AI

Artificial Intelligence

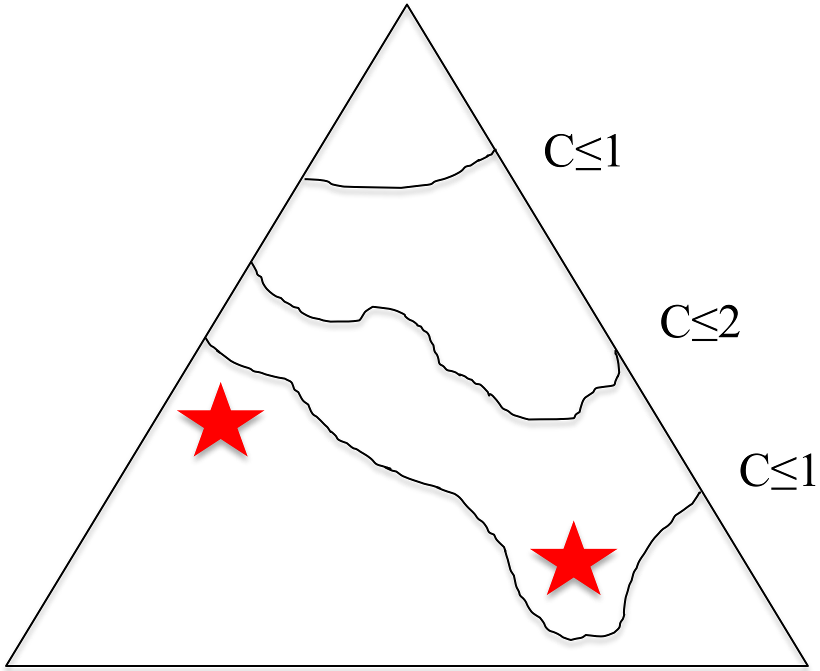
8.2.3
Problem Solving and Searching
(Chapter 4)

Outline

- Best-first search
  - Greedy best-first search
  - A^* search
- Heuristics

Drawbacks of UCS

- It has no concept of where the goal state is



Review: Tree search

```
function TREE-SEARCH( problem, fringe) returns a solution, or failure
    fringe ← INSERT(MAKE-NODE(INITIAL-STATE[problem]), fringe)
    loop do
        if fringe is empty then return failure
        node ← REMOVE-FRONT(fringe)
        if GOAL-TEST[problem] applied to STATE(node) succeeds return node
        fringe ← INSERTALL(EXPAND(node, problem), fringe)
```

- Basic idea:
  - offline, simulated exploration of state space by generating successors of already-explored states (a.k.a.~expanding states)
- A search strategy is defined by picking the order of node expansion

Best-first search

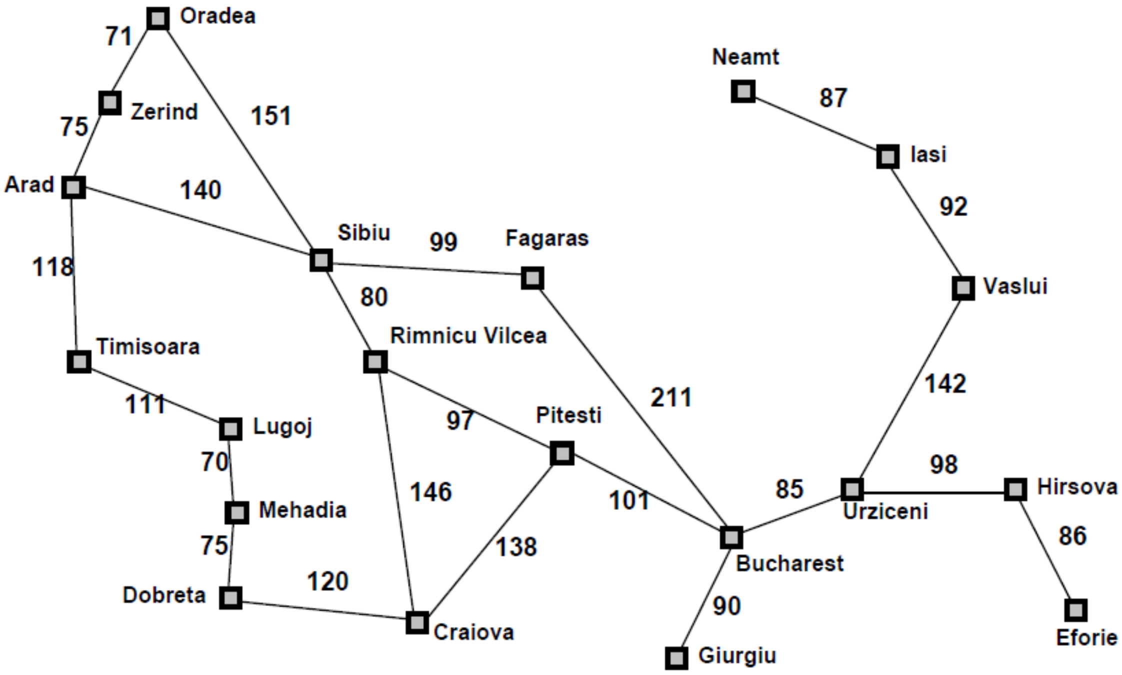
- Idea: use an evaluation function *f(n)* for each node
  - estimate of "desirability"
  - → Expand most desirable unexpanded node
- Implementation:
  Order the nodes in fringe in decreasing order of desirability
- Special cases:
  - Greedy best-first search
  - A^* search

Greedy best-first search

- Evaluation function *f(n)* = *h(n)* (heuristic)

  = estimate of cost from *n* to *goal*
- For example:
  - $h_{SLD}(n)$ = straight-line distance from *n* to Bucharest
- Greedy best-first search expands the node that appears to be closest to goal

Romania with step costs in km



Straight−line distance to Bucharest

Arad	366
Bucharest	0
Craiova	160
Dobreta	242
Eforie	161
Fagaras	178
Giurgiu	77
Hirsova	151
Iasi	226
Lugoj	244
Mehadia	241
Neamt	234
Oradea	380
Pitesti	98
Rimnicu Vilcea	193
Sibiu	253
Timisoara	329
Urziceni	80
Vaslui	199
Zerind	374

Greedy best-first search example

Greedy best-first search example

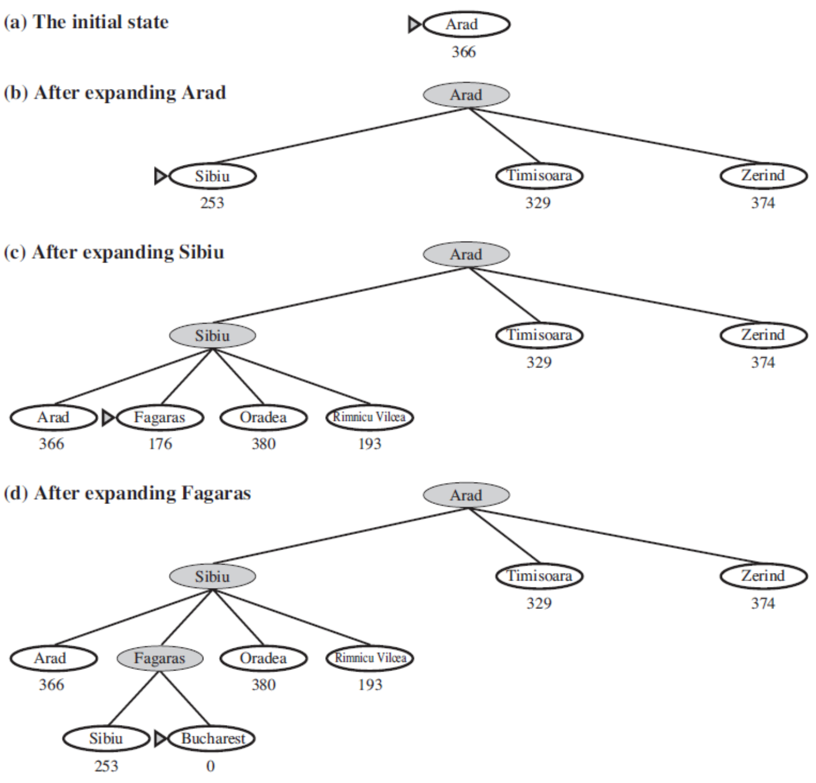


Figure 3.23 Stages in a greedy best-first tree search for Bucharest with the straight-line distance heuristic h_{SLD}. Nodes are labeled with their h-values.

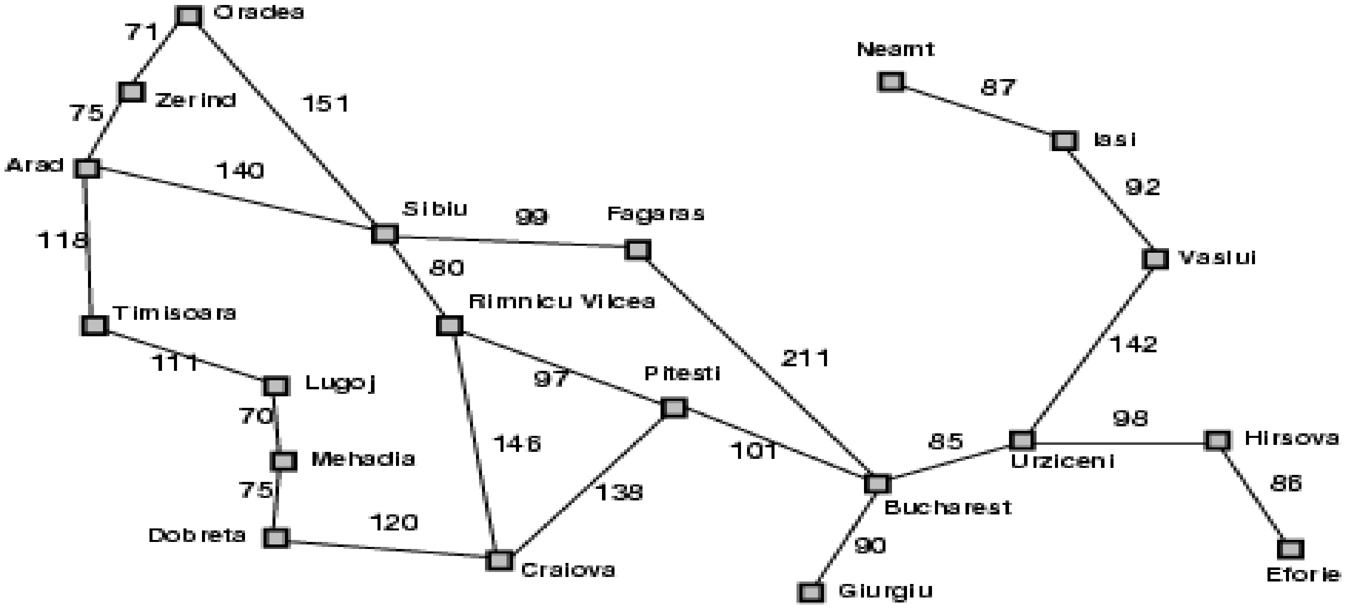
Properties of greedy best-first search

- Complete?
  - No – can get stuck in loops, e.g., Iasi → Neamt → Iasi → Neamt →
- Time?
  - $O(b^m)$, but a good heuristic can give dramatic improvement
- Space?
  - $O(b^m)$ -- keeps all nodes in memory
- Optimal?
  - No

A^* search

- Idea: Avoid expanding paths that are already expensive
- [Hart, Nilsson, Raphael 1968]
- Evaluation function $f(n) = g(n) + h(n)$
  - $g(n)$ = cost so far to reach n
  - $h(n)$ = estimated cost from n to goal
  - $f(n)$ = estimated total cost of path through n to goal

Romania with step costs in km



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SLD Values to Bucharest

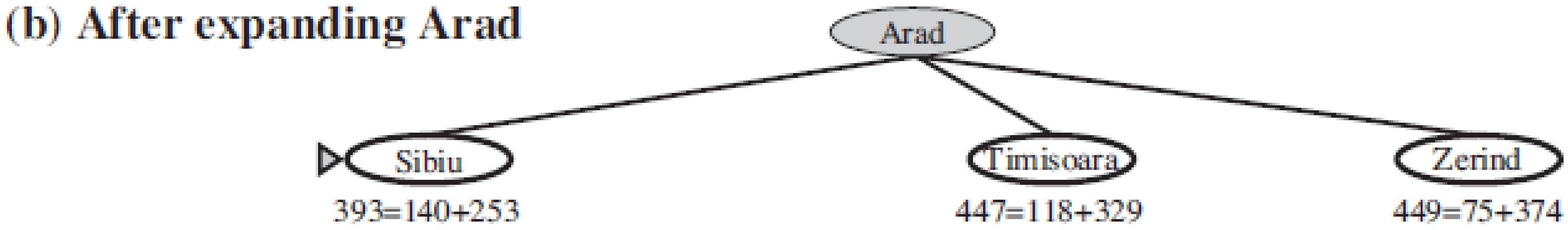
Arad	366	Mehadia	241
Bucharest	0	Neamt	234
Craiova	160	Oradea	380
Drobeta	242	Pitesti	100
Eforie	161	Rimnicu Vilcea	193
Fagaras	176	Sibiu	253
Giurgiu	77	Timisoara	329
Hirsova	151	Urziceni	80
Iasi	226	Vaslui	199
Lugoj	244	Zerind	374

Figure 3.22 Values of h_{SLD}—straight-line distances to Bucharest.

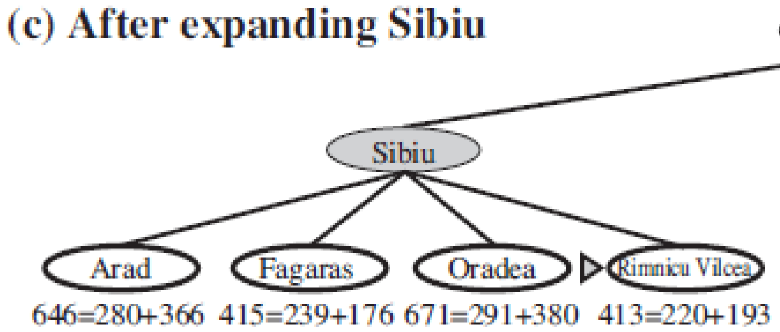
A^* search example

(a) The initial state

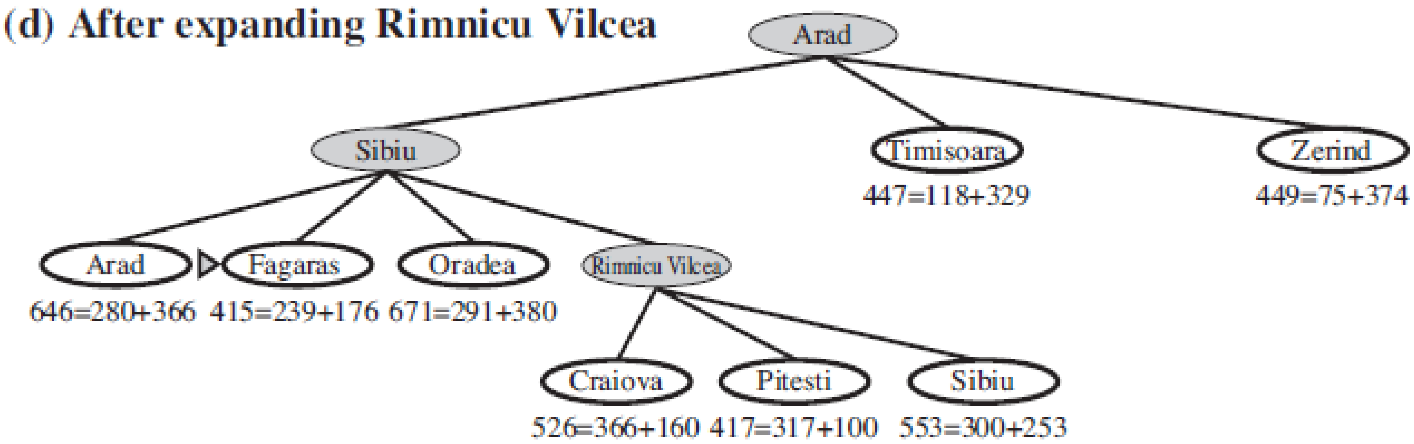
A^* search example



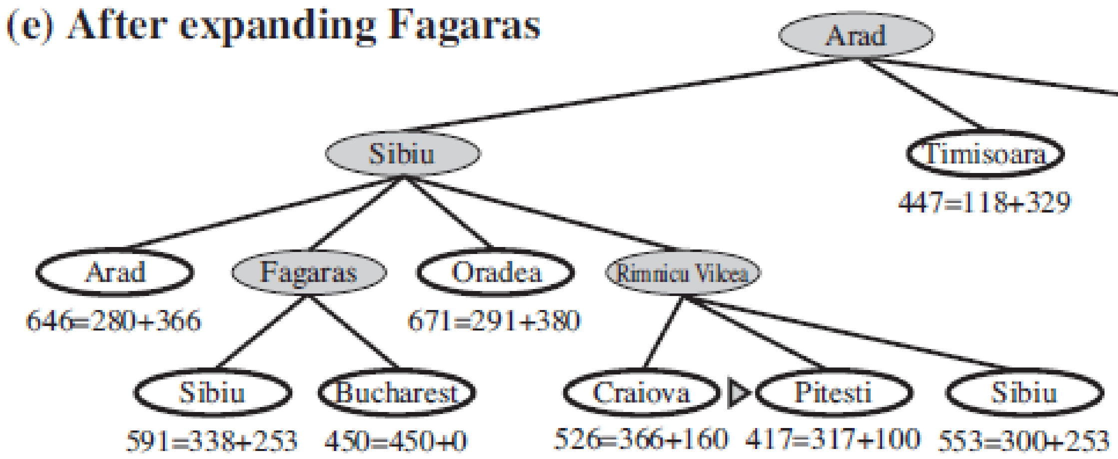
A* search example



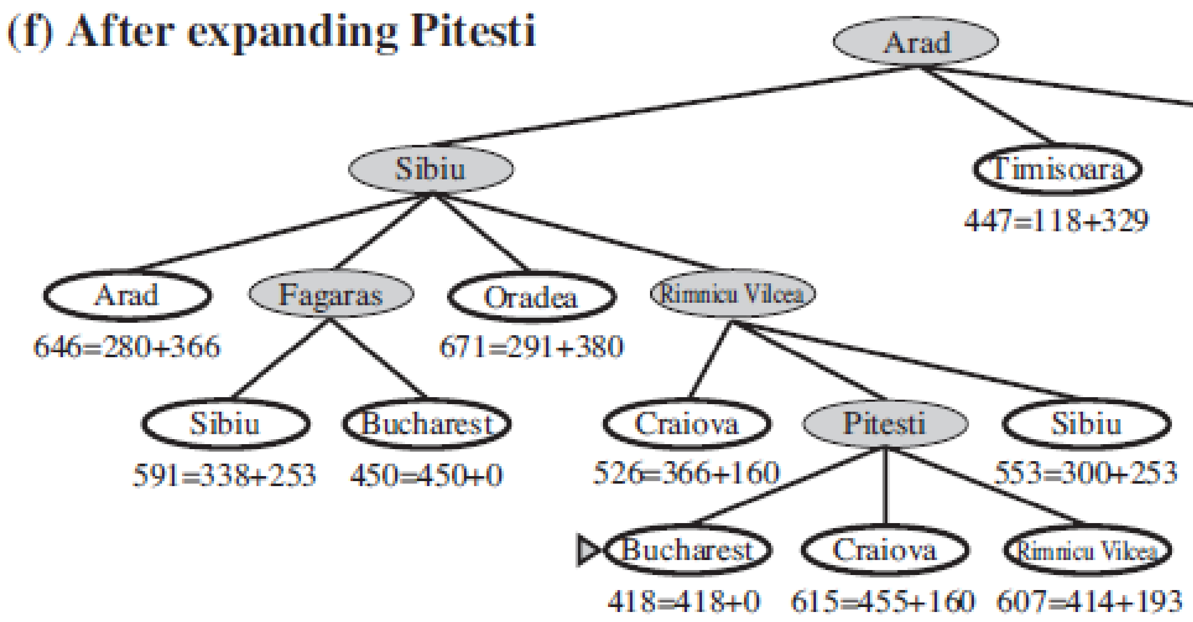
A* search example



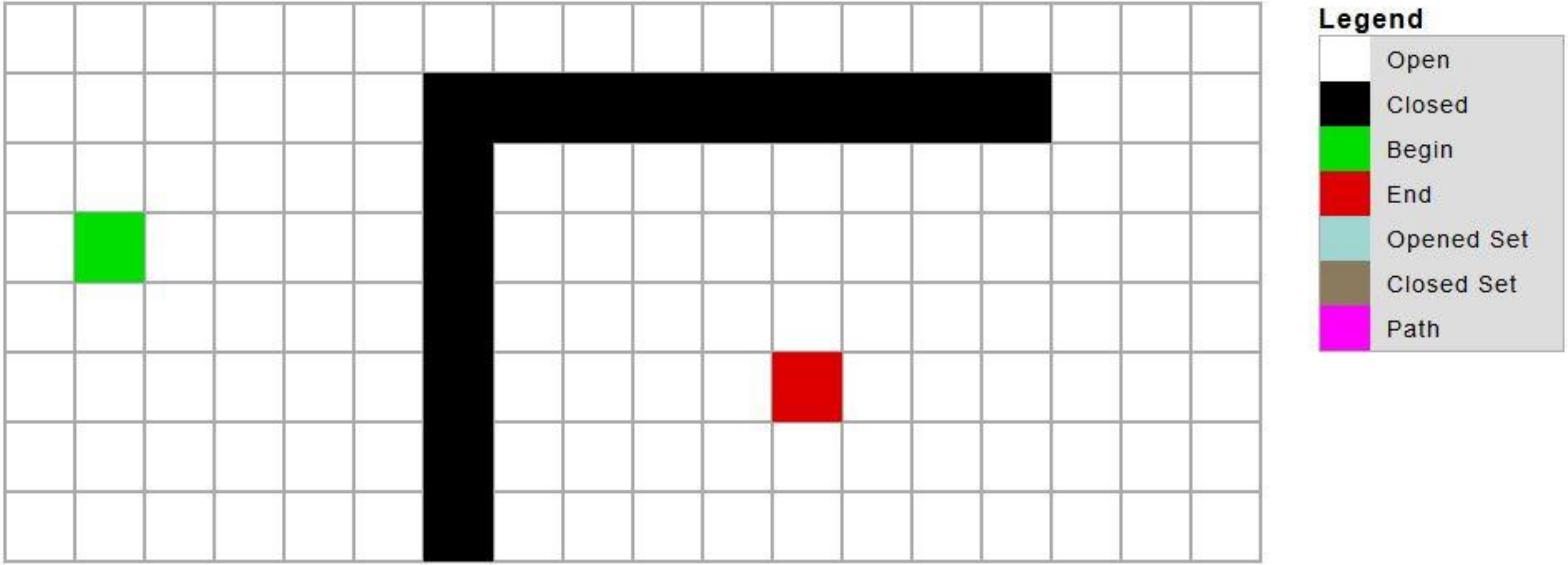
A^* search example



A* search example

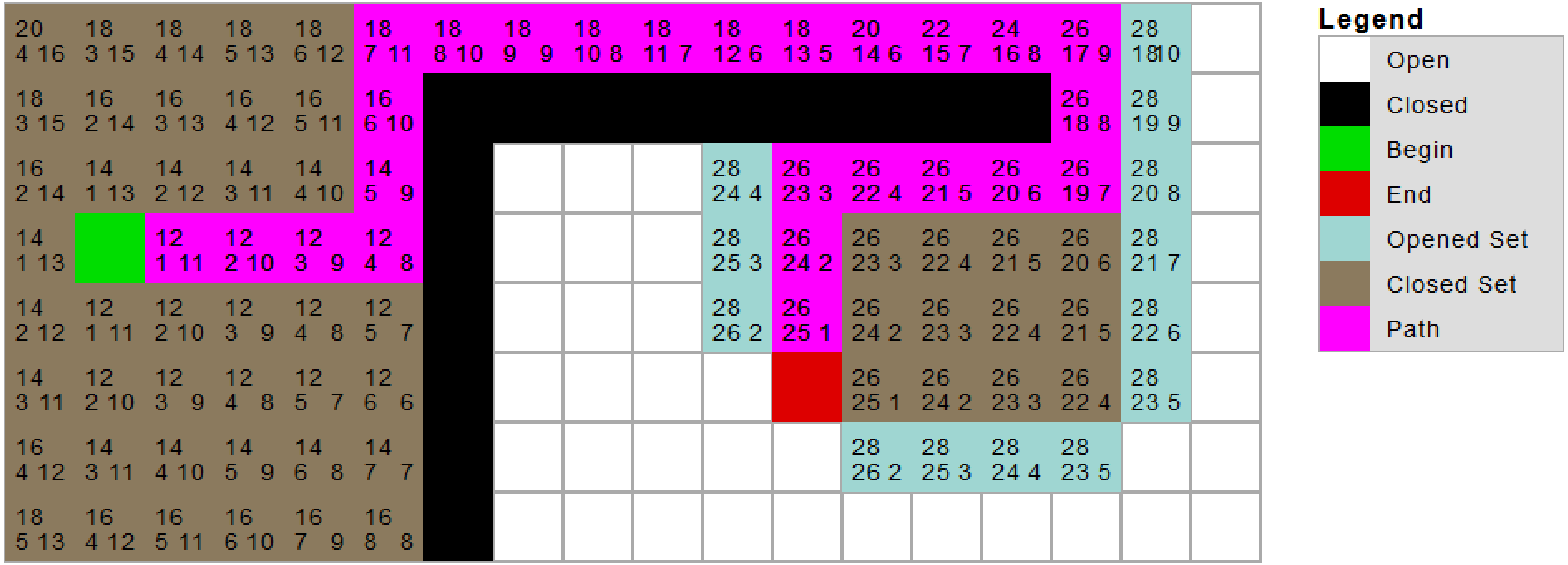


Demo



http://ashblue.github.io/javascript-pathfinding/

Demo



http://ashblue.github.io/javascript-pathfinding/

Other Demos

- https://qiao.github.io/PathFinding.js/visual/
- https://briangrinstead.com/blog/astar-search-algorithm-in-javascript/
- http://www.policyalmanac.org/games/aStarTutorial.htm

Admissible heuristics

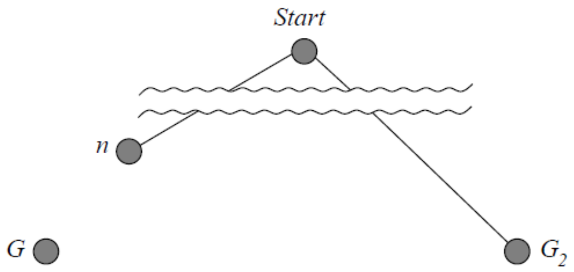
- A heuristic $h(n)$ is admissible if for every node n,

$h(n) \leq h^*(n)$, where $h^*(n)$ is the true cost to reach the goal state from n.

- An admissible heuristic never overestimates the cost to reach the goal, i.e., it is optimistic
  - Example: $h_{SLD}(n)$ (never overestimates the actual road distance)
- Theorem: If $h(n)$ is admissible, A* using `TREE-SEARCH` is optimal

Optimality of A* (proof)

- Suppose some suboptimal goal G_2 has been generated and is in the fringe. Let n be an unexpanded node in the fringe such that n is on a shortest path to an optimal goal G.



- $g(G_2) > g(G)$ since G_2 is suboptimal
- $f(G_2) = g(G_2)$ since $h(G_2) = 0$
- $f(G) = g(G)$ since $h(G) = 0$
- $f(G_2) > f(G)$ from above

Optimality of A^* (proof)

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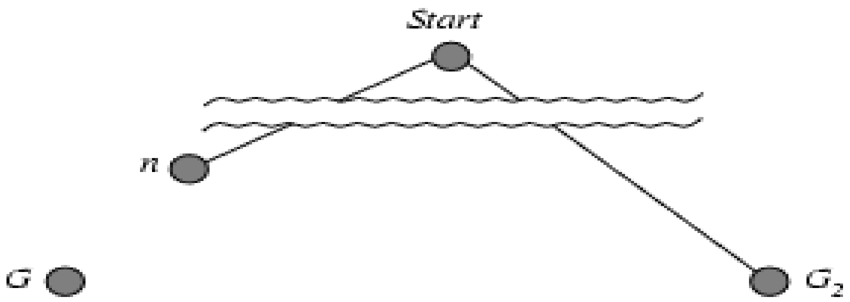


• $f(G_2)$	$> f(G)$	from above
• $h(n)$	$\le h^*(n)$	since h is admissible
• $g(n) + h(n)$	$\le g(n) + h^*(n)$	
• $f(n)$	$\le f(G)$	

Hence $f(G_2) > f(n)$, and A^* will never select G_2 for expansion

Consistent heuristics

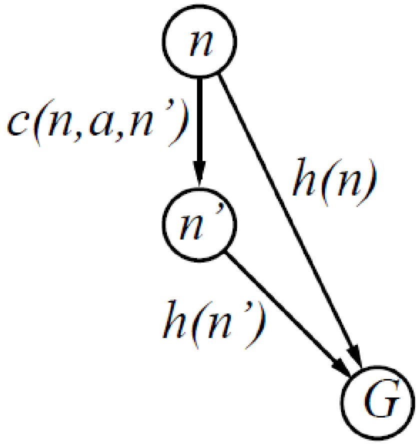
- A heuristic is consistent if, for every node n, every successor n' of n generated by any action a,

  $h(n) \leq c(n,a,n') + h(n')$

- If h is consistent, we have

$$\begin{aligned} f(n') &= g(n') + h(n') \\ &= g(n) + c(n,a,n') + h(n') \\ &\geq g(n) + h(n) \\ &= f(n) \end{aligned}$$

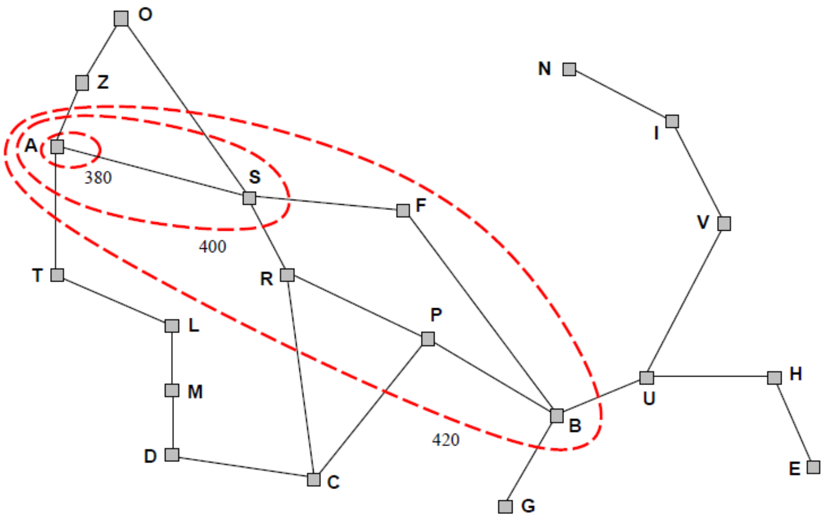
- i.e., $f(n)$ is non-decreasing along any path.



- Theorem: If $h(n)$ is consistent, A* using GRAPH−SEARCH is optimal

Optimality of A*

- A^* expands nodes in order of increasing f value
- Gradually adds "f-contours" of nodes
- Contour i has all nodes with $f=f_i$, where $f_i < f_{i+1}$



Properties of A*

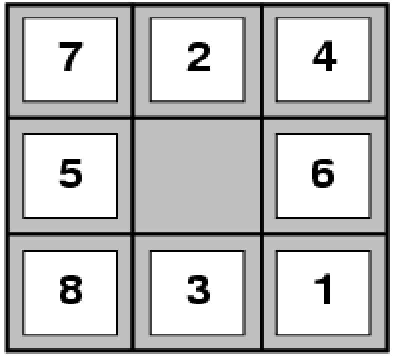
- Complete?
  - Yes (unless there are infinitely many nodes with f ≤ *f(G)*)
- Time?
  - Exponential
- Space?
  - Keeps all nodes in memory
- Optimal?
  - Yes

Admissible heuristics

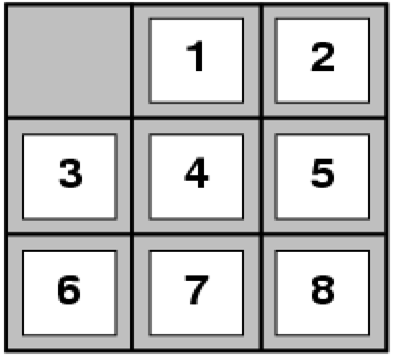
E.g., for the 8-puzzle:

- $h_1(n)$ = number of misplaced tiles
- $h_2(n)$ = total Manhattan distance

(i.e., no. of squares from desired location of each tile)



Start State



Goal State

- $h_1(S) = ?$
- $h_2(S) = ?$

Admissible heuristics

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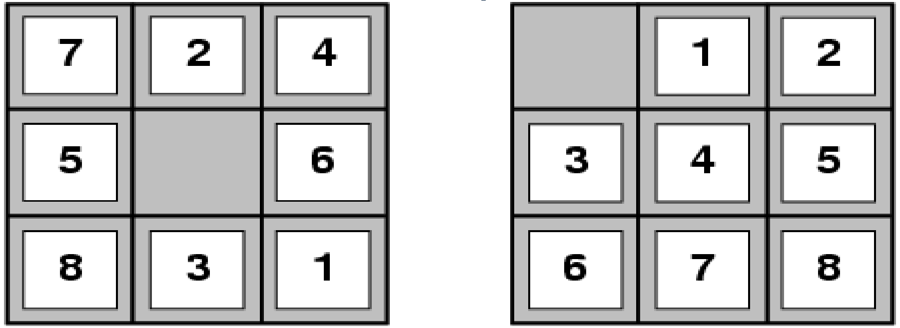


Start State   Goal State

- $h_1(S)$ = ? 8
- $h_2(S)$ = ? 3+1+2+2+2+3+3+2 = 18

Finding a route from the East Coast to LA

https://en.wikipedia.org/wiki/File:A*_Search_Example_on_North_American_Freight_Train_Network.gif

Dominance

- If $h_2(n) \geq h_1(n)$ for all n (both admissible)
- then h_2 dominates h_1
- h_2 is better for search

- Typical search costs (average number of nodes expanded):

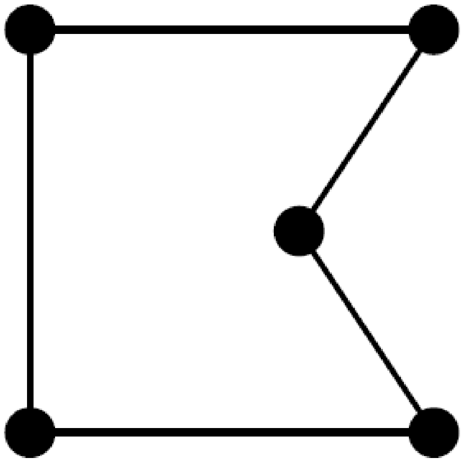
- $d=12$ IDS = 364,404 nodes
  $A^*(h_1)$ = 227 nodes
  $A^*(h_2)$ = 73 nodes
- $d=24$ IDS = too many nodes
  $A^*(h_1)$ = 39,135 nodes
  $A^*(h_2)$ = 1,641 nodes

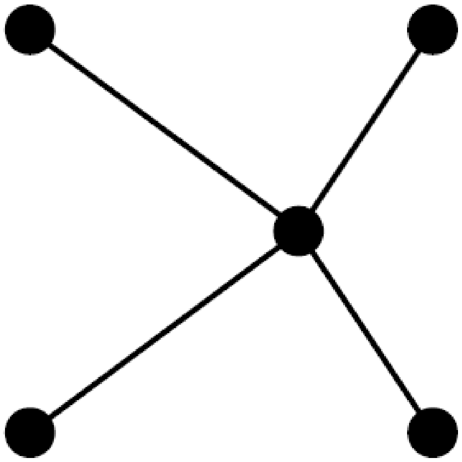
Relaxed problems

- A problem with fewer restrictions on the actions is called a relaxed problem
- The cost of an optimal solution to a relaxed problem is an admissible heuristic for the original problem
- If the rules of the 8-puzzle are relaxed so that a tile can move anywhere, then $h_1(n)$ gives the shortest solution
- If the rules are relaxed so that a tile can move to any adjacent square, then $h_2(n)$ gives the shortest solution

Relaxed problems contd.

Well-known example: travelling salesperson problem (TSP)
Find the shortest tour visiting all cities exactly once





Minimum spanning tree can be computed in $O(n^2)$
and is a lower bound on the shortest (open) tour

Summary

- Heuristic functions estimate costs of shortest paths
- Good heuristics can dramatically reduce search cost
- Greedy best-first search expands lowest h
  - incomplete and not always optimal
- A* search expands lowest $g + h$
  - complete and optimal
  - also optimally efficient (up to tie-breaks, for forward search)
- Admissible heuristics can be derived from exact solution of relaxed problems

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