Lec 3:

$PV_{asset} = PV_{payment\ Receive}$

$$P_{bond} = \frac{C}{1+i} + \frac{C}{(1+i)^2} + \cdots + \frac{C}{(1+i)^n} + \frac{F}{(1+i)^n}$$

i: yield to maturity

⇒ Relationship between ① yield and ② price of bond

How i is determined?

Two Model:

① Bond Market Model [classical]

Results:

① interest rate ↓ during recession

② Fisher Effect: $\pi_e \uparrow \rightarrow i \uparrow$

② liquidity preference Framework [Keynesian]

Money supply and interest rate

↓

liquidity Effect

Question: Same maturity, different interest rate?

Reason:
① risk
② Liquidity
③ info cost
④ Taxation

Question: Different Maturity → Different interest rate?

Explanation: Theory
1> Expectation Theory
2> Segmented Market Theory
3> Liquidity Premium Theory

1. Risk Structure of Interest Rate

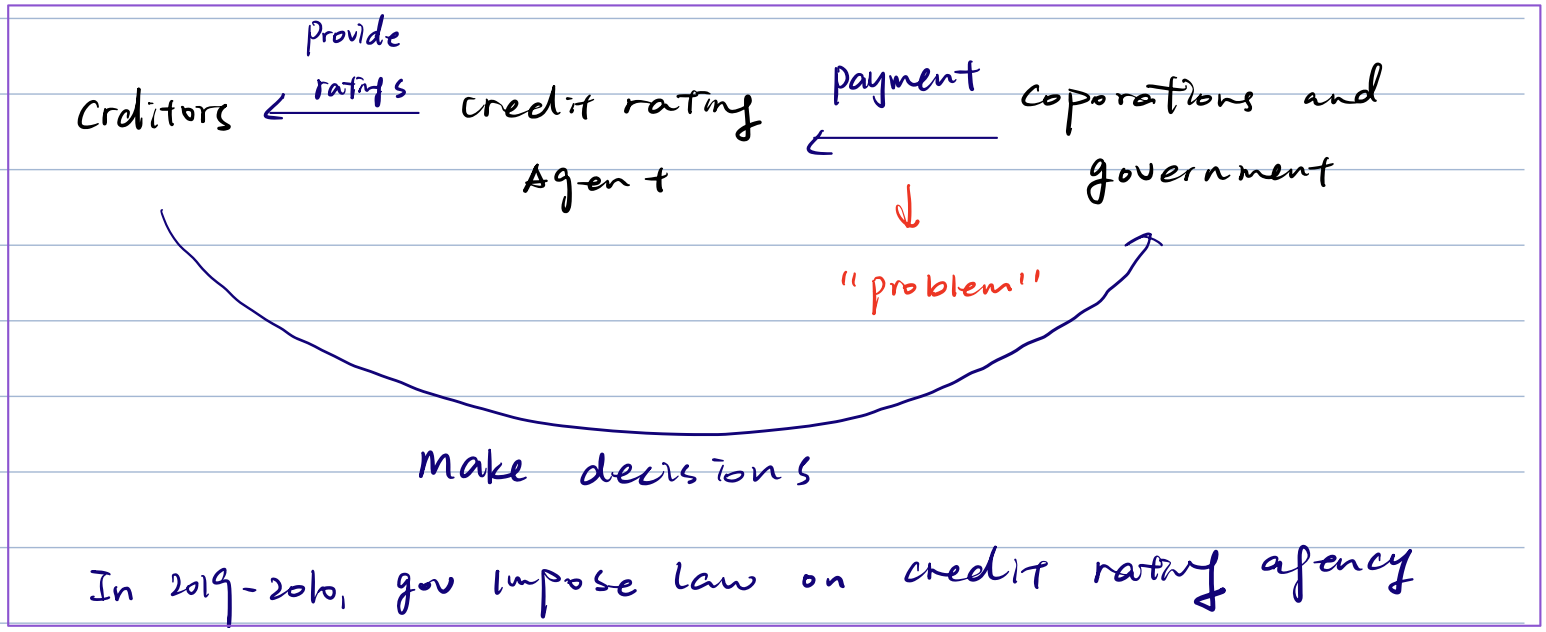
Questions: Why Bond has different maturity have different interest Rate ?

① Risk
② Liquidity
③ Information cost
④ Taxation

Reason 1 = Risk

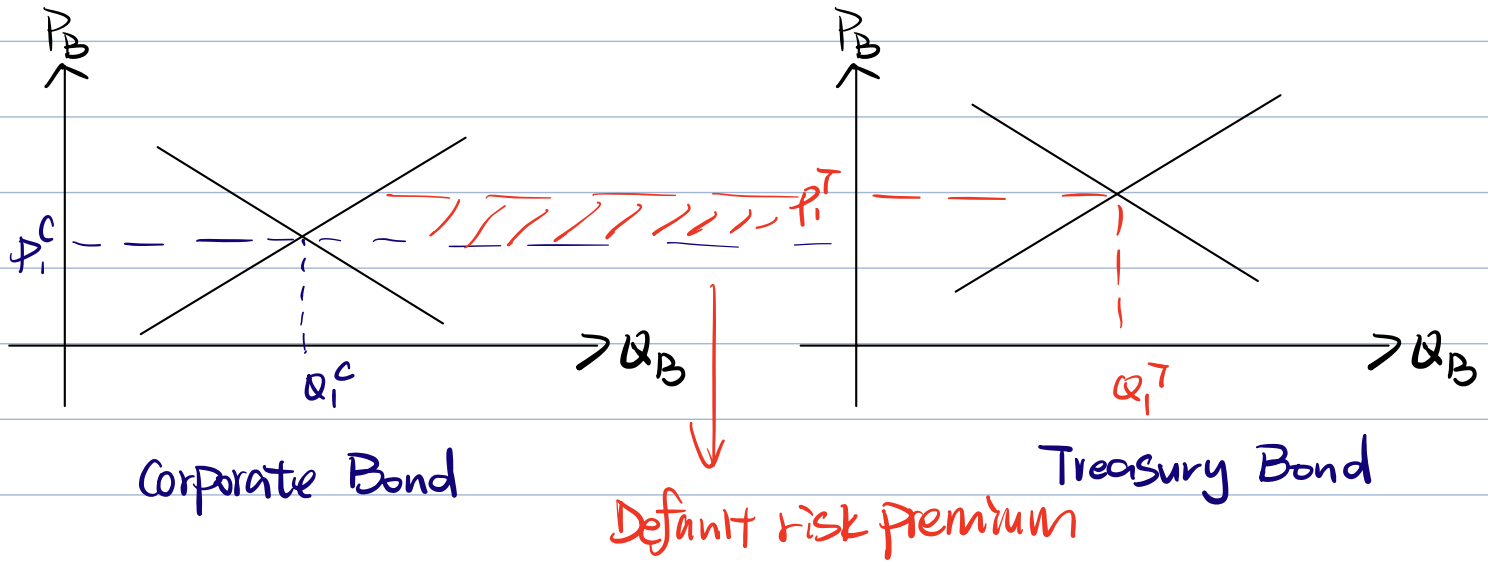
The Risk structure of interest Rate
→ Explains bonds have different characteristics with the same maturity

How is risk Measured? By credit rating



In 2019-2010, gov impose law on credit rating afency

Default Risk: Interest rate − Interest Rate
Premium on a Bond on Treasury Notes

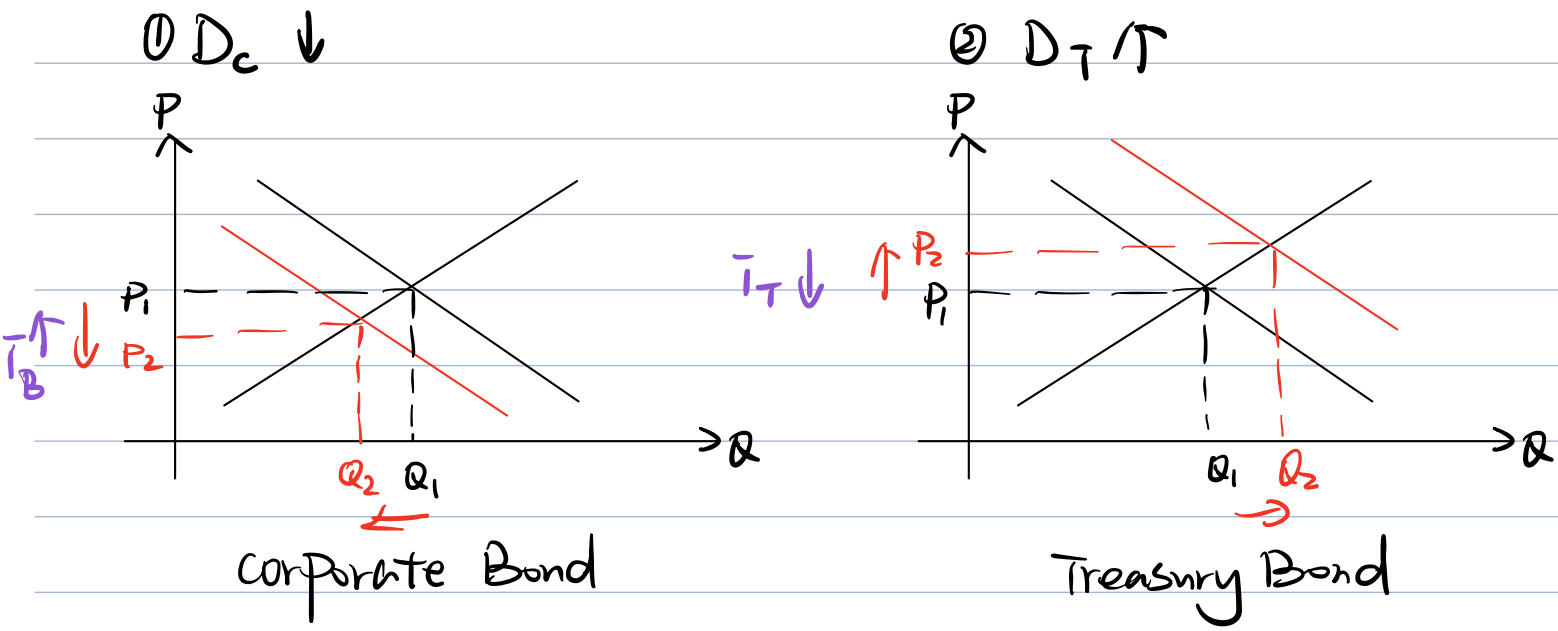


Since $i_{\text{corporate Bond}} > i_{\text{Treasury Bond}}$

$\Downarrow$

$P_C < P_T$

Default Risk ↑ ⇒ this happens during recession



Q: Why we need to have Risk Premium?

Investors require extra return to compensate them for a high level of risk on the Baa corporate bond

Reason 2: Liquidity

↓

Investors are willing to accept a lower interest Rate on more liquid investment

Reason 3: Info cost

↓

- Spending time and money acquiring info reduces a bond's Expected Return

Reason 4: Taxation

Investors care about the return they receive left after paying their taxes.

Recall the two types of income from owning bonds:

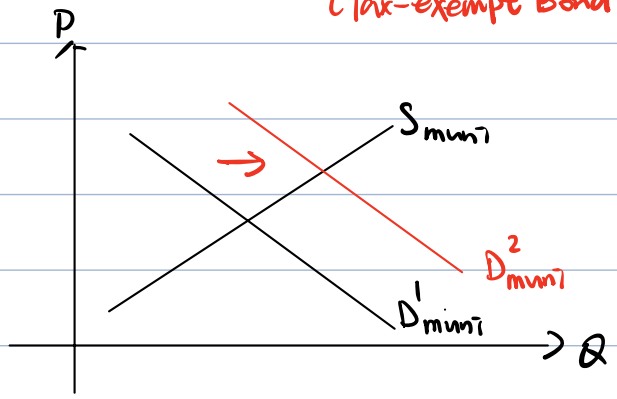
① • interest income from coupons – taxed at the same rates as wage and salary income.

② • capital gains or losses from price changes on the bonds – taxed at a lower rate than interest rate, and taxed only if they are realized, i.e., a bold sold at a higher price than paid for.

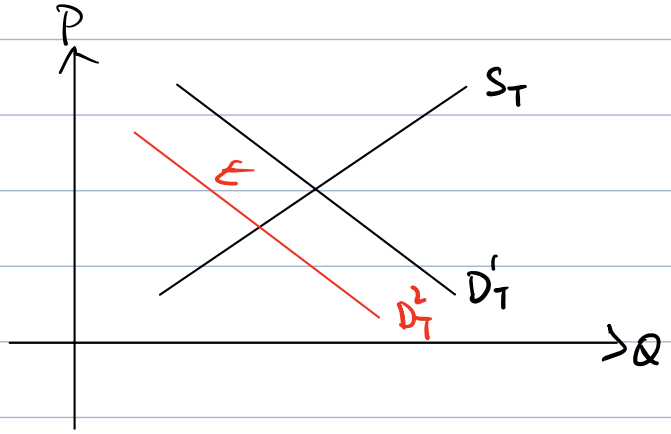
↓

Case 1: Federal Income Tax ↑

① Demand for municipal Bond shifts right
(Tax-exempt Bond)



② Demand for Treasury Bond shifts left



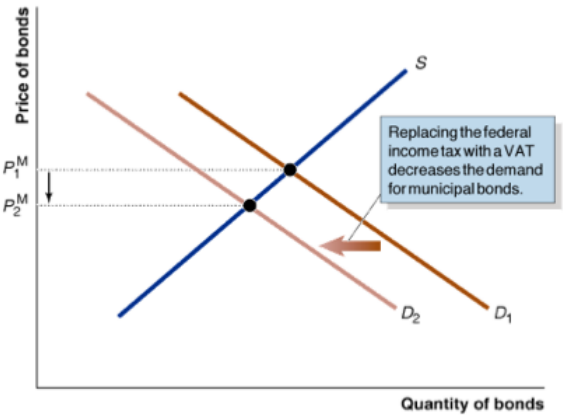
Case 2: Shift Federal Income Tax to Value-Added Tax (VAT)

① Demand for manicipal bond shifts left

② Demand for corporate and gov Bond shift right

↓

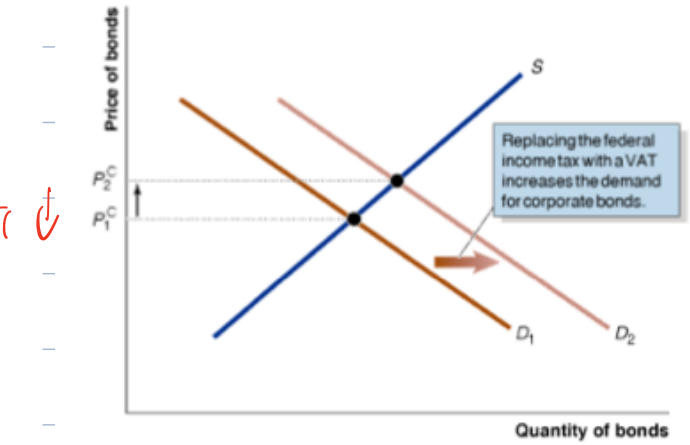
Switch from Federal Income Tax → VAT

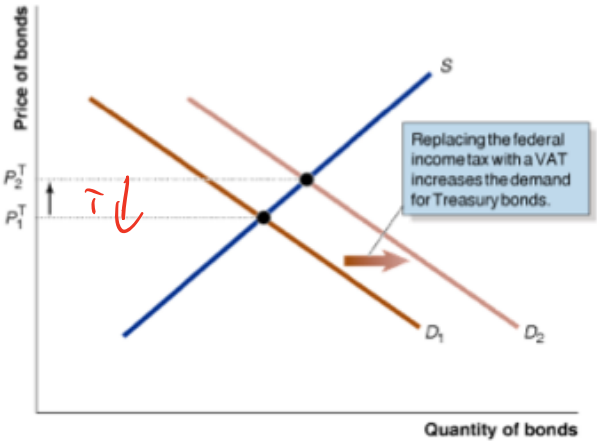


← Demand for tax-exempt manicipal bonds ↓

Demand for corporate / gov bonds ↑

↓





Causes affect yield to maturity

An increase in a bond's...	causes its yield to...	because...
default risk ↑	rise ↑	Investors must be compensated for bearing additional risk
liquidity ↑	fall ↓	Investors incur lower costs in selling the bond
information costs ↑	rise ↑	Investors must spend more resources to evaluate the bond
tax liability ↑	rise ↑	Investors care about after-tax returns and must be compensated for paying higher taxes

Q: Bonds with different interest Rate and maturity

Term structure

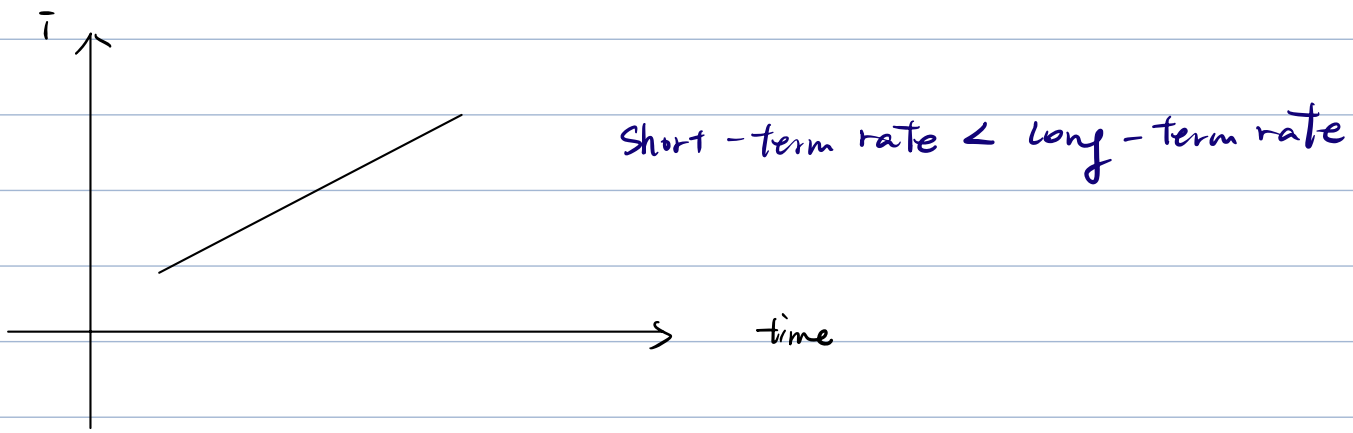
The term structure of interest rates is the relationship among the interest rates on bonds that are otherwise similar but that have different maturities.

Yield curve

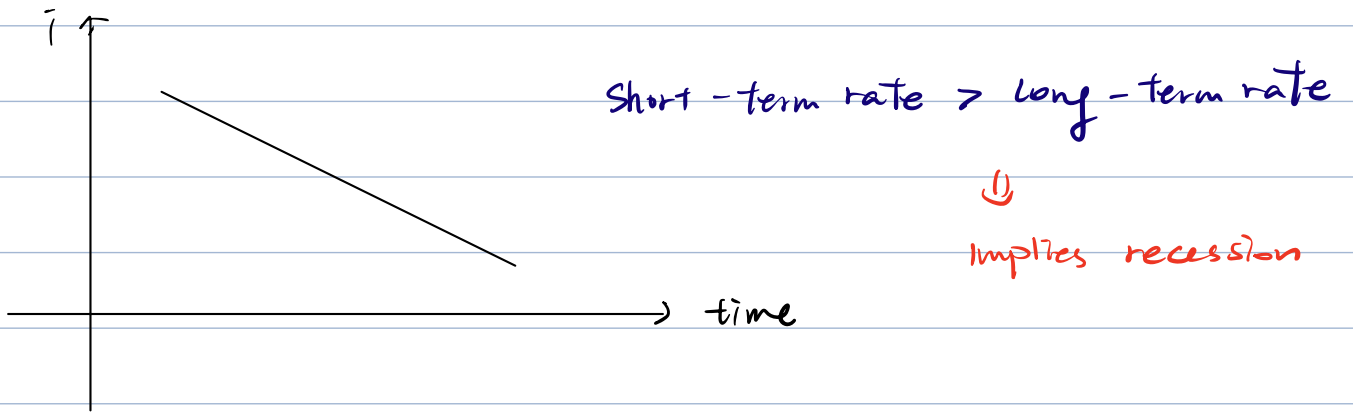
The Treasury yield curve shows the relationship among the interest rates on Treasury bonds with different maturities.

↓

graph of yield curve



Short-term rate < long-term rate



Short-term rate > long-term rate

↓

Implies recession

3 Facts regarding term structure :

① Long-term rate > Short-term rate

② Long-term rate < Short-term rate

③ Long-term rate = Short-term rate

Economists have advanced three theories to explain these facts:

① expectations theory
② segmented markets theory
③ liquidity premium theory or preferred habitat theory

1> Expectation theory

Interest Rate on long-term bonds = the average of the interest Rates investors expect on short-term bond over the lifetime of long-term bonds

2 Assumptions

① Investors have the same objective
② Long-term and short-term bonds are perfect substitutes

Ex: Two Strategy

① Buy and hold strategy

$\$1(1+i_{2t})(1+i_{2e})$

i_{2t}: interest rate on 2 yr bond

② Roll-over Strategy

$\$1(1+i_t)(1+i^e_{t+1})$

i^e_{t+1}: interest rate on 1yr bond expected for next period

Expected Return =

why? = uncertainty

$\$1(1+i_{2t})(1+i_{2e})-1$

$= 2i_{2t} + (i_{2t})^2$ (ignore)

$\approx 2i_{2t}$

$\$1(1+i_t)(1+i^e_{t+1})-1$

$= 1+i_t+i^e_{t+1}+i_t(i^e_{t+1})-1$

$= i_t + i^e_{t+1} + i_t(i^e_{t+1})$ (ignore)

$\approx i_t + i^e_{t+1}$

Hold Bonds only if the following holds =

$$ER^{\text{buy and hold}} = ER^{\text{Roll over}}$$

$$2i_{2t} = i_t + i^e_t + 1$$

$$i_{2t} = \frac{i_t + i^e_{t+1} + 1}{2}$$

Generalise this formula for n years:

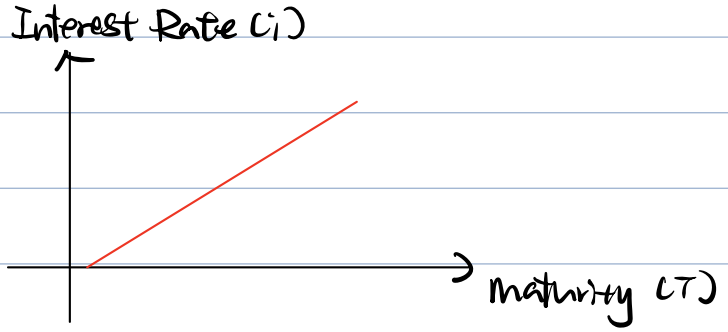
$$i_{nt} = \frac{i_t + i^e_{t+1} + \cdots i^e_{t+n-1}}{n}$$

The expectation Theory implies =

① A upward sloping yield curve :

expected future short-term rate > current short-term Rate

$$i^e_{t+1} > i_t$$



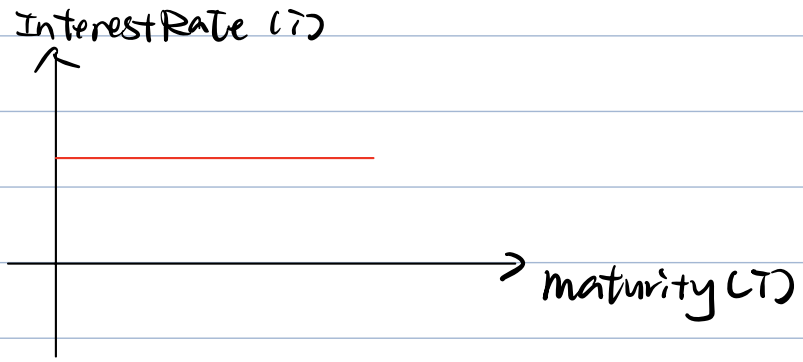
$$\frac{i_t + i^e_{t+1}}{2} = i_{2t}$$

long-term interest rate ↑

② Flat field curve

expected future short-term rate = current short-term Rate

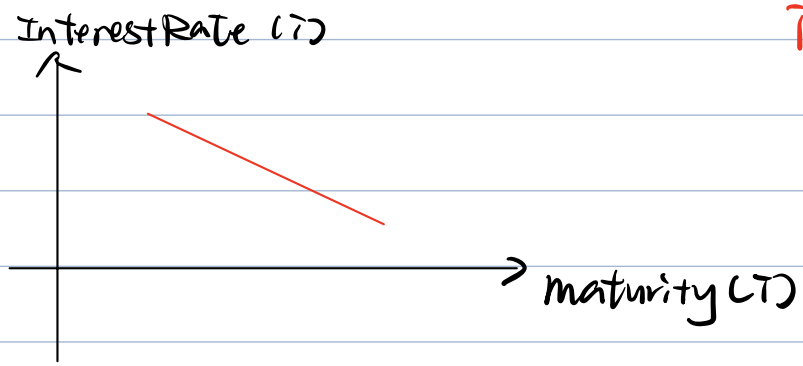
$$i^e_{t+1} = i_t$$



③ A Downward sloping field curve

expected future short-term rate < current short-term Rate

$$i^e_{t+1} < i_t$$



$$\frac{i_t + i^e_{t+1}}{2} = i_t$$

long-term interest rate will ↓

Expectation Theory

① Slope of the yield curve
- ① upward $i^e_{t+1} > i_t$
- ② flat $i^e_{t+1} = i_t$
- ③ downward $i^e_{t+1} < i_t$

② Short-term and long-term interest Rate moves together

Long term interest rate = average of short-term interest Rate investors expect

⊗ does not explain why yield curve is usually upward-sloping

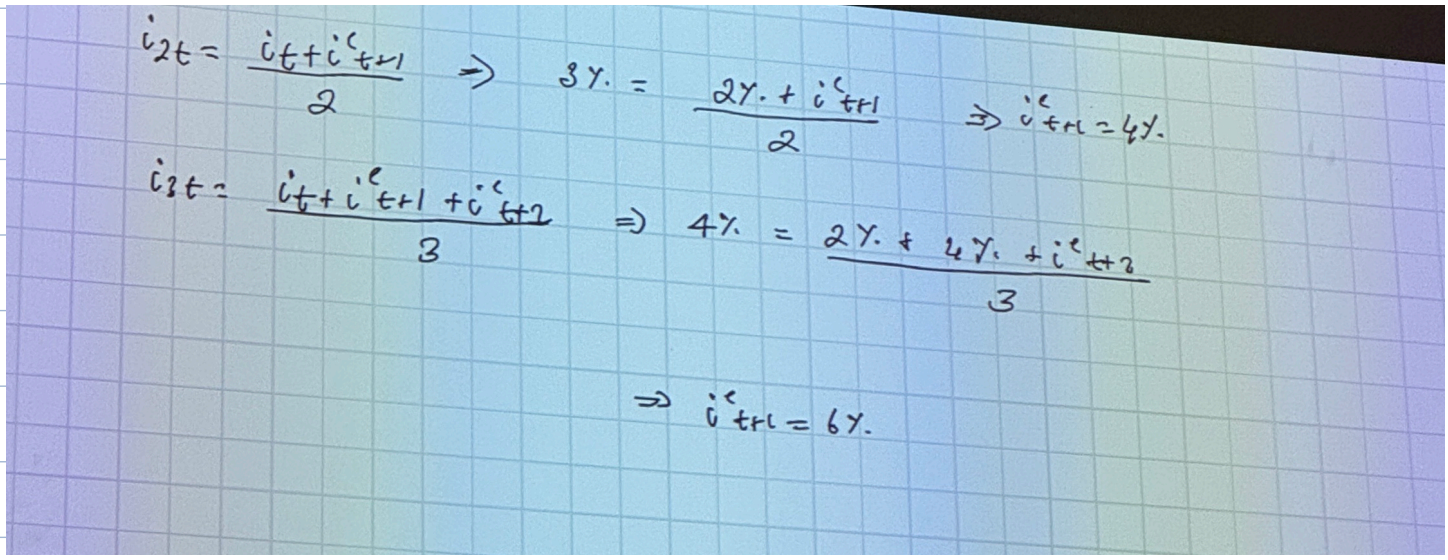
↳ short-term rate are about equal likely to fall and rise

EX: $i_t = 2\%$ (year $t = 2024$)

$i_{2t} = 3\%$ (interest rate on 2 yr bond in 2024)

$i_{3t} = 4\%$ 3 yr bond in 2024)

$i^e_{t+1} = ?$ (Expected 1 yr interest rate in 2025)



>> Segmented Market Theory

- The interest Rate on a bond is determined by the demand and supply of bond of that maturity

Observation:

① Investors in the bond market do not all have the same objective

② Investors do not see bonds of different maturities as perfect substitutes

⇓

Investors in the bond market of one maturity do not participate in markets for bond of other maturities

Theory: Long-term Bonds are less liquid

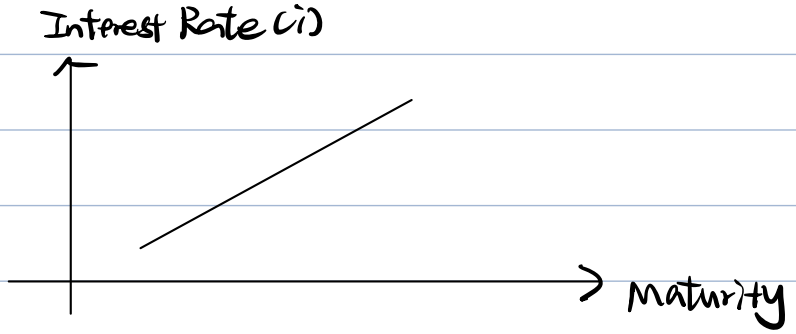
⇓

Subject to interest rate risk

⇓

Therefore, Investors are in the market for short-term bond

$P_{short} > P_{long}$ $i_{short} < i_{long}$



Ⓥ upward-sloping yield curve

Ⓧ downward-sloping yield curve

Ⓧ why Bonds of different maturities move together

3> The Preferred Habitat Theory (PHT)

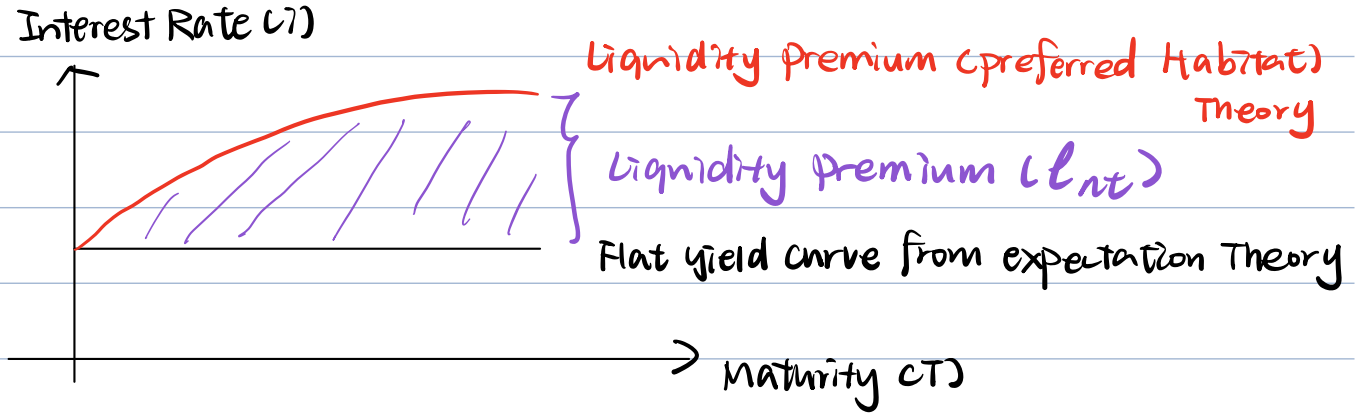
- Investors have preference for Bonds of one maturity over another

↓

Investors are likely to prefer short-term bonds over Long-term bonds

⇓

they will buy bonds of different maturities only if they earn a somewhat higher expected Return



Assumption: Investors view Bonds of different maturity as substitutes for each other but not perfect substitutes

$$i_{nt} = \frac{i_t + i^e_{t+1} + i^e_{t+2} + \cdots + i^e_{t+(n-1)}}{n} + l_{nt}$$

↓

① Interest rate of different maturities move together

② upward sloping ($i_{long} > i_{short}$) ⇒ liquidity premium

③ downward sloping ($i_{short} > i_{long}$)

$i^e_{t+1} < i_t \Rightarrow \frac{i_t + i^e_{t+1}}{2} = i_t$ ⇒ low expected Return

↓

1 year	2 year	3 year
1.25%	2.00%	2.50%

- Given the data and the liquidity premium theory, what did investors expect the interest rate to be on the one-year Treasury bill two years from that time if the term premium on a two-year Treasury note was 0.20% and the term premium on a three-year Treasury note was 0.40%?.

we know $i_t = 1.25\%$ $i_{2t} = 2\%$ $i_{3t} = 2.50\%$

step 1: Find i^e_{t+1}

$$i_{2t} = \frac{i_t + i^e_{t+1}}{2} + 0.2\%$$

$$2\% = \frac{1.25\% + i^e_{t+1}}{2} + 0.2\%$$

$$i^e_{t+1} = 2.35\%$$

Step 2: Find i^e_{t+2}

$$i_{3t} = \frac{i_t + i^e_{t+1} + i^e_{t+2}}{3}$$

$$2.5\% = \frac{2\% + 2.35\% + i^e_{t+2}}{3}$$

$$i^e_{t+2} = 2.7\%$$

Theory	Assumptions	Predictions	What the theory explains
Expectations	Investors have the same investment objectives, and, for a given holding period, investors view bonds of different maturities as perfect substitutes for each other.	The interest rate on a long-term bond equals the average of the interest rates expected on the one-year bonds during this period.	Explains the slope of the yield curve and why interest rates on short-term and long-term bonds move together. Drawback: Does not explain why the yield curve is usually upward sloping.
Segmented markets	Investors in the bond market do not all have the same objectives, and investors do not see bonds of different maturities as being substitutes for each other.	Interest rates on bonds of different maturities are determined in separate markets.	Explains why the yield curve is usually upward sloping. Drawback: Does not explain why the yield curve should ever be downward sloping or why interest rates on bonds of different maturities should move together.
Liquidity premium	Investors view bonds of different maturities as substitutes for each other—but not as perfect substitutes.	The interest rate on an *n*-year bond equals the average of the interest rates expected on the *n* one-year bonds during these *n* years plus a term premium.	Explains all three important facts about the term structure.

Summary of Lec 3:

Interest rate (i)

Risk Structure

Bonds { 1. Same maturity
2. diff in risk structure

↓

Default Premium:

$= i_{bond} - i_{safe\ assets}$

compensation for holding risky bonds

Term Structure

Bonds { 1. Same risk structure
2. diff in maturity

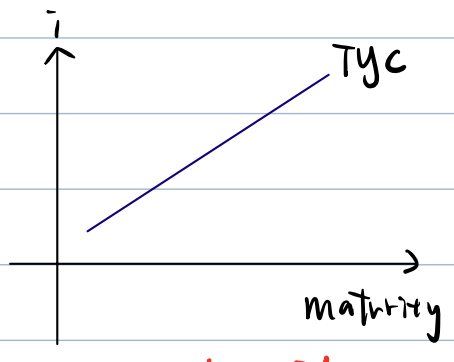
↓

Yield Curve

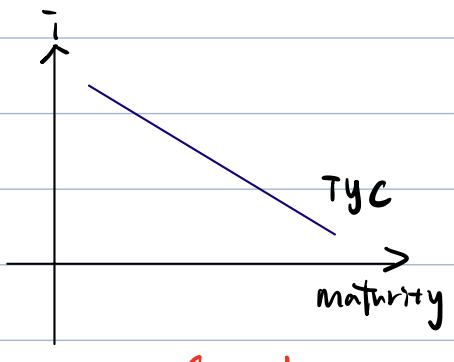
describe relationship between interest rate (i) and maturity (T)

Default risk ↑ $\Rightarrow D_{bond}$ ↓ $\Rightarrow P_{bond}$ ↓ $\Rightarrow$ i ↑

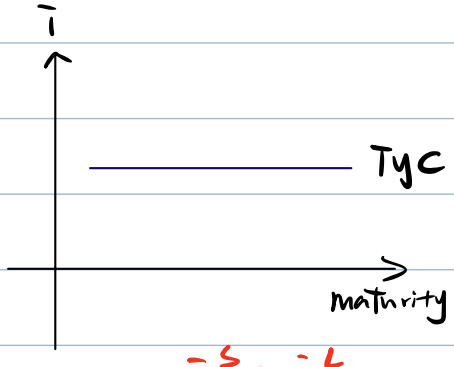
3 Types of Treasury yield curve:



$i^S < i^L$



$i^S > i^L$



$i^S = i^L$

3 Facts

① $i^{\text{long term}} > i^{\text{short Term}}$

② Sometimes $i^{\text{short term}} < i^{\text{long term}}$

③ Comovement

3 Theorem :

① Explanation Theory

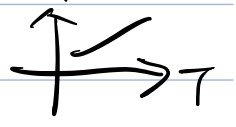
$$i_{2t} = \frac{i_t + i^e_{t+1}}{2}$$

✓ Explain comovements

✗ upward, downward, flat yield curve are equal likely

✗ does not explain why

Fact 1: $i^l > i^s$



② Segmented Market Theorem

✓ Explain why $i^{\text{long-term}} > i^{\text{Short-term}}$

③ liquidity preminm

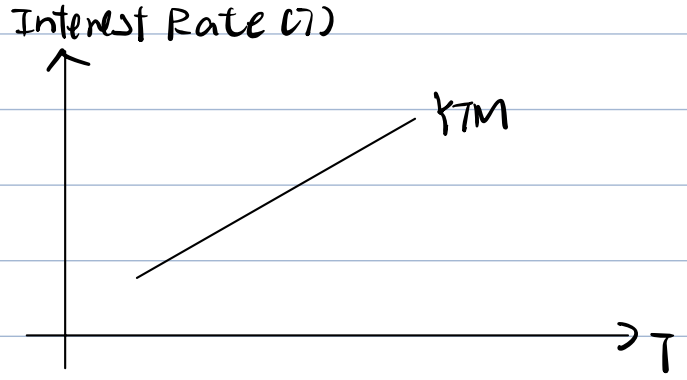
✓ Explain all types

$$i_{2t} = \frac{i_t + i^e_{t+1}}{2} + \underbrace{l_t}_{\text{liquidity premium}}$$

Suppose = (a upward yield curve)

⇓

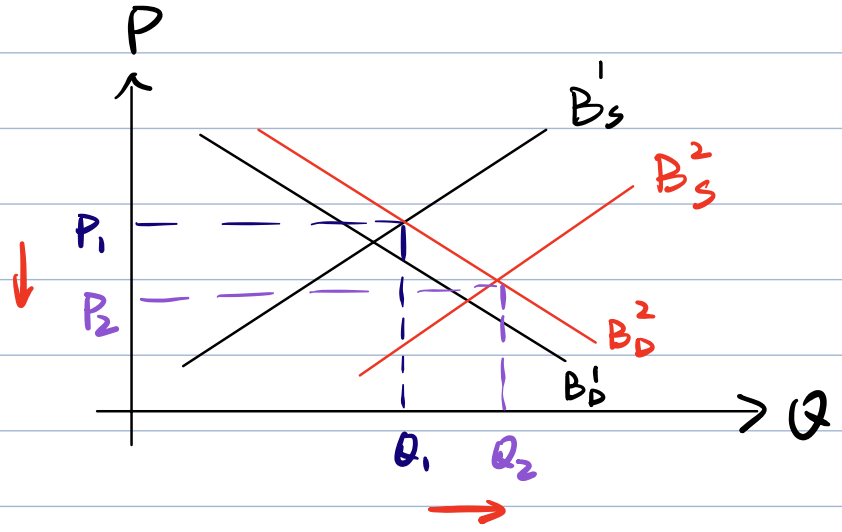
the future short-term interest Rate ↑



$i^S < i^L$

① firm will issue more bonds to borrow money for investment
B^S ↑

② investors will buy more bonds B^D ↑



Equilibrium = { ① price (P) ↓ ⇒ ③ interest rate (i) ↑
② quantity (Q) ↑