# Systematic Unvertainty Note

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중력가속도 실험에 대한 Systematic Uncertainty Estimation
Based on Monte-Carlo Simulation

## 1 Equation of Datapoints

**Basic Equation**

$$h = \frac{1}{2}gt^2 \tag{1}$$

$$t = \sqrt{2\frac{g}{h}} \tag{2}$$

**Basic Equation with uncertainty**

$$h + \delta h = \frac{1}{2}g\left(t + \delta t\right)^2 \tag{3}$$

$$t = \sqrt{2\frac{g}{h + \delta h}} + \delta t \tag{4}$$

with following random distribution

| Random Variable | Distribution | Reason |
|---|---|---|
| $\delta h$ | Gaussian Disribution | Measured by human-eye |
| $\delta t$ | Uniform Disribution | Measured by mechanical clock |

With overlapped uncertainty (depending on measuring methods), the $\delta h$ has different value for each datapoints.

## 2 Case Overlapped Length Dependencies

If measuring length for each data-points has dependencies with each previous data-point. Uncertainty for each data-points is needed to declared especially.

## 2.1 Error Propagation

**Basic Equation of uncertainty**

$$\delta f = \sqrt{\sum_i \left(\frac{\partial f}{\partial x_i}\delta x_i\right)^2} \tag{5}$$

**Overlapped Uncertainty, Propagated for Summation**

$$\delta(f+g) = \sqrt{(\delta g)^2 + (\delta f)^2} \tag{6}$$

For independent 2 formula $f$ and $g$.

Overlapping uncertainties. With measuring length period $l$, number $i$, single measuring uncertainty $\delta l_i$, Initial offset $b$ and its uncertainty $\delta b$

First term

$$h_0 = b \tag{7}$$
$$h_1 = b + l + \delta l_1 \tag{8}$$
$$h_2 = b + 2l + \delta l_1 + \delta l_2 \tag{9}$$

with regularity, following correlation can be assumed.

$$h_n = b + n \cdot l + \sum_i \delta l_i \tag{10}$$

Uncertainty is

$$\delta h_i = \sqrt{\sum_i (\delta l)^2} \tag{11}$$

$\delta l$ is constant for each $n$,

$$\delta h_n = \sqrt{n}\delta l \tag{12}$$