2026 Lec 2

Part 1: Interest Rate and Rate of Return

1.1 Why do lenders charge interest on Loans?

- Interest Rate should cover the opportunity cost of supplying credit, therefore interest Rate should include
  1. Compensation for inflation
  2. Compensation for default risk
  3. Compensation for the opportunity cost of waiting to spend the Money

1.2 The interest Rate, Present Value, Future Value

- Compounding: $FV = PV(1+i)^n$

  Compounding is a way of earning interest as saving accumulates over time

- Discounting: $PV = FV/(1+i)^n$

  Due to Time value of Money:

  value of payment changes depending on when payment is received

  ⇓

  Therefore: Funds in future is worth less than Funds in the present

  ⇓

  Funds have to be discounted to find their present value

1.3 Debt instrument and their prices

- PV of a financial asset = PV of payment received from owning it

- Debt instrument / credit market instrument / Fixed Income Asset

Eg: Simple loans, fixed payment loans, discount bonds, coupon bonds

1.4 Equity = claim to part ownership of a firm

Eg: Common stock issued by corporation

1.5 Yield to maturity:

Yield to maturity is the interest Rate that makes the PV of an asset = the asset's price today

Debt instrument Type 1 = Simple Loan

- Receive principal from lender, and agree to pay back lender the principal + interest
- Interest Rate = Yield to maturity

• For a simple loan $10,000 to pay $11,000 in one year

↓

Value today = PV of future payment

$10,000 = 11,000 / (1+i)

i = 10%

$PV = \frac{FV}{(1+i)^n}$

Debt instrument Type 2: Fixed payment loan

- requires borrower to make regular periodic payment to lender
- Loan value = $\frac{FP}{(1+i)} + \frac{FP}{(1+i)^2} + \cdots + \frac{FP}{(1+i)^n}$

Debt instrument Type 3: Discount Bond

- borrowers repays the amount of loan in a single payment at maturity but receives less than the face value of the bond initially
- $YTM = FV - P / P$

Ex: $\$10,000$ one year discount Bond with value today $\$9,200$

FV P

$$YTM = \frac{FV - P}{P} = \frac{10,000 - 9,200}{9,200} = 8.7\%$$

Debt instrument Type 4: Conpon Bond

- requires multiple payment of interest on a regular Basis, and a payment of the face-value at maturity
- Coupon Rate $(c) = \frac{C}{FV}$ (C conpon payment) (future value)
- $P = \frac{C}{1+i} + \frac{C}{(1+i)^2} + \cdots + \frac{C}{(1+i)^n} + \frac{FV}{(1+i)^n}$
- Price of conpon Bond (P) is negatively related to YTM
- $\begin{cases} P = FV \Rightarrow YTM = \text{conpon rate} \Rightarrow \text{current yield} = \text{Conpon Rate} \\ P < FV \Rightarrow YTM > \text{conpon rate} \\ P > FV \Rightarrow YTM < \text{conpon rate} \end{cases}$

↓

Special Case: Conpon Bond with Perpetuity

- perpetuity does not mature
- $P = \frac{C}{i}$

Ex: perpetuity with conpon \$25 and price \$500

$i = C / i = 25 / 500 = 5\%$

Formulas =

① Simple Loans: $PV = \frac{FV}{(1+i)^n}$

② Fixed payment loan:

Loan value $= \frac{FP}{(1+i)} + \frac{FP}{(1+i)^2} + \cdots + \frac{FP}{(1+i)^n}$

③ Discount Bond:

$YTM = FV - P \,/\, P$

④ Compon Bond:

$$P = \frac{C}{1+i} + \frac{C}{(1+i)^2} + \cdots + \frac{C}{(1+i)^n} + \frac{FV}{(1+i)^n}$$

$$i = \frac{C + \frac{FV-P}{n}}{\frac{FV+P}{2}}$$

1-6 Distinction Between Interest Rate and Return

- Return = Securities total earning
  Rate of return = return on a security as a % of initial pric

⇒ Return for holding Coupon Bond =

$$R = \boxed{\frac{C}{P_t}} + \boxed{\frac{P_{t+1} - P_t}{P_t}}$$

current yield capital gain / loss

- Current Yield = value of coupon expressed as a percentage of the current price

$$i_c = \frac{C}{P_t}$$

- capital gain / loss = market price increases / decreases
  Rate of capital gain: $g = \frac{P_{t+1} - P_t}{P_t}$

Facts regarding Interest Rate and Return

- $R = YTM$ if holding period = Time to maturity
- If a bond is not hold to maturity

  ↑

  Interest Rate (i) ↑

  ⇓

  Price (P) ↓ ⇒ resulting in capital loss $g = \frac{P_{t+1} - P_t}{P_t}$

- The more distant a bond's maturity, with an increase in Interest Rate (i)↑ the lower the rate of Return (R)↓

- Even if a bond has substantial initial interest Rate, it's Return can be negative (-R) if interest Rate (i)↑
- Interest Rate is not always positive (Eg. Japan)

1.7 Interest Rate Risk

- prices and Returns for Long-term bonds are more volatile than short-term bond
- ✗ No Interest Risk if Time to maturity = Holding Period

1.8 Distinction Between Real and Nominal interest Rate

- Nominal Interest Rate: not include inflation
- Real interest Rate: adjusted for inflation (change in price)

Eg: ① ante real interest Rate: 事前实际利率
adjusted for expected change in price level

② post real interest Rate: 事后实际利率
adjusted for actual changes in price level

↓

Lenders and Borrowers don't know the actual interest Rate will be during the Loan ⇒ they estimate expected real interest Rate

Part 2: Determine interest rate

2.1 What is Asset? Why is asset important In Economy?

- Asset is anything that can be owned and has value.
- Assets are important because Economic agents hold a variety of different Assets

2.2 Determinant of Asset Demand

① Wealth =
total resources owned by Individuals, Including all Assets

② Expected Return:
the return expected over the next period on One asset relative to alternative Asset

Expected Return = (Probability of event 1 × Value of event 1) + (Probability of event 2 × Value of event 2)

③ Risk:
the degree of uncertainty associated with the return on One asset relative to alternative asset

- Facts regarding risk of Investor

1> Risk-Averse = Asset with lowest risk when have Same expected Return

2> Risk-loving = prefer to hold risky Assets that give maximum Returns

3> Risk-Neutral: do not care about Risk, only care about returns

④ Liquidity:
the ease and speed which an asset can be turned into cash relative to alternative Assets

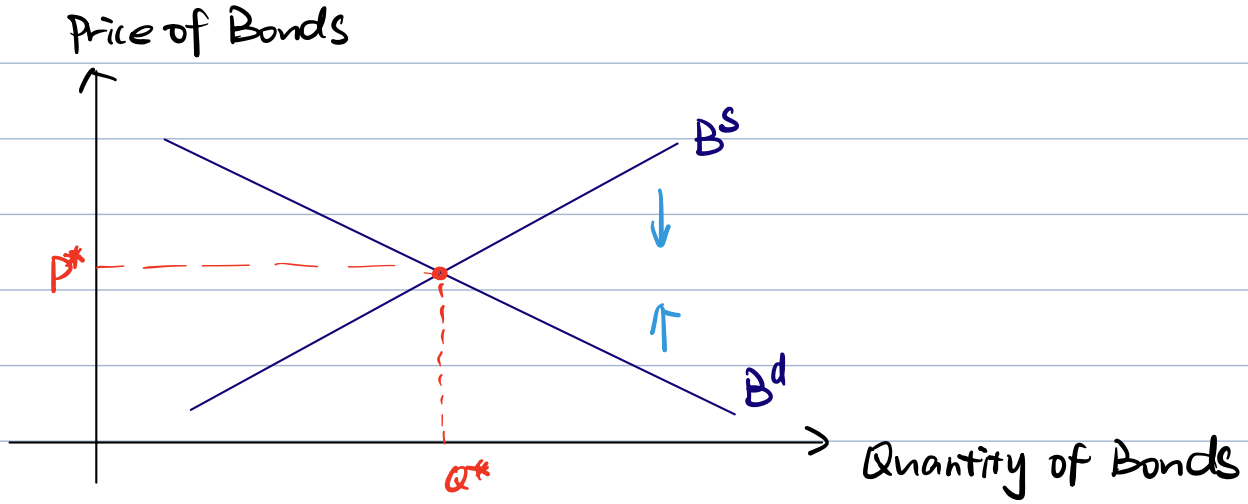
2.3 Theory of Portfolio Choice

- The quantity demanded of an asset is positively related to
  - ① Wealth ⇑
  - ② Expected Return Relative to alternative Assets ↑ ⇒ D^A ↑
  - ③ Liquidity Relative to alternative Assets ↑
- The quantity demanded of an asset is negatively related to
  - Risk Relative to alternative Assets ↑ ⇒ D^A ↓

2.4 Interest Rate Vs Supply and demand in Bond Market

- Low price (High Interest Rate) (i) ↑
  - ① Low Demand for Bonds (D^B) ↓
  - ② High supply for Bonds (S^B) ↑

2.5 Bond Market



- with excess supply = The Bond price falls ↓ to P^*
- with excess Demand = The Bond price increases ↑ to P^*

Bond Market Equilibrium price and interest Rate

$B^d = B^s$

Not In Equilibrium =

- $B^d > B^s \Rightarrow$ Excess Demand = $P\uparrow \Rightarrow i\downarrow$
- $B^d < B^s \Rightarrow$ Excess supply = $P\downarrow \Rightarrow i\uparrow$

2.6 Shift to Bond Demand / Supply when Interest Rate changes?

1> Shift in Demand curve

- Shift Demand curve for Bonds right

Increase in Interest Rate (i) ↑ Increases

① wealth ↑ (In expansion)

② Liquidity of bonds ↑

- Shift Demand curve for Bonds Left

Increase in Interest Rate (i) ↑ decreases

① Expected Return

Higher expected interest Rate in future lower the expected Return for Long-term Bonds

why? people can buy Bonds at a Higher interest Rate in future

↳ will not buy Long-term Bond

- If investors Buy Long-term bonds currently

$$P = \frac{C}{(1+i)} + \frac{C}{(1+i)^2} + \cdots + \frac{C}{(1+i)^n} + \frac{FV}{(1+i)^n}$$

(i ↑ ⇒ P ↓)

Bond price will decrease

⇓

capital loss: $\frac{P_{t+1} - P_t}{P_t}$ ($P_{t+1}\downarrow$)

⇓

Lower the expected Return for current Long-term Bonds

② Expected Inflation ↑

Why? people think interest rate ↑ will increase future price level ↑

therefore increase expected inflation

$R = i - \pi^e$

R ↓, π^e ↑

⇓

Lower the expected Return for Bonds

③ Risk ↑ ⇒ investors don't want to hold bond

2) Shift in Supply curve

Increase in Interest Rate (i) ↑

① Expected profitability of investment opportunity ↑

Supply more Bond to gather fund to invest

② Expected Inflation ↑

$R = i - \pi^e$ ⇒ cost of borrowing ↓

R ↓, π^e ↑

⇓

issue Bond is actually borrow money from bond holder

③ Government budget deficit ↑

Why? when interest rate (i) ↑ ⇒ ① gov need to pay for a higher coupon to issue a new bond

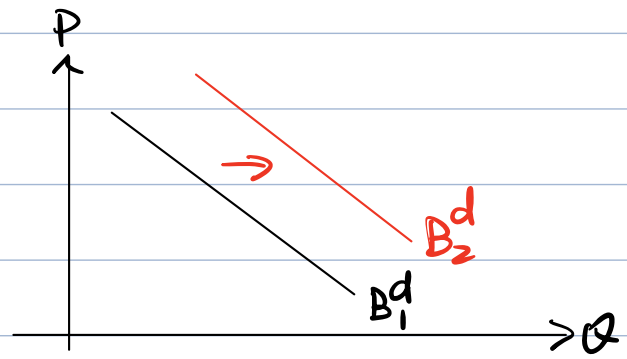
② cost of borrowing for investment ↑

⇓

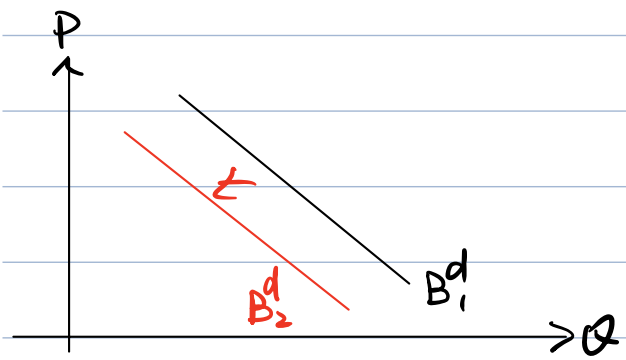
gov will Supply more bonds to cover its budget deficit

Summary of shift in Bond Demand and supply

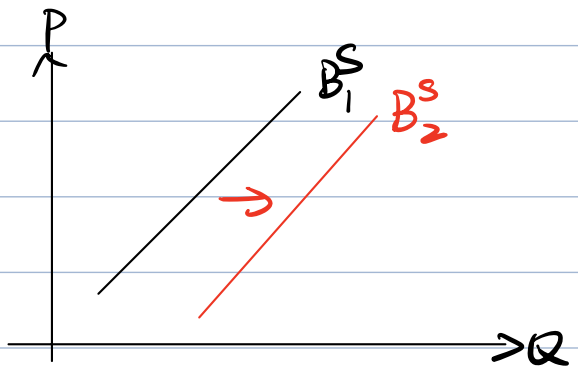
① Shift B^d right ⇒ $r\uparrow$ { ① wealth ↑ ② liquidity ↑



② Shift Demand left ⇒ $i\uparrow$ { ① Expected Return ② Expected inflation ③ Risk



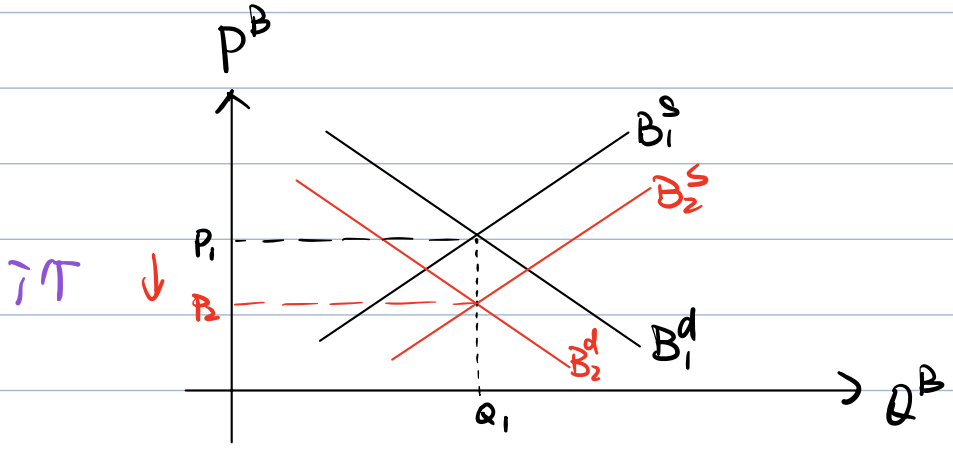
③ Shift Supply right ⇒ $i\uparrow$ { ① profitability of investment opportunity ② Expected inflation ③ Government deficit



2-7 Bond Market Equilibrium response to

case 1: Expected Inflation will Increase

① shift B^d to the left
② shift B^s to the right

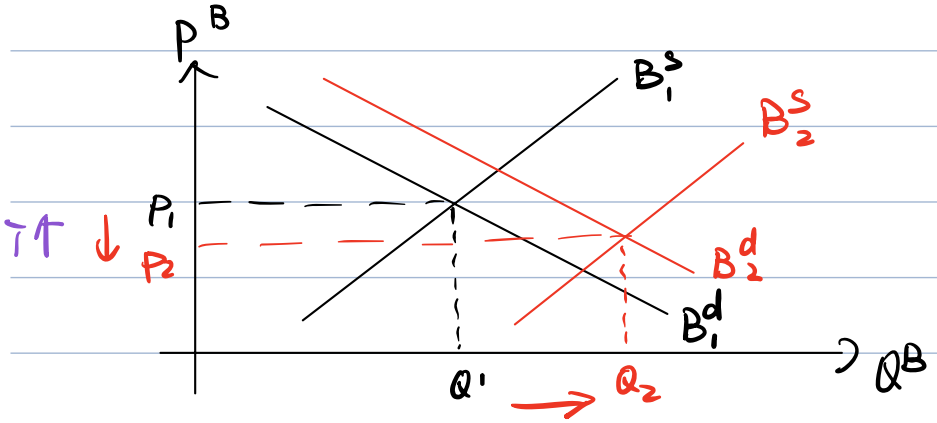


Result:
① Equilibrium price ↓
② Equilibrium interest Rate ↑

case 2: Bussiness cycle expansion

⇓

① wealth ↑ ⇒ B^D shift right
② profitability of Investment opportunity ↑ ⇒ B^S shift right
$\Delta B^S > \Delta B^D$



Result:
① Equilibrium price ↓
② Equilibrium quantity ↑
③ Interest Rate ↑

2-8 Supply / demand in the Money Market: Liquidity Framework

"货币市场中的供求关系：流动性偏好框架"（Supply and Demand in the Market for Money: The Liquidity Preference Framework）是经济学家约翰·梅纳德·凯恩斯（John Maynard Keynes）提出的一种理论模型，用来解释利率的决定。该框架基于人们对货币的需求与供给之间的相互作用，重点关注流动性偏好在货币市场中的角色。具体而言，它探讨了在利率、货币供给和货币需求之间的关系。

- Keynesian Model determines that equilibrium interest Rate in terms of the supply and demand for the money

⇓

Equilibrium Interest Rate is determined by

① Money Market
② Bond market

Why? people use ① Money ② Bond to store their assets

- Suppose Total wealth in the economy

$$B^S + M^S = B^D + M^D$$

Rearrange:

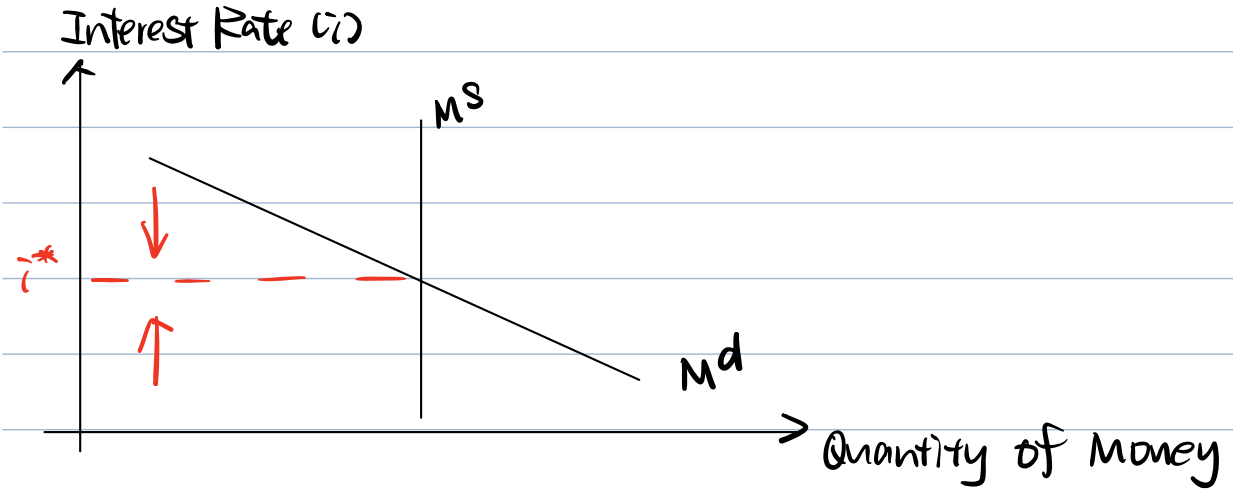
$$B^S - B^D = M^S - M^D$$

Conclusion:

If money market is in equilibrium $M^S = M^d$

then Bond market is also in equilibrium $B^S = B^D$

Equilibrium in Money Market



{ With excess supply: interest rate ↓ to i^*
With excess Demand: interest rate ↑ to i^*

Demand for money in the Liquidity Preference Framework

1> Shift Money Demand to the left

Interest rate (i) ↑ lead to

{ ① opportunity cost of holding money increases
↑↑ ⇒ invest in Bond Market gives a higher return
② the relative expected return of money decreases
↑↑ ⇒ the expected return compared to other investment ↓

⇓

Money Demand ↓ ⇒ shift M^d to the left

2> Shift Money Demand to the right

① Income effect: Higher Income causes the M^d at each interest rate ↑

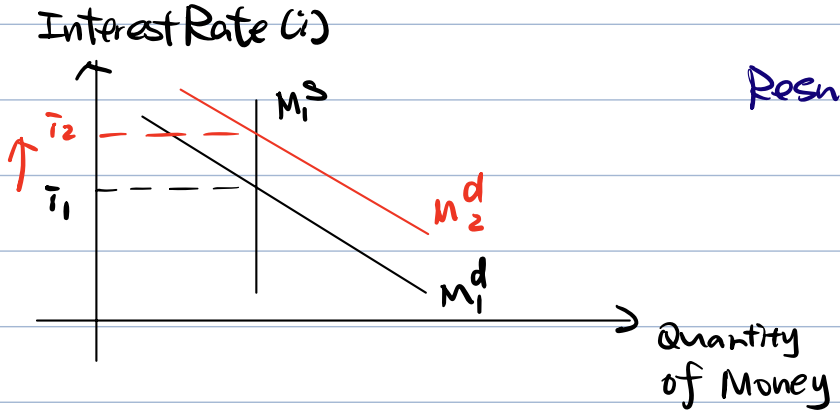
② price level effect: A rise in the price level causes M^d at each interest Rate to increase

Supply for money in the Liquidity Preference Framework

① Money Supply controlled by central Bank ↑

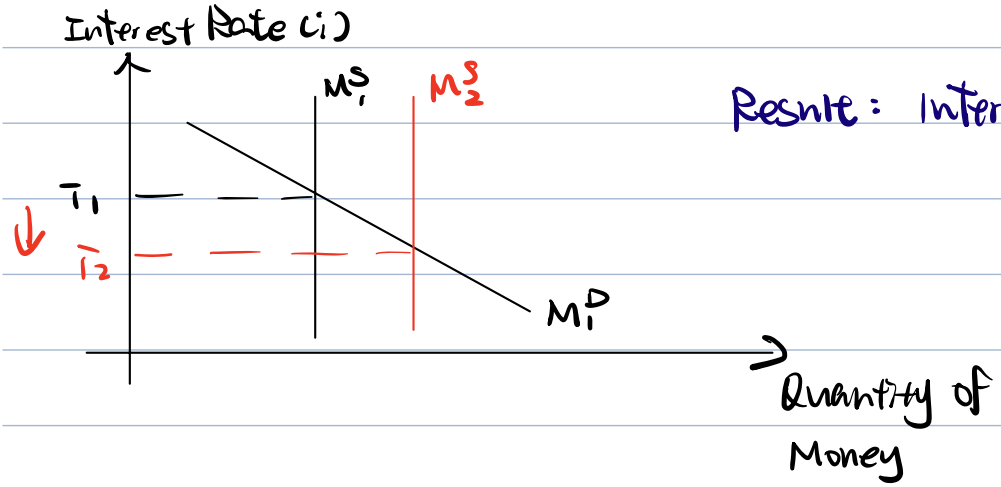
Summary: changes in Equilibrium interest Rate in the Liquidity preference Framework

① shift M^d right { ① Income effect ② A rise in price level



Result: interest rate ↑

② Shift in Money Supply



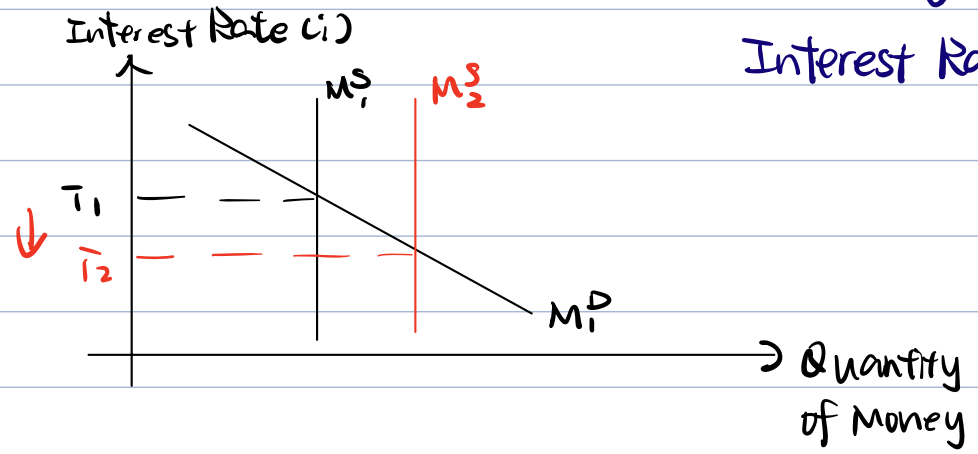
Result: Interest Rate ↓

2-9 what happens to interest Rate if Money Supply increases

① From liquidity preference Framework:

⇓

Interest Rate will fall



② From { ① price level Effect ⇒ interest rate will rise
② Expected inflation effect

- price level Effect: Increase Money Supply will lead to rise in price level

this implies inflation

According to $i = R + \pi$

(nominal) (real) (inflation)

⇓

Nominal interest Rate ↑

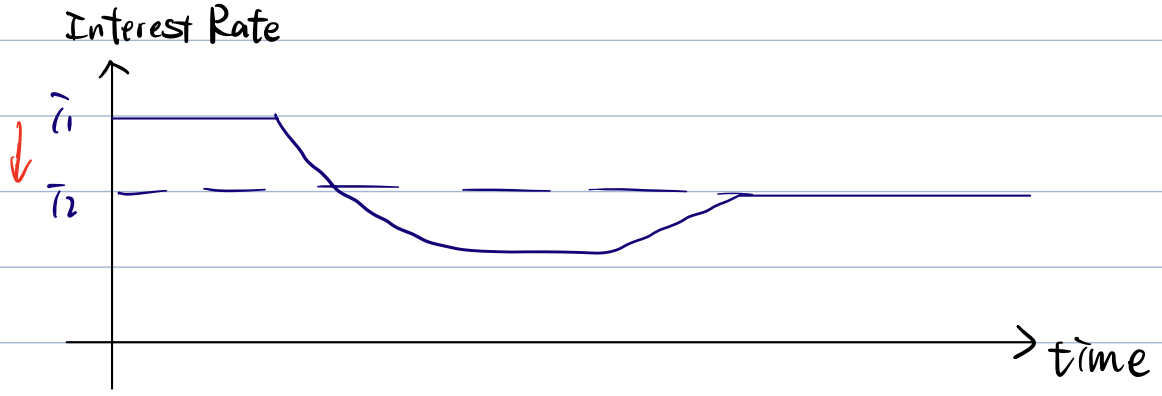
The price level remains even after prices stop rising

- Expected - Inflation Effect = As price level continues to rise, people expect a high inflation in the future

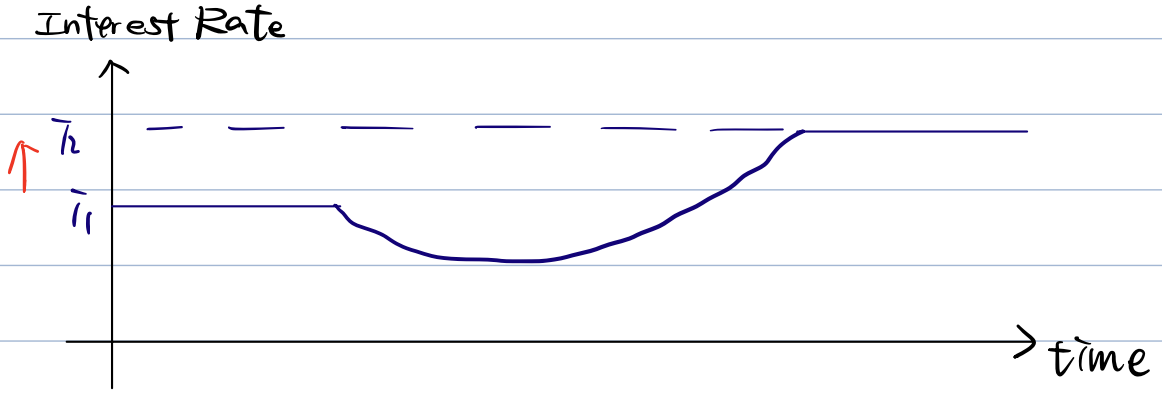
(the Demand curve will shift to the right)

2.10 Response over time to an Increase in Money Supply

① Liquidity preference > other Effect ⇒ Interest Rate ↓



② Liquidity preference < other Effect ⇒ Interest Rate ↑
+ slow adjustment of expected inflation



③ Liquidity preference < other Effect ⇒ Interest Rate ↑
+ fast adjustment of expected inflation