One-holed Contexts from Derivative of Domain Equation

I. DeFrain

1 λ-calculus

The grammar for the λ-calculus

$$M, N \ ::= \ x \mid \lambda x.M \mid MN$$

has the domain equation for types T parametric in X

$$T[X] = X + (X \times T[X]) + (T[X] \times T[X])$$

Introducing the recursion variable R into the domain equation, we get

$$T[X, R] = X + (X \times R) + (R \times R)$$

Differentiating w.r.t. R, we get

$$\frac{dT[X, R]}{dR} = (X \times 1) + (1 \times R) + (R \times 1)$$

and evaluating at the fixed point $R = T[X, R]$, we get

$$\left.\frac{dT[X, R]}{dR}\right|_{R=T[X,R]} = (X \times 1) + (1 \times T) + (T \times 1)$$

One-holed contexts correspond to the derivative of the domain equation, i.e.

- $X \times 1 \leftrightarrow X \times K$ corresponds to the *abstraction context*
- $1 \times T \leftrightarrow K \times T$ and $T \times 1 \leftrightarrow T \times K$ correspond to the two versions of *application context*

Therefore, the one-holed λ-calculus contexts are of the form

$$K \ ::= \ [\,] \mid \lambda x.K \mid KM \mid MK$$

Note: [ ] is a one-holed context in any grammar

2 ρ-calculus

The grammar for the ρ-calulus

$$P, Q ::= \ 0 \mid \text{for}(y \leftarrow x)P \mid x!(Q) \mid P|Q \mid {}^{*}x$$

$$x, y ::= \ @P$$

has the domain equation for types P parametric in X

$$P[X] = 1 + (X \times X \times P[X]) + (X \times P[X]) + (P[X] \times P[X]) + X$$

and is the least fixed point

$$RP = P[RP]$$

Introducing the recursion variable R into the domain equation, we get

$$P[X, R] = 1 + (X \times X \times R) + (X \times R) + (R \times R) + X$$

Differentiating (naively) w.r.t. R, we get

$$\frac{dP[X, R]}{dR} = (X \times X \times 1) + (X \times 1) + (1 \times R) + (R \times 1)$$

and evaluating at the fixed point $R = P[X, R]$, we get

$$\left.\frac{dP[X, R]}{dR}\right|_{R=P[X,R]} = (X \times X \times 1) + (X \times 1) + (1 \times P) + (P \times 1)$$

One-holed contexts correspond to the derivative of the domain equation, i.e.

- $X \times X \times 1 \leftrightarrow X \times X \times K$ corresponds to the *input-guarded context*
- $X \times 1 \leftrightarrow X \times K$ corresponds to the *output conext*
- Both $1 \times P \leftrightarrow K \times P$ and $P \times 1 \leftrightarrow P \times K$ correspond to the *parallel context* but we only need one instance because $P|Q \equiv Q|P$

Therefore, the one-holed ρ-calculus contexts are of the form

$$K \ ::= \ [\,] \mid \text{for}(y \leftarrow x)K \mid x!(K) \mid P|K$$