

AI

Artificial Intelligence

8.2.6.

Advanced Search (Ch 4) Part 2

Outline

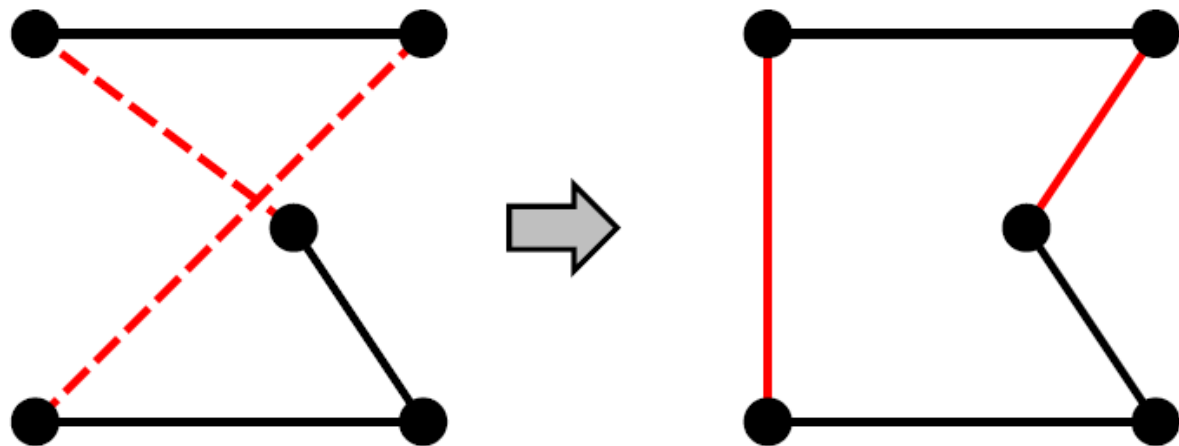
- Local search algorithms
 - Hill-climbing search
 - Simulated annealing search
 - Local beam search
- Genetic algorithms

Local search algorithms

- In many optimization problems, the **path** to the goal is irrelevant; the goal state itself is the solution
- State space = **set of "complete" configurations**
 - Find configuration satisfying constraints, e.g., n-queens
- In such cases, we can use **local search algorithms**
 - keep a single "current" state, try to improve it

Example: Travelling Salesperson Problem

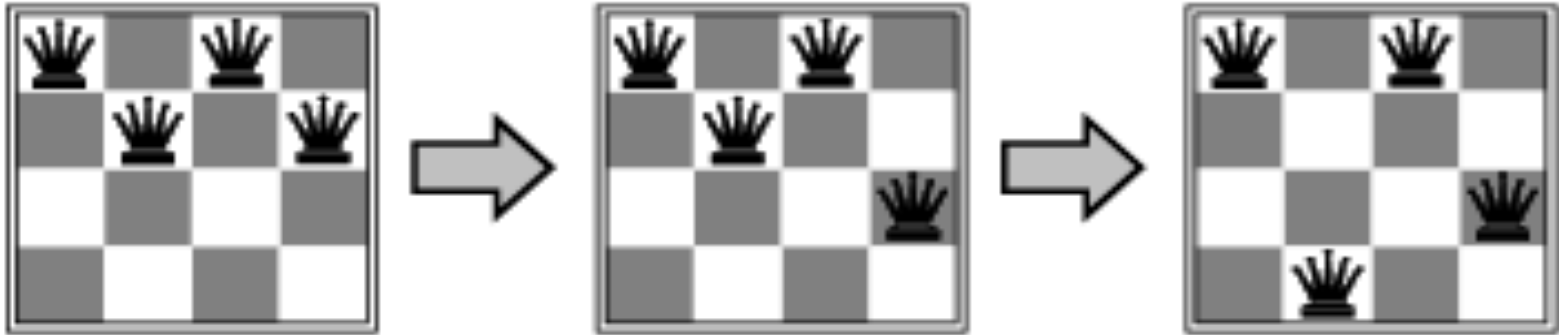
Start with any complete tour, perform pairwise exchanges



Variants of this approach get within 1% of optimal very quickly with thousands of cities

Example: n -queens

- Put n queens on an $n \times n$ board with no two queens on the same row, column, or diagonal



Hill-climbing search

- "Like climbing Everest in thick fog with amnesia"

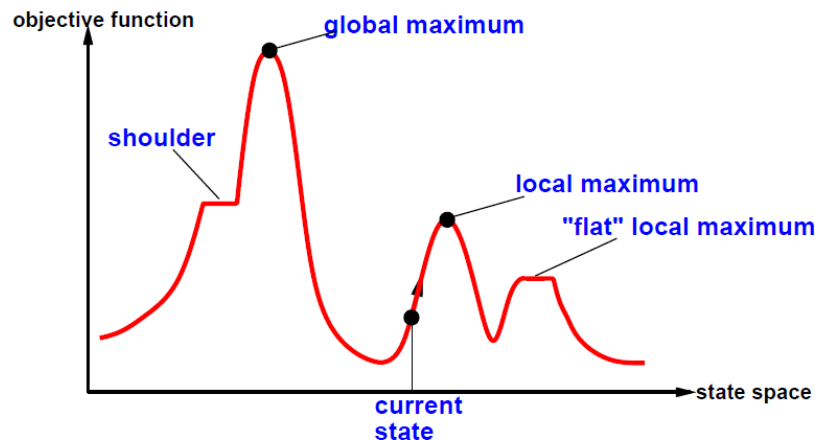
```
function HILL-CLIMBING(problem) returns a state that is a local maximum
  inputs: problem, a problem
  local variables: current, a node
                  neighbor, a node

  current ← MAKE-NODE(INITIAL-STATE[problem])
  loop do
    neighbor ← a highest-valued successor of current
    if VALUE[neighbor] ≤ VALUE[current] then return STATE[current]
    current ← neighbor
  end
```

Hill-climbing search

- Problem: depending on initial state, can get stuck in local maxima

Useful to consider state space landscape



Random-restart hill climbing overcomes local maxima—trivially complete

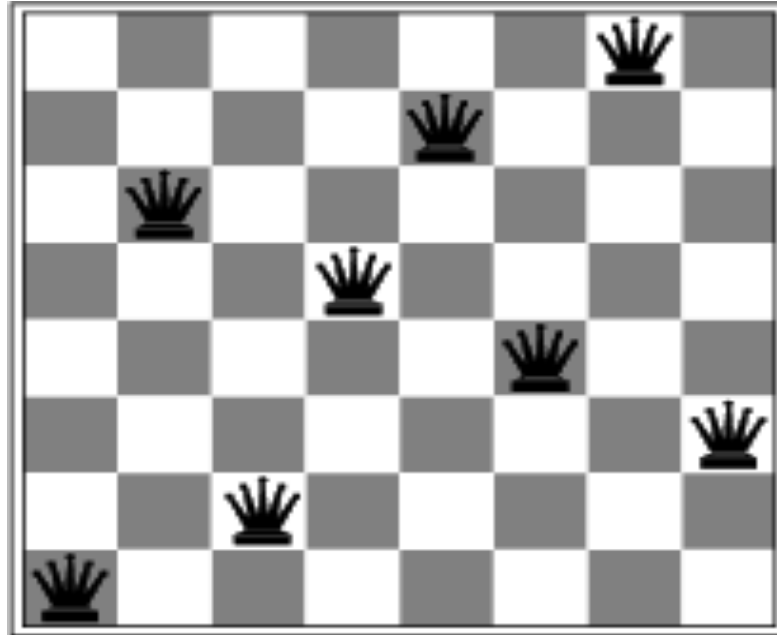
Random sideways moves 🤪 escape from shoulders 🚫 loop on flat maxima

Hill-climbing search: 8-queens

18	12	14	13	13	12	14	14
14	16	13	15	12	14	12	16
14	12	18	13	15	12	14	14
15	14	14	♚	13	16	13	16
♚	14	17	15	♚	14	16	16
17	♚	16	18	15	♚	15	♚
18	14	♚	15	15	14	♚	16
14	14	13	17	12	14	12	18

- h = number of pairs of queens that are attacking each other, either directly or indirectly
- $h = 17$ for the above state

Hill-climbing search: 8-queens



- A local minimum with $h = 1$

Demos

- <http://eightqueen.becher-sundstroem.de/>
- http://cliplab.org/~jfran/ptojs/queens_ui/queens_ui.html

Simulated annealing search

- Idea: escape local maxima by allowing some "bad" moves but **gradually decrease** their frequency

```
function SIMULATED-ANNEALING(problem, schedule) returns a solution state
  inputs: problem, a problem
           schedule, a mapping from time to "temperature"
  local variables: current, a node
                   next, a node
                   T, a "temperature" controlling prob. of downward steps

  current ← MAKE-NODE(INITIAL-STATE[problem])
  for t ← 1 to ∞ do
    T ← schedule[t]
    if T = 0 then return current
    next ← a randomly selected successor of current
     $\Delta E \leftarrow \text{VALUE}[\textit{next}] - \text{VALUE}[\textit{current}]$ 
    if  $\Delta E > 0$  then current ← next
    else current ← next only with probability  $e^{\Delta E/T}$ 
```

Properties of simulated annealing search

At fixed “temperature” T , state occupation probability reaches Boltzman distribution

$$p(x) = \alpha e^{\frac{E(x)}{kT}}$$

T decreased slowly enough \implies always reach best state x^*
because $e^{\frac{E(x^*)}{kT}} / e^{\frac{E(x)}{kT}} = e^{\frac{E(x^*) - E(x)}{kT}} \gg 1$ for small T

Is this necessarily an interesting guarantee??

Devised by Metropolis et al., 1953, for physical process modelling

Widely used in VLSI layout, airline scheduling, etc.

Local Beam Search

- Keep track of k states rather than just one
- k is called the **beam width**
- Start with k randomly generated states
- At each iteration, all the successors of all k states are generated
- If any one is a goal state, stop; else select the k best successors from the complete list and repeat.

Genetic algorithms

- A successor state is generated by combining two parent states
- Start with k randomly generated states (**population**)
- A state is represented as a string over a finite alphabet (often a string of 0s and 1s)
- Evaluation function (**fitness function**). Higher values for better states.
- Produce the next generation of states by **selection**, **crossover**, and **mutation**.

Genetic algorithms contd.

- What is the fitness function?
- How is an individual represented?
 - Using a string over a finite alphabet.
 - Each element of the string is a **gene**.
- How are individuals selected?
 - Randomly, with probability of selection proportional to fitness
 - Usually, selection is done with **replacement**.
- How do individuals reproduce?
 - Through crossover and mutation

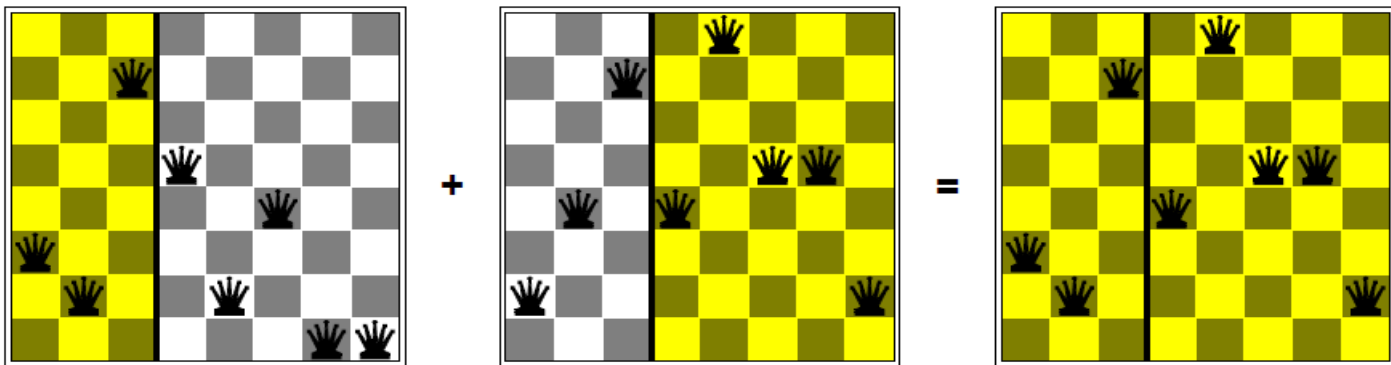
GA Pseudocode

- Choose initial population (usually random)
- Repeat (until terminated)
 - Evaluate each individual's fitness
 - Select pairs to mate
 - Replenish population (next-generation)
 - Apply crossover
 - Apply mutation
 - Check for termination criteria

Genetic Algorithms

GAs require states encoded as strings (GPs use programs)

Crossover helps **iff** substrings are meaningful components

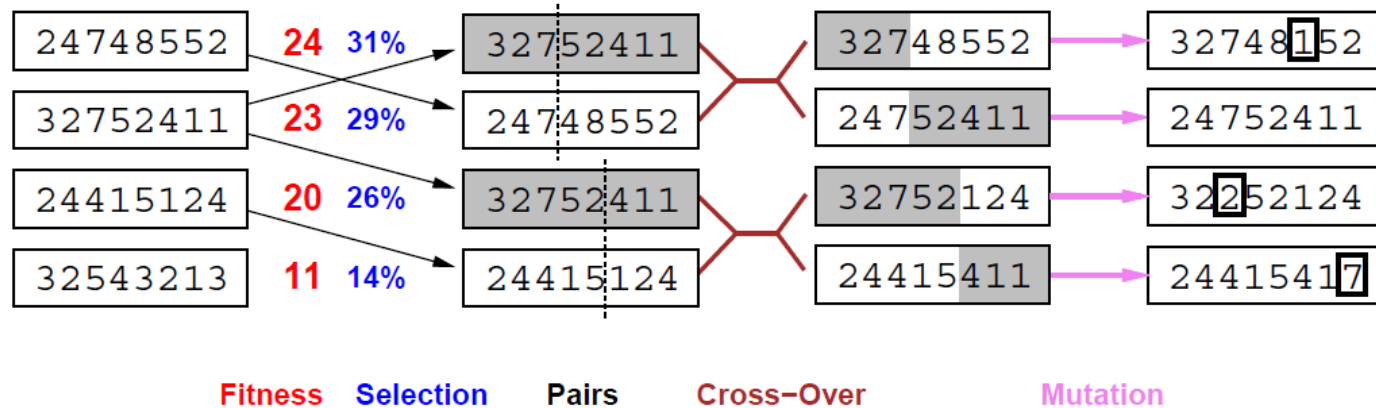


GAs \neq evolution: e.g., real genes encode replication machinery!

$$\begin{array}{|c|c|c|c|c|c|c|c|} \hline 3 & 2 & 7 & 5 & 2 & 4 & 1 & 1 \\ \hline \end{array} + \begin{array}{|c|c|c|c|c|c|c|c|} \hline 2 & 4 & 7 & 4 & 8 & 5 & 5 & 2 \\ \hline \end{array} = \begin{array}{|c|c|c|c|c|c|c|c|} \hline 3 & 2 & 7 & 4 & 8 & 5 & 5 & 2 \\ \hline \end{array}$$

Genetic algorithms

= stochastic local beam search + generate successors from **pairs** of states



- Fitness function: number of non-attacking pairs of queens (min = 0, max = $8 \times 7/2 = 28$)
- $24/(24+23+20+11) = 31\%$
- $23/(24+23+20+11) = 29\%$ etc.

Replacement

- Simple or generational GAs replace entire population
- Steady state or online GAs use different replacement schemes:
 - Replace worst
 - Replace best
 - Replace parent
 - Replace random
 - Replace most similar

Demo

```
% python ga.py 10 4 | more
```

Initial Population

```
individual bitarray('010011001000') fitness 4  
individual bitarray('000001001000') fitness 2  
individual bitarray('001000000000') fitness 2  
individual bitarray('000010000011') fitness 4  
individual bitarray('010011001001') fitness 3  
individual bitarray('011010001001') fitness 2  
individual bitarray('001001011010') fitness 3  
individual bitarray('000010000011') fitness 4  
individual bitarray('011010010001') fitness 3  
individual bitarray('010000010000') fitness 4
```

average 3.1

...

average 3.7

...

average 3.9

...

average 4.2

...

```
parent_x bitarray('001011000010')
```

```
parent_y bitarray('011011000010')
```

```
crossover_pt 2
```

```
child bitarray('001011000010')
```

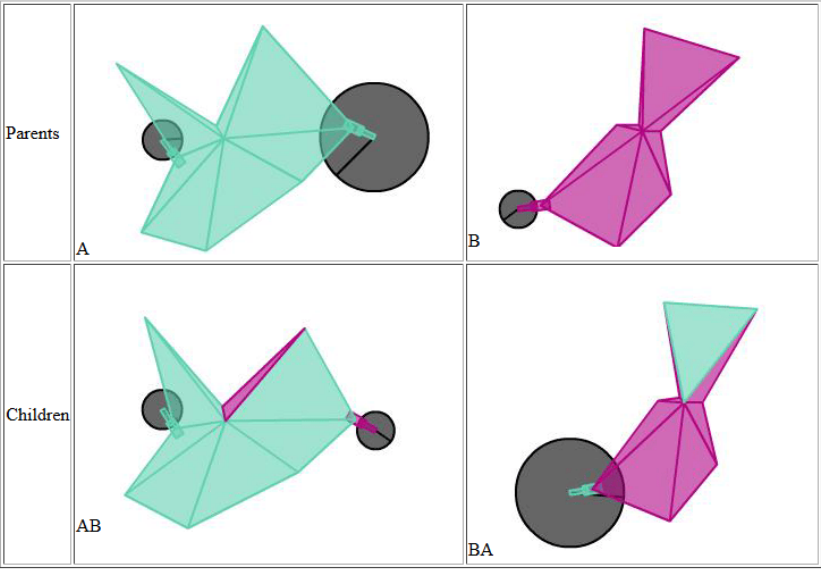
individual bitarray('001011000010') fitness 6

2 4 1 3

Car Design Simulation

- <http://www.boxcar2d.com/>
- <http://www.boxcar2d.com/about.html>

Crossover



Here are the associated chromosomes for the cars shown above:

Car	Angle0	Mag0	Angle1	Mag1	WheelVertex0	AxleAngle0	WheelRadius0	WheelVertex1	AxleAngle1	WheelRadius1	
A	0.769	2.614	0.584	0.319	0.278	2.883	0.666	1.13	0.305	2.752	0.376	2.507	0.814	1.963	0.392	2.872	3	5.284	0.434	7	2.625	1.191
B	0.535	2.682	0.732	2.256	0.422	0.149	0.676	0.578	0.709	2.774	0.592	2.623	0.519	1.531	0.924	0.404	-1	0.704	0.122	4	0.167	0.409
AB	0.535	2.682	0.584	0.319	0.278	2.883	0.666	1.13	0.305	2.752	0.376	2.507	0.814	1.963	0.392	2.872	3	5.284	0.434	7	2.625	0.409
BA	0.769	2.614	0.732	2.256	0.422	0.149	0.676	0.578	0.709	2.774	0.592	2.623	0.519	1.531	0.924	0.404	-1	0.704	0.122	4	0.167	1.191

Genetic Algorithms Links

- <http://math.hws.edu/eck/jsdemo/jsGeneticAlgorithm.html>
- <http://www.theprojectspot.com/tutorial-post/simulated-annealing-algorithm-for-beginners/6>
- <http://www.theprojectspot.com/tutorial-post/applying-a-genetic-algorithm-to-the-travelling-salesman-problem/5>
- <http://www.theprojectspot.com/tutorials/page/2>
- <http://genetic-algorithms-explained.appspot.com/>

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