已知每条河宽度 S_i

每条河流速 V_i

人的固定游泳速度 v

游泳的总时间 T

求沿河流方向最大距离时，各河流游泳角度a_i

设有沿河方向游泳距离函数$h(a_1, a_2, ..., a_n)$

则需求：

$$minh(a_1, a_2, a_3, ..., a_n)$$
$$s.t. \sum_{i=1}^{n} \frac{S_i}{v \cos a_i} = T$$

以下将$h(a_1, a_2, ..., a_n)$略写为$h(a_i)$

$$h(a_i) = \sum_{i=1}^{n} (V_i + v \sin a_i) \frac{S_i}{v \cos a_i}$$

Lagrange Multiplier:

$$F(a_1, a_2, ..., a_n, \lambda) = \sum_{i=1}^{n} \frac{(V_i + v \sin a_i) S_i}{v \cos a_i} + \lambda (\sum_{i=1}^{n} \frac{S_i}{v \cos a_i} - T)$$

对上式各变量求偏导有：

$$\frac{\partial F}{\partial a_i} = (\frac{((V_i + \lambda) \sin a_i + v) S_i}{v \cos^2 a_i}) \tag{1}$$

$$\frac{\partial F}{\partial \lambda} = (\sum_{i=1}^{n} \frac{S_i}{v \cos a_i}) - T \tag{n+1}$$

最大值时，各偏导数为0，且依照常识，取值范围有：

$$S_i > 0$$
$$a_i \in (0, \frac{\pi}{2})$$
$$v > 0$$

因此有：

$$\frac{S_i}{v \cos^2 a_i} > 0$$

所以：

$$(V_i + \lambda) \sin a_i + v = 0$$

方程组：

$$(V_1+\lambda)\sin a_1 + v = 0 \tag{1}$$
$$(V_2+\lambda)\sin a_2 + v = 0 \tag{2}$$
$$\dots$$
$$(V_n+\lambda)\sin a_n + v = 0 \tag{n}$$
$$(\sum_{i=1}^{n}\frac{S_i}{v\cos a_i}) = T \tag{n+1}$$

根据以上前n个方程组解得各a_i：

$$a_i = \arcsin\frac{-v}{V_i+\lambda}$$

带入(n+1)式可得λ值，进而可求得每个a_i的值

下附$\frac{\partial F}{\partial a_i}$求解过程：

$$\frac{\partial F}{\partial a_1} = \frac{\partial}{\partial a_1}\left(\frac{V_1S_1 + V_pS_1\sin a_i + \lambda S_i}{V_p\cdot\cos a_i}\right)$$

$$= \frac{\partial}{\partial a_1}\left(\frac{V_1S_1+\lambda S_1}{V_p\cdot\cos a_1} + S_i \times \tan a_i\right)$$

$$= \frac{\partial}{\partial a_1}\left(\left[\frac{V_1S_1+\lambda S_1}{V_p}\cdot\cos a_1^{-1}\right] + S_i\cdot\tan a_i\right)$$

$$= \frac{V_1S_1+\lambda S_1}{V_p}\cdot\cos a_1^{-2}\cdot\sin a_1 + S_i\cdot\cos a^{-2}$$

$$\frac{\partial F}{\partial a_i} = \left(\frac{V_iS_i+\lambda S_i}{V_p}\right)\cdot\frac{\sin a_i}{\cos^2 a_i} + \frac{S_i}{\cos^2 a_i} = 0$$