24/05 610 FINANCIAL ECONOMETRICS-CRT2: Qtn4

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Gaussian Copula:

A copula is a statistical tool used to describe the dependence structure between multiple random variables, independently of their marginal distributions (Durante & Sempi, 2015) This notebook defines a Gaussian copula, which is a type of copula that assumes normal marginal distributions and models dependence using a correlation coefficient.I will use synthetic data to illutrate this.

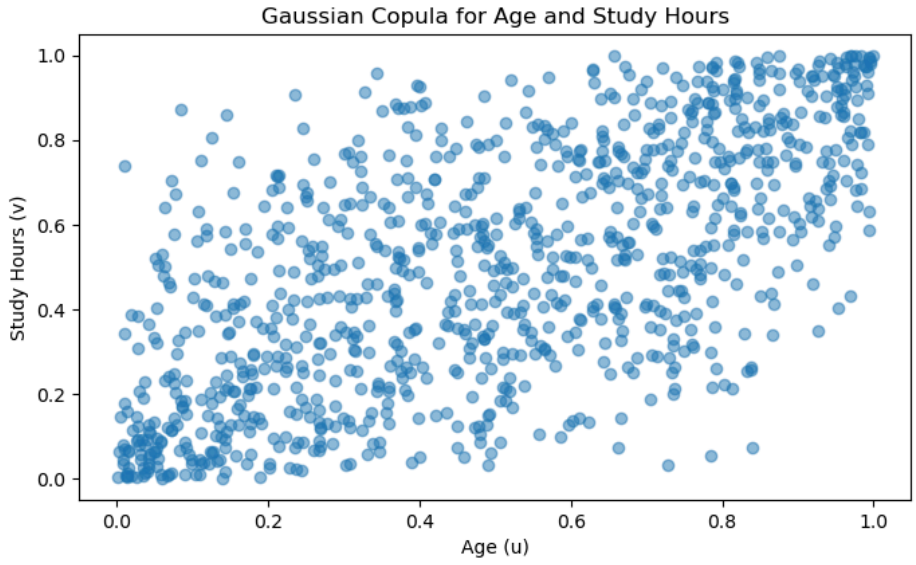
In [1]:
```
# Importing necessary dependencies
import numpy as np
import matplotlib.pyplot as plt
from scipy.stats import norm
```

In [2]:
```
# Assume correlation cofficient between age and study hours
rho = 0.7
# Random data for the ages of students
np.random.seed(42)
num_students = 1000
age_mean = 28
age_std = 4
study_hours_mean = 25
study_hours_std = 5

# Constructing the covariance matrix
cov_matrix = [[age_std**2, rho * age_std * study_hours_std],
              [rho * age_std * study_hours_std, study_hours_std**2]]
# Samples from a bivariate normal distribution
age, study_hours = np.random.multivariate_normal([age_mean, study_hours_mean], cov_matrix, num_students).T
# Transform samples to uniform marginals using the inverse CDF
u = norm.cdf(age, np.mean(age), np.std(age))
v = norm.cdf(study_hours, np.mean(study_hours), np.std(study_hours))
```

In [4]:
```
# Plot the Gaussian copula
print('Figure 1:The Gaussian copula showing the transformed samples uniform variables (u) and dependent variables (v).')
plt.figure(figsize=(8,4.5))
plt.scatter(u, v, alpha=0.5)
plt.title('Gaussian Copula for Age and Study Hours')
plt.xlabel('Age (u)')
plt.ylabel('Study Hours (v)')
plt.show()
```

```
Figure 1:The Gaussian copula showing the transformed samples uniform variables (u) and dependent variables (v).
```



The plot shows the relationship between two variables: transformed age (u) and transformed study hours (v). Each point represents a pair of values from the copula with uniform marginals. Given the correlation coefficient of 0.7, we expect a moderate positive correlation between age and study hours. This means older students tend to study more hours. which is indeed showed by the graph above

Reference:

Durante, F., & Sempi, C. (2015). Principles of Copula Theory. DOI: 10.1201/b18674.