Reproducing images with splines

Numerical Methods

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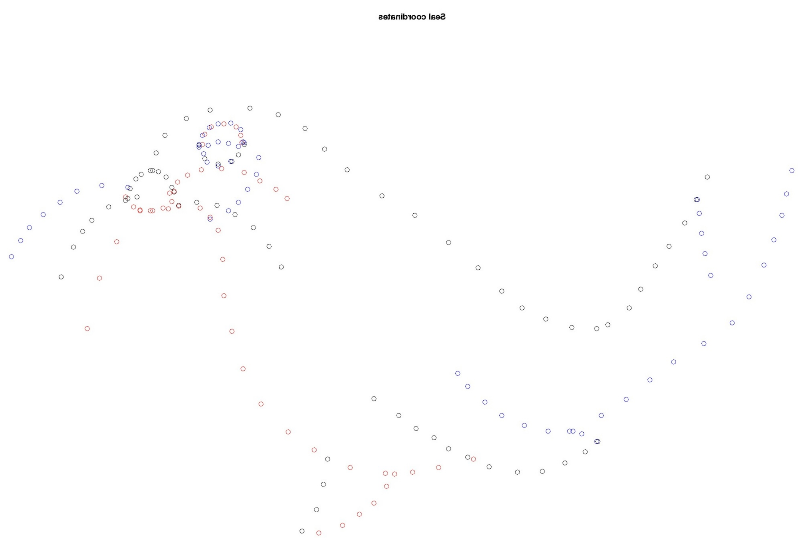
Fall 2023



3^{rd} Project

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MA 377

Introduction

The use of cubic splines is favored over other interpolation techniques due to their ability to maintain smoothness and continuity in the first and second derivatives at the data points. Unlike global polynomial equations, which tend to oscillate and overshoot, cubic splines strike a balance between precision and smoothness. Furthermore, in numerous applications, the ability of cubic splines to approximate the value of a function within known data points is a valuable property, as it does not require knowledge of the function itself. Cubic splines, as defined by Timo Denk. (2017), are a specific category of splines that are intentionally constructed to traverse a predetermined set of controlled points.Denk They can be found across the STEM disciplines, which include science, technology, engineering, and mathematics.

Our group is made up of students majoring in pure mathematics and digital art, applied mathematics and business, and engineering management. Despite any differences in degree requirements, the application of splines is substantial across all of our disciplines. Cubic splines play a crucial role in mathematics by facilitating the computation of integrals and derivatives, as well as aiding in numerical analysis and interpolation. They are also widely used in computer graphics and geometric modeling to create smooth curves and motion paths. In engineering, cubic splines are utilized to approximate values between known data points, making them particularly valuable in domains like chemical engineering. These applications independently interpolate each coordinate using a unique parameter using cubic spline functions. A new interpolation method has been proposed in the domain of engineering that combines the non-overshooting characteristics of linear interpolation with the smooth curve attributes of spline interpolation. It is crucial in numerous chemical engineering applications that overshoot at intermediate points be avoided in order to achieve the desired consistency. A detailed look at this method, including how it is put together mathematically and the specific algorithms that make it work, along with a discussion of how it might be better than other methods like spline or linear interpolation, can help us understand how important it is.

We reconstructed the seal drawing after our inability to reproduce an image of Among Us. This image was found on an online resource that provides drawing instructions called "All About Drawings.com." Godsall We thought it was adorable, and unlike the image from the video game Among Us, we did not have many pointed corners. Therefore, avoiding overcomplications in the process of reconstruction. Interpolation using cubic splines is used, as instructed in the project manual, to generate multiple segments of the shape. The methodology utilized included utilizing the ginput function in Matlab to trace the image and cubic splines to approximate the image's curves.Dolor

Methods

To reconstruct the drawing of the seal, we first needed to understand how a cubic spline works. A cubic spline is defined in mathematics as "a piecewise cubic function that interpolates a set of data points and guarantees smoothness at the data points" McClarren. While we understood the concept of a cubic spline, we also needed to understand the matrix construction for cubic spline interpolation. One of our references that provided background information on how to construct a cubic spline interpolation was from Lab 10, which we recently completed. In this lab, we used interpolation to generate a spline from the bird data that was provided to us. The information we were given in the lab helped us create the cubic spline of the bird data; thus, we used the knowledge and function file created at the end of Lab 10 to recreate our chosen image.

Our initial step was to draw the seal image using Adobe Fresco and then extract the corresponding points at various spots on the seal drawing. Without these corresponding locations, it would have been difficult to replicate the sketch using MATLAB since we would have needed to install extra image recognition tools. Additionally, although we could have created the corresponding points on grid paper, MATLAB includes a tool called ginput that simplifies this process for us. The ginput function is one of the functions used for tracing points in an image. We learned that it is critical to know the total number of points required for tracing a line or curve, as well as the specific coordinates of the start and end points, after running this command. MathWorks Inc To draw a line or curve, at least two or three points must be traced, respectively. The sketch was divided into 20 continuous segments to avoid any sharp edges and ensure that the points traced per segment did not result in dysfunctional behavior. Subsequently, the traced points were copied onto an Excel spreadsheet. This was necessary due to the problem encountered in Google Sheets, where numerical values were stored as strings when uploading onto Matlab. The file was then imported into Matlab, where it was automatically loaded along with the column headers. The data was then extracted into a matrix using the importdata function. Afterwards, similar to our procedure in Lab 10, we obtained the x and y coordinates for each segment.

We used the cubic spline with a curvature adjusted after obtaining the evaluation points. To carry out this method, the data was interpolated from the seal drawing coordinates by locating the min and max points of the x coordinates. A constant right-hand vector and a coefficient matrix were formed using the cubic-spline-interpolant function file, and the spline was constructed. For this purpose, the variables gamma and delta are used in the function for storing the evaluation points. The cubic-spline-interpolant file produced the main diagonal, super diagonal, sub diagonal, start and end points, and constant coefficients in detail. There are three methods that could have been utilized to calculate the c coefficient: Newton's method, Gauss-Seidel, and Gaussian elimination. In our opinion, the most efficient method for making the seal drawing is Gaussian elimination. The method is built in MatLab and has previously been proven to be effective in Lab 10. We constructed appropriate curvature splines that produced a recognized seal drawing by considering the curvature addicted to u = 0.

A weakness of cubic adjusted splines is their inability to accurately represent extremely sharp curves or sudden changes in direction, as evidenced by our initial image (Among Us Dressed as Santa). The reason for this is that the spline interpolated minimizes the appearance of these sudden characteristics, leading to a decrease in precision. One advantage of cubic adjusted splines is their ability to create a smooth and continuous curve that accurately fits the provided data points. This makes them excellent for situations where a visually appealing and well-behaved curve needs to be produced. Furthermore, cubically adjusted splines can efficiently handle large amounts of data without significant computational overhead.

Results & Discussion

We chose to recreate a seal drawing from an online resource that offers drawing instructions. The image's simplicity influenced our decision over the more complex ones we tried initially, such as Among Us. Adobe Fresco was used to draw the seal image, and corresponding points were retrieved at various positions on the drawing via MATLAB. The coordinates were then utilized to generate an image cubic spline interpolation. The end result includes the original seal drawing, the coordinates created from the extracted points, and a cubic spline Seal drawing.

Figures

Figure 1: Original Seal Drawing

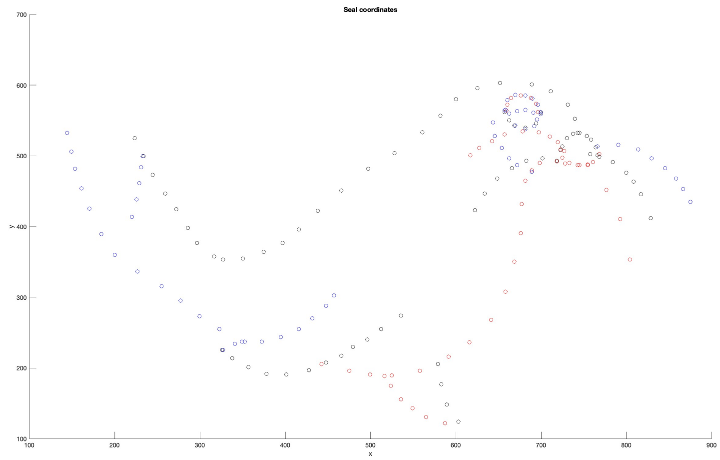

Figure 2: Extracted Points

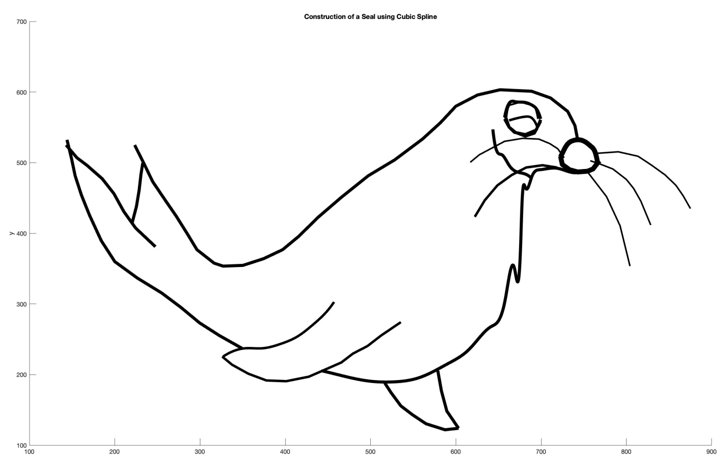

Figure 3: Cubic Spline Seal Drawing

The ability of cubic splines to maintain continuity and smoothness in the first and second derivatives at the data points justifies their use in this project. As described by Timo Denk (2017), cubic splines are specifically constructed to traverse a predetermined set of controlled points.Denk In our case, these controlled points are the extracted coordinates from the seal drawing. As previously stated, cubic splines' benefits lay in their ability to achieve a compromise between accuracy and smoothness. In our spline reproduction of the seal drawing, the cubic spline efficiently captures the overall shape and curvature of the seal, providing a recognizable representation. It's not perfect, but it's recognizable.

Although cubic adjusted splines have difficulties with extremely sharp curves, they demonstrate exceptional performance in generating a smooth and accurate curve that precisely fits the given data points. Ultimately, the cubic spline interpolation effectively replicates the seal drawing with a visually pleasing and smooth curve, demonstrating the efficacy of cubic splines in image reconstruction.

Conclusion

As the results suggest, cubic-spline interpolation is seen as an effective technique for reproducing the seal drawing. There are numerous limitations associated with this method that may arise if the implementation is not executed appropriately.One primary constraint is that its application is restricted to continuous data points. Extreme outliers or points with significant error may contribute to the appearance of curves that lack accuracy in representing the data. Additionally, cubic spline interpolation is unsuitable for estimating values beyond the provided data points Aewinter.

Roles

Everyone

We all helped each other out when we needed it and got together once a week on the weekends to talk about how the project was going. Dylan took notes during the meetings, Bryant planned out the tasks that needed to be done before the meetings, and Isaac gave a full report afterward. We used MATLAB to make plots and then wrote the report in Latex.

Bryant

- Responsible for overseeing the overall progress and timeline of the project

Dylan

- Provided an engineering viewpoint and contributed to the project's research.

Isaac

- Responsible for overseeing the technical coding and proofreading tasks.

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