Are Rats Smart?

Numerical Methods

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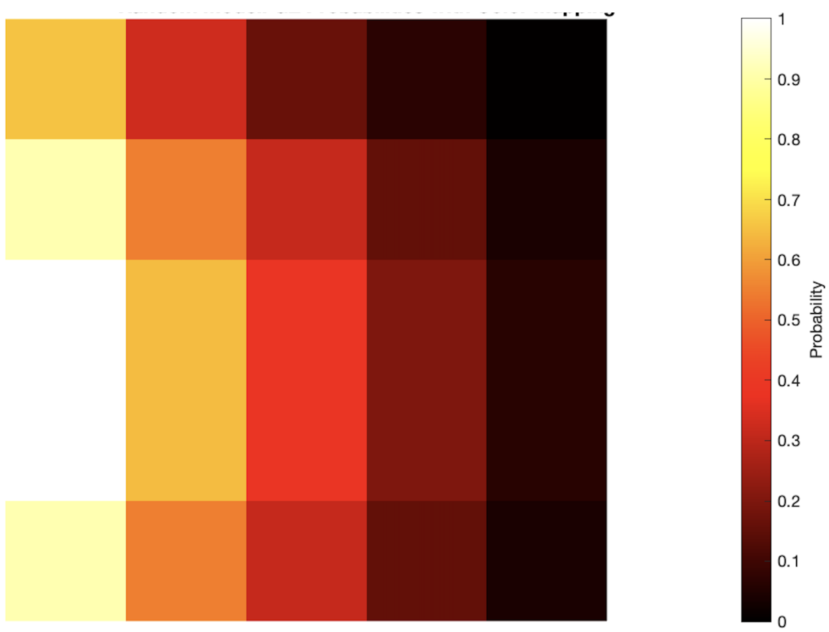

2^{rd} Project

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Introduction

In several experimental studies involving rats, the utilization of mazes has been explored as a means to monitor and analyze various outcomes related to their participation in maze-based tasks. For instance, in one experiment, scientists positioned food in different sections of a maze and monitored whether or not the rats were able to locate and consume the food. The Maze Patrolling experiment is done for rodent species, but more notably for rat species. The rat belongs to the class Mammalia and is classified as one of several species of rodents. Rats depend on scent as a primary sensory modality for locating food sources. They have an exceptionally sensitive nose and, if needed, can cover a distance of 50 to 300 feet to seek food. This distance is not often traveled, as they tend to build their nest next to a reliable food source. *Guide* Rat species have been extensively utilized in laboratory tests, particularly those pertaining to the fields of psychology and behavioral studies. There are several reasons why researchers prefer to conduct experiments with rats, including their cognitive abilities and physiological similarity to humans.

Rat species have played a significant role in maze experiments since the establishment of "mazes to test rat intelligence" dates back a century, followed by the breeding of the initial albino Wistar rats in close succession. *Animalinformation.com* Researchers opt to utilize rats as experimental subjects for tasks such as maze trials owing to their distinctive brain structure. Their brains are significantly larger than those of other laboratory animals, such as mice. In addition to possessing an advantage over mice, the rat species has a brain structure that "resembles the more primitive elements of human brains, and hence they can be used to model some human behaviors." *University of Cambridge* Previous studies have conducted trials using rats navigating through a maze, including both food-rewarded and non-rewarded locations. According to Fitzgerald, Isler, Roseberg, Oettinger, and Battig's research, rats who exhibit hunger reduce the amount of time it takes them to find food sources in the Maze Patrolling by Rats With and Without Food experiment. Fitzgerald et al.

Drawing upon the insights acquired from prior studies done on rats, our initial claim is rats should be able to obtain food throughout the maze in our project. Moreover, it is expected that the rats will exhibit a greater efficacy in obtaining food in the non-random model as opposed to the random model as a result of the limits imposed. In order to investigate these possibilities, we initially construct algorithms for both models for the probability of movement. Finally, the techniques of Gaussian Elimination and Gauss-Seidel method are implemented using the MATLAB software. MathWorks Inc

Methods

The main aim of this project is to investigate the behavioral patterns shown by rats within a maze grid, with a particular focus on examining both random and non-random models. We then aim to utilize numerical techniques, such as Gaussian elimination, Gauss-Seidel, Newton's methods, or other relevant numerical methods, to address the task of solving systems of equations. Additionally, the chosen approaches will be compared to one another in terms of speed and accuracy. This will allow us to determine the probabilities related to each potential direction that the rat is inclined to pursue. Through this approach, our goal is to evaluate the intellectual capacity demonstrated by the rat in both models. LibreTexts

0.1 ***Random Model***

The Random Model is defined by a precisely specified formula that is commonly used for studying the behavior of rats displaying random movement patterns as described in the project description. A grid is built in which the rat is presented with four distinct choices at each given location. The probability of the rat's movement is determined by the following numerical formula:

$$P_{i,j} = \frac{1}{4}P_{i-1,j} + \frac{1}{4}P_{i+1,j} + \frac{1}{4}P_{i,j-1} + \frac{1}{4}P_{i,j+1} \tag{1}$$

Equation (1) describes the likelihood of the rat's movement from any given position inside the maze's grid in order to seek food. The probability, denoted as P_{ij}, of the rat locating food at a certain location (i, j), where i denotes the rows and j denotes the columns, is equivalent to the likelihood of the rat moving to an adjacent location and then finding food there. Given the positioning of food at the left exits and the inability of the rat to go backward and retrace its steps upon reaching an exit, the resulting probabilities were observed to be true: P_{0j}, P_{7j}, and P_{i6} are all equivalent to 0, indicating instances where the rat departs without food. In contrast, P_{i0} is equal to 1, corresponding to the event of an exit associated with the food. A set of 30 equations were created by hand using the equation (1). Subsequently, the equations were solved and restructured into a matrix with dimensions of 30 by 30.

It was observed that all 30 equations generated were linear equations.Thus, a system of linear equations with 30 equations and 30 unknowns, with the unknowns as the transition probabilities. ($Ap = b$) was formed. The probabilities described in Equation (1) determine the coefficients of the linear system. "A" denotes the coefficient that is linked to the transition probability P_{ij}. Matrix A is a representation of the probability related to the rat's ability to reach exits, with or without food, from different locations inside the maze. The constant vector b, located on the right-hand side of the linear system, is defined by the circumstances present at the maze exits. The values 0 or $\frac{1}{4}$ represent these instances. In order to solve for the variable p within the linear system, the methods of Gaussian elimination and Gauss-Seidel iterative were utilized. These methods were implemented using the latest MATLAB version: 23.2.0. MathWorks Inc Furthermore, the condition number and computation times were computed using MATLAB in order to make easier comparisons.

0.2 ***Non-Random Model***

Similarly, the system of equations for the Non-random Model is derived from the behavior of the rat, which includes the condition that the rat never moves in a direction opposite to the location of food. The probabilities for movement are explicitly defined: "*There is a 0.5 probability of moving straight towards the food and a 0.25 chance of moving in any other direction.*" The equations for the transition probabilities P_{ij} in the maze are constructed using these criteria. The system of equations may be represented in matrix form as $Ap = b$, similar to the Random Model. Here, A denotes the coefficient matrix, p represents the vector of transition probabilities, and b represents the constant vector that represents the event at the maze exits.

The following algorithm was used to calculate the likelihood that a rat will go from a certain position (i, j) in the maze to a location where it will find food before it reaches an exit:

$$P_{ij} = \frac{1}{2}P_{i,j-1} + \frac{1}{4}P_{i-1,j} + \frac{1}{4}P_{i+1,j} \tag{2}$$

- P_{ij} represents the probability associated with an event of food at the particular location (i, j).

The probabilities that are shown on the right-hand side of the equation relate to the different possible moves that may occur from the current location to the ones that are adjacent to it.

- $P_{i,j-1}$ represents the probability of the rat moving to the left.
- $P_{i-1,j}$ represents the probability of the rat moving to upwards.
- And $P_{i+1,j}$ is the probability of the rat moving to downwards.

Furthermore, we organized the system of linear equations $Ap = b$ in a Google Sheet and subsequently imported it into MATLAB. We utilized ChartGPT to gain knowledge on the functioning of imshow plots. The modifications made to the plots were accomplished through the use of markup, a feature provided by Apple in its Photos application that facilitates the process of overlaying content onto any image. Markup: Highlight & Annotate

0.3 *Numerical Methods*

The process of choosing numerical methods for solving a linear system of size 30 by 30 requires careful consideration of both efficiency and accuracy. The utilization of Gaussian elimination is selected as a way to transform the matrix into its reduced form, hence demonstrating its efficacy. However, the likelihood of significant round-off errors in situations involving a large number of equations demands the use of the Gauss-Seidel method. The use of this dual-method approach is employed in order to leverage the benefits offered by both methodologies. The Gaussian elimination method is well recognized for its stability and accuracy, particularly when applied to well-conditioned systems. However, it is known that this method can become inefficient when dealing with large matrices. In contrast, the Gauss-Seidel method offers enhanced control over round-off errors, hence improving accuracy in situations that may involve numerical instability. The major strength of this system lies in its ability to effectively handle large and sparse systems. However, it is important to use it with caution, since particular arrangements have the potential to result in slow convergence. To check if the linear system is stable, we figured out the condition number $cond(A)$. This is a very important parameter for figuring out what problems might come up during the Gaussian elimination process. An understanding of the sensitivity of the numerical solution to variations in input data is crucial for making informed decisions on the selection of appropriate numerical methods that results accurate and reliable results.

Results and discussion:

The linear system was initially formulated as 30 equations and 30 unknowns, reflecting each point on the matrix. By utilizing the Google Sheet platform and inputting each data entry into the sheet, it was observed that the random model matrix displayed a sparse matrix structure. As indicated in the Method section, sparse matrices have significant importance in numerical methods due to their efficiency and storage capabilities. In the random model, the matrix exhibited a unique structural pattern, with entries positioned both above and below the main diagonal. Particularly inside this matrix, there were repeatedly two entries of $\frac{-1}{4}$ above and below the diagonals when the variable b equaled zero. All the remaining entries in the matrix were recorded as zeros. This implies that the matrix in the random model is strictly diagonally dominant. Furthermore, this particular structure aids in reducing computational time after storing the matrix as a sparse in MATLAB.

The matrix structure of the Non-random Model was similar to that of the Random Model. It was still a transition matrix that represents the rat's likelihood of movement. Furthermore, it should be noted that the condition of the model is not ill-conditioned, similar to the random model. The computed condition number of matrix A for the Non-random was found to be 21.56 as indicated in table 2, in contrast to the condition number of 16.17 for the random model in table 1. Being strictly diagonally dominant (with the main diagonals equal to one) guarantees that the matrix can be factored into a product of lower and upper triangular matrices without pivoting. The preceding attribute, combined with its tridiagonal structure, significantly improves computing efficacy for determining transition probabilities.

0.4 Findings

The computational efficiency of Gaussian Elimination can be seen by the significantly shortened time required for execution, measuring $5.325e - 05$ seconds as seen in *table* 1. In contrast, the Gauss-Seidel method exhibited a greater degree of precision, as evidenced in *table* 1 by an attained absolute error of 0.0005 following 22 iterations. Nevertheless, it required a longer computational duration, which is 0.00289 seconds. Despite the fact that the random model's matrix yielded a lower condition number $cond(A)$, it is important to acknowledge that this matrix is of greater size. Therefore, in the context of this project, we consider accuracy to be crucial over speed, as we aim to obtain more accurate results. It doesn't even take a second to calculate the transition probabilities using either methods. Therefore, the Gauss-Seidel method was selected as a relatively superior method (which is also utilized in analyzing non-random probabilities).

Table 2 shows that the non-random model produced a fast convergence for Gaussian elimination. In the Gauss-Seidel method, an absolute error of 0.0005 was achieved after 22 iterations. The process of solving this one, as shown in *table* 2 , requires a slightly longer duration. Given our primary focus on the Gauss-Seidel method to solve for the non-random model of the transition probabilities of the rat, it is evident that the current computational time required to solve for the transition probabilities p is 0.0047 seconds, which is roughly double the time it took to solve random transition probabilities.

0.5 *Potential Rat's Paths*

The figures for the random model indicate that both Gaussian Elimination and Gauss-Seidel methods resulted in transition probabilities that demonstrate a relatively small discrepancy. Both cases involved the rat successfully locating food from a given location with a probability exceeding 0.7. It was expected for the rat to find food at least once in the maze. Figure 1 illustrates two potential paths that the rat may undertake in order to successfully locate food. The straight path, denoted by the blue track, is built on the assumption that the rat will travel in a manner that optimizes its likelihood of finding food. The red path represents an alternative path, in which the rat makes a decision to pursue the second direction that offers the highest likelihood of finding food. This trace was built with the aim of observing the potential path that the rat would take. In Figure 2, the rat follows similar paths, but with the adjustment that it strictly chooses directions that maximize its likelihood of locating food.

A significant observation in our study of the non-random model is that the rat's optimal strategy for locating food results in a singular path represented by the color red. In this scenario, it is necessary to assume that the rat, from the beginning, strictly begins its path from the rightmost position, which has the utmost probability of transition. Furthermore, it is important to note that in the non-random model, the transition probabilities exceed a value of 1. In general, probabilities are constrained within the range of 0 and 1, denoting the probability of an event taking place. However, this is not the case in the non-random model as a result of the imposed constraints. The justification for transition probabilities exceeding a value of 1 in a random model lies in the assumptions or rules established for the model.

Initially, the team had concerns regarding the obtained results. However, further examination and review of the underlying assumption indicated that the probabilities are aligned with the anticipated behavior of the rat within the maze and validate the formulated equations.

0.6 *Discussion*

The transition probabilities that have been calculated are consistent with the assumptions regarding rat behavior as outlined in the project. The project proceeds under the assumption that rats possess a particular probability of navigating in different directions within a maze. The transition probabilities, in turn, indicate the likelihood of a rat successfully reaching an exit containing food prior to reaching an exit without food. The findings, as presented in the tables and figures, offer valuable insights into rats' decision-making processes within the maze.

In the Random Model, it was observed that both the Gaussian Elimination and Gauss-Seidel methods yielded transition probabilities that suggest a comparatively elevated probability of the rat effectively locating food from several locations. The illustrated paths in the figures conform to the anticipated outcome that, when using a random model, the rat will have at least one valid path to locate food within the maze. And the findings in the Non-Random Model align with the underlying assumptions that the rat exhibits a predetermined likelihood of moving towards food and a comparatively reduced likelihood of moving in alternative directions. The calculated probabilities exceeding 1 in the non-random model are a result of the imposed constraints and are in line with the expectations outlined in the project description.

The findings from the random model suggest that rats, despite exhibiting random movement patterns, possess a considerable likelihood of successfully obtaining food. The presence of multiple potential paths indicates a significant level of ability to adapt in their decision-making process, enabling them to locate food from different locations within the maze. The Non-Random Model demonstrates a specific tactic in which the rat consistently selects the direction with the highest probability of successfully reaching the food source. Hence, the rat took a singular path, implying that the rat may display more predictable patterns of decision-making under specific assumptions.

0.7 *Tables*

Parameters	**Values**
Condition Number (Cond(A))	16.1670
Iterations (0.5*10^{-3} error)	22
Time (Gaussian Elimination)	**5.325e-05 sec**
Time (**Gauss-Seidel**)	0.0028913 sec

Table 1: **Summary of the Random Model**

Parameters	**Value**
Condition Number (Cond(A))	21.5627
Iterations (0.5*10^{-3} error)	**22**
Time (Gaussian Elimination)	0.004401 sec
Time (Gauss-Seidel)	**0.0046597 sec**

Table 2: **Summary of the Non-Random Model**

0.8 *Plots*

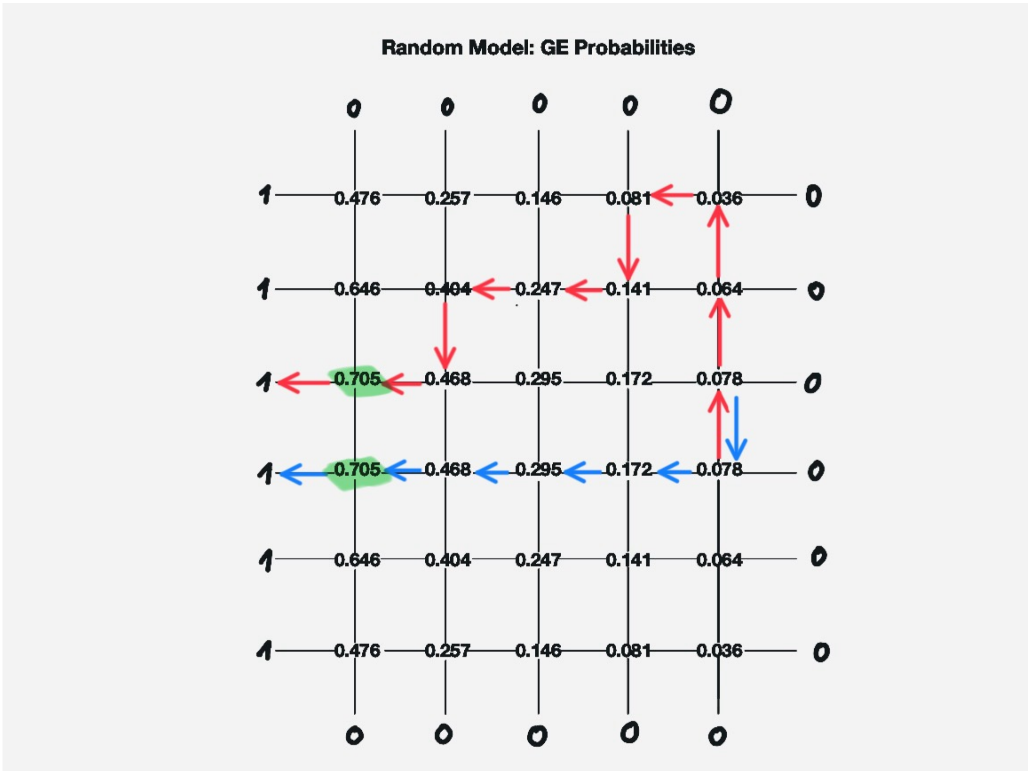

Figure 1: Random-GE

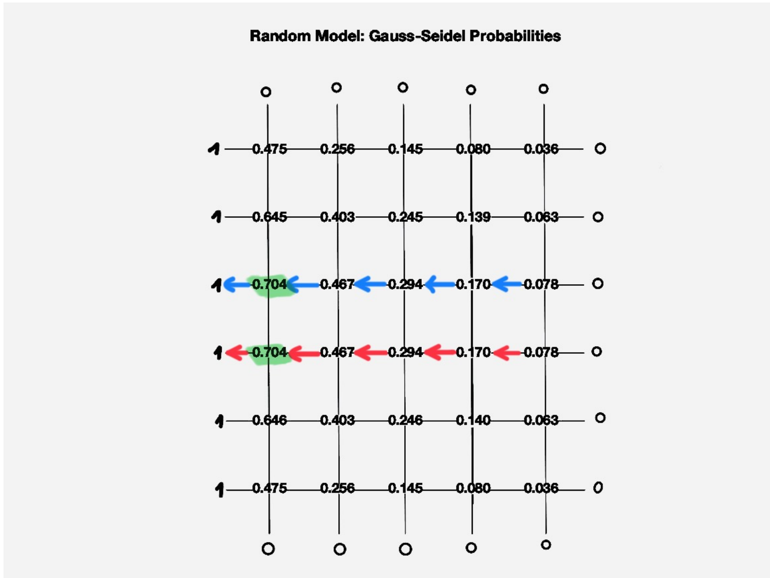

Figure 2: Random-Gauss-Seidel

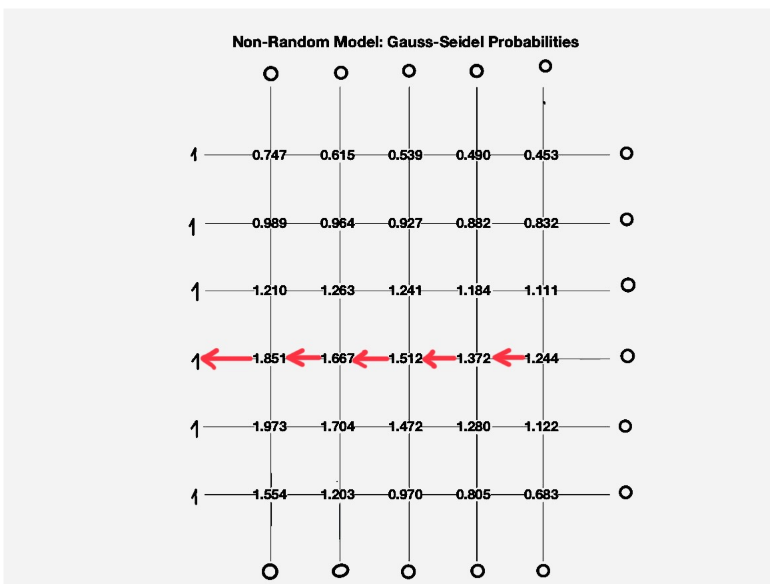

Figure 3: Non-Random-Gauss-Seidel

Conclusion

In order to enhance the rat's brainpower in maze navigation and food collection, it is possible to introduce additional rules that mimic complex brain functions. For example, taking into account sensory information as indicated by our research findings By incorporating sensory inputs that impact the rat's decision-making process, there is a potential increase in the likelihood of the rat's movement towards areas with stronger scent cues, thus replicating its use of its smell sense. An additional potential rule could be the inclusion of a learning process where the rat retains and recalls its previous experiences within the maze. In this particular example, the rat has the ability to allocate higher probabilities to pathways that have previously resulted in food, thereby modeling an elementary type of memory. To replicate social interactions, incorporate the introduction of additional rats into the maze environment. The decision-making process of rats may be subject to the influence of other rats, indicating the possibility of social learning. Including additional rules in the model would improve its intelligence by integrating elements of smell, learning, and decision-making that more accurately capture the complex nature of rat behavior. However, these modifications would increase the computational complexity. Nonetheless, they have the potential to offer a deeper understanding of intelligent decision-making during maze navigation.

In general, the study relies on a number of simplifying assumptions pertaining to the behavior of rats. The maze model fails to fully capture the complexities of real-world scenarios, in which rats may make advanced decisions based on sensory cues, memory, and learning.

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