$u \perp v \iff u \circ v = 0$

$(\theta = 90^\circ$

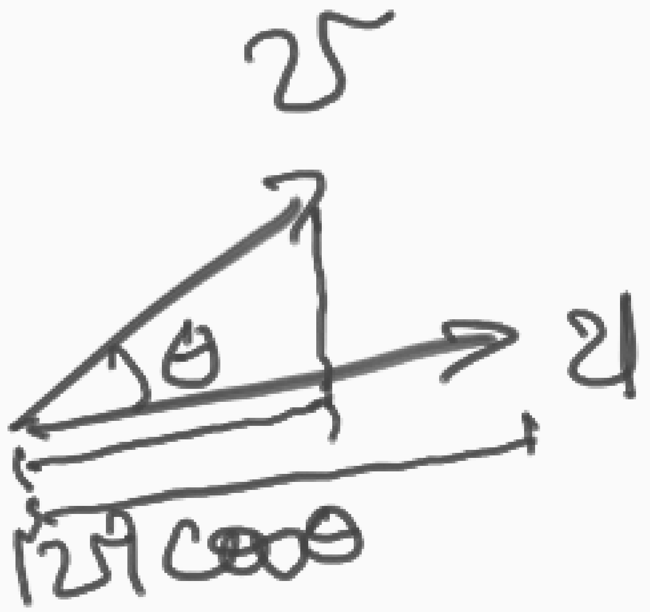
$u, v \in R^n$

$\underline{R^n}$ R^2

$u, v \in R^2$

$u = (u_x, u_y)$

$v = (v_x, v_y)$



$\underline{u \circ v} = u_x \circ v_x + u_y \circ v_y$

inner product

$u \circ v = \underline{|u|} \, |v| \, \underline{\cos\theta}$.

$u = (u_0, u_1, \cdots u_{n-1})$

$v = (v_0, v_1, \cdots, v_{n-1})$

$u \circ v = u_0 v_0 + \cdots + u_{n-1} v_{n-1}$

$= \sum_{i=0}^{n-1} u_i v_i$

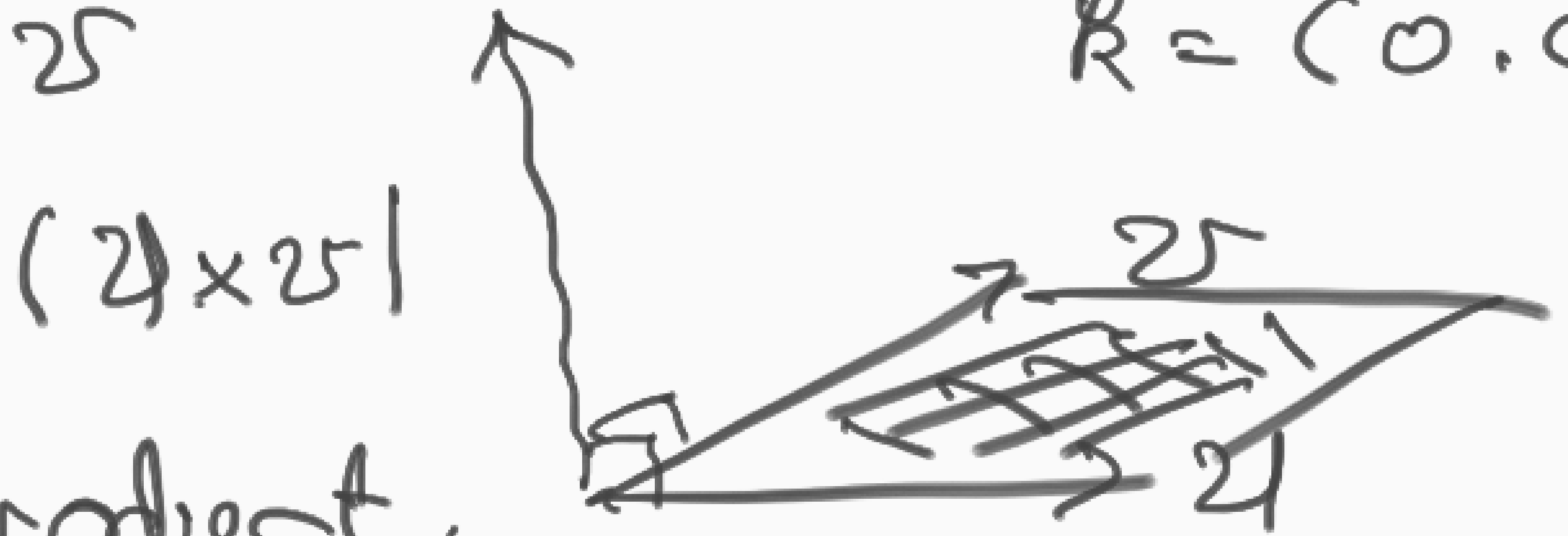
$u, v \in R^3$

$u = (u_x, u_y, u_z)$

$v = (v_x, u_y, v_z)$

$|u \times v| = $ area of u, v

$u \times v \perp u, v$



cross product.

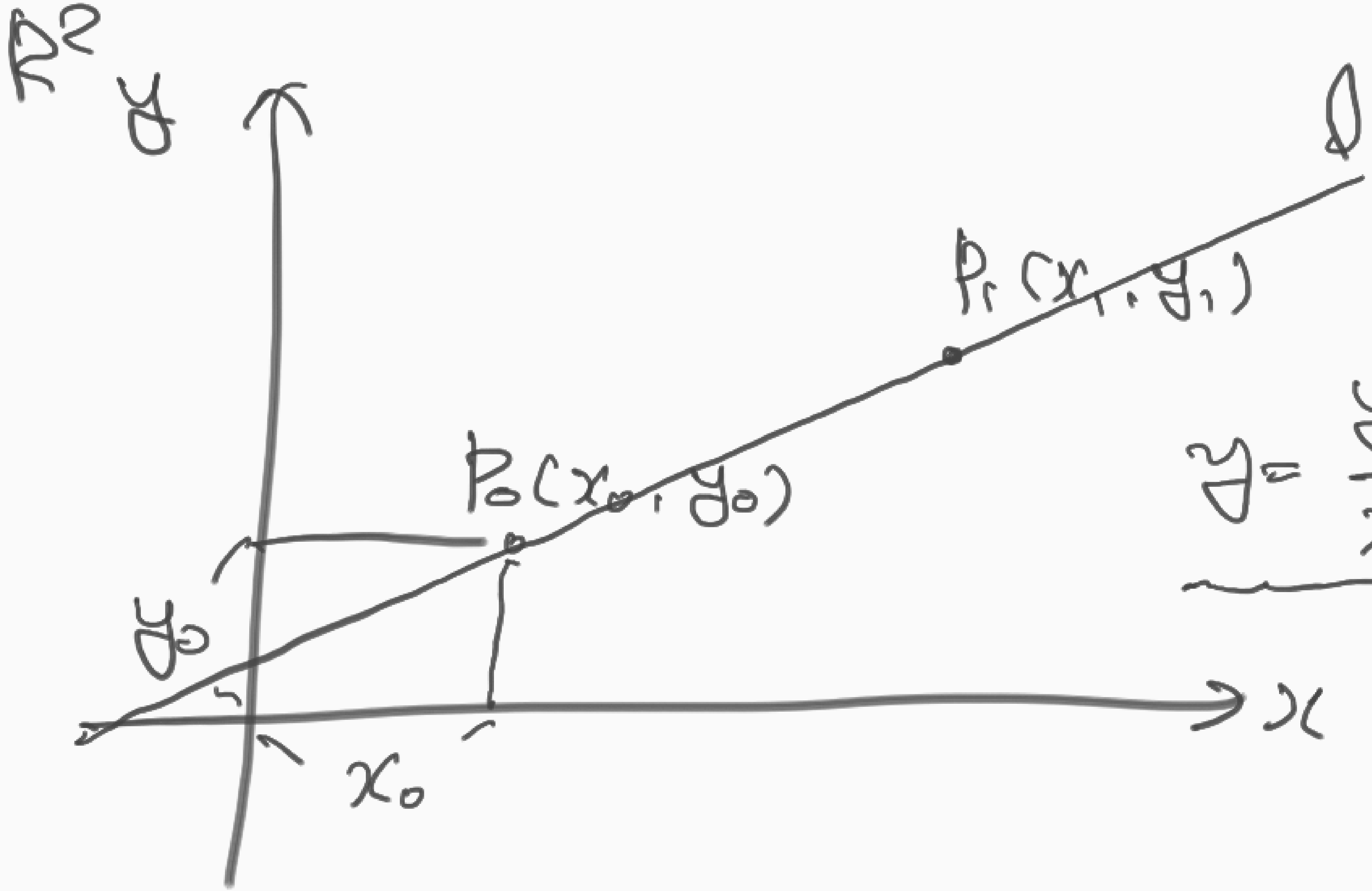
$$u \times v = \begin{vmatrix} i & j & k \\ u_x & u_y & u_z \\ v_x & v_y & v_z \end{vmatrix} \in R^3$$

$i = (1, 0, 0)$

$j = (0, 1, 0)$

$k = (0, 0, 1)$

Euclidean coordinate system



in computer programming

$$y = \frac{y_1 - y_0}{x_1 - x_0}(x - x_0) + y_0$$

?

$\mathbb{R}^3$

P

$u(u_x, u_y, u_z)$

$P_0(x_0, y_0, z_0)$

$\mathbb{R}^2$

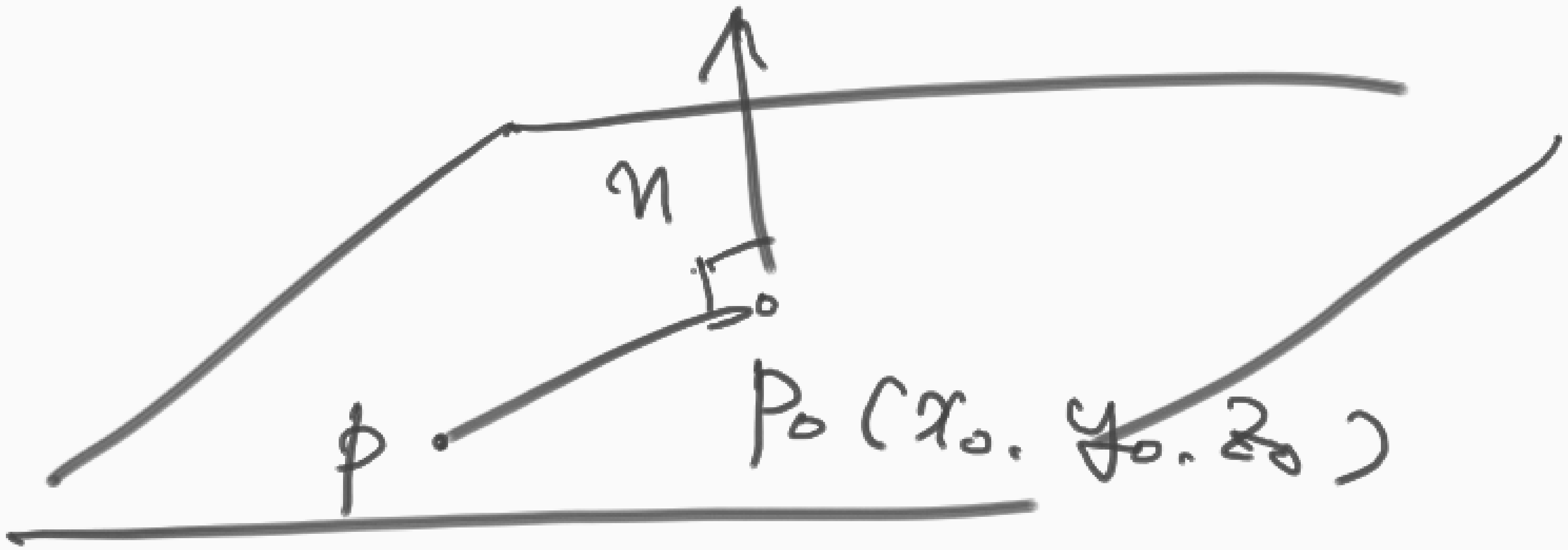
$$y = \underbrace{\frac{y_1 - y_0}{x_1 - x_0}}(x - x_0) + y_0$$

$$\underbrace{P = P_0 + u \cdot t} \qquad t \in \mathbb{R},\ P_0, u \in \mathbb{R}^3$$

$$\begin{cases} x = x_0 + u_x \cdot t \\ y = y_0 + u_y \cdot t \\ z = z_0 + u_z \cdot t \end{cases}$$

R^3

plane equation. $\begin{cases} \underline{(p-p_0)\cdot n = 0} \\ p-p_0 \perp n \end{cases}$



line eq

$$\begin{cases} (p-p_0)\cdot n_0 = 0 \\ (p-p_1)\cdot n_1 = 0 \end{cases}$$

4

Coordinate Free System

Barycentric Coordinate System

3

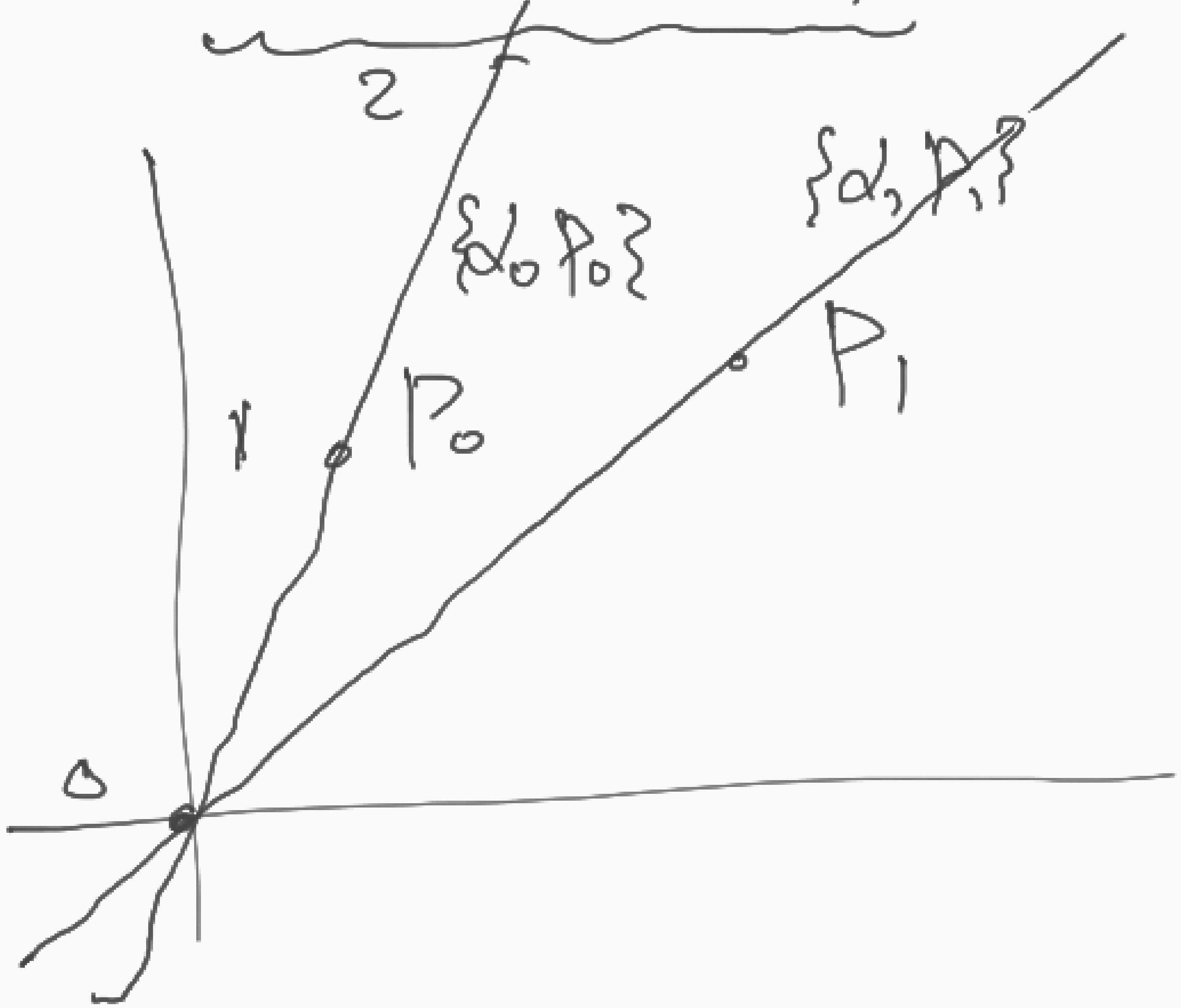
$\{\alpha_1 P_1\}$

$\{\alpha_0 P_0 + \alpha_1 P_1\}$?

R^2

$\underbrace{\alpha_0 P_0 + \alpha_1 P_1}_{2}$

$\alpha_0, \alpha_1 \in R$, $P_0, P_1 \in R^2$



$\{\alpha_0 P_0\}$ $\alpha_0 \in R^2$

if $\alpha_0 = 0$ $\quad 0 \cdot P_0 = 0$

$\alpha_0 = 1$ $\quad 1 \cdot P_0 = P_0$

$\alpha_0 = 2$

$$P = \alpha_0 P_0 + \alpha_1 P_1$$

$$\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} x_0 & y_0 \\ x_1 & y_1 \end{pmatrix} \begin{pmatrix} \alpha_0 \\ \alpha_1 \end{pmatrix}$$

$$P = (x, y)$$

$$P_0 = (x_0, y_0)$$

$$P_1 = (x_1, y_1)$$

$$\Rightarrow \begin{pmatrix} x_0 & y_0 \\ x_1 & y_1 \end{pmatrix}^{-1} \begin{pmatrix} \underline{\underline{x}} \\ \underline{\underline{y}} \end{pmatrix} = \begin{pmatrix} \underline{\underline{\alpha_0}} \\ \underline{\underline{\alpha_1}} \end{pmatrix}$$

$$\{ P = \underline{\underline{\alpha_0}} P_0 + \underline{\underline{\alpha_1}} P_1 \} = R^2$$

$$\underline{\underline{\{ \alpha_0 P_0 \}}} \quad \underline{\underline{\{ \alpha P_1 \}}} : \text{line}$$

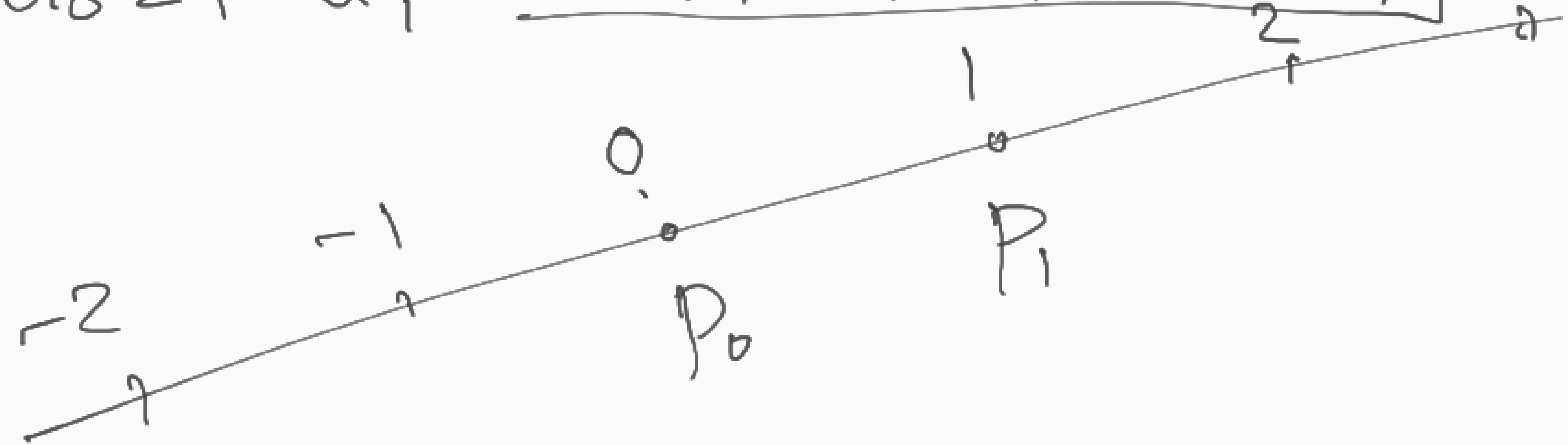
$$\{\alpha_0 P_0 + \alpha_1 P_1\} = R^2$$

$$\alpha_0 + \alpha_1 = 1$$

$$\alpha_0 = 1 - \alpha_1$$

$$(1-\alpha_1) P_0 + \alpha_1 P_1$$

$$\boxed{(1-t) P_0 + t P_1}$$



t	$(1-t)P_0 + tP_1$
-2	$3P_0 - 2P_1$
-1	$2P_0 - 1\cdot P_1$
0	$1\cdot P_0 + 0\cdot P_1$
1	$0\cdot P_0 + 1\cdot P_1$
2	$-1\, P_0 + 2P_1$
3	$-2\, P_0 + 3P_1$

$$\{(1-t) P_0 + t P_1\} \quad t \in R$$

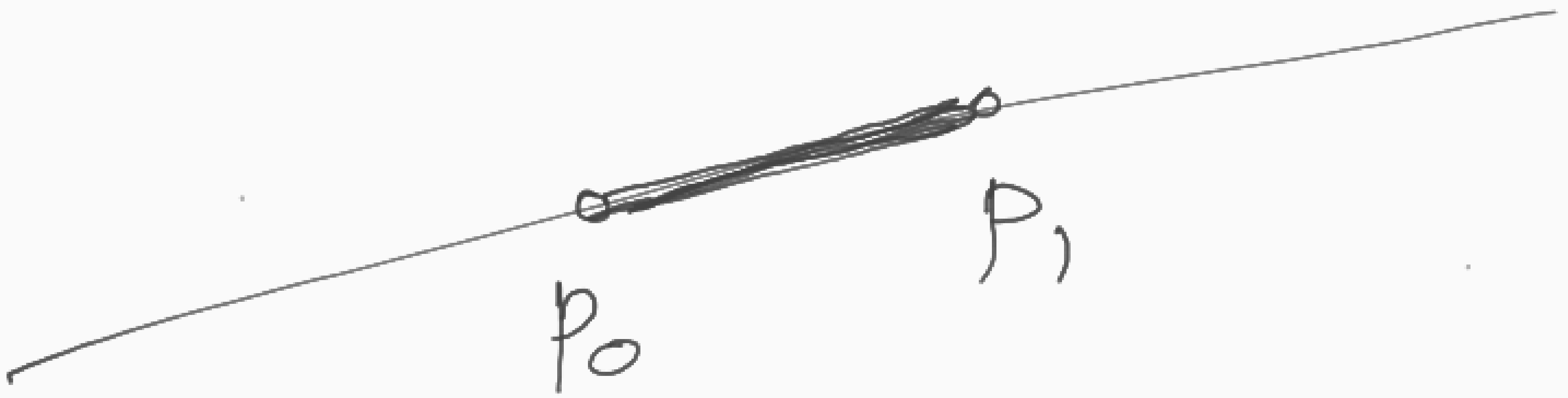
$\{(1-t)P_0 + tP_1 : t \in R\}$: line eq p- t P_0 and P_1

R^2 $\quad \alpha_0 P_0 + \alpha_1 P_1$: R^2 space.

with $\underline{\alpha_0 + \alpha_1 = 1}$: line eg p.t $\underline{P_0}$ and $\underline{P_1}$

with $\boxed{\alpha_0 , \alpha_1 \geq 0}$ line segment

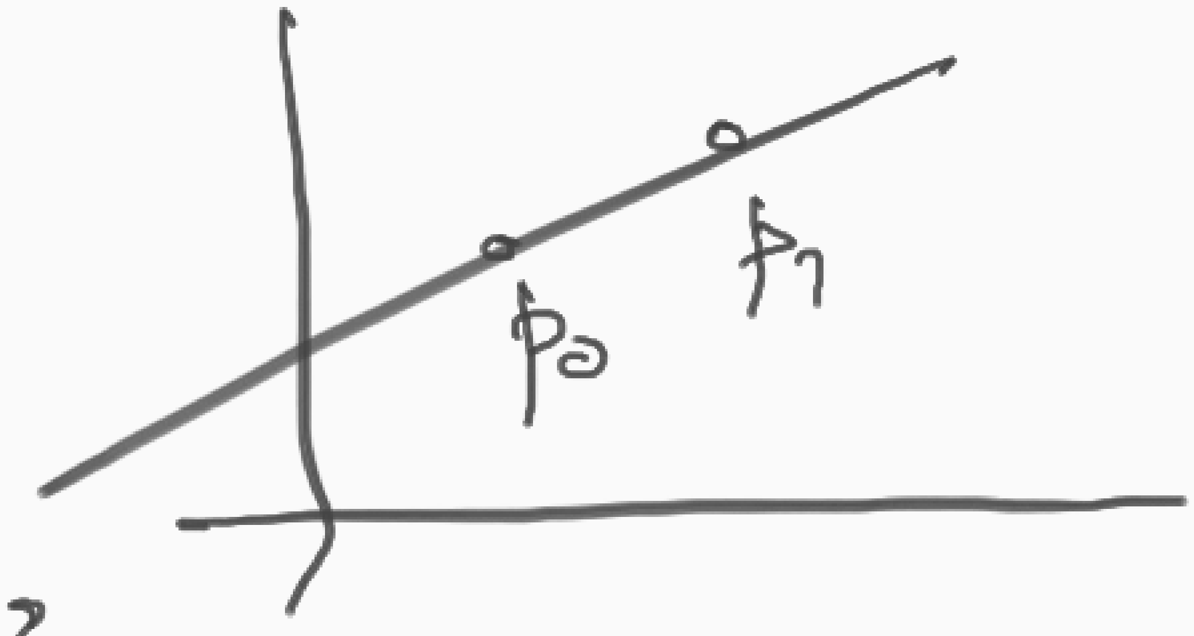
$(\underline{1-t}) P_0 + t P_1$



$\underline{1-t > 0}$, $t > 0$

$t < 1$ $\quad 0 < t < 1$

$\mathbb{R}^2$

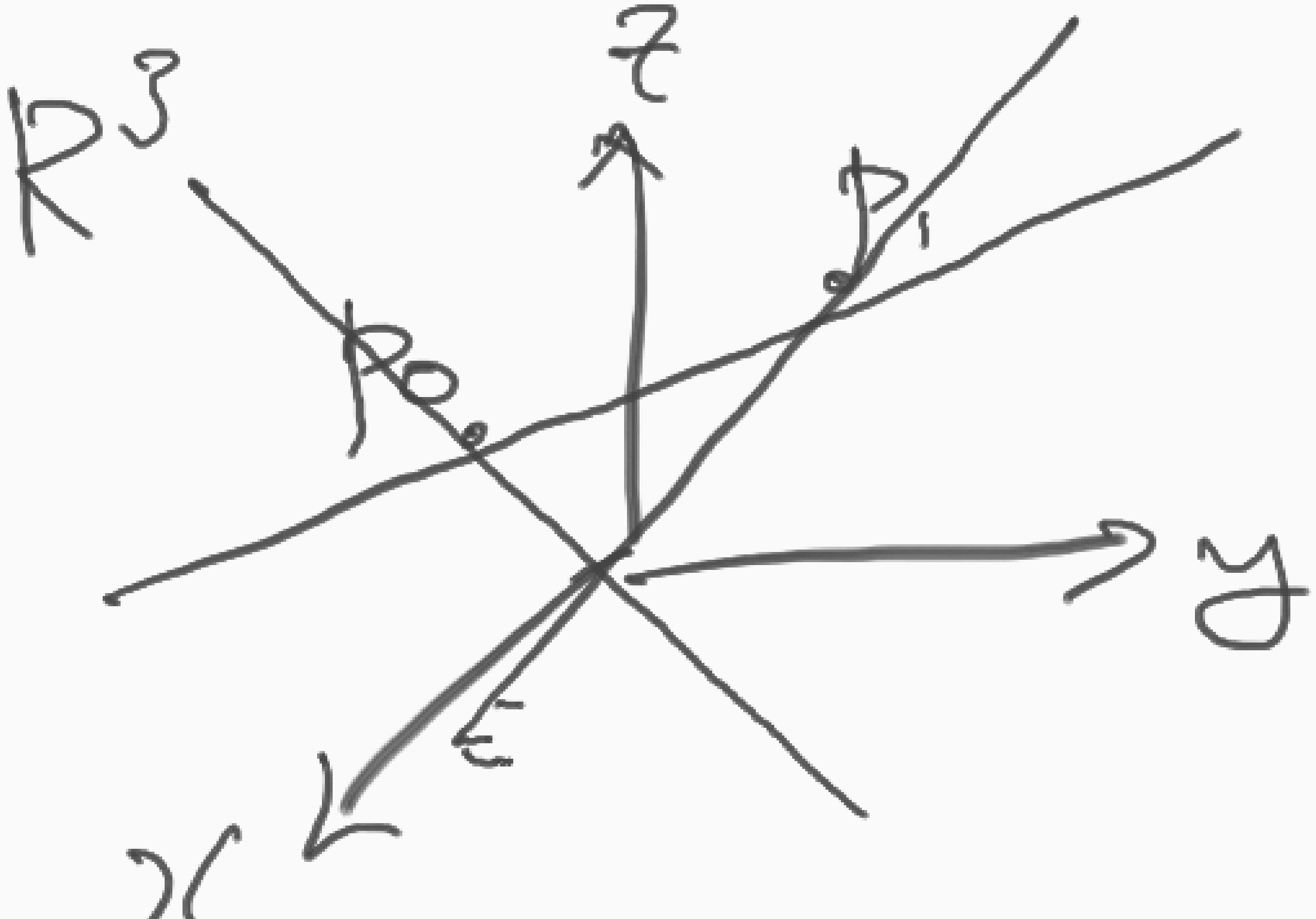


$\alpha_0 p_0 + \alpha_1 p_1$: $\mathbb{R}^2$

$\alpha_0 + \alpha_1 = 1$: line eq

$\alpha_0, \alpha_1 \geq 0$: line segment

$\mathbb{R}^3$



p_0, p_1, O

$\{\alpha_0 p_0 + \alpha_1 p_1\}$: plane eq

$\{\alpha_0 p_0\}$: line eq p.t. p_0 and Origin.

$\{\alpha_1 p_1\}$: $\alpha_0 + \alpha_1 = 1$

$\mathbb{R}^3$, $\mathbb{R}^2$

$\{\alpha_0 P_0 + \alpha_1 P_1\}$ plane eg p.t. Origin, P_0, P_1

with $\alpha_0 + \alpha_1 = 1$ line eg p.t P_0, P_1

with $\alpha_0, \alpha_1 \geq 0$ line segment P_0, P_1

R^3

$P_0, P_1, P_2 \in R^3, \alpha_0, \alpha_1, \alpha_2 \in R$

$\{\underline{\underline{\alpha_0}} P_0 + \underline{\underline{\alpha_1}} P_1 + \underline{\underline{\alpha_2}} P_2\} : R^3$

$P = \alpha_0 P_0 + \alpha_1 P_1 + \alpha_2 P_2$

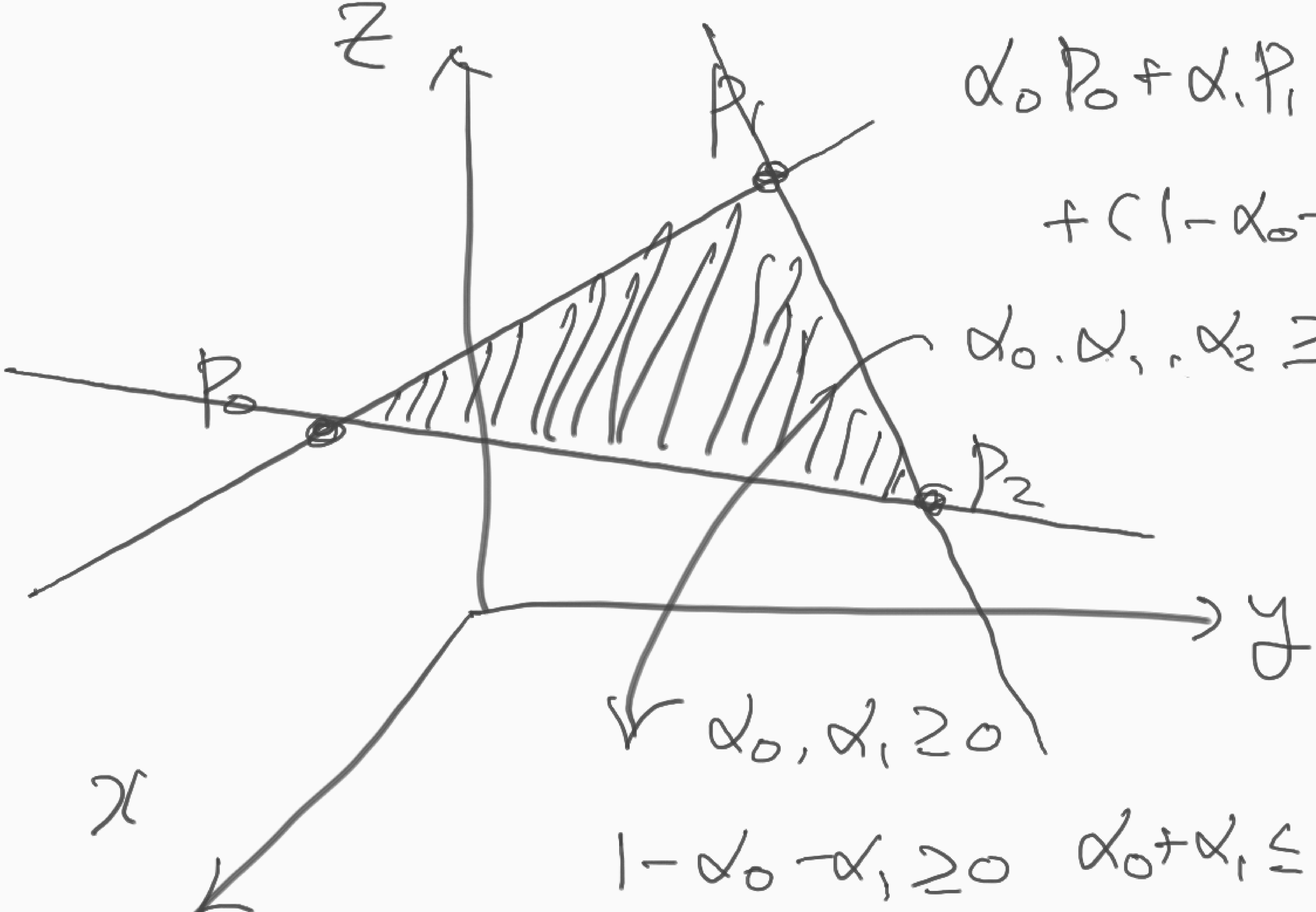
$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} x_0 & x_1 & x_2 \\ y_0 & y_1 & y_2 \\ z_0 & z_1 & z_2 \end{pmatrix} \begin{pmatrix} \alpha_0 \\ \alpha_1 \\ \alpha_2 \end{pmatrix}$$

$$\begin{cases} x = \alpha_0 x_0 + \alpha_1 x_1 + \alpha_2 x_2 \\ y = \alpha_0 y_0 + \alpha_1 y_1 + \alpha_2 y_2 \\ z = \alpha_0 z_0 + \alpha_1 z_1 + \alpha_2 z_2 \end{cases}$$

$$\{\alpha_0 P_0 + \alpha_1 P_1 + \alpha_2 P_2\} = R^3$$

with $\alpha_0 + \alpha_1 + \alpha_2 = 1$

$$\{\alpha_0 P_0 + \alpha P_1 + (1 - \alpha_0 - \alpha_1) P_2\}$$



$\alpha_0 P_0 + \alpha_1 P_1$

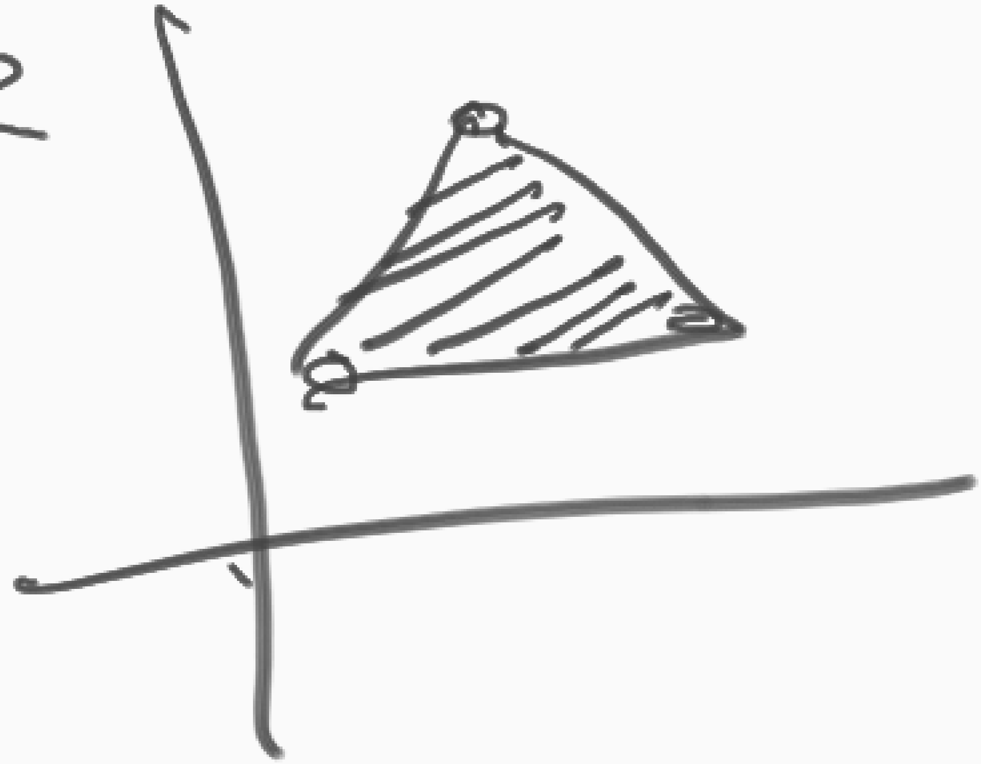
$+ (1 - \alpha_0 - \alpha_1) P_2$

$\alpha_0, \alpha_1, \alpha_2 \geq 0$

$\alpha_0, \alpha_1 \geq 0$

$1 - \alpha_0 - \alpha_1 \geq 0 \quad \alpha_0 + \alpha_1 \leq 1$

R^2 $\qquad \underline{\alpha_0 P_0 + \alpha_1 P_1 + \alpha_2 P_2}$ $\qquad R^2$



with $\alpha_0 + \alpha_1 + \alpha_2 = 1$

$\underline{\alpha_0} P_0 + \underline{\alpha_1} P_1 + (1 - \alpha_0 - \alpha_1) P_2 \qquad R^2$

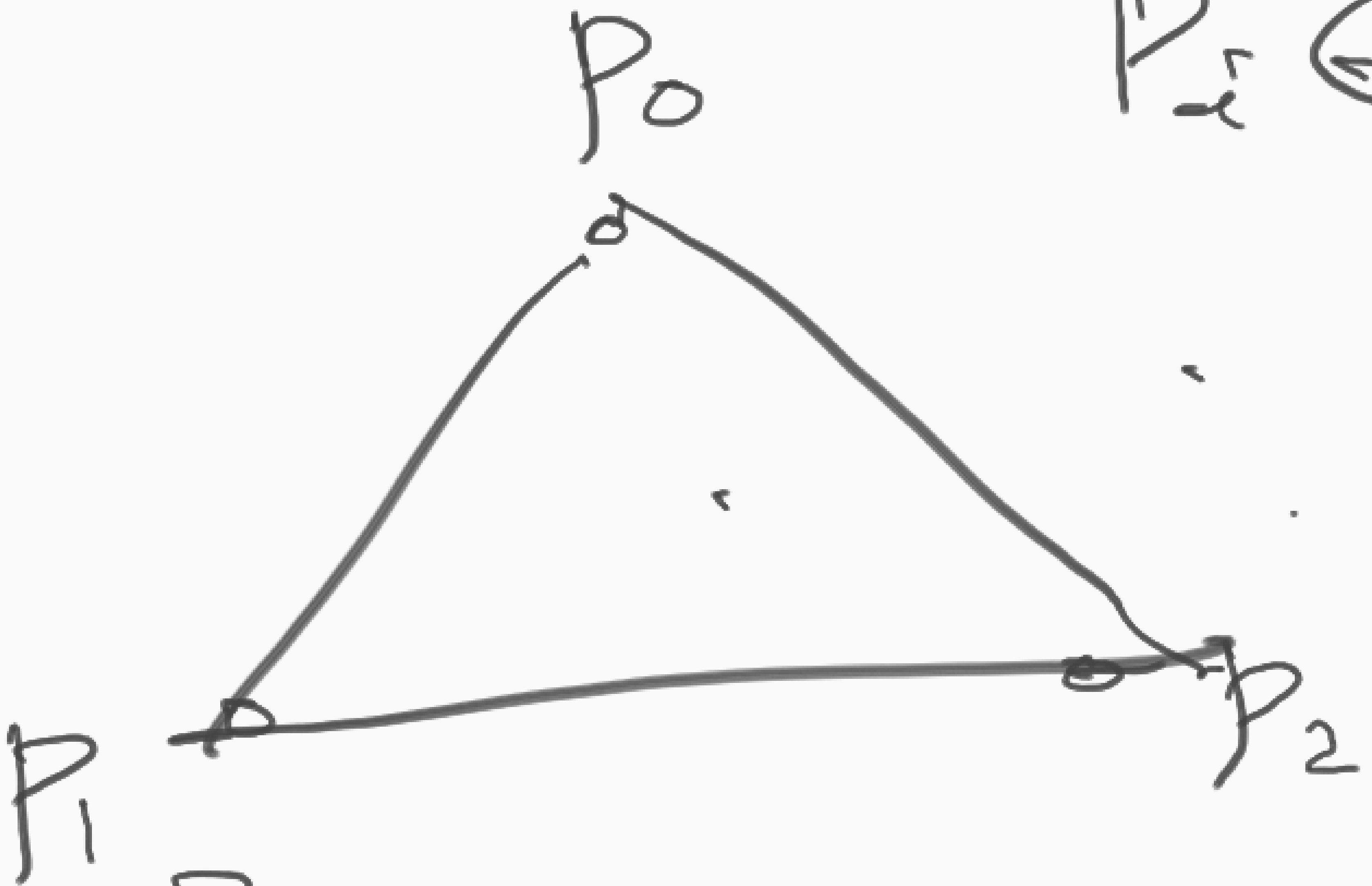
with $\alpha_0, \alpha_1, \alpha_2 \geq 0$

$\alpha_0, \alpha_1 \geq 0, \; \alpha_0 + \alpha_1 \leq 1$

Computational Geometry

Problem

$P_i \in R^2$



$P_{(x, y)}$

$P \in \Delta P_0 P_1 P_2$?

linear combination

P_0 : point

p, q : point

α, β, λ : scalar $\in \mathbb{R}$

control point

$c_0 \quad c_1 \quad \dots$

$$c = \sum_{j=0}^{n} \alpha_j c_j$$

$$\boxed{\alpha_0 c_0 + \alpha_1 c_1 + \alpha_2 c_2}$$

$\mathbb{R}^2$

$\alpha_j \geq 0$

with $\alpha_0 + \alpha_1 + \alpha_2 = 1$

convex $\longrightarrow$ Barycentric combin.

$\mathbb{R}^2$

Barycentric Coordinate System