AI

Artificial Intelligence

8.2.1
Problem Solving and Search (Ch. 3)

Outline

- Problem-solving agents
- Problem types
- Problem formulation
- Example problems
- Basic search algorithms

Cops and Robbers

- Three cops and three robbers come to a river and find a boat that holds two. If the robbers ever outnumber the cops on either bank, the robbers will overpower the cops and escape.
- What is the strategy to cross the river without letting the robbers escape?

Cops and Robbers

- State representation?
- Start State?
- End State?
- Transitions?

Problem-solving agents

```
function SIMPLE-PROBLEM-SOLVING-AGENT(percept) returns an action
    static: seq, an action sequence, initially empty
            state, some description of the current world state
            goal, a goal, initially null
            problem, a problem formulation

    state ← UPDATE-STATE(state, percept)
    if seq is empty then
        goal ← FORMULATE-GOAL(state)
        problem ← FORMULATE-PROBLEM(state, goal)
        seq ← SEARCH(problem)
    action ← RECOMMENDATION(seq, state)
    seq ← REMAINDER(seq, state)
    return action
```


0
40
80 miles
0
60
120 km.
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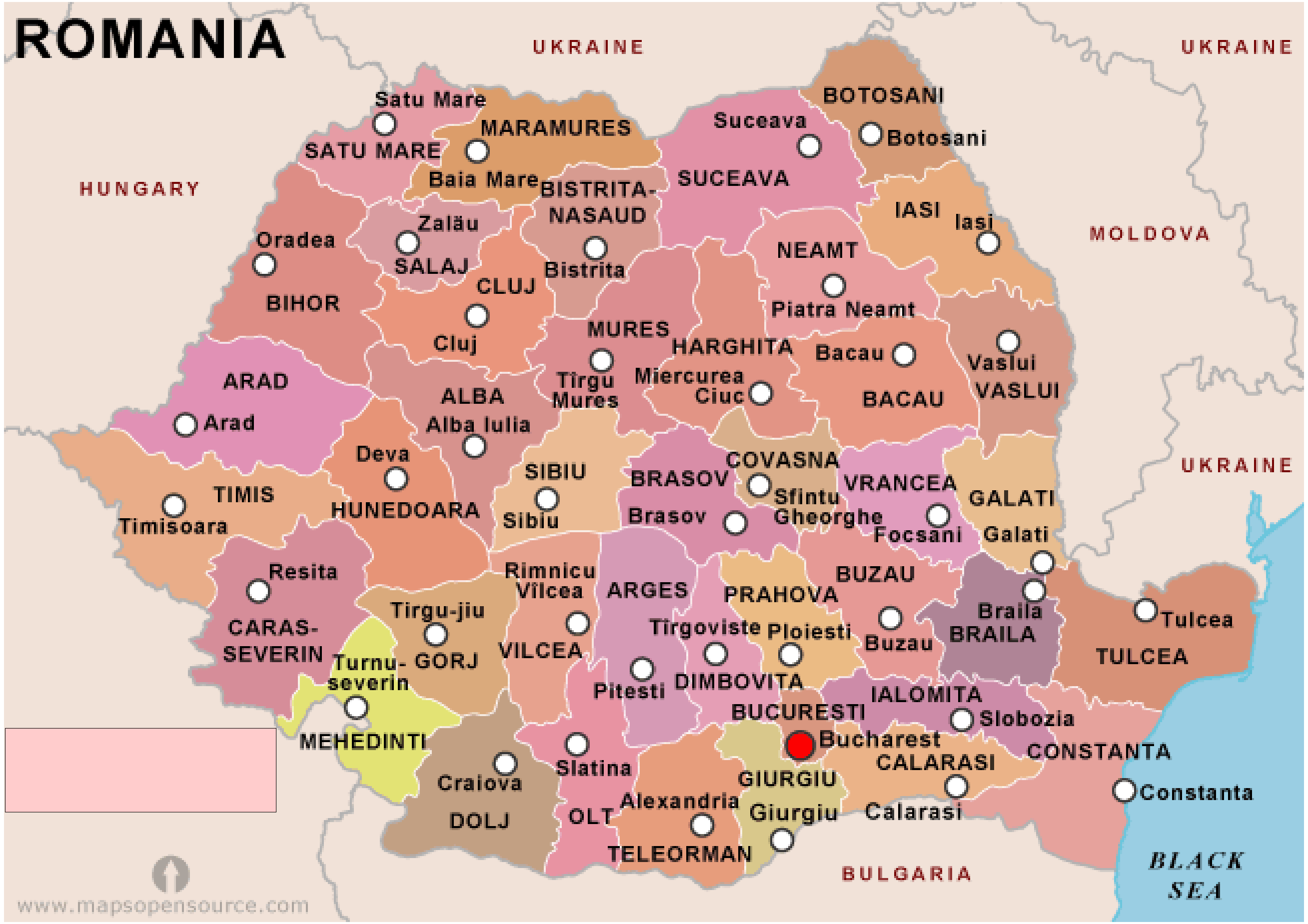
ROMANIA
UKRAINE
UKRAINE
HUNGARY
MOLDOVA
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BULGARIA
BLACK SEA
Satu Mare
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Baia Mare
BOTOSANI
Botosani
Suceava
SUCEAVA
BISTRITA-NASAUD
Bistrita
Zalău
SALAJ
Oradea
BIHOR
CLUJ
Cluj
IASI
Iasi
NEAMT
Piatra Neamt
MURES
Tirgu Mures
HARGHITA
Miercurea Ciuc
Bacau
BACAU
Vaslui
VASLUI
ARAD
Arad
ALBA
Alba Iulia
Deva
HUNEDOARA
TIMIS
Timisoara
SIBIU
Sibiu
BRASOV
Brasov
COVASNA
Sfintu Gheorghe
VRANCEA
Focsani
GALATI
Galati
Resita
CARAS-SEVERIN
Rimnicu Vîlcea
VILCEA
ARGES
Pitesti
PRAHOVA
Ploiesti
BUZAU
Buzau
Braila
BRAILA
Tulcea
TULCEA
Tirgu-jiu
GORJ
Turnu-severin
MEHEDINTI
Tirgoviste
DIMBOVITA
BUCURESTI
Bucharest
IALOMITA
Slobozia
CONSTANTA
Constanta
CALARASI
Calarasi
GIURGIU
Giurgiu
Craiova
DOLJ
Slatina
OLT
Alexandria
TELEORMAN
www.mapsopensource.com

Example: Romania

- On holiday in Romania; currently in Arad.
- Flight leaves tomorrow from Bucharest
- Formulate goal:
  - be in Bucharest
- Formulate problem:
  - states: various cities
  - actions: drive between cities
- Find solution:
  - sequence of cities, e.g., Arad, Sibiu, Fagaras, Bucharest

Example: Romania

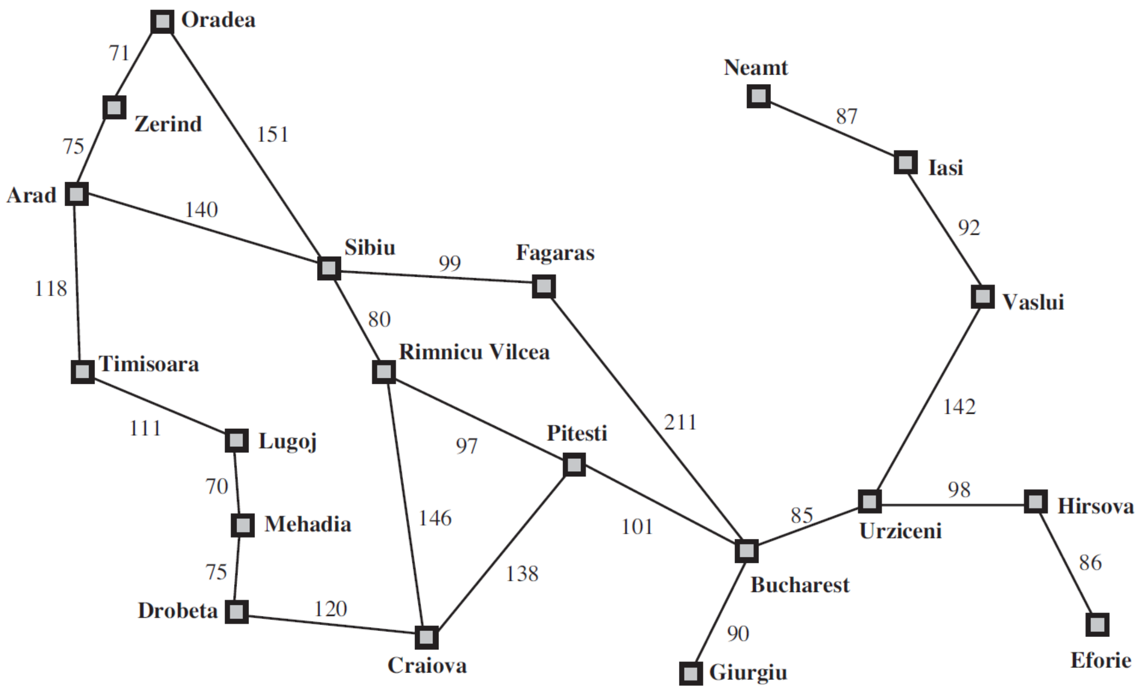


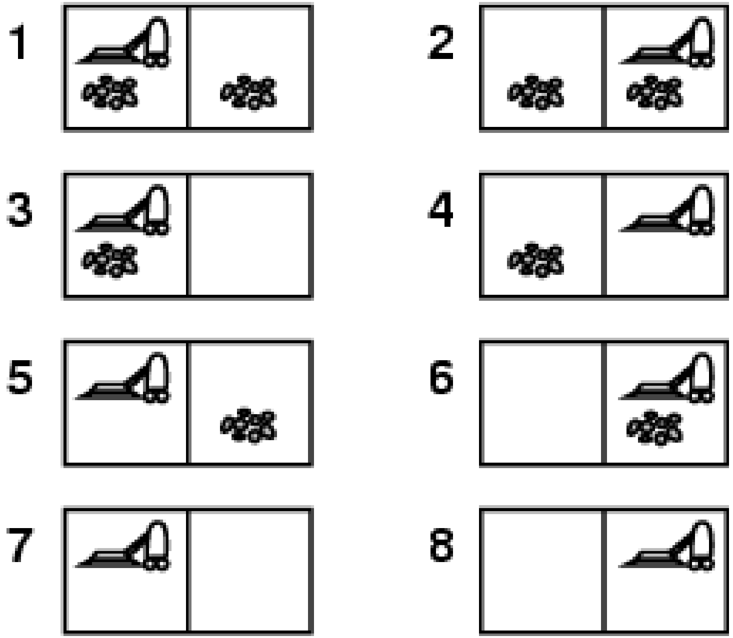
Figure 3.2 A simplified road map of part of Romania.

Problem Types

- Deterministic, fully observable → single-state problem
  - Agent knows exactly which state it will be in; solution is a sequence
- Non-observable → sensorless problem (conformant problem)
  - Agent may have no idea where it is; solution is a sequence
- Nondeterministic and/or partially observable → contingency problem
  - percepts provide new information about current state
  - often interleave search, execution
- Unknown state space → exploration problem

Example: Vacuum World

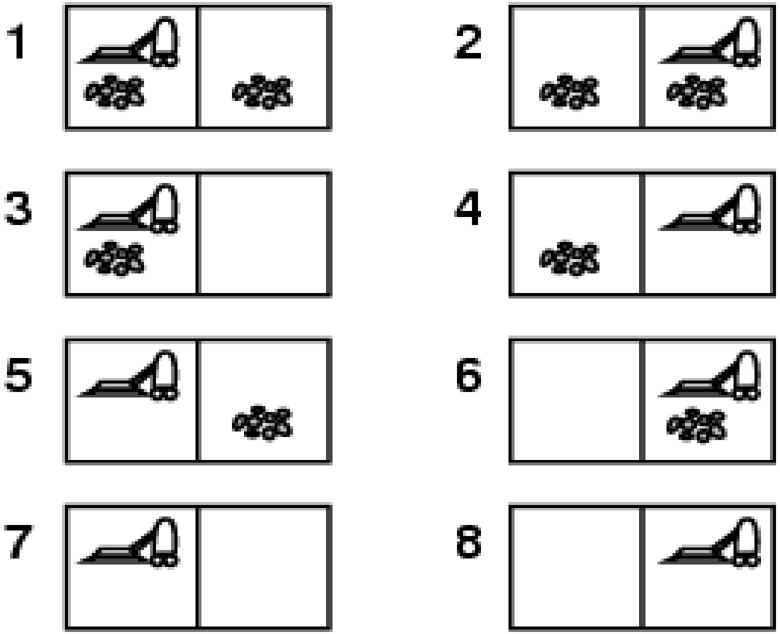
- Single-state, start in #5.
  Solution?



Example: Vacuum World

- Single-state, start in #5.
  Solution? *[Right, Suck]*

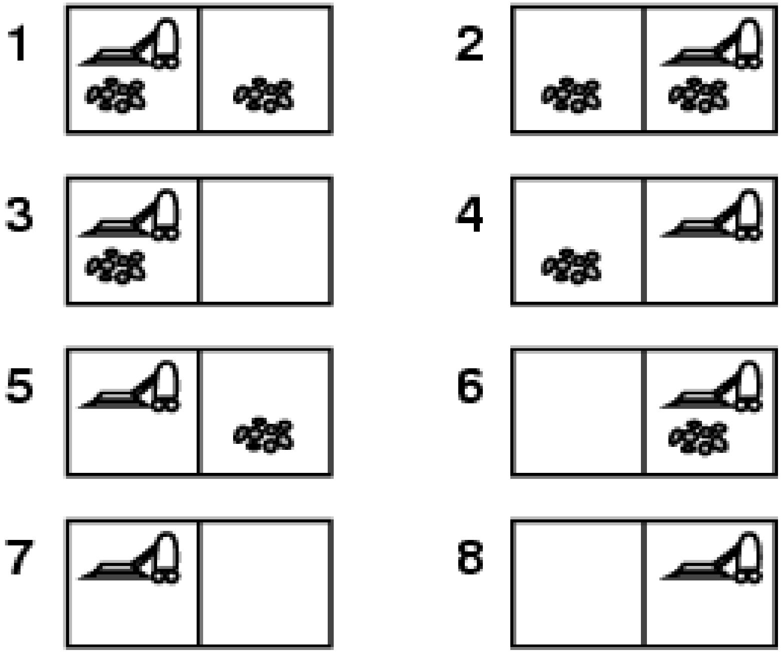
- Sensorless, start in {*1,2,3,4,5,6,7,8*} e.g.,
  Right goes to {*2,4,6,8*}
  Solution?



Example: Vacuum World

- Sensorless, start in {*1,2,3,4,5,6,7,8*} e.g., *Right* goes to {*2,4,6,8*} Solution? *[Right,Suck,Left,Suck]*

- Contingency
  - Nondeterministic: *Suck* may dirty a clean carpet
  - Partially observable: location, dirt at current location.
  - Percept: *[L, Clean],* i.e., start in #5 or #7 Solution?



Example: Vacuum World

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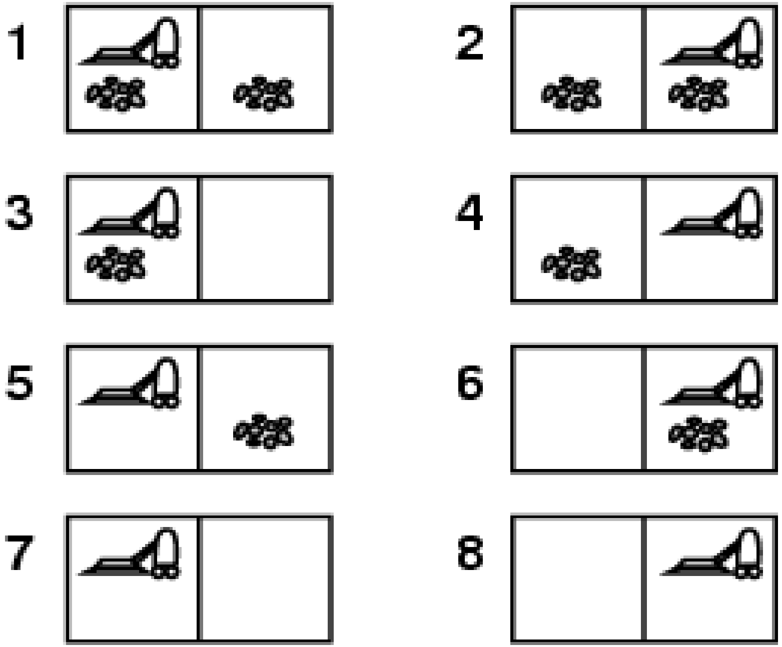
  - Nondeterministic: *Suck* may dirty a clean carpet
  - Partially observable: location, dirt at current location.
  - Percept: *[L, Clean],* i.e., start in #5 or #7 Solution? *[Right,* ***if*** *dirt* ***then*** *Suck]*



Single-state problem formulation

A problem is defined by four items:

1. initial state e.g., "at Arad"
2. actions or successor function $S(x)$ = set of action–state pairs
   – e.g., $S(Arad) = \{<Arad \rightarrow Zerind, Zerind>, \ldots\}$
3. goal test, can be
   – explicit, e.g., x = "at Bucharest"
   – implicit, e.g., $Checkmate(x)$
4. path cost (additive)
   – e.g., sum of distances, number of actions executed, etc.
   – $c(x,a,y)$ is the step cost, assumed to be ≥ 0

- A solution is a sequence of actions leading from the initial state to a goal state

Selecting a State Space

- Real world is absurdly complex

  → state space must be abstracted for problem solving
- (Abstract) state = set of real states
- (Abstract) action = complex combination of real actions
  - e.g., "Arad → Zerind" represents a complex set of possible routes, detours, rest stops, etc.
- For guaranteed realizability, any real state "in Arad“ must get to some real state "in Zerind"
- (Abstract) solution = set of real paths that are solutions in the real world
- Each abstract action should be "easier" than the original problem

Reflex Agent Solution

- Precompute all paths from any start state to the final state
- Store these paths in a table
- When faced with a new instance of the problem, look up the answer in the table

Cops and Robbers

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- What is the strategy to cross the river without letting the robbers escape?

Cops and Robbers

- State representation?
- Start State?
- End State?
- Transitions?

Cops and Robbers

- State representation?
  - Number of cops on the source bank
  - Number of robbers on the source bank
  - Number of boats on the source bank
- Start State?
- End State?
- Transitions?
- How many states total?
- Some states are forbidden
- Some states are unreachable

Cops and Robbers

- Search space

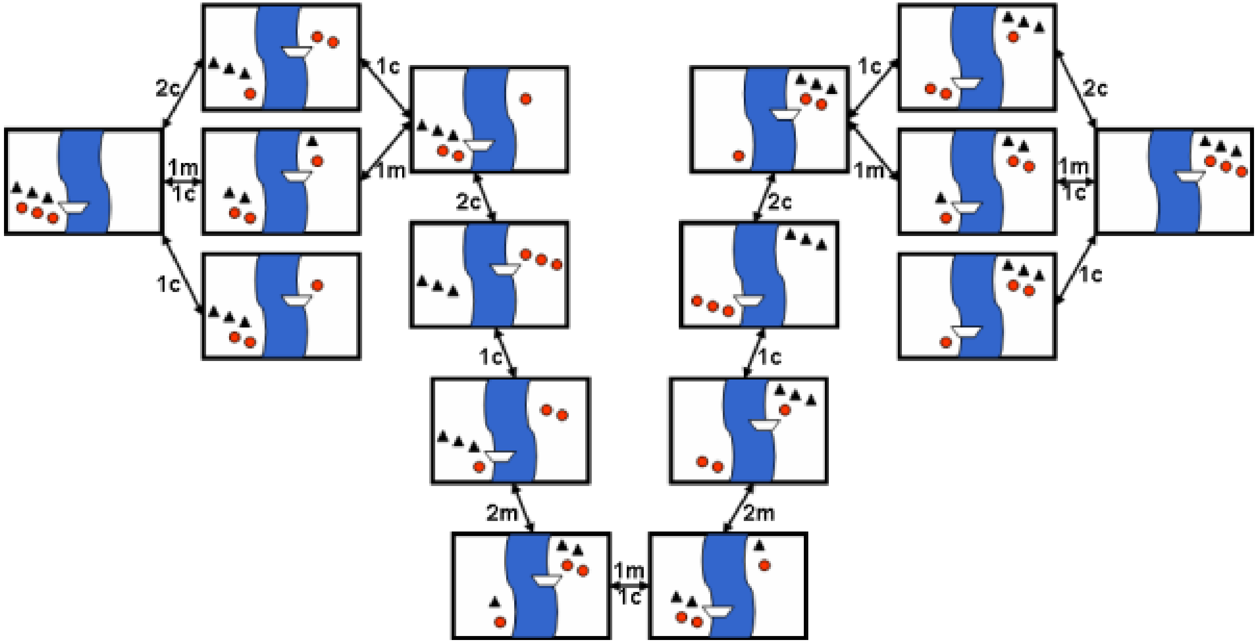
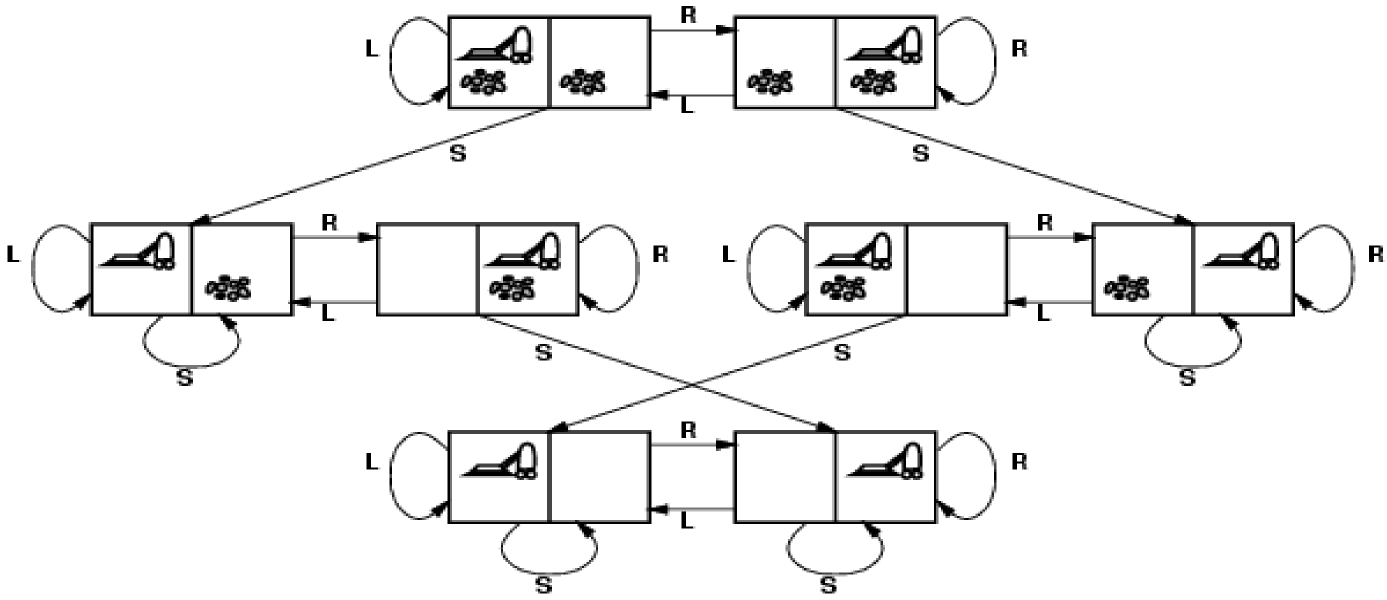


Image from Gerhard Wickler

Cops and Robbers

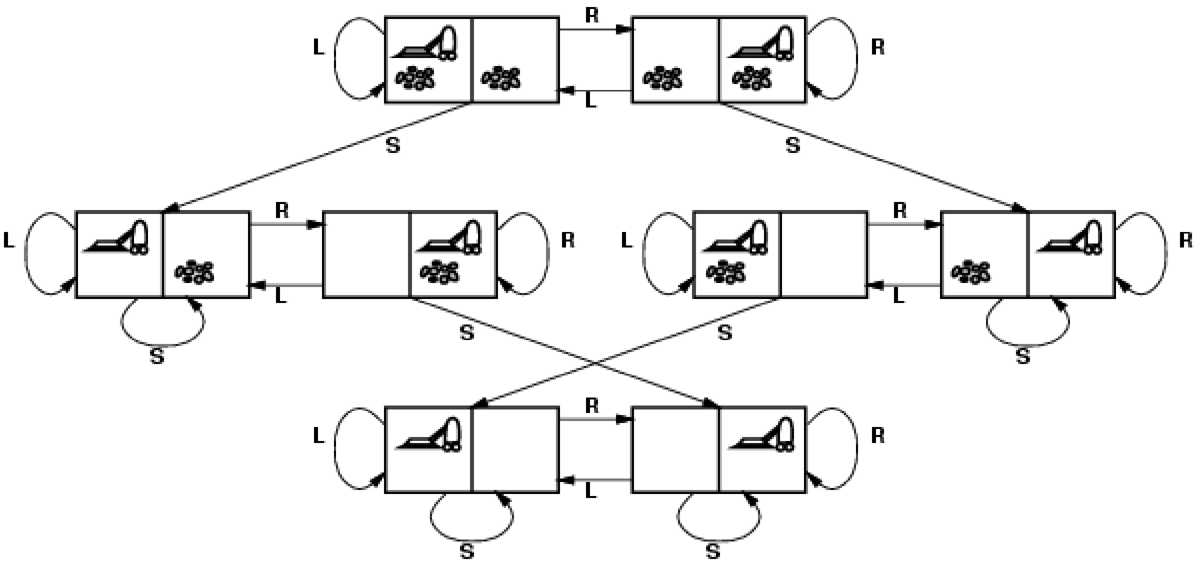
- Solution $331 \rightarrow 310 \rightarrow 321 \rightarrow 300 \rightarrow 311 \rightarrow 110 \rightarrow 221 \rightarrow 020 \rightarrow 031 \rightarrow 010 \rightarrow 021 \rightarrow 000.$
- Is a reflex agent the right method for this problem?
- Idea: separate algorithm/model from problem description
- Idea: planning agent

Vacuum world state space graph



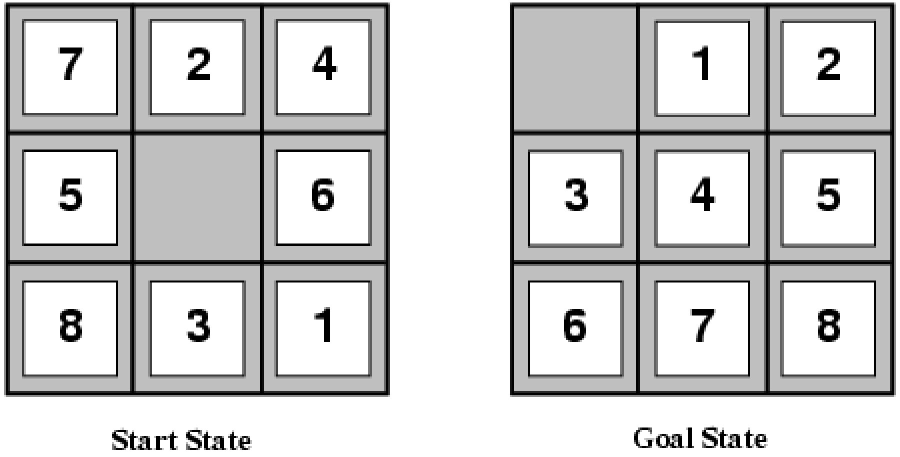
- states?
- actions?
- goal test?
- path cost?

State Space Graph



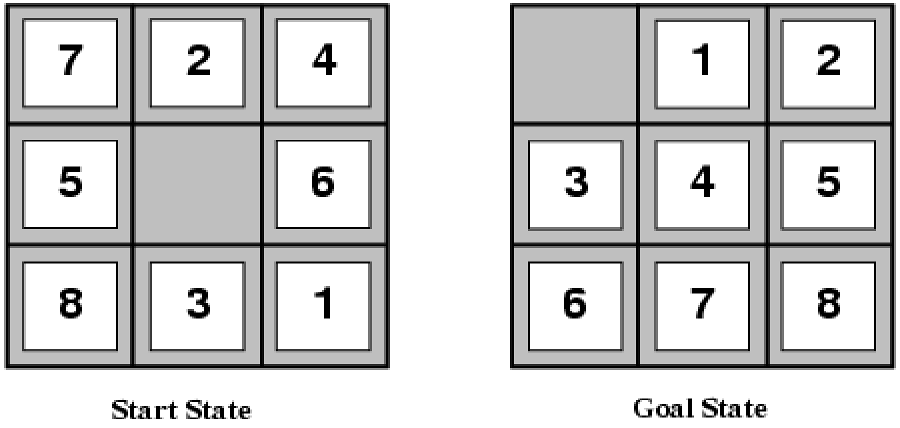
- states? integer dirt and robot location
- actions? *Left*, *Right*, *Suck*
- goal test? no dirt at all locations
- path cost? 1 per action

Example: The 8-puzzle



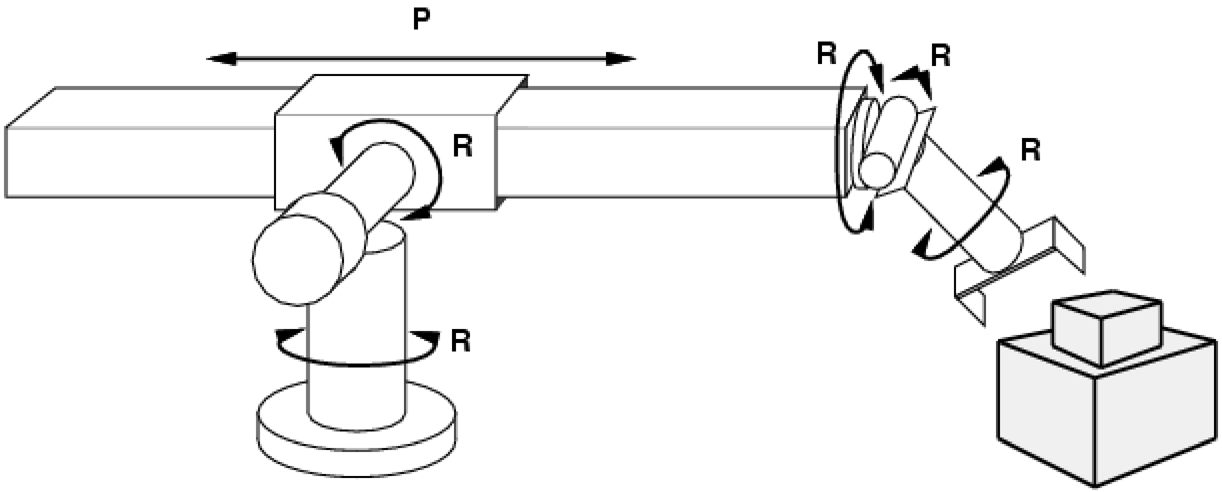
- states?
- actions?
- goal test?
- path cost?

Example: The 8-puzzle



- states? locations of tiles
- actions? move blank left, right, up, down
- goal test? = goal state (given)
- path cost? 1 per move
- [Note: optimal solution of *n*-Puzzle family is NP-hard]

Example: Robotic Assembly



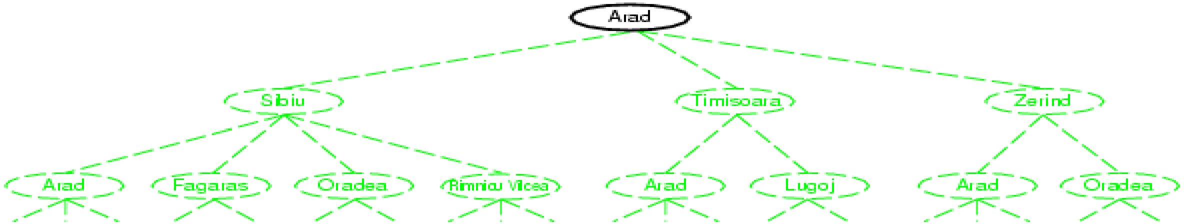
- states?: real-valued coordinates of robot joint angles parts of the object to be assembled
- actions?: continuous motions of robot joints
- goal test?: complete assembly
- path cost?: time to execute

Tree Search Algorithms

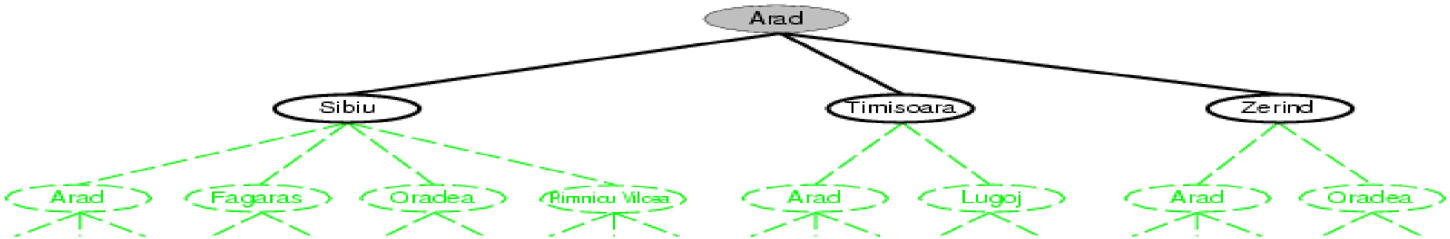
- Basic idea:
  - offline, simulated exploration of state space by generating successors of already-explored states (a.k.a.~expanding states)

```
function TREE-SEARCH( problem, strategy) returns a solution, or failure
    initialize the search tree using the initial state of problem
    loop do
        if there are no candidates for expansion then return failure
        choose a leaf node for expansion according to strategy
        if the node contains a goal state then return the corresponding solution
        else expand the node and add the resulting nodes to the search tree
```

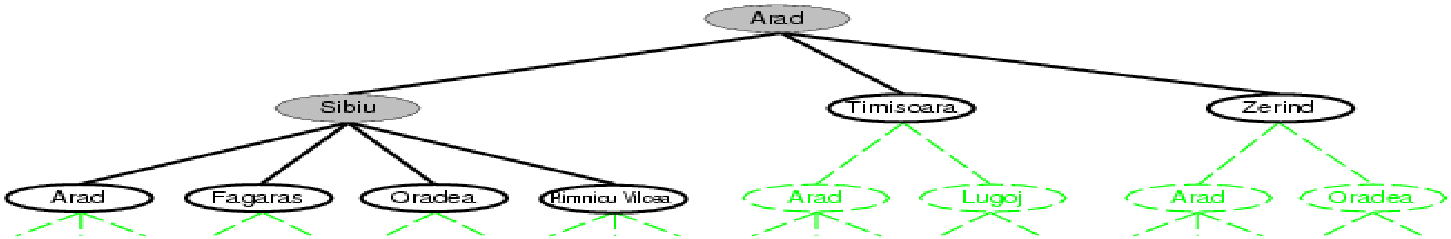
Tree search example



Tree search example



Tree search example



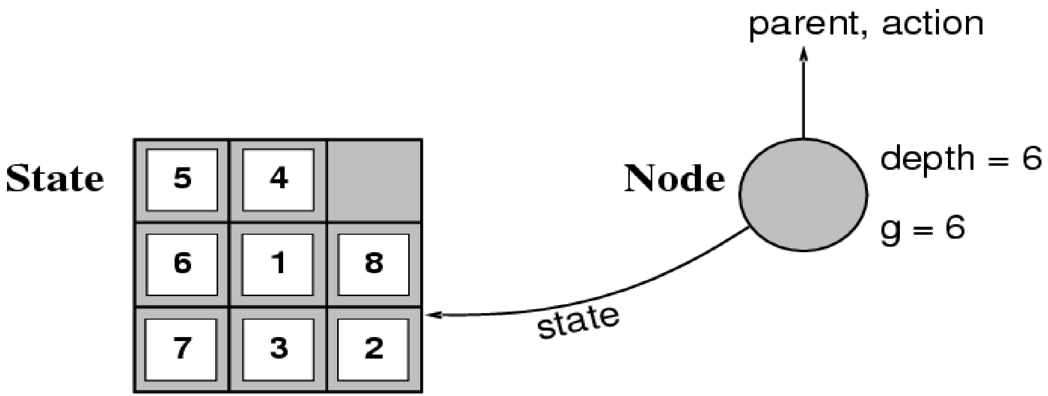
Implementation: general tree search

```
function TREE-SEARCH( problem, fringe) returns a solution, or failure
    fringe ← INSERT(MAKE-NODE(INITIAL-STATE[problem]), fringe)
    loop do
        if fringe is empty then return failure
        node ← REMOVE-FRONT(fringe)
        if GOAL-TEST[problem](STATE[node]) then return SOLUTION(node)
        fringe ← INSERTALL(EXPAND(node, problem), fringe)
```

```
function EXPAND( node, problem) returns a set of nodes
    successors ← the empty set
    for each action, result in SUCCESSOR-FN[problem](STATE[node]) do
        s ← a new NODE
        PARENT-NODE[s] ← node;   ACTION[s] ← action;   STATE[s] ← result
        PATH-COST[s] ← PATH-COST[node] + STEP-COST(node, action, s)
        DEPTH[s] ← DEPTH[node] + 1
        add s to successors
    return successors
```

Implementation: states vs. nodes

- A state is a (representation of) a physical configuration
- A node is a data structure constituting part of a search tree includes state, parent node, action, path cost *g(x)*, depth



- The `Expand` function creates new nodes, filling in the various fields and using the `SuccessorFn` of the problem to create the corresponding states.

Search Strategies

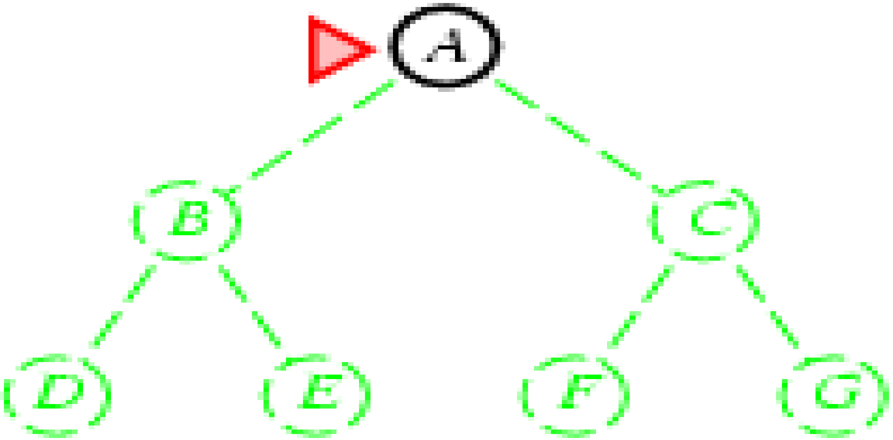
- A search strategy is defined by picking the order of node expansion
- Strategies are evaluated along the following dimensions:
  - completeness: does it always find a solution if one exists?
  - time complexity: number of nodes generated
  - space complexity: maximum number of nodes in memory
  - optimality: does it always find a least-cost solution?
- Time and space complexity are measured in terms of
  - b: maximum branching factor of the search tree
  - d: depth of the least-cost solution
  - m: maximum depth of the state space (may be ∞)

Uninformed search strategies

- Uninformed search strategies use only the information available in the problem definition
  - Breadth-first search
  - Uniform-cost search
  - Depth-first search
  - Depth-limited search
  - Iterative deepening search

Breadth-first search

- Expand shallowest unexpanded node
- Implementation:
  - *fringe* is a FIFO queue, i.e., new successors go at end



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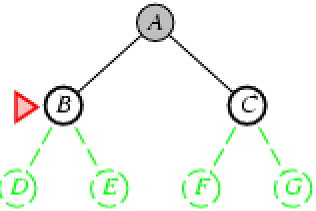

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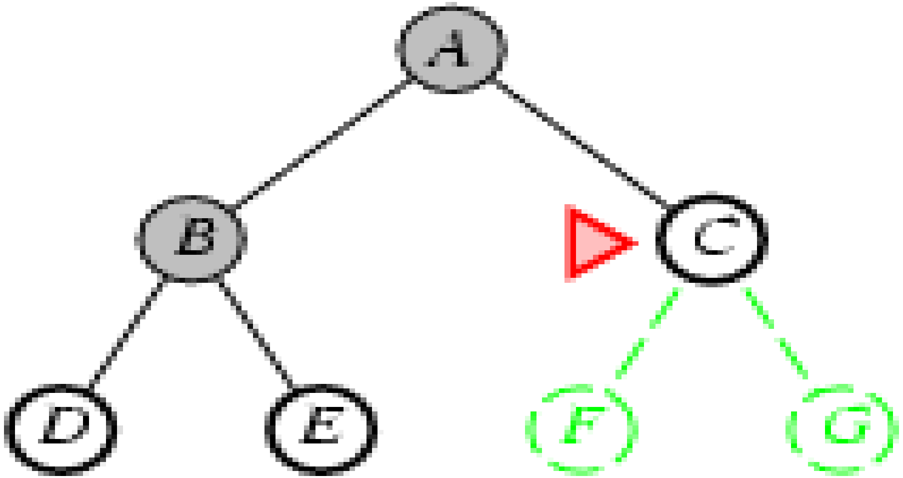

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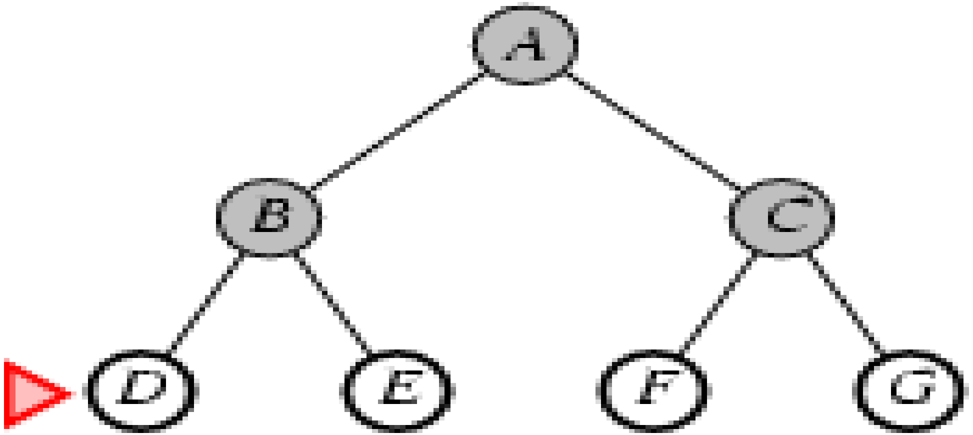


Properties of breadth-first search

- Complete? Yes (if b is finite)
- Time? $1+b+b^2+b^3+\ldots +b^d + b(b^d-1)$ = $O(b^{d+1})$
- Space? $O(b^{d+1})$ (keeps every node in memory)
- Optimal? Yes (if cost = 1 per step)

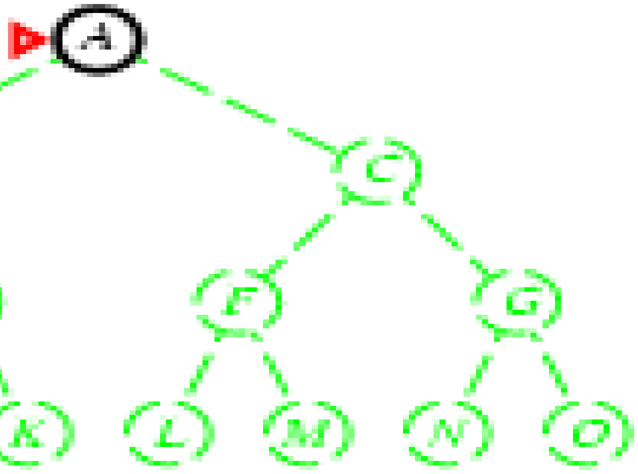
- Space is the bigger problem (more than time)

Uniform-cost search

- Expand least-cost unexpanded node
- Implementation:
  - *fringe* = queue ordered by path cost
- Equivalent to breadth-first if step costs all equal
- Complete? Yes, if step cost ≥ ε
- Time? # of nodes with *g* ≤ cost of optimal solution, $O(b^{ceiling(C^*/ \varepsilon)})$ where C^* is the cost of the optimal solution
- Space? # of nodes with *g* ≤ cost of optimal solution, $O(b^{ceiling(C^*/ \varepsilon)})$
- Optimal? Yes – nodes expanded in increasing order of *g(n)*

Depth-first search

- Expand deepest unexpanded node
- Implementation:
  - *fringe* = LIFO queue, i.e., put successors at front



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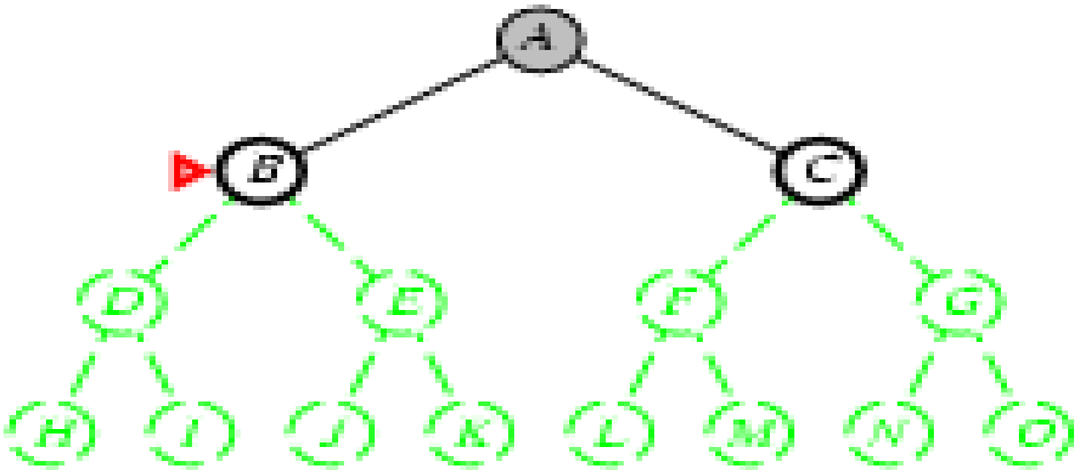

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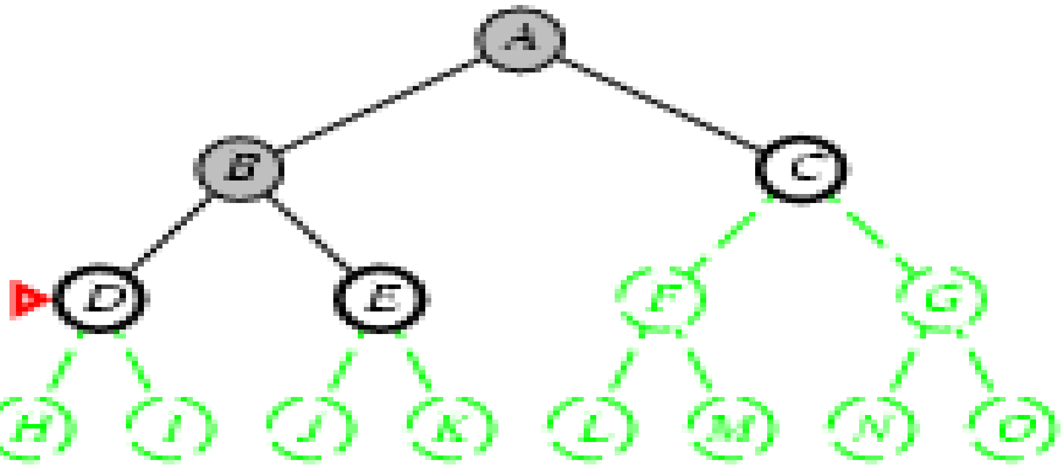

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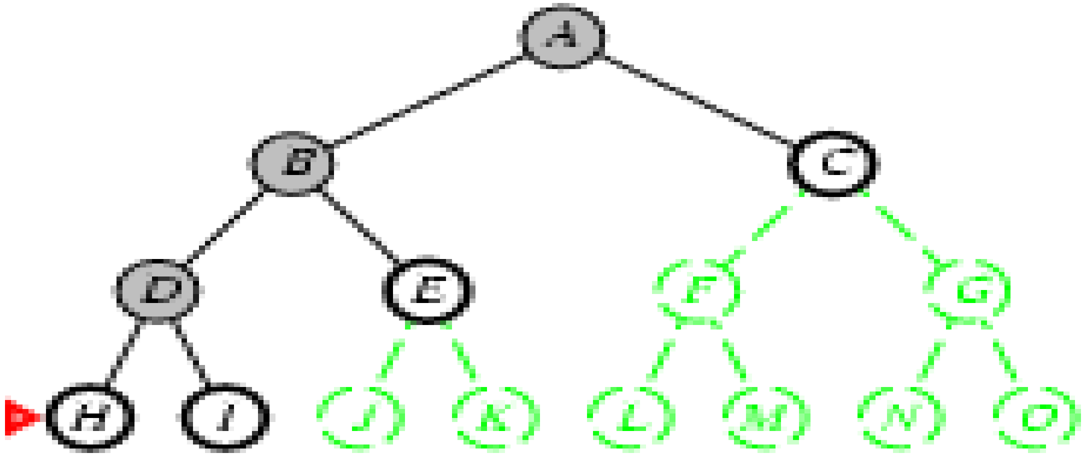

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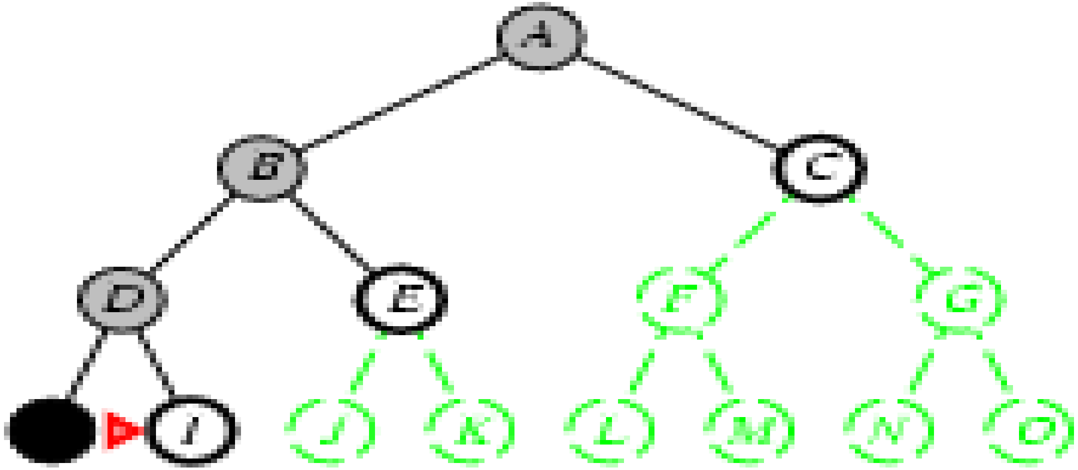

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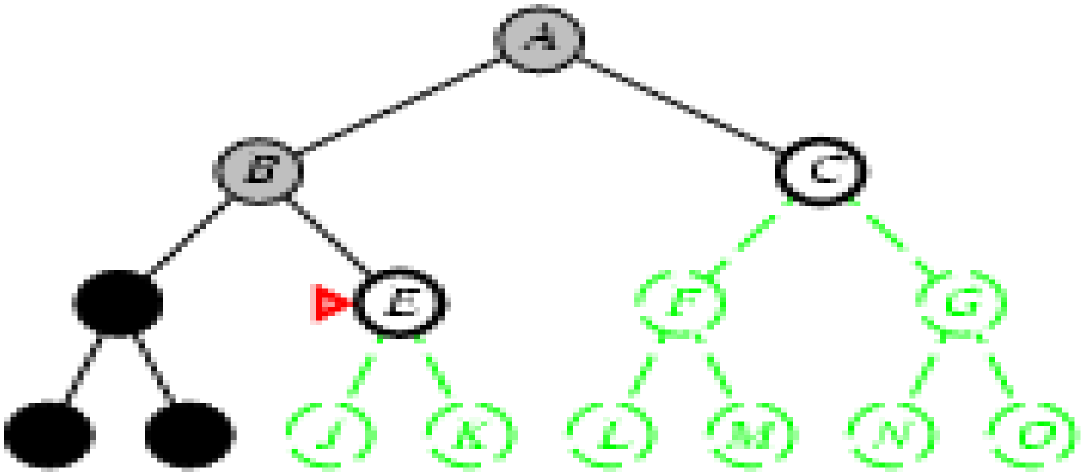

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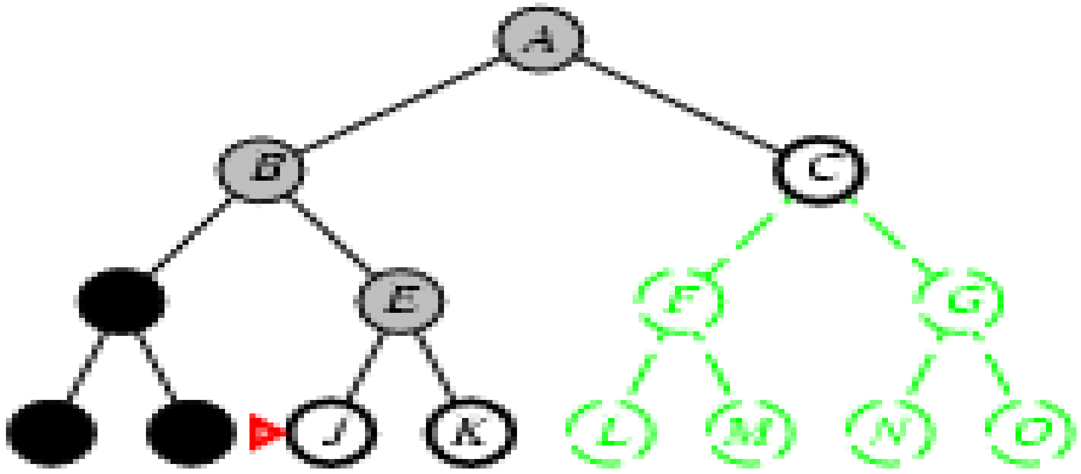

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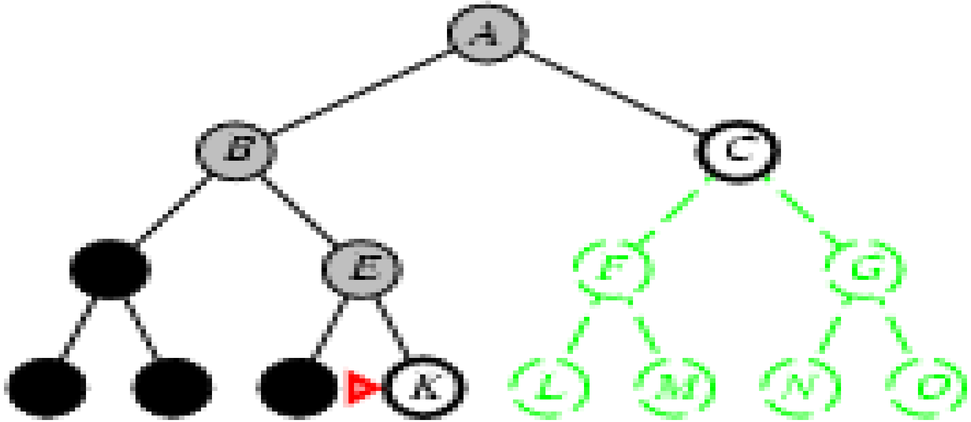

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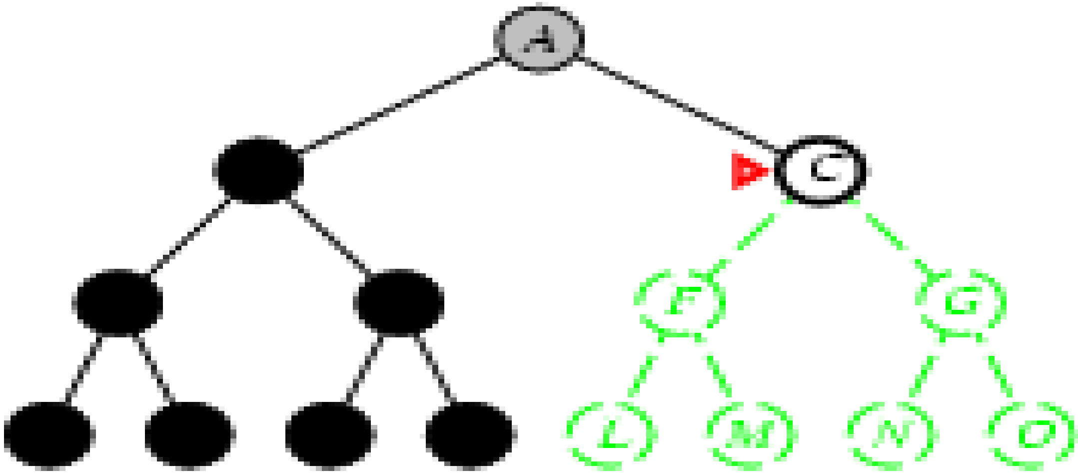

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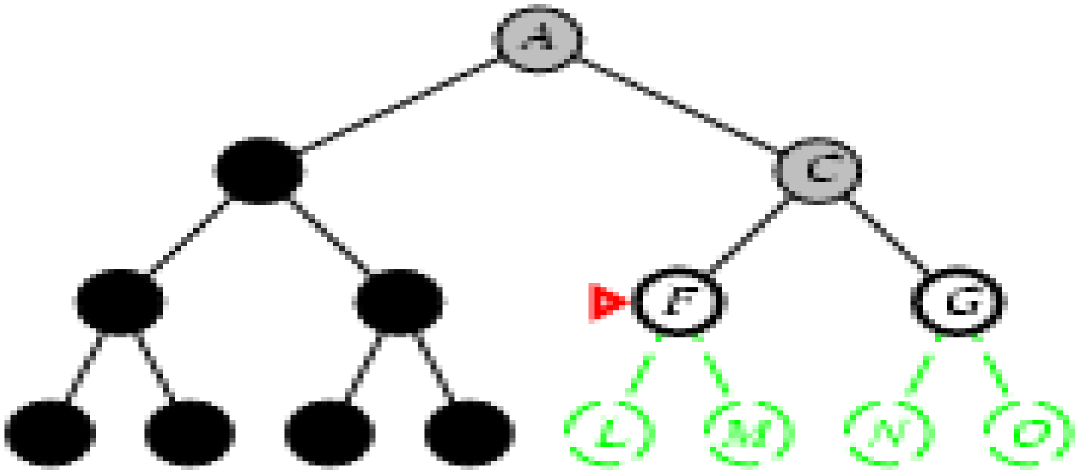

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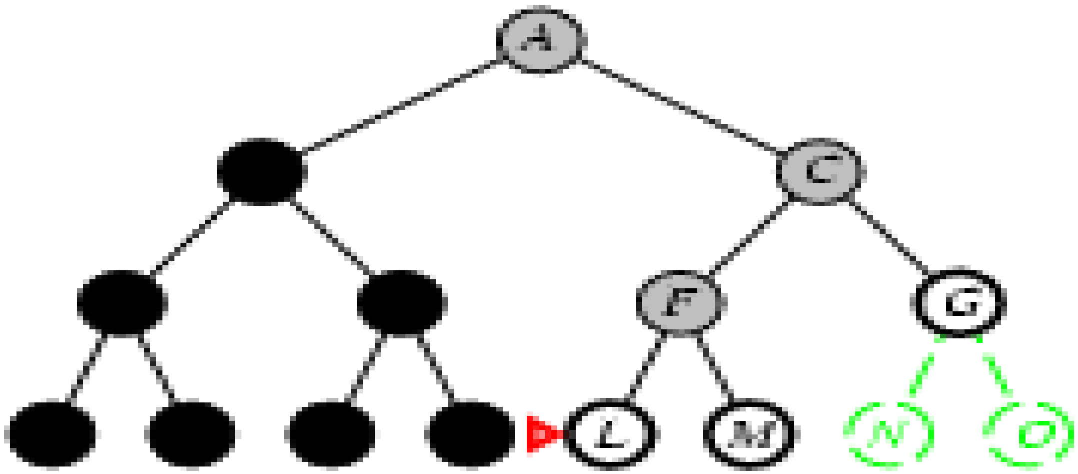

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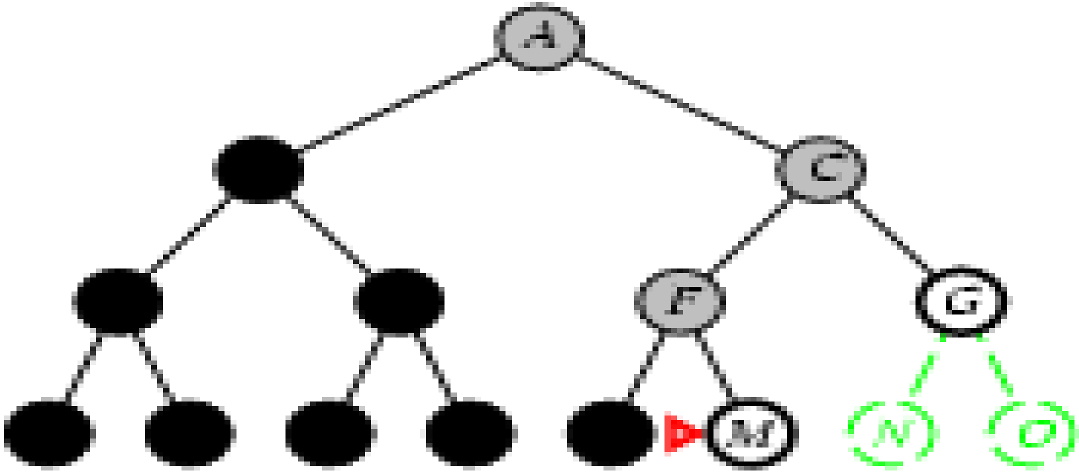


Properties of depth-first search

- Complete? No: fails in infinite-depth spaces, spaces with loops
  - Modify to avoid repeated states along path

    → complete in finite spaces
- Time? $O(b^m)$: terrible if m is much larger than d
  - but if solutions are dense, may be much faster than breadth-first
- Space? $O(bm)$, i.e., linear space!
- Optimal? No

Demos

- https://www.cs.usfca.edu/~galles/visualization/DFS.html
- https://www.cs.usfca.edu/~galles/visualization/BFS.html
- https://www.cs.usfca.edu/~galles/visualization/Algorithms.html
- http://www.redblobgames.com/pathfinding/a-star/introduction.html
- http://www.greenfoot.org/scenarios/4900

Depth-limited search

= depth-first search with depth limit *l*,

i.e., nodes at depth *l* have no successors

- Recursive implementation:

```
function DEPTH-LIMITED-SEARCH( problem, limit) returns soln/fail/cutoff
   RECURSIVE-DLS(MAKE-NODE(INITIAL-STATE[problem]), problem, limit)

function RECURSIVE-DLS(node, problem, limit) returns soln/fail/cutoff
   cutoff-occurred? ← false
   if GOAL-TEST[problem](STATE[node]) then return SOLUTION(node)
   else if DEPTH[node] = limit then return cutoff
   else for each successor in EXPAND(node, problem) do
        result ← RECURSIVE-DLS(successor, problem, limit)
        if result = cutoff then cutoff-occurred? ← true
        else if result ≠ failure then return result
   if cutoff-occurred? then return cutoff else return failure
```

Iterative deepening search

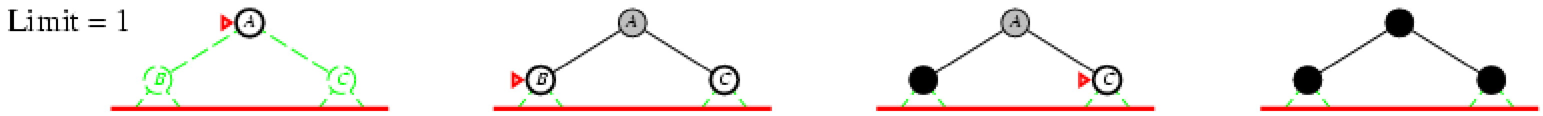
```
function ITERATIVE-DEEPENING-SEARCH( problem) returns a solution, or fail-
ure
    inputs: problem, a problem

    for depth ← 0 to ∞ do
        result ← DEPTH-LIMITED-SEARCH( problem, depth)
        if result ≠ cutoff then return result
```

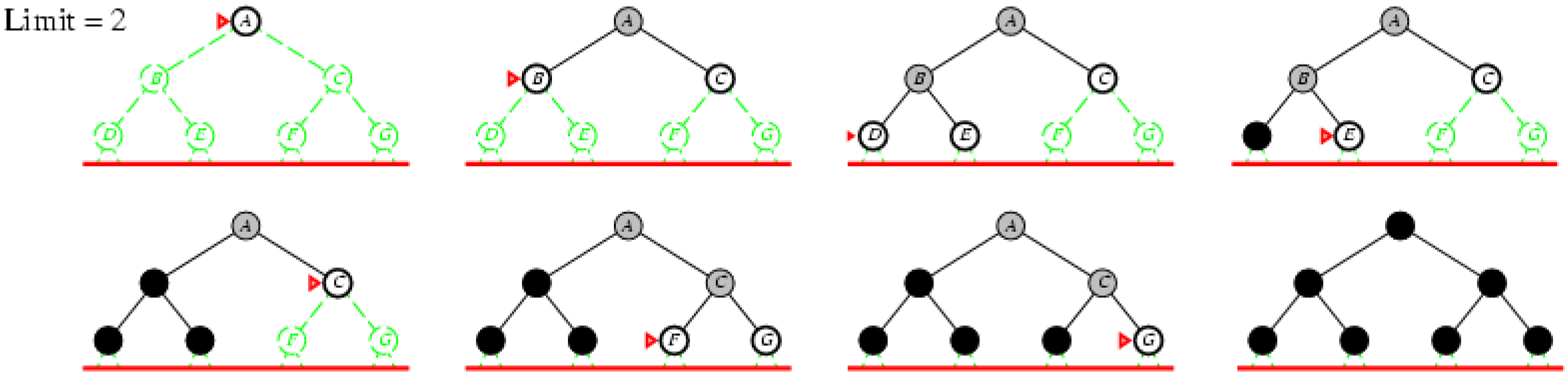
Iterative deepening search l =0

Limit = 0

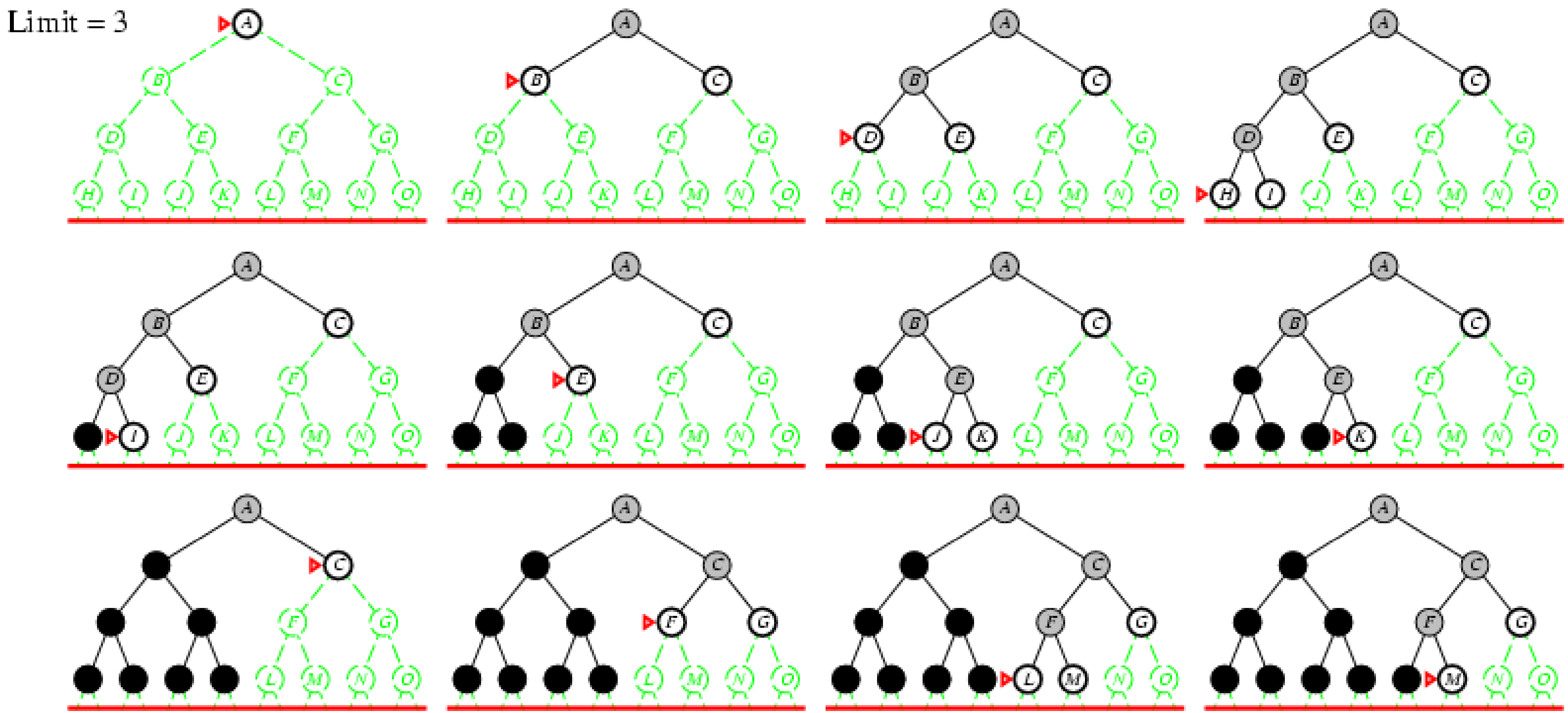
Iterative deepening search l =1



Iterative deepening search *l* =2



Iterative deepening search l =3



Iterative deepening search

- Number of nodes generated in a depth-limited search to depth *d* with branching factor *b*:

$$N_{DLS} = b^0 + b^1 + b^2 + \ldots + b^{d-2} + b^{d-1} + b^d$$

- Number of nodes generated in an iterative deepening search to depth *d* with branching factor *b*:

$$N_{IDS} = (d+1)b^0 + d\ b\text{^}1 + (d-1)b\text{^}2 + \ldots + 3b^{d-2} + 2b^{d-1} + 1b^d$$

- For $b = 10$, $d = 5$,
  - $N_{DLS} = 1 + 10 + 100 + 1{,}000 + 10{,}000 + 100{,}000 = 111{,}111$
  - $N_{IDS} = 6 + 50 + 400 + 3{,}000 + 20{,}000 + 100{,}000 = 123{,}456$

- Overhead = (123,456 - 111,111)/111,111 = 11%

Properties of iterative deepening search

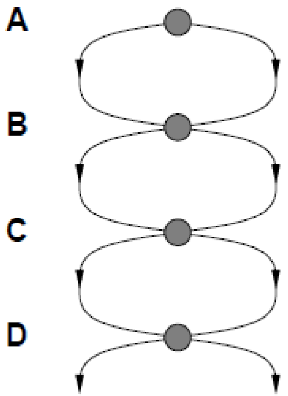
- Complete? Yes
- Time? $(d+1)b^0 + d\, b^1 + (d-1)b^2 + \ldots + b^d = O(b^d)$
- Space? $O(bd)$
- Optimal? Yes, if step cost = 1

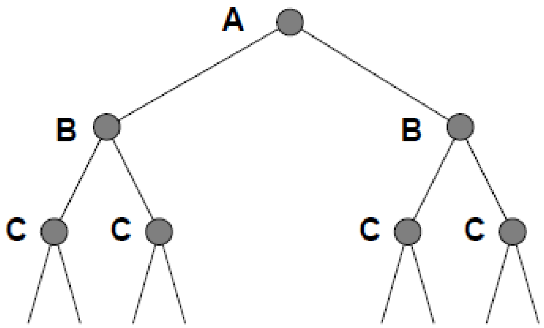
Summary of algorithms

Criterion	Breadth-First	Uniform-Cost	Depth-First	Depth-Limited	Iterative Deepening
Complete?	Yes*	Yes*	No	Yes, if $l \geq d$	Yes
Time	b^{d+1}	$b^{\lceil C^*/\epsilon \rceil}$	b^m	b^l	b^d
Space	b^{d+1}	$b^{\lceil C^*/\epsilon \rceil}$	bm	bl	bd
Optimal?	Yes*	Yes	No	No	Yes*

Repeated states

- Failure to detect repeated states can turn a linear problem into an exponential one!





Dynamic Programming

- Split the problem of computing the total cost from Start state to End state into:
  - Past cost to current state S_i
  - Future cost from S_i to end state
- The future cost is equal to 0 at the final state
- In other states:
  - Consider all possible actions a_j
  - Pick the one that minimizes
    - Cost (S_i, a_j) + Future Cost $(successor(S_i, a_j))$

Dynamic Programming

- How to avoid exponential costs?
  - By storing all relevant history in the description of the current state
- Romania example:
  - The future cost of a node doesn't depend on the path that got us there
  - In this case, the current node only needs to store the best cost that got us there
  - DP rule: if the best past cost has been computed for a node, don't compute it again

Dynamic Programming Demos

- https://www.cs.usfca.edu/~galles/visualization/DPFib.html
- https://www.cs.usfca.edu/~galles/visualization/DPLCS.html
- https://blast.ncbi.nlm.nih.gov/Blast.cgi?PAGE_TYPE=BlastSearch&PROG_DEF=blastn&BLAST_PROG_DEF=blastn&BLAST_SPEC=GlobalAln&LINK_LOC=BlastHomeLink
- http://experiments.mostafa.io/public/needleman-wunsch/
- https://zhanglab.ccmb.med.umich.edu/NW-align/
- https://zhanglab.ccmb.med.umich.edu/FASTA/
- http://www.cs.yale.edu/homes/radev/nlpclass/slides2017/313.pdf

Sample Protein Sequence

>gi|186681228|ref|YP_001864424.1| phycoerythrobilin:ferredoxin oxidoreductase
MNSERSDVTLYQPFLDYAIAYMRSRLDLEPYPIPTGFESNSAVVGKGKNQEEVVTTSYAFQTAKLRQIRA
AHVQGGNSLQVLNFVIFPHLNYDLPFFGADLVTLPGGHLIALDMQPLFRDDSAYQAKYTEPILPIFHAHQ
QHLSWGGDFPEEAQPFFSPAFLWTRPQETAVVETQVFAAFKDYLKAYLDFVEQAEAVTDSQNLVAIKQAQ
LRYLRYRAEKDPARGMFKRFYGAEWTEEYIHGFLFDLERKLTVVK

Nucleic acid codes:

A --> adenosine M --> A C (amino) C --> cytidine S --> G C (strong) G --> guanine W --> A T (weak) T --> thymidine B --> G T C U --> uridine D --> G A T R --> G A (purine) H --> A C T Y --> T C (pyrimidine) V --> G C A K --> G T (keto) N --> A G C T (any)

Amino acid codes:

A ALA alanine P PRO proline B ASX aspartate or asparagine Q GLN glutamine C CYS cystine R ARG arginine
D ASP aspartate S SER serine E GLU glutamate T THR threonine F PHE phenylalanine U selenocysteine
G GLY glycine V VAL valine H HIS histidine W TRP tryptophan I ILE isoleucine Y TYR tyrosine K LYS lysine
Z GLX glutamate or glutamine L LEU leucine X any M MET methionine * translation stop N ASN asparagine
- gap of indeterminate length

Uniform Cost Search

- Dynamic programming only works when the graph is acyclic
- The assumption is that we will compute the past cost of a state before computing the past cost of its successors.
- This doesn't work for general graphs

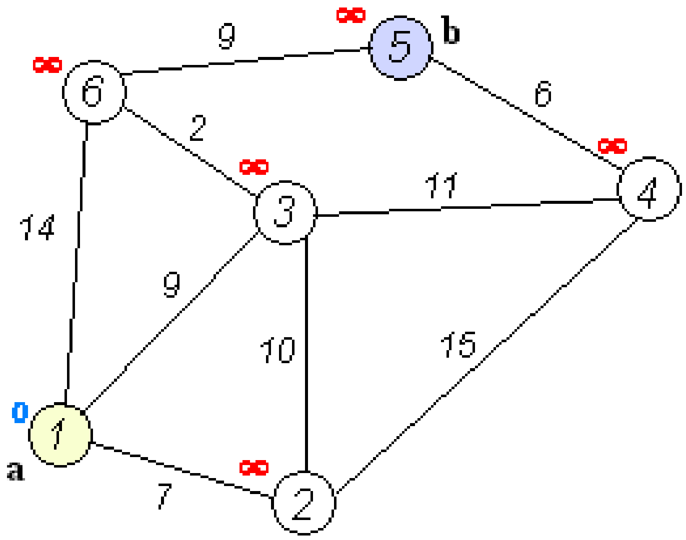
Uniform Cost Search

- The idea of uniform cost search is to order the states by past cost
- Not the same as Dijkstra’s algorithm because the graph may not be fully specified at search time
- Furthermore, UCS is intended to find the best cost path only from the start state to the goal state, not between all pairs of states.

Uniform Cost Search

- Explored states – cost known
- Frontier states – known states but cost is unknown
- Unexplored states – the rest

Dijkstra Algorithm



https://en.wikipedia.org/wiki/Dijkstra%27s_algorithm#/media/File:Dijkstra_Animation.gif

By Ibmua - Work by uploader., Public Domain, https://commons.wikimedia.org/w/index.php?curid=6282617

Graph Search

```
function GRAPH-SEARCH( problem, fringe) returns a solution, or failure
    closed ← an empty set
    fringe ← INSERT(MAKE-NODE(INITIAL-STATE[problem]), fringe)
    loop do
        if fringe is empty then return failure
        node ← REMOVE-FRONT(fringe)
        if GOAL-TEST[problem](STATE[node]) then return SOLUTION(node)
        if STATE[node] is not in closed then
            add STATE[node] to closed
            fringe ← INSERTALL(EXPAND(node, problem), fringe)
```

Dijkstra Demos

- https://www.cs.usfca.edu/~galles/visualization/Dijkstra.html
- https://qiao.github.io/PathFinding.js/visual/
- https://visualgo.net/de/sssp?slide=1

Learning the Costs

- The solution depends on the costs
- If the costs are given, fine
  - What if they are not?
- This is a learning problem (the inverse of search)
  - Observed sequence of actions -> costs
- Sample algorithm
  - Structured perceptron (later)

Summary

- Problem formulation usually requires abstracting away real-world details to define a state space that can feasibly be explored
- Variety of uninformed search strategies
- Iterative deepening search uses only linear space and not much more time than other uninformed algorithms

AI