Yikes!"
14 Achieved a perfect score on
15 Obstinate reply
16 African country bordering 12-Down
17 Socialite who inspired 1950's "Call Me Madam"
19 Texts, e.g.: Abbr.
20 Fossil fuel
21 Sulu and Uhura, e.g., on the Enterprise
23 1957 hit covered by Creedence Clearwater Revival in 1968
25 First word in many newspaper names
26 "___ you for real?"
27 ___ Dhabi
28 Stale-smelling
31 Like Old Norse writing
33 Workplace communication
35 Letters before an alias
36 Not eat eagerly
37 Pastrami and salami
40 Vietnam War weapon
43 Make a goof
44 Sea of Tranquillity, for the Apollo 11 astronauts
48 Car fuel additive
49 Army NCOs
51 Site of a church kneeler
52 ___-la-la
53 McKellen who played Magneto in the "X-Men" films
55 One of a 1980s demographic
57 Run out of energy
61 Martinique et Guadeloupe
62 Many a car on the autobahn
63 Groups hired for high-profile cases
66 Carpentry spacer
67 Sky-blue
68 Where Vietnam is
69 Roget's listings: Abbr.
70 Theater reservations
71 1930s art style, informally

DOWN

1 Clothing chain with a "Baby" division
2 Drink cooler
3 Like a game with equal winners and losers
4 Old politico Stevenson
5 "Garfield" cartoonist Davis
6 Leaves dumbstruck
7 Deluxe
8 It's really something
9 ___ infection
10 Jane Austen heroine
11 What's filled up in a fill-up
12 African country bordering 16-Across
13 Cut up, as a frog
18 Kind of school after nursery school: Abbr.
22 Boy Scout ___ badge
23 Uncle ___
24 Hesitation about something
29 Slalom, say
30 Breaks . . . or an anagram of the ends of five Across answers in this puzzle
32 Golden State school up the coast from L.A.
34 Old jazz icon Anita
36 The "P" of G.O.P.
38 Perry of fashion
39 Part of a joule
40 Business setback recorded on Schedule C
41 Wither away
42 Add by degrees
45 Mollify
46 Earth-shaking
47 Animal whose name sounds like a pronoun
49 It goes "Ah-h-h-choo!"
50 Zoot ___
54 Mythical strong man
56 Beg
58 State trees of North Dakota and Massachusetts
59 Spanish water
60 Many a service station adjunct
64 "___ Misérables"
65 ___ Paulo

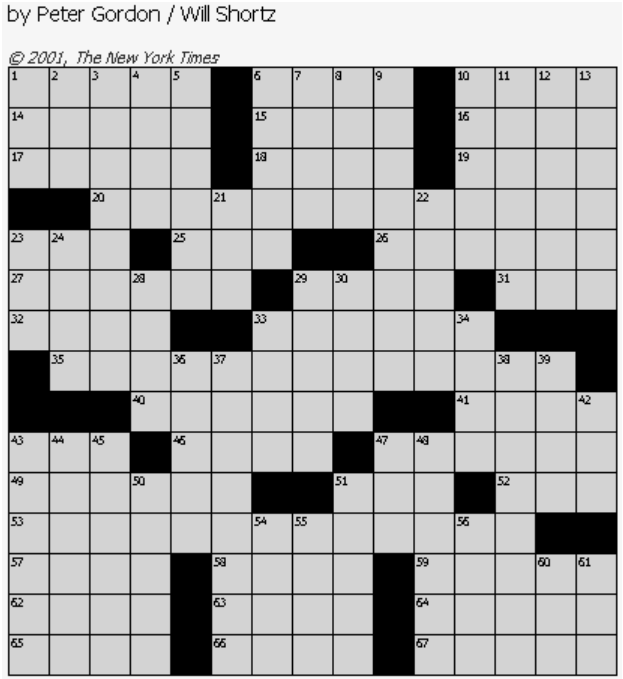


9/1/14

ANSWER TO PREVIOUS PUZZLE

L	O	M	B	A	R	D		G	O	G	O	B	A	R
A	Z	O	B	L	U	E		O	V	E	R	A	T	E
Z	Z	Z	Q	U	I	L		G	A	N	G	S	T	A
A	F	I		I	N	T	E	G	R	A		S	I	D
R	E	L	Y		G	O	L	L	Y		M	A	M	E
U	S	L	A	W		I	K	E		B	A	L	E	R
S	T	A	S	H	E	D		S	E	E	R	E	S	S
			M	I	X				D	I	Y			
F	I	T	I	N	T	O		I	U	N	K	A	R	T

Crossword puzzles

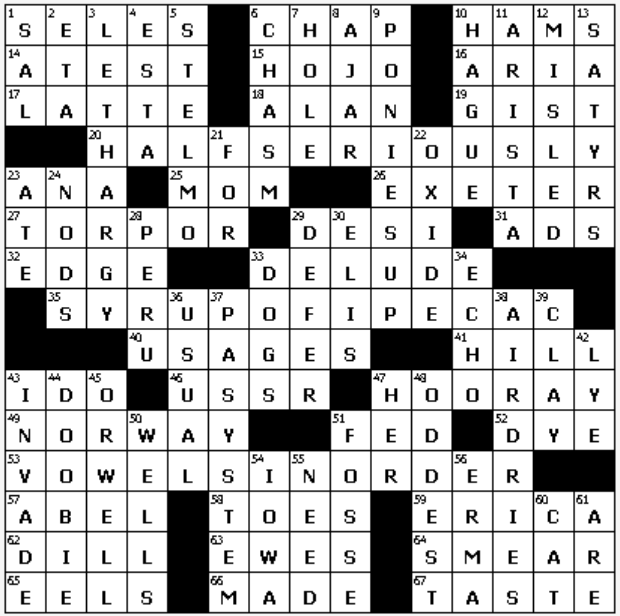


Across

1. Tennis champ Monica
6. Fellow
10. Radio operators
14. Alamogordo event
15. Orange-roofed hotel, for short
16. Met song
17. Starbucks order
18. Harvard's Dershowitz
19. Main idea
20. In a semi-joking way
23. Santa ___, Calif.
25. May honoree
26. New England prep school town
27. Listlessness
29. Ricky's portrayer
31. What Consumer Reports lacks
32. Brink
33. Dupe
35. Emergency medicine for infants
40. Customary practices
41. Sledding spot
43. Nuptial agreement
46. Cold war initials
47. "Yippee!"
49. Lillehammer's country
51. G-man
52. Batiking need
53. Feature of 20- and 35-Across, forward and backward
57. Noted fratricide victim
58. Low digits
59. "All My Children" character
62. Pickle flavoring
63. She sheep
64. Window washer's mistake
65. Sushi fish
66. Earned
67. Sample

Down

1. Mule of song
2. Fraternity letter
3. Listlessness
4. "Cómo ___ usted?"
5. Sailor's patron
6. Abyss
7. Ace place?
8. Not quite shut
9. Pays
10. Noted court site, with "The"
11. Popular record label
12. Deceived
13. Lechers
21. In place of
22. Nitrous ___ (laughing gas)
23. Took a meal
24. Silent assents
28. Home for llamas
29. Put off
30. Yale players
33. Follows persistently
34. Reverberate
36. Typical
37. It's used for announcements
38. Hangs on the line
39. Potter's supply
42. Drain cleaner ingredient
43. Intrude upon
44. The ___ Brothers of 70's-80's rock
45. "Animal Farm" author
47. Boating pronoun
48. Most bizarre
50. "The War of the Worlds" author
51. Choreographer Bob
54. Part of the Corn Belt
55. Reason for a donation
56. Witty Bombeck
60. Garfield, e.g.
61. Exist


1 S 2 E 3 L 4 E 5 S 6 C 7 H 8 A 9 P 10 H 11 A 12 M 13 S
14 A T E S T 15 H O J O 16 A R I A
17 L A T T E 18 A L A N 19 G I S T
20 H A L 21 F S E R I 22 O U S L Y
23 A 24 N A 25 M O M 26 E X E T E R
27 T O R 28 P O R 29 D 30 E S I 31 A D S
32 E D G E 33 D E L U D 34 E
35 S Y R 36 U 37 P O F I P E C 38 A 39 C
40 U S A G E S 41 H I L 42 L
43 I 44 D 45 O 46 U S S R 47 H 48 O O R A Y
49 N O R 50 W A Y 51 F E D 52 D Y E
53 V O W E L S 54 I 55 N O R D 56 E R
57 A B E L 58 T O E S 59 E R I 60 C 61 A
62 D I L L 63 E W E S 64 S M E A R
65 E E L S 66 M A D E 67 T A S T E

Word 1

- Sense 1
  - –It has bark but no bite
  - –Its bark is silent
- Sense 2
  - –Branch location
- Sense 3
  - –Fort locale
  - –Type of house
  - –House for kids
- Sense 4
  - –Lineage display
  - –Every family has one
  - –Family chart
- Sense 5
  - –Cobbler's need
  - –Shoe stretcher
- Others
  - –It leaves in the spring
  - –Where to get dates
  - –Site of many a cat rescue
  - –Dendrophobe's fear
  - –Forbidden fruit source
  - –Golf-course obstacle
  - –Ring holder
  - –Leaves home
  - –Newspaper source
  - –K-I-S-S-I-N-G place

TREE

Words 2 and 3

- Sense 1
  - –Flood survivor
  - –Ararat lander
  - –Rider of the lost ark
  - –Biblical helmsman
  - –Guy who believed in “take two”
  - –Noted couples protector
  - –Early matchmaker
  - –Captain of a famous cruise for couples
  - –Life preserver
  - –Person known for double takes
- Sense 2
  - –First name in lexicography
  - –One of the Websters
- Sense 3
  - –Actor Wyle

NOAH

- Sense 1
  - –Houston Colt 45 today
  - –Enron Field player, once
  - –Houston player
  - –Texas leaguer
  - –Nolan Ryan, notably
  - –2005 world series participant
  - –Player under a dome
  - –Pirate battler, at times
- Sense 2
  - –Cartoon dog
  - –Space age hound
  - –Jetson canine
  - –Animated pooch
  - –Elroy’s pet
- Sense 3
  - –Bygone Chevrolet van

ASTRO

Evaluation

PH**I**L..SCUBA.IMP	PH**I**L..SCUBA.IMP
OEA-O.NANAS.DOE	OU**T**ED.NANAS.DOE
WRM-SR-BBIT.ORE	**WHITERABBIT**.ORE
..IMOUT.ROOSTER	..SUNUP.ROOSTER
GA**S**P.ITNO..AONE	GA**S**P.ITNO..AONE
ET**A**.KNOCKONWOOD	ET**A**.KNOCKONWOOD
AM**I**SH..RELAY...	AM**I**SH..RELAY...
RE**D**CAPS.NANETTE	RE**D**CAPS.NANETTE
...OKIES..CREED	...OKIES..CREED
AMERICAN**P**IE.LAD	AMERICAN**P**IE.LAD
NAVE..WO**O**L.ALLY	NAVE..WO**O**L.ALLY
ICEDTEA.**RETRO**..	ICEDTEA.**TETRA**..
MAN.SILV**E**RBELLS	MAN.SILV**E**RBELLS
AWE.ARLE**N**.ANITA	AWE.ARLE**N**.ANITA
LSD.RESE**T**..AERO	LSD.RESE**T**..AERO

Crossword clues

- Pyramid city close to Cairo (4)
- __ Dhabi (3)
- McKellen who plays Magneto in the “X-Men” franchise (3)
- Martinique et Guadeloupe (4)
- Many a car on the Autobahn (4)
- Mythical strong man (5)

Crossword clues

- 19th out of 24 (3) TAU
- The continents, e.g., (6) SEPTET
- Made bats (9) DRIVENMAD
- Runners in the cold (5) NOSES
- Quit stalling (7) DOITNOW
- Oktoberfest quaff (4) BIER
- Out-of-the way way (6) DETOUR
- Rubber from Arabia (7) ALADDIN
- One inspiring love of poetry (5) ERATO
- Lab report (3) ARF
- Year John Dryden died (4) MDCC
- One spinning one's wheels (6) POTTER
- It's bigger than a family (5) ORDER
- Some loaves (4) RYES
- The Beatles' "P.S. I Love You", e.g. (5) BSIDE
- Maximally hip (7) COOLEST

Crossword clues

- Early third-century year (4) CCIV
- When the day's done, to Denis (4) NUIT
- Cover with new shingles (6) REROOF
- Puncture preceder (3) ACU
- Suffix with miss and dismiss (3) IVE
- Nurse (3) SIP
- Politician with a like button? (3) IKE
- Beyond piqued (5) ANGRY
- Ones trapped in boxes of their own making? (5) MIMES
- Quince, e.g., (6) NUMERO
- "Celeste Aida," for one (4) ARIA
- Take a shot? (6) IMBIBE
- Long sentence (4) LIFE
- Hyundai and Kia (5) AUTOS
- Suffix with shepherd (3) ESS
- "Thank you," in Hawaii (6) MAHALO

Crossword clues

- 2,502, to ancient Romans (5) MMDII
- ___ Beach, S.C. (6) MYRTLE
- Sierra ___ (5) LEONE
- Canoodling in a restaurant, e.g. (abbr.) (3) PDA
- New Orleans-to-Detroit dir. (3) NNE
- Harry Potter's Hedwig, e.g., (3) OWL
- Mothers with pride? (9) LIONESSES
- Platinum, for example (5) METAL
- Work undercover, in a way (3) SPY
- Helper, in brief (4) ASST
- Intro to physics? (4) META
- Cronus and Hyperion (6) TITANS
- Shrek or Fiona (5) OGRES
- Electrican, often (5) WIRER
- Lion, sometimes (6) ROARER

Crossword clues

- Plasm preceder (4) ECTO
- Prefix with scope (6) STETHO
- Overshoot, say (4) MISS
- Gait (4) PACE
- Medical suffix (3) OMA
- Fox tail? (4) TROT
- Honda model (6) ACCORD
- What losses do to fans (6) SADDEN
- Largest city in Nebraska (5) OMAHA
- "Owner of a Lonely Heart" band (3) YES
- Fire antonym (4) HIRE
- Chemists' org (3) AIC
- "Titanic actor Billy (4) ZANE
- Former South African president (5) BOTHA

Constraint Satisfaction Problem

- (X, D, C)
  - X is set of variables = (X_1, … , X_k)
  - D is set of domains = (D_1, … , D_k)
    - Each D_i is set of allowed values for X_i
  - C is set of constraints
    - C_i = <participants, relation>

Example by Steven Wilson

For the Crosswords Problem

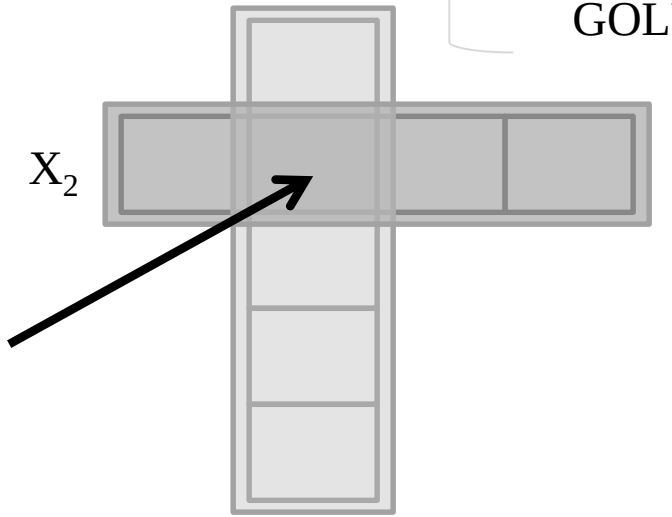
- (X, D, C)
  - X is the set of entries that must be filled
  - D is set of candidate answers
  - C enforces intersections

D_1= FRODO, MERRY, GOLUM

X_1

D_2= DRAW, FIRE, SHOT

X_2



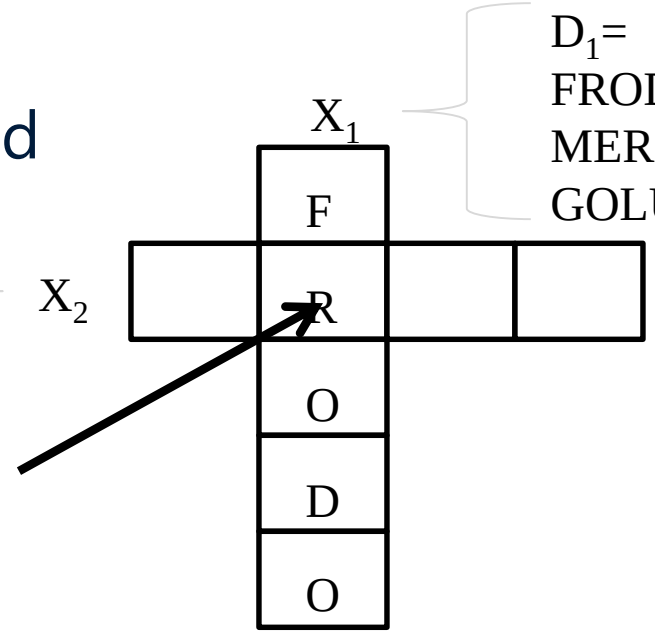
C = (<X_1,X_2>, $X_1[1]=X_2[1]$ OR $X_1[1]=$“” OR $X_2[1]=$“”)

Minimum Conflicts Heuristic

- Choose v ∈ D_i with lowest number of conflicts
  - Otherwise randomly
- Continue to reassign conflicted variables

D_2= DRAW, FIRE, SHOT

D_1= FRODO, MERRY, GOLUM



C = (<X_1,X_2>, $X_1[1]=X_2[1]$ OR $X_1[1]=$“” OR $X_2[1]=$“”)

What to do with conflicts?

- Back off one answer at a time
  - Choose x = answer with lowest ranking
  - Set x to blank
  - Repeat until no conflicts left

A crossword puzzle CSP

- Variables: the entries of the puzzle (“1A”, etc).
- Domains: possible words for an entry. Generated by NLP components.
- Neighbors: entries sharing an intersection.
- Constraint: entries that share an intersection must have the same letter in that intersection.

Example by Andrew Mercer-Taylor

Solving the crossword puzzle

- Initially, all entries are set to “-”*, which indicates missing values. This means our CSP is technically solved from the beginning.
- Instead of looking for a solution that just fulfills the constraints, we search for one that maximizes the likelihood of the words.
- Every candidate word has some score reflecting how likely it is for the given clue. These are provided by the NLP components.

Dr. Fill Procedure 6.4

Given a crossword puzzle CSP, a partial solution S, best solution so far B, and previously pitched assignments P:

solve(CSP, S, B, P):

if S assigns every variable, return B or S, whichever is better

S' ← S with the addition of the most likely assignment not in P to an unassigned variable

CSP' ← propagate(CSP, S')

if propagation succeeded, B ← solve(CSP', S', B, P)

if P contains all values, return B

P' ← P with the addition of the assignment we tried in S'

B ← solve(CSP, S, B, P')

Postprocessing

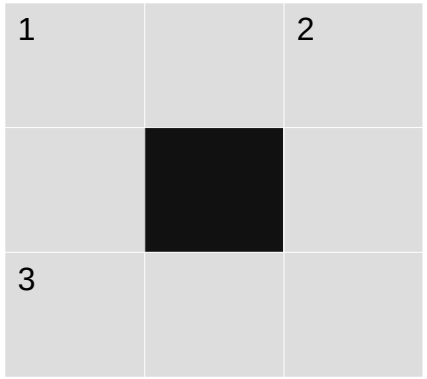
In practice, this algorithm results in many cases where a single letter is missing. We therefore perform the following postprocessing procedure:

incompleteEntries ← a list of entries with missing letters, ordered by the number of letters missing

for entry e in incompleteEntries:

    w ← a word that matches the length and the existing letters of entry, None if none found

    if w is not None, assign e to w



Across:

1) Man's best friend

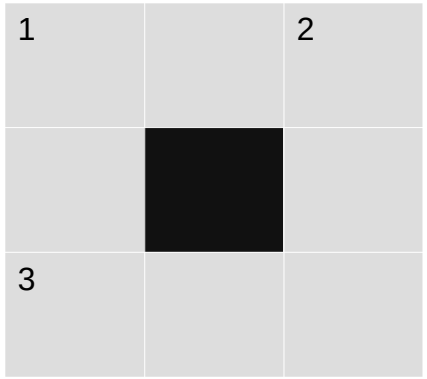
3) Application format

Down:

1) To be paid

2) ETS exam

Variable	Domain (ranked by score)
1A	DOG, PET, PAL
1D	DUE, IOU, FEE, OWE, PRO
2D	SAT, GRE
3A	DLL, EXE, BAT, MSI, ZIP, SYS



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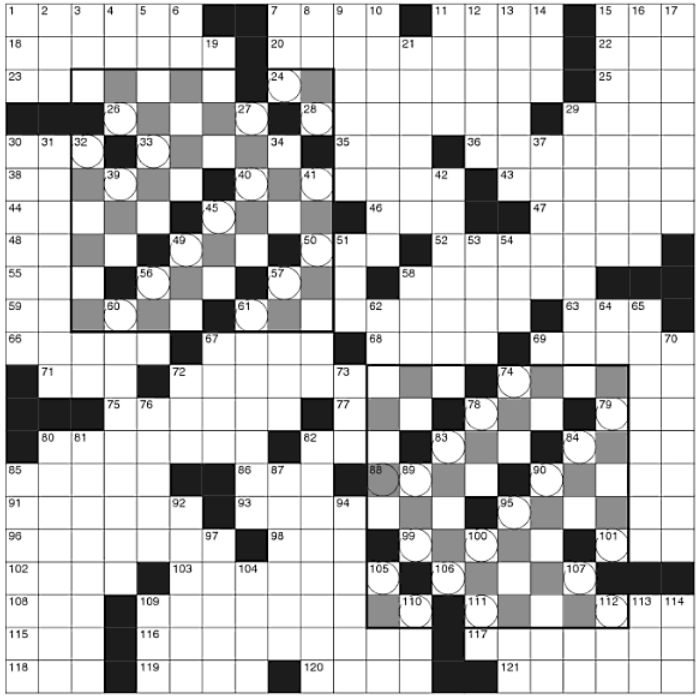
1 D	O	2 G
U		R
3 E	X	E

Solving a crossword CSP

- Instead of looking for a solution that just fulfills the constraints, we search for one that maximizes the likelihood of the words.
- Every candidate word has some score reflecting how likely it is for the given clue. These are provided by the NLP components.



by Derrick Niederman

ACROSS

1 Panhandle site
7 In ___ rush
11 Two-wheeler
15 Farm mother
18 Defendants' protests
20 Health resorts
22 Call ___ day
23 Chaim Weizmann, for one
24 Orbit
25 Shook hands, say
26 Capital of Jamaica [black]
28 Beloved British figure [black]
29 District of Colombia?
30 Platoon V.I.P.
33 Noted Twain portrayer [black]
35 N.Y.C. subway
36 Chloe's love, in myth
38 Military pilots
40 Fortress
43 April, May and June, to Daisy Duck
44 Misdirect, maybe
45 Suffix with 44-Across
46 Long time
47 Yellowish shade
48 When schoolkids go to bed early [black]
49 Actress Ullmann
50 U.K.'s locale
52 Common hospital name
55 Baseball Hall-of-Famer Roush
56 Grown-up kit
57 Writer/humorist Joe [white]
58 Basic belief
59 Eying [white]
61 The sixth of Henry VIII
63 Big inits. in bowling
66 Rotten
67 Solitary
68 Easy two-pointer
69 ___-Turkish War
71 Hwy.
72 Abandoned child
74 Postwinter flood
75 Author Gordimer
77 Famed bluesman [black]
78 Cry on "The Simpsons"
79 Dog in "Dondi" [black]
80 Hobo
82 Newspaper adv. unit
83 Criticize
84 Bankrolls
85 Russian emperor in 1800
86 Tokyo, formerly
88 Bidding site
90 Send
91 Freed
93 Way of thinking
95 100 centimos
96 Begin to flourish
98 Excellent service
99 Ship's mooring aid [black]
101 Med. land
102 Actress Lillian
103 Some fuel-carrying ships
106 Old ___, Conn. [black]
108 Actress Sue ___ Langdon
109 Colonial craft [white]
111 Rotten
115 ___ culpa
116 What black and white squares do on a chessboard
117 Leather factory
118 Round no., maybe
119 Predicament
120 Husky vehicle
121 Stowe character

DOWN

1 Relative of a chisel
2 Circular greeting
3 Year abroad
4 Kind of feeling
5 Given name among old Chinese leaders
6 Old Dodgers manager called "Smokey"
7 Enzyme suffix
8 Spring festival honoree
9 Geritol target
10 Unruliness
11 One of the three B's
12 Annoyed
13 John of radio's "Information, Please!"
14 Tuck in
15 Follower of samedi
16 Studios
17 "The Piano Lesson" painter
19 Small runway craft, for short
21 Walk shakily
27 First World Series color broadcaster, 1955
29 Result of black's move from the upper-left board to the lower-right board
30 Proved pleasing
31 Emergency safeguard
32 Yelled
34 Beginner
37 Devout
39 R&B singer Brian
41 Picked up the check
42 Some corporal punishment
45 Early morning hour
49 Chance
51 Durham sch.
53 Fun race
54 Prevaricate
56 Wray of "King Kong"
57 Kind of chair
58 Link aggregation, in computing
60 "Proper" speech
61 Expression of numerical certainty
62 Food
64 Box office draws
65 Smoothness
67 Butcher's offering
69 I, in Berlin
70 Poor, filmwise
72 Medicine approval agcy.
73 Wall Street deal, in brief
74 Charged particle
76 Sign of spring
78 ___ job
80 United and Allied, e.g.
81 It's typically made of leather or plush
82 Responsibilities
83 Trails
84 N.B.A.'s Unseld
85 Darts, e.g.
87 Old phone user
89 Dub "sir"
90 Singer Leon
92 Mild-mannered
94 Skin-related
95 Online purchase facilitator
97 Sheds
100 Toward dawn
104 Mad. and Lex.
105 Glut
107 Cheating
109 "Cheers" role
110 Like a double-decker checker
112 Celtic Neptune
113 Afore
114 ___ job

Crossword puzzles

- OneAcross – Littman
- Dr. Fill – Ginsberg
- **Our corpus**
  - **174,641** words with **1,692,482** clues
  - Highest density words:
  - **Ear:** 1808 clues
  - **Sea:** 1707 clues
  - **Tree:** 1578 clues
  - **Era:** 1578 clues
  - **Aria:** 1428 clues
  - ….

Generated	Actual Solution
LASERBEAM.IDOLS	CLICKBAIT.ANGST
E---RO--E.RAMOS	HENRIETTA.NORAH
AUSTRALIA.ESTER	INSOMNIAC.DRIVE
DOIN..ELLEN.TAB	RAPS..MLKJR.NAB
EOOYLOU..BETSEY	APOSTLE..LEANNE
R---OP.RUE.NINA	CETERA.AGO.NINA
...-PUSI-.JONAS	...DOTTIE.JONAS
MOVIESTOOKSIXTY	FIVETHIRTYEIGHT
AGNES.EJ----...	ADAYS.RESEWN...
NINE.RPA.LOOSES	CORE.RED.LETSAT
TENANT..ALLENDE	ENDSIT..ALLENDE
INN.AELSS..ISEE	STA.KELPS..DAVE
L--LF.OHTEA----	OLLIE.OUTSPOKEN
L--ET.SORNC----	FIONA.STRAINERS
ARENA.STODE----	FESTS.STOPPEDBY

Components

1 Missing last name

- Titanic actor Billy (4) ZANE
- James who wrote "A Death in the Family" (4) AGEE
- Cesar who played the Joker (6) ROMERO
- Cheri formerly of "S.N.L." (5) OTERI

2 Missing first name

- “Wrecking Ball" singer Cyrus (5) MILEY
- Playwright O'Neill (6) EUGENE
- Stuntman Knievel (4) EVEL
- "Morning Joe" co-host Brzezinski (4) MIKA
- Yiddish author Aleichem (6) SHOLOM

3 Roman years

- Year John Dryden died (4) MDCC
- Early third-century year (4) CCIV
- 2,502, to ancient Romans (5) MMDII

4 Common French words

- When the day's done, to Denis (4) NUIT

5 Prefix

- Puncture preceder (3) ACU
- Intro to physics? (4) META
- Plasm preceder (4) ECTO
- Prefix with scope (6) STETHO

6 Suffix

- Suffix with miss and dismiss (3) IVE
- Suffix with shepherd (3) ESS
- Medical suffix (3) OMA
- Fox tail? (4) TROT

7 Direction

- New Orleans-to-Detroit dir. (3) NNE

8 World capitals

- Samoa's capital (4) APIA

9 Common French words

- Black: Fr. (4) NOIR
- Mrs., in Marseille (3) MME

10 Common German words

11 Common Spanish words

12 Common Portuguese, Italian, Hawaiian, etc. words

- "Thank you," in Hawaii (6) MAHALO

13 Brands

- Hyundai and Kia (5) AUTOS

14 Competitors

- Marriott alternative (4) OMNI

15 Acronym

- Canoodling in a restaurant, e.g. (abbr.) (3) PDA
- Chemists' org (3) AIC

16 Hypernym

- Platinum, for example (5) METAL
- Cronus and Hyperion (6) TITANS

17 Synonym

- Overshoot, say (4) MISS
- Gait (4) PACE

18 Roles

- 1963 Elizabeth Taylor role (9) CLEOPATRA

19 Products

- Honda model (6) ACCORD

20 Cities

- Largest city in Nebraska (5) OMAHA

21 Songs and performers

- "Owner of a Lonely Heart" band (3) YES

22 Famous person

- Former South African president (5) BOTHA

23 Partner

- Gentleman's partner (4) LADY

24 Dictionary Definitions

- Coffee dispenser (3) URN

25 Two+ word phrases

- On task (4) ATIT
- Drink a little here, drink a little there... (6) BARHOP
- Went without a copilot (8) FLEWSOLO
- An operator may help place one (9) PHONECALL
- Flown into a rage (13) GONEBALLISTIC
- Wine-producing area of SE France (11) RHONEVALLEY
- Food, warmth or a cozy bed (15) CREATURECOMFORT
- Money available for nonessentials (9) SPARECASH

26 Counterpart

- Bull's counterpart (4) BEAR

27 Alternative

- Reebok alternative (4) NIKE

28 Musical key

- Key of Mozart's Symphony No. 40 (6) GMINOR

29 Movies

- Tom Cruise film set in Chicago (13) RISKYBUSINESS

30 Co-star

- Tom Cruise's 'Risky Business' co-star (15) REBECCADEMORNAY

31 Characters

- Judy's brother on the Jetsons (5) ELROY

32 Fill in the blank (for parts of names or words, rather than phrases in general)

- ___ Beach, S.C. (6) MYRTLE
- Boston ___ Party (3) TEA

What is a DOG?

- 272 definitions!

- Utter failure, in slang crosswordheaven.com/clues/utter-failure-in-slang
- Track crosswordheaven.com/clues/track
- Follow crosswordheaven.com/clues/follow
- Dyslexic's deity? crosswordheaven.com/clues/dyslexics-deity
- Greyhound, e.g. crosswordheaven.com/clues/greyhound-eg
- Pluto, for one crosswordheaven.com/clues/pluto-for-one
- Mexican hairless, for one crosswordheaven.com/clues/mexican-hairless-for-one
- It's found in a pound crosswordheaven.com/clues/its-found-in-a-pound
- Terrier or retriever crosswordheaven.com/clues/terrier-or-retriever
- One may sit for a master crosswordheaven.com/clues/one-may-sit-for-a-master
- Tail crosswordheaven.com/clues/tail
- Frank crosswordheaven.com/clues/frank
- Pug or boxer crosswordheaven.com/clues/pug-or-boxer
- "Man's best friend" crosswordheaven.com/clues/mans-best-friend
- Puggle, e.g. crosswordheaven.com/clues/puggle-eg
- Pointer, e.g. crosswordheaven.com/clues/pointer-eg
- Follow relentlessly crosswordheaven.com/clues/follow-relentlessly
- Husky or hound crosswordheaven.com/clues/husky-or-hound

- Follow everywhere crosswordheaven.com/clues/follow-everywhere
- One enrolled in obedience school crosswordheaven.com/clues/one-enrolled-in-obedience-school
- Goofy, e.g. crosswordheaven.com/clues/goofy-eg
- Hound crosswordheaven.com/clues/hound
- Follow closely crosswordheaven.com/clues/follow-closely
- Lab or boxer crosswordheaven.com/clues/lab-or-boxer
- Beethoven, for one crosswordheaven.com/clues/beethoven-for-one
- Seeing Eye trainee crosswordheaven.com/clues/seeing-eye-trainee
- One may be fetching crosswordheaven.com/clues/one-may-be-fetching
- "You lucky ___!" crosswordheaven.com/clues/you-lucky-___
- Put on the ___ (be ostentatiously elegant) crosswordheaven.com/clues/put-on-the-___-be-ostentatiously-elegant
- Pooch crosswordheaven.com/clues/pooch
- Borzoi, for one crosswordheaven.com/clues/borzoi-for-one
- Tail tirelessly crosswordheaven.com/clues/tail-tirelessly
- Follow tirelessly crosswordheaven.com/clues/follow-tirelessly
- German shepherd, for one crosswordheaven.com/clues/german-shepherd-for-one
- Komondor, for one crosswordheaven.com/clues/komondor-for-one
- Pointer, for one crosswordheaven.com/clues/pointer-for-one

- Bark source crosswordheaven.com/clues/bark-source
- One with a bone appetite crosswordheaven.com/clues/one-with-a-bone-appetite
- Hound or husky crosswordheaven.com/clues/hound-or-husky
- Bo or Barney of the White House crosswordheaven.com/clues/bo-or-barney-of-the-white-house
- Pit bull or poodle, e.g. crosswordheaven.com/clues/pit-bull-or-poodle-eg
- Worthless thing, in slang crosswordheaven.com/clues/worthless-thing-in-slang
- Woofer? crosswordheaven.com/clues/woofer
- Barker crosswordheaven.com/clues/barker
- Plague crosswordheaven.com/clues/plague
- Labrador, e.g. crosswordheaven.com/clues/labrador-eg
- Brittany, e.g. crosswordheaven.com/clues/brittany-eg
- See 22-Across crosswordheaven.com/clues/see-22-across
- Labradoodle, e.g. crosswordheaven.com/clues/labradoodle-eg
- Boxer or pug crosswordheaven.com/clues/boxer-or-pug
- Fetch player crosswordheaven.com/clues/fetch-player
- Follow persistently crosswordheaven.com/clues/follow-persistently
- Stay close behind crosswordheaven.com/clues/stay-close-behind

- Lab, for one crosswordheaven.com/clues/lab-for-one
- Underperforming investment, slangily crosswordheaven.com/clues/underperforming-investment-slangily
- Lab or peke crosswordheaven.com/clues/lab-or-peke
- Loaf, with "it" crosswordheaven.com/clues/loaf-with-it
- Man's best friend crosswordheaven.com/clues/mans-best-friend
- Spaniel or setter crosswordheaven.com/clues/spaniel-or-setter
- ''Man's best friend'' crosswordheaven.com/clues/mans-best-friend
- Weimaraner or Pomeranian crosswordheaven.com/clues/weimaraner-or-pomeranian
- Pluto or Pongo crosswordheaven.com/clues/pluto-or-pongo
- Fetching one crosswordheaven.com/clues/fetching-one
- Boxer that can lick anyone? crosswordheaven.com/clues/boxer-that-can-lick-anyone
- ''A ___ of Flanders'' crosswordheaven.com/clues/a-___-of-flanders
- Every one has its day crosswordheaven.com/clues/every-one-has-its-day
- Basenji, for one crosswordheaven.com/clues/basenji-for-one
- Lady or Beethoven crosswordheaven.com/clues/lady-or-beethoven
- Bone exhumer crosswordheaven.com/clues/bone-exhumer
- "Every ___ has his day" crosswordheaven.com/clues/every-___-has-his-day

- Friend of the man? crosswordheaven.com/clues/friend-of-the-man
- 111 Across, e.g. crosswordheaven.com/clues/111-across-eg
- Setter or shepherd, e.g. crosswordheaven.com/clues/setter-or-shepherd-eg
- Trail crosswordheaven.com/clues/trail
- Cat chaser crosswordheaven.com/clues/cat-chaser
- Snoopy, for one crosswordheaven.com/clues/snoopy-for-one
- Flop crosswordheaven.com/clues/flop
- Closely follow crosswordheaven.com/clues/closely-follow
- "A ___ of Flanders" crosswordheaven.com/clues/a-___-of-flanders
- Corgi or collie crosswordheaven.com/clues/corgi-or-collie
- Benji, or a Basenji crosswordheaven.com/clues/benji-or-a-basenji
- Malinois or Malamute crosswordheaven.com/clues/malinois-or-malamute
- Inferior item crosswordheaven.com/clues/inferior-item
- Marmaduke, e.g. crosswordheaven.com/clues/marmaduke-eg
- Poodle, e.g. crosswordheaven.com/clues/poodle-eg
- Homie crosswordheaven.com/clues/homie

New Search

Clue: | Pattern: | Go 1

Current Search

Clue: capital | Pattern: ?????? | Go 2

Score	Match	Source
★★★★	BEIRUT	similar to known clue *(capital)*
★	BISSAU	similar to known clue *(capital)*
★	OTTAWA	similar to known clue *(capital)*
★	TIRANA	similar to known clue *(capital)*
★	HELENA	similar to known clue *(capital)*
★	ANKARA	similar to known clue *(capital)*

Dr. Fill

- http://www.pri.org/stories/2014-09-24/dr-fill-vies-crossword-solving-supremacy-still-comes-short
- “It’s also improving quickly. For the past three years, Dr. Fill has competed “informally” at the American Crossword Puzzle Tournament, organized by Will Shortz, the crossword editor at the New York Times. The first year, Dr. Fill came in 141st out of about 600 competitors. It did a little better the second-year; last year it was 65th. “I get a little smarter so it gets a little smarter each year — and I’m inching my way up there,” Ginsberg says.”

Beam Search

- Based on Breadth-First Search
- Expand a node based on heuristic cost
- Beam size determines how many nodes to keep in memory
- Not complete
- Not optimal
- Used for large search spaces, e.g., in machine translation

6.7 Consider the following logic puzzle: In five houses, each with a different color, live five persons of different nationalities, each of whom prefers a different brand of candy, a different drink, and a different pet. Given the following facts, the questions to answer are "Where does the zebra live, and in which house do they drink water?"

The Englishman lives in the red house.
The Spaniard owns the dog.
The Norwegian lives in the first house on the left.
The green house is immediately to the right of the ivory house.
The man who eats Hershey bars lives in the house next to the man with the fox.
Kit Kats are eaten in the yellow house.
The Norwegian lives next to the blue house.
The Smarties eater owns snails.
The Snickers eater drinks orange juice.
The Ukrainian drinks tea.
The Japanese eats Milky Ways.
Kit Kats are eaten in a house next to the house where the horse is kept.
Coffee is drunk in the green house.
Milk is drunk in the middle house.

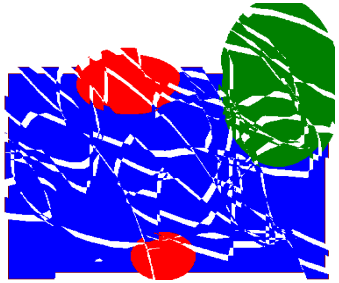
Discuss different representations of this problem as a CSP. Why would one prefer one representation over another?

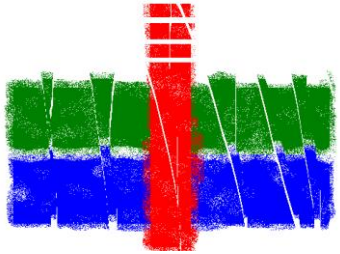
6.7 The "Zebra Puzzle" can be represented as a CSP by introducing a variable for each color, pet, drink, country, and cigarette brand (a total of 25 variables). The value of each variable is a number from 1 to 5 indicating the house number. This is a good representation because it easy to represent all the constraints given in the problem definition this way. (We have done so in the Python implementation of the code, and at some point we may reimplement this in the other languages.) Besides ease of expressing a problem, the other reason to choose a representation is the efficiency of finding a solution. here we have mixed results—on some runs, min-conflicts local search finds a solution for this problem in seconds, while on other runs it fails to find a solution after minutes.

Another representation is to have five variables for each house, one with the domain of colors, one with pets, and so on.

Linear programs: example

- We make reproductions of two paintings





- Painting 1 sells for $30, painting 2 sells for $20
- Painting 1 requires 4 units of blue, 1 green, 1 red
- Painting 2 requires 2 blue, 2 green, 1 red
- We have 16 units blue, 8 green, 5 red

maximize $3x + 2y$

subject to

$4x + 2y \leq 16$

$x + 2y \leq 8$

$x + y \leq 5$

$x \geq 0$

$y \geq 0$

Solving the linear program graphically

maximize $3x + 2y$

subject to

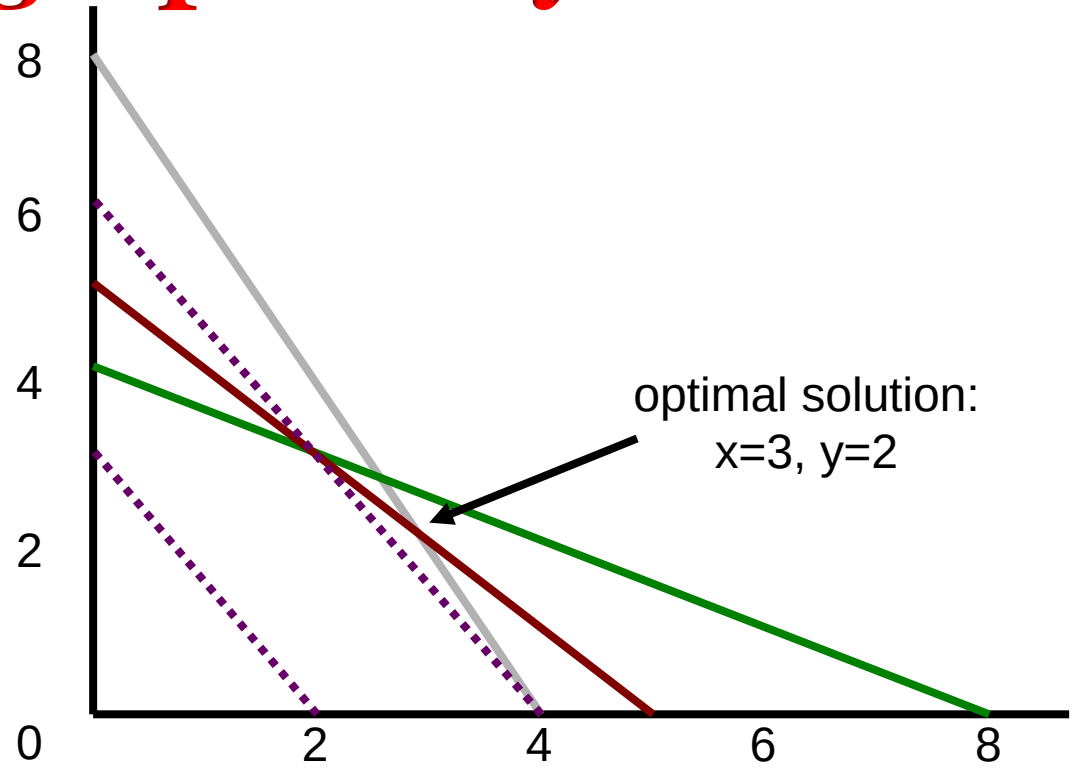
$4x + 2y \leq 16$

$x + 2y \leq 8$

$x + y \leq 5$

$x \geq 0$

$y \geq 0$



Modified LP

maximize $3x + 2y$

subject to

$4x + 2y \leq 15$

$x + 2y \leq 8$

$x + y \leq 5$

$x \geq 0$

$y \geq 0$

Optimal solution: x = 2.5, y = 2.5

Solution value = 7.5 + 5 = 12.5

Half paintings?

Integer (linear) program

maximize 3x + 2y

subject to

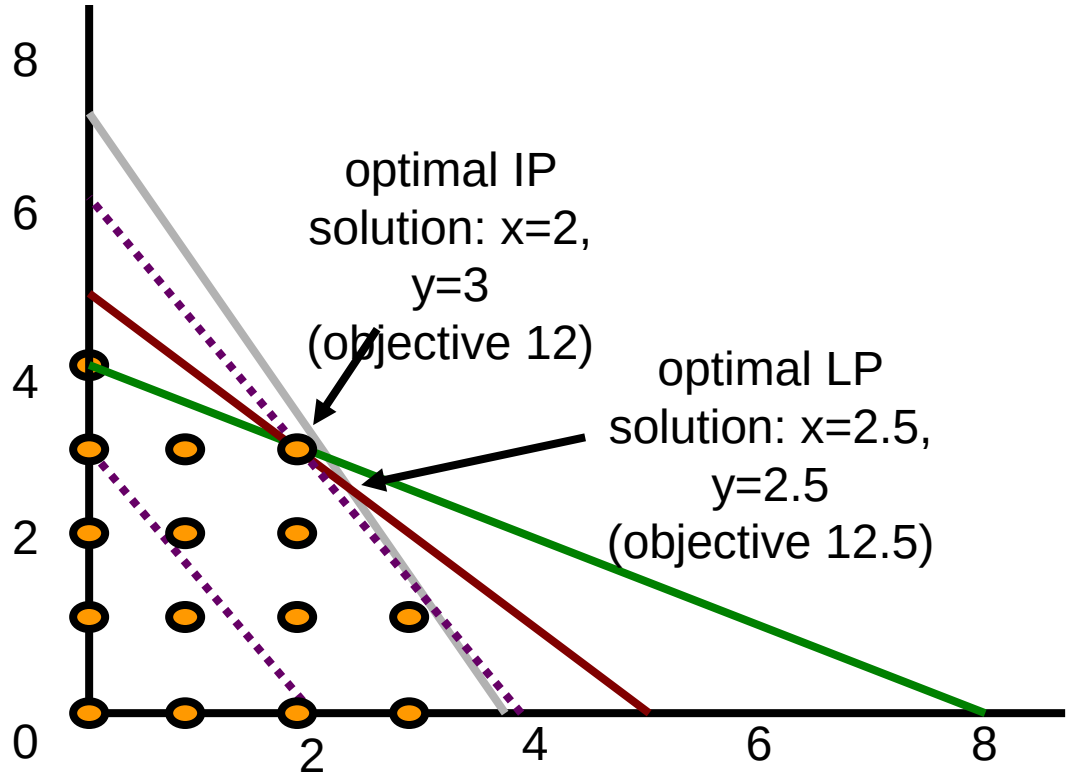
4x + 2y ≤ 15

x + 2y ≤ 8

x + y ≤ 5

x ≥ 0, integer

y ≥ 0, integer



Mixed integer (linear) program

maximize $3x + 2y$

subject to

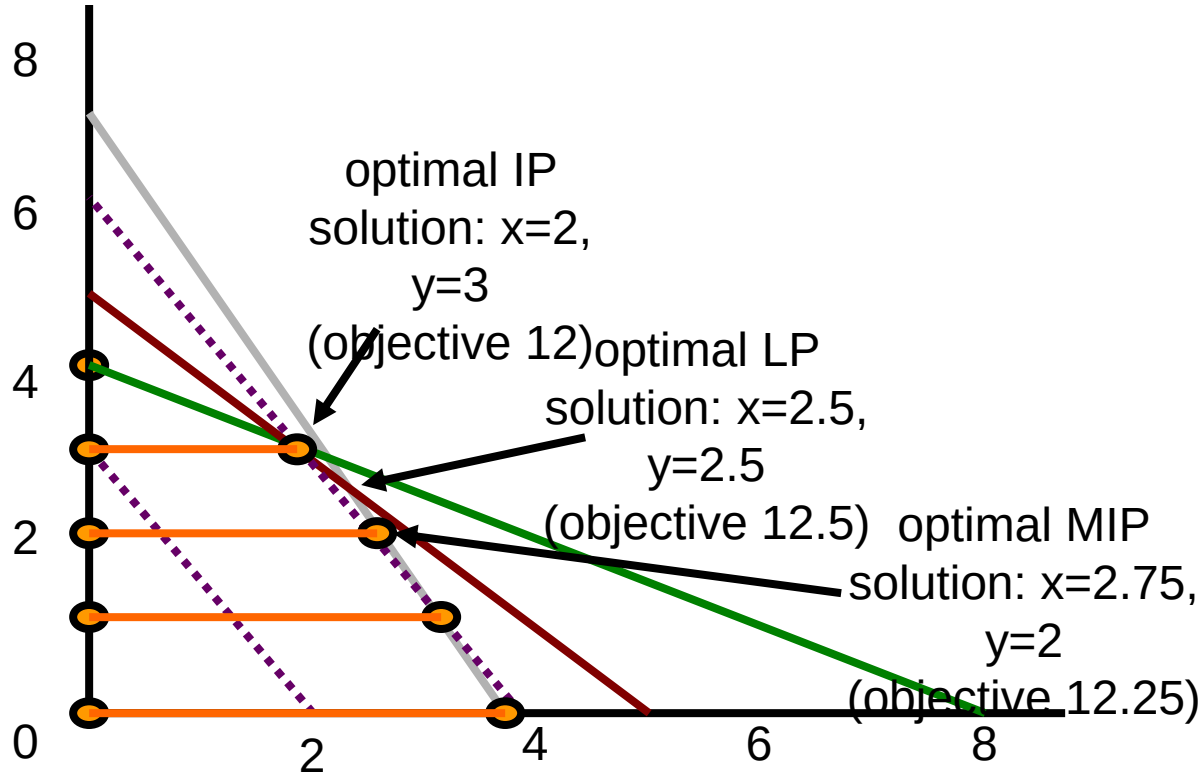
$4x + 2y \leq 15$

$x + 2y \leq 8$

$x + y \leq 5$

$x \geq 0$

$y \geq 0$, integer



Solving linear/integer programs

- Linear programs can be solved efficiently
  - Simplex, ellipsoid, interior point methods…
- (Mixed) integer programs are NP-hard to solve
  - Quite easy to model many standard NP-complete problems as integer programs (try it!)
  - Search type algorithms such as branch and bound
- Standard packages for solving these
  - GNU Linear Programming Kit, CPLEX, …
- LP relaxation of (M)IP: remove integrality constraints
  - Gives upper bound on MIP (~admissible heuristic)

Satisfiability as an integer program

(x_1 OR x_2 OR NOT(x_4)) AND (NOT(x_2) OR NOT(x_3)) AND …

becomes

for all x_j, $0 \leq x_j \leq 1$, x_j integer (shorthand: x_j in {0,1})

$x_1 + x_2 + (1-x_4) \geq 1$

$(1-x_2) + (1-x_3) \geq 1$

…

Solving integer programs is at least as hard as satisfiability, hence NP-hard (we have reduced SAT to IP)

Try modeling other NP-hard problems as (M)IP!

Solving the integer program with DFS branch and bound

trick: for integer x and k, either $x \leq k$ or $x \geq k+1$

maximize $3x + 2y$
subject to
$4x + 2y \leq 15$
$x + 2y \leq 8$
$x + y \leq 5$

LP solution: x=2.5, y=2.5, obj = 12.5

maximize $3x + 2y$
subject to
$4x + 2y \leq 15$
$x + 2y \leq 8$
$x + y \leq 5$
$x \geq 3$

LP solution: x=3, y=1.5, obj = 12

maximize $3x + 2y$
subject to
$4x + 2y \leq 15$
$x + 2y \leq 8$
$x + y \leq 5$
$x \leq 2$

LP solution: x=2, y=3, obj = 12

maximize $3x + 2y$
subject to
$4x + 2y \leq 15$
$x + 2y \leq 8$
$x + y \leq 5$
$x \geq 3$
$y \geq 2$

LP solution: infeasible

maximize $3x + 2y$
subject to
$4x + 2y \leq 15$
$x + 2y \leq 8$
$x + y \leq 5$
$x \geq 3$
$y \leq 1$

LP solution: x=3.25, y=1, obj = 11.75

maximize $3x + 2y$
subject to
$4x + 2y \leq 15$
$x + 2y \leq 8$
$x + y \leq 5$
$x \geq 3$
$y \leq 1$
$x \geq 4$

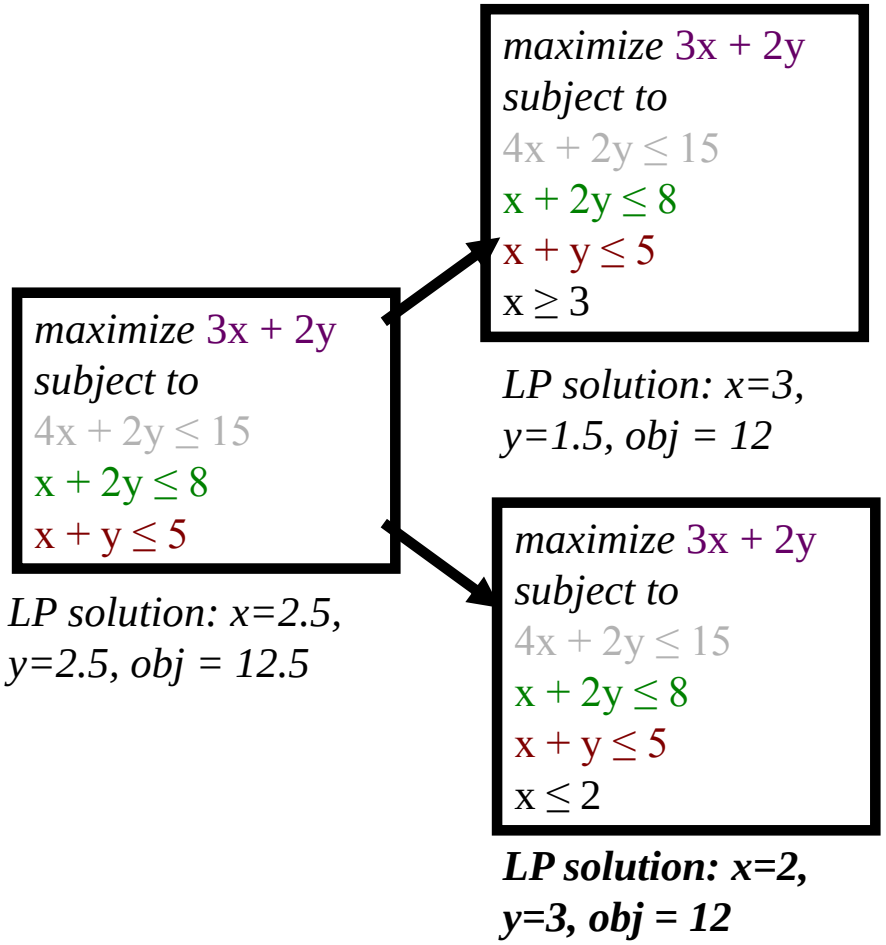
LP solution: infeasible

maximize $3x + 2y$
subject to
$4x + 2y \leq 15$
$x + 2y \leq 8$
$x + y \leq 5$
$x \geq 3$
$y \leq 1$
$x \leq 3$

LP solution: x=3, y=1, obj = 11

if LP solution is integral, we are done

Again with a more fortunate choice



done!

Summary

- CSPs are a special kind of problem:
  - states defined by values of a fixed set of variables
  - goal test defined by constraints on variable values
- Backtracking = depth-first search with one variable assigned per node
- Variable ordering and value selection heuristics help significantly
- Forward checking prevents assignments that guarantee later failure
- Constraint propagation (e.g., arc consistency) does additional work to constrain values and detect inconsistencies
- Iterative min-conflicts is usually effective in practice


AI