

AA: Project-1

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1 Introduction

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2 Usage

The standard script (i.e. everything but the l2 stuff) can be run by the command 'python3 cache-sim.py standard flags' with Python 3.8.10 or Python 3.12.2. For faster runs do 'pip install numba'.

The l2 program is in the two-level-cache directory and can also be run by 'python3 cache-sim.py standard flags' where the flags effects the l1 cache.

3 Correctness

Correctness tests can be ran by adding the -t, -p, and -a algorithm flags to the end of the cache-sim.py script.

$$\mathbf{mxm_blocked} = \begin{bmatrix} 30 & 36 & 42 \\ 84 & 108 & 132 \\ 138 & 180 & 222 \end{bmatrix}$$

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$$\mathbf{daxpy} = [0 \ 5 \ 10 \ 15 \ 20 \ 25 \ 30 \ 35 \ 40]$$

4 Associativity

My results in Table 1 suggest that this is a reasonable result, as long as 8-way associativity does not increase CPI or complexity significantly over 2-way, since both had almost identical results.

| Cache Associativity | Instructions | Read Hits | Read Misses | Read Miss % | Write Hits | Write Misses | Write Miss % |
|--------------------------|--------------|-----------|-------------|-------------|------------|--------------|--------------|
| 1 | 449971200 | 222216486 | 2423514 | 1.08 | 3630720 | 516480 | 12.45 |
| 2 | 449971200 | 223698840 | 941160 | 0.42 | 4060800 | 86400 | 2.08 |
| 4 | 449971200 | 223747200 | 892800 | 0.40 | 4060800 | 86400 | 2.08 |
| 8 | 449971200 | 223747200 | 892800 | 0.40 | 4060800 | 86400 | 2.08 |
| 16 | 449971200 | 223747200 | 892800 | 0.40 | 4060800 | 86400 | 2.08 |
| 1024 (Fully Associative) | 449971200 | 223747200 | 892800 | 0.40 | 4060800 | 86400 | 2.08 |

Table 1: Associativity table

5 Memory Block Size

In table 2 the read and write miss rates drop until 256 and then start increasing again. This is exactly as expected since with a 32x32 tiling scheme of 8-byte doubles, each tile row fits exactly into a 256 byte block.

If you use a smaller block size, you will have multiple misses per row which is what we see here with the read miss rate having every time we double block size up until 256 since we need double the number of reads from memory per row. If you use a larger block size, you will gain no benefits in terms of pre-loading elements, and you will loose out on the number of sets which will cause increased conflict and capacity misses.

| Block Size (bytes) | Instructions | Read Hits | Read Misses | Read Miss % | Write Hits | Write Misses | Write Miss % |
|--------------------|--------------|-----------|-------------|-------------|------------|--------------|--------------|
| 8 | 449971200 | 217110720 | 7529280 | 3.35 | 3456000 | 691200 | 16.67 |
| 16 | 449971200 | 220875360 | 3764640 | 1.68 | 3801600 | 345600 | 8.33 |
| 32 | 449971200 | 222757680 | 1882320 | 0.84 | 3974400 | 172800 | 4.17 |
| 64 | 449971200 | 223698840 | 941160 | 0.42 | 4060800 | 86400 | 2.08 |
| 128 | 449971200 | 224169420 | 470580 | 0.21 | 4104000 | 43200 | 1.04 |
| 256 | 449971200 | 224404710 | 235290 | 0.10 | 4125600 | 21600 | 0.52 |
| 512 | 449971200 | 222728738 | 1911262 | 0.85 | 3734992 | 412208 | 9.94 |
| 1024 | 449971200 | 219352185 | 5287815 | 2.35 | 2932456 | 1214744 | 29.29 |

Table 2: Memory Block size table

6 Total Cache Size

I found that 32KB was just barely above a read miss rate of 0.5%, while 64 KB was below it. To achieve a 0.5% miss rate with 32KB, I simply doubled the block size (as per my last section) and achieved a read miss rate of 0.32

7 Problem Size and Cache Thrashing

1. Cache performance does vary a lot throughout Table 4. This is due to conflict misses in B, where either the fact that $n*n$ in the case of naive mxm or n in the case of tiled mxm is a multiple of the number of sets and therefore causes each element of B to be loaded n times.

| Cache Size (bytes) | Instructions | Read Hits | Read Misses | Read Miss % | Write Hits | Write Misses | Write Miss % |
|-------------------------|--------------|--------------|----------------|----------------|---------------|-----------------|-----------------|
| 4096 | 449971200 | 109688400 | 114951600 | 51.17 | 0 | 4147200 | 100.00 |
| 8192 | 449971200 | 206585790 | 18054210 | 8.04 | 0 | 4147200 | 100.00 |
| 16384 | 449971200 | 222907245 | 1732755 | 0.77 | 3225600 | 921600 | 22.22 |
| 32768 | 449971200 | 223321204 | 1318796 | 0.59 | 3946112 | 201088 | 4.85 |
| 32768 (128 byte blocks) | 449971200 | 223913696 | 726304 | 0.32 | 3989312 | 157888 | 3.81 |
| 65536 | 449971200 | 223698840 | 941160 | 0.42 | 4060800 | 86400 | 2.08 |
| 131072 | 449971200 | 223736248 | 903752 | 0.40 | 4060800 | 86400 | 2.08 |
| 262144 | 449971200 | 224019377 | 620623 | 0.28 | 4060800 | 86400 | 2.08 |
| 524288 | 449971200 | 224133224 | 506776 | 0.23 | 4060800 | 86400 | 2.08 |

Table 3: Total Cache size table

- 512 destroys the performance of tiled matrix multiplication due to conflict misses. Specifically, since it is the only one the number of sets (1024) is a multiple of the dimension, every other element in a column maps to the same set which causes conflict misses to erase all of the gains we got from using tiled matrix multiplication, and causes every element of B to be loaded n times (same as naive).
- I noticed only small improvements in miss rates for the 8-way associative version in Table 5 which makes sense since mxm and mxm tiled cause O(-d) and O(-f) conflicts in B, respectively. These are both way bigger than 8 conflicts and therefore we see basically no gain. Fully associative in Table 6 on the other hand fixes the problem entirely since it increases the associativity past the number of elements trying to map to each set in each calculation of a C[i,j]. However, in real life fully-associative data caches are often not popular due to the complexity of implementation and the penalty paid in CPI.
- Option 1: Pad the matrices with an extra row and column of 0s, and then either not calculate or remove this row and column from C (the result) after the calculation. This slight alteration makes it so that n/nxn is not a divider/multiple of the number of sets, and therefore you will not run into significant conflict misses. Option 2 is to only store the rows in continuous memory, but have the matrix be an array of n pointers to those rows (which are scattered in memory). Both of these should fix the conflict issue.

| Matrix Dimension | MxM Method | Blocking Factor | Instructions | Read Hits | Read Misses | Read Miss % | Write Hits | Write Misses | Write Miss % |
|---------------------|---------------|--------------------|--------------|--------------|----------------|----------------|---------------|-----------------|-----------------|
| 480 | Regular | - | 443520000 | 107236881 | 114177519 | 51.57 | 604800 | 316800 | 34.38 |
| 480 | Blocked | 32 | 449971200 | 223698840 | 941160 | 0.42 | 4060800 | 86400 | 2.08 |
| 488 | Regular | - | 466047808 | 217673225 | 14993463 | 6.44 | 863272 | 89304 | 9.38 |
| 488 | Blocked | 8 | 494625088 | 244625824 | 2329504 | 0.94 | 15151912 | 89304 | 0.59 |
| 512 | Regular | - | 538181632 | 134120448 | 134577152 | 50.08 | 0 | 1048576 | 100.00 |
| 512 | Blocked | 32 | 546045952 | 137084928 | 135544832 | 49.72 | 0 | 4980736 | 100.00 |

Table 4: Matrix Multiply Problem size table – Associativity = 2

| Matrix Dimension | MxM Method | Blocking Factor | Instructions | Read Hits | Read Misses | Read Miss % | Write Hits | Write Misses | Write Miss % |
|------------------|------------|-----------------|--------------|-----------|-------------|-------------|------------|--------------|--------------|
| 480 | Regular | - | 443520000 | 107317995 | 114096405 | 51.53 | 604800 | 316800 | 34.38 |
| 480 | Blocked | 32 | 449971200 | 223747200 | 892800 | 0.40 | 4060800 | 86400 | 2.08 |
| 488 | Regular | - | 466047808 | 218080372 | 14586316 | 6.27 | 863272 | 89304 | 9.38 |
| 488 | Blocked | 8 | 494625088 | 244751209 | 2204119 | 0.89 | 15151912 | 89304 | 0.59 |
| 512 | Regular | - | 538181632 | 134121472 | 134576128 | 50.08 | 688128 | 360448 | 34.38 |
| 512 | Blocked | 32 | 546045952 | 137101312 | 135528448 | 49.71 | 688128 | 4292608 | 86.18 |

Table 5: Matrix Multiply Problem size table – Associativity = 8

| Matrix Dimension | MxM Method | Blocking Factor | Instructions | Read Hits | Read Misses | Read Miss % | Write Hits | Write Misses | Write Miss % |
|------------------|------------|-----------------|--------------|-----------|-------------|-------------|------------|--------------|--------------|
| 480 | Regular | - | 443520000 | 207532802 | 13881598 | 6.27 | 835200 | 86400 | 9.38 |
| 480 | Blocked | 32 | 449971200 | 223747200 | 892800 | 0.40 | 4060800 | 86400 | 2.08 |
| 488 | Regular | - | 466047808 | 218080370 | 14586318 | 6.27 | 863272 | 89304 | 9.38 |
| 488 | Blocked | 8 | 494625088 | 245079944 | 1875384 | 0.76 | 15151912 | 89304 | 0.59 |
| 512 | Regular | - | 538181632 | 251854850 | 16842750 | 6.27 | 950272 | 98304 | 9.38 |
| 512 | Blocked | 32 | 546045952 | 271548416 | 1081344 | 0.40 | 4882432 | 98304 | 1.97 |

Table 6: Matrix Multiply Problem size table – Fully Associative

8 Replacement Policy

In Table 7 we can see that LRU has the best performance in terms of both read and write hit rates. FIFO is behind it, and Random is in last place.

| Replacement Policy | Instructions | Read Hits | Read Misses | Read Miss % | Write Hits | Write Misses | Write Miss % |
|--------------------|--------------|-----------|-------------|-------------|------------|--------------|--------------|
| Random | 449971200 | 223494091 | 1145909 | 0.51 | 4037137 | 110063 | 2.65 |
| FIFO | 449971200 | 223629200 | 1010800 | 0.45 | 4059340 | 87860 | 2.12 |
| LRU | 449971200 | 223698840 | 941160 | 0.42 | 4060800 | 86400 | 2.08 |

Table 7: Replacement policy table

9 Second-Level Cache Extension

| Config | L1 Read Miss % | L1 Write Miss % | L2 Read Miss % | L2 Write Miss % |
|---------|----------------|-----------------|----------------|-----------------|
| L1 Only | 0.20 | 0.04 | N/A | N/A |
| L1 + L2 | 0.20 | 0.04 | 0.05 | 0.00 |

Table 8: Miss rates at each cache level

| Config | RAM Accesses | Read Hits (L1+L2) | Write Hits (L1+L2) |
|---------------|---------------------|--------------------------|---------------------------|
| L1 Only | 1002300 | 223724100 | 4060800 |
| L1 + L2 | 122400 | 224604000 | 4579200 |

Table 9: Memory access totals and hits with and without L2

10 Limitations

The only limitation that I am aware of is that my L2 data cache's statistics are somewhat confusing. Specifically since during l1 write misses, l1 first loads the data before writing it to cache, l2 will always load the data before l1 writes to l2 so it is impossible for l2 to have a write miss. This also means that the loads done by l2 aren't just during l1 load misses, but also l1 write misses. Because of this it is somewhat hard to interpret the statistics from l2.