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Multi-start heuristics for the profitable tour problem

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A B S T R A C T

This paper is concerned with an interesting variant of the traveling salesman problem (TSP) called a profitable tour problem (PTP). Unlike TSP, in PTP there is no need to visit all the cities, and each city is associated with a profit which the salesman gets in case he visits that city. Like TSP, a travel cost is incurred in visiting a city that depends on the city visited last before visiting the city in consideration. The goal of the problem is to maximize the total profit minus total travel cost. In this paper, we have proposed three methods, viz. a multi-start hyper-heuristic (MSHH), a multi-start iterated local search (MS-ILS) and a multi-start general variable neighborhood search (MS-GVNS) to solve the PTP. MSHH uses eight different low level heuristics, whereas MS-ILS and MS-GVNS utilize variable neighborhood descent search over five different neighborhoods for local search. To evaluate the performance of the proposed approaches, a set of benchmark instances is generated based on the publicly available TSPLIB instances. Computational results on these instances show the effectiveness of our proposed approaches.

1. Introduction

The profitable tour problem (PTP) is a variant of the traveling salesman problem (TSP). Given a set of n cities, the TSP is to find a minimum length tour that visits all the n cities. Unlike TSP, in PTP, these n cities have associated profits, and there is no need to visit all the cities. The salesman gets a profit associated with a city by visiting that city. A travel cost is also incurred to visit a city that depends on the city visited immediately before visiting the city in consideration. The salesman always starts and ends its tour at a base city (depot), assumed to have zero profit. The tour always includes at least one city in addition to the depot. The objective of the PTP is to find a tour that maximizes the total profit minus total travel cost. Note that visiting all the n cities may not maximize the objective, and an optimal tour can contain any number of cities between 2 to n. The PTP comes under the category of traveling salesman problem with profits. *Feillet* et al. [18] classified the traveling salesman problem with profits into three generic problems based on the two values, viz. collected profit and travel cost incurred. These two values can be either part of the objective function or as a constraint. Three generic problems are as follows.

1. Both values are part of the objective function: the goal is to find a tour that maximizes the collected profit minus travel cost. Alternatively, it can be considered as minimizing the travel cost minus collected profit.
2. The value of travel cost as a constraint: the goal is to find a tour that maximizes collected profit in such a way that the travel cost does not exceed a predefined value.
3. The value of profit as a constraint: the goal is to find a tour that minimizes travel cost in such a way that the collected profit should not be less than a predefined value.

These three generic problems have appeared with many names in the literature. However, these three problems are usually referred to as profitable tour problem [14], orienteering problem [38] and prize collecting traveling salesman problem [1] respectively. In this paper, we address the profitable tour problem, which was first defined by Dell'Amico *et al.* [14]. The PTP is also addressed as an augmented traveling salesman problem in [34]. PTP, being a generalization of the TSP, is also $\mathcal{NP}$-hard [21]. The PTP finds applications in logistics and home delivery services.

Several variants of the PTP exist in the literature. A variant of the PTP, profitable arc tour problem was introduced in [17], where the profits are associated with arcs instead of cities or vertices. Sun *et al.* [36] introduced a time dependent capacitated PTP with time windows and precedence constraints. The objective of this problem is also to maximize the profit collected minus the total travel cost. The authors proposed two methods, viz. an exact solution method called tailored labeling algorithm and a restricted dynamic programming heuristic algorithm. The exact solution method's computational results show that most cases of up to 75 requests can be optimally resolved within the

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specified two weeks time period and some cases remain unresolved. The restricted dynamic programing heuristic algorithm can find the good quality solutions for all the cases in short execution times. Lera-Romero *et al.* [25] studied the time dependent PTP with resource constraints, and, proposed a mixed integer linear programming formulation and a tailored branch-and-cut algorithm. Bruni *et al.* [2] introduced a fascinating variant of PTP with stochastic costs under a risk-averse perspective. To solve this problem, the authors proposed two metaheuristics approaches, viz. a genetic algorithm(GA) and a tabu search (TS) algorithm.

Zhang *et al.* [46] introduced a probabilistic version of PTP. This version can be used to resolve a scenario in which a collection of consumers, each with a probability of requesting service, is presented and only a subset must be chosen and served. The authors provided a non-linear mathematical programming formulation and developed a genetic algorithm to solve this problem. Chentli *et al.* [8] introduced PTP with simultaneous pickup and delivery services. The authors provided a mathematical formulation and developed an extension of the adaptive large neighborhood search heuristic called selective adaptive large neighborhood search (sALNS) heuristic for solving it. Another interesting variant of this problem called multi-vehicle profitable pickup and delivery problem is studied in [20]. Chentli *et al.* [6] addressed a well-known vehicle routing problem variant called capacitated profitable tour problem (CPTP). The authors presented a hybrid iterative local search (ILS) heuristic, which uses as local search a large neighborhood search (LNS) heuristic and a variable neighborhood descent with random neighborhood ordering (RVND). The authors presented the impact of iterated local search heuristic hybridization on CPTP in [7]. Cortés-Murcia *et al.* [9] showed a generalization of CPTP called the electric capacitated profitable tour problem with mandatory stops (ECPTPMS). To solve this problem, the authors proposed a mathematical programming formulation and developed a branch-and-price algorithm. Bulhões *et al.* [3] introduced the vehicle routing problem with service level constraints, which can be considered as an extension of CPTP. To solve this problem, a compact mathematical programming formulation, a branch-and-price algorithm and a hybrid genetic algorithm with population management have been proposed in [3].

Of late, there is an emerging trend to solve the combinatorial optimization problems utilizing machine learning based methods. These machine learning based methods can learn the appropriateness of different heuristics in different circumstances, can automatically produce a new heuristic depending on the problem instance at hand, and can be employed to solve the complete problem or a part of it. TSP and its variants are no exception, and several machine learning based approaches have been proposed for them, e.g., [24,43–45].

Compared to other variants of the TSP, the PTP did not get that much attention from the researchers. Still, researchers from different fields of study including heuristics and metaheuristics need to explore the basic version of PTP, though a few heuritic and metaheuristic approaches exist in the literature for variants of PTP as mentioned in previous paragraphs. Developing heuristics and metaheuristics for PTP is harder than TSP as one has to deal with deciding which cities to visit in addition to deciding the ordering among cities to be visited, and both aspects are equally vital in order to find a good solution. In this paper, we have proposed a multi-start hyper-heuristic (MSHH) approach, a multi-start iterated local search (MS-ILS) and a multi-start general variable neighborhood search (MS-GVNS) for the PTP. Hyper-heuristics can be considered as high-level strategies that manage a set of low-level heuristics and work either by selecting a heuristic from available heuristics or generating a new heuristic from components of available heuristics at each decision point in the search process and applying the heuristic selected/generated [4,5]. Hence, to deal with a wide range of instances of PTP, a hyper-heuristic is used. This hyper-heuristic is designed keeping in mind the specific needs of PTP, and, utilizes several low-level heuristics, each catering to different characteristics of PTP. Our multi-start hyper-heuristic will be referred to as *MSHH* subsequently. Iterated local search is a simple and powerful metaheuristic for solving combinatorial optimization problems [26,27]. We have developed a multi-start iterated local search *MS-ILS* for the PTP which utilizes variable neighborhood descent [22] for local search. Variable neighborhood search (VNS) is one among the most successful metaheuristic techniques that utilizes systemic changes to the neighbourhood structure within a local search to solve the global optimization problems. We have developed a multi-start general variable neighborhood search *MS-GVNS*. General variable neighborhood search (GVNS)[22] is a variant of VNS where the variable neighborhood descent (VND) method is used as the local search. Computational results on 77 benchmark instances under different scenarios show the effectiveness of our proposed approaches.

The remainder of this paper is organized as follows. Section 2 introduces the notational conventions, formally defines the PTP and analyzes its search space. Section 3 provides an overview of hyper-heuristics. Section 4 describes the proposed multi-start hyper-heuristic approach. Section 5 presents multi-start iterated local search (MS-ILS) approach. Section 6 presents multi-start general variable neighborhood search (MS-GVNS) approach. Section 7 presents the computational results and their analysis. Finally, Section 8 outlines some concluding remarks regarding contributions made and directions for future research.

2. Formal definition of PTP

2.1. Problem definition and notational conventions

Given a complete directed graph $G = (V, E)$, where $V = \{1, 2, \dots, n\}$ is the set of cities, $E = \{(i, j) | i, j \in V\}$ is the set of edges, a distance d_{ij} is associated with each edge $(i, j) \in E$, a profit p_i is associated with each city $i \in V$, a designated city $h \in V$ with zero profit (p_h=0) known as base city or depot where the salesman has to start and end his tour, and the travel cost per unit of distance is α. The cities that are part of the tour are termed visited cities, whereas the remaining cities are termed unvisited cities. The objective of the PTP is to find a tour that has at least one city in addition to the depot and that maximizes the net profit gained, which means the total profit collected from the visited cities after deducting the total travel cost incurred in visiting these cities. By introducing binary variables y_i to indicate whether city i is visited ($y_i = 1$) or not ($y_i = 0$), and another binary variable x_{ij} to indicate whether edge (i, j) is part of the tour ($x_{ij} = 1$) or not ($x_{ij} = 0$), an integer programming model for the PTP can be formulated as follows:

$$\text{Maximize} \quad N_{PTP} = \sum_{i \in V} P_i * y_i - \alpha * C_{PTP} \tag{1}$$

$$C_{PTP} = \sum_{i \in V} \sum_{j \in V} d_{ij} x_{ij} \tag{2}$$

subject to:

$$\sum_{i \in V} x_{hi} = 1 = \sum_{i \in V} x_{ih} \tag{3}$$

$$\sum_{(i,j) \in E} x_{ij} + \sum_{(k,i) \in E} x_{ki} = 2y_i \quad \forall i, j, k \in V \tag{4}$$

$$\sum_{i \in S} \sum_{j \in S} x_{ij} \leq |S| - 1, \quad \forall S \subset \{j : (j \in V) \wedge (y_j = 1)\} \subseteq V \tag{5}$$

$$x_{ij}, y_i \in \{0, 1\} \quad \forall (i, j) \in E, i \in V. \tag{6}$$

Eq. 1 is the objective function for the PTP and it maximizes the net profit, i.e., the total profit collected after deducting the travel cost as computed in Eq. 2. Eq. 3 ensures that a tour must start and end at the depot (i.e., node h), and has at least one city other than the depot. Eq. 4 satisfies the constraints of the indegree and outdegree of the visited cities. Eq. 5 represents the sub tour elimination constraint. 6 enforces the binary nature of the decision variables x_{ij}, and y_i. Throughout this paper, we will use node and city interchangeably.

It is to be noted that a tour can have any number of cities between 2 and n including depot, and, as the value of α increases, the net profit reduces. PTP involves decision making about which cities to visit and in what order. Hence, PTP has the characteristics of subset selection and permutation. On the other hand, TSP has the characteristic of permutation only as all cities need to be visited and we have to decide the ordering among cities only.

2.2. Solution space size

In this section, we compute the solution space (set of all feasible solutions) size for the PTP.

Theorem 1. *The solution space size of PTP on n cities is* $\sum_{k=2}^{n} \binom{n}{k} \times (k-1)!$

Proof. The PTP can possibly have $2 \le k \le n$ cities. These k cities can be chosen from given n cities in $\binom{n}{k}$ ways. And the chosen k cities can be ordered in $(k-1)!$ different ways in a tour. Hence, the solution space size of PTP is $\sum_{k=2}^{n} \binom{n}{k} \times (k-1)!$ □

3. Overview of Hyper-Heuristics

For a decade or so, hyper-heuristics are receiving growing attention from the researchers owing to their ability to swiftly adapt as per the problem instance at hand, thereby, ensuring good quality solutions over a wide range of instances of a problem [4]. The term *hyper-heuristic* was first used in a technical report by Denzinger *et al.* [15] as a strategy to combine a range of artificial intelligence methods for automated theorem proving, and does not provide any definition of hyper-heuristics. However, the basic idea of automating the design and/or selection of heuristics is proposed in the early 1960s by Fisher *et al.* [19] and Crowston *et al.* [12]. In Cowling *et al.* [10], hyper-heuristics are described as the heuristics to choose the heuristics in the context of combinatorial optimization. The hyper-heuristics can be considered as high-level strategies which manage a set of low-level heuristics, and, work either by selecting a heuristic from available heuristics or generating a new heuristic from components of available heuristics at each decision point in the search process and applying the heuristic selected/generated [4,5]. Consequently, a hyper-heuristic operates in the search space of heuristics, selecting and applying a single or a combination of low-level heuristics from a given set of heuristics. This is the fundamental difference between a hyper-heuristic and a metaheuristic as the latter directly operates over a search space of solutions to the problem under consideration. In other words, a hyper-heuristic searches for a good heuristic to solve the problem at hand, whereas a metaheuristic searches for a good solution to the problem at hand [5]. The motivation for the development of hyper-heuristics comes from the fact that the performance of different heuristics may vary significantly depending on the specific characteristics of the problem instance under consideration. Moreover, individual heuristics may be particularly effective at certain stages in the search process (for example, when exploration is more important than exploitation), while performing poorly at other stages. Therefore, it is fair to expect that several heuristics applied in an appropriate manner may produce better solutions than individually using any single heuristic. It is to be noted that a metaheuristic may also be used as a low-level heuristic in a hyper-heuristic or a hyper-heuristic may be used inside a metaheuristic for local search / neighborhood search.

Based on their nature, the hyper-heuristics can be classified into two categories.

- Heuristic selection: methodologies for choosing from existing heuristics.
- Heuristic generation: methodologies for generating new heuristics from the components of existing heuristics.

Our hyper-heuristic approach for the PTP, which is described in the next section, falls in the former class, i.e., it selects one or more low level heuristics as per the selection policy and applies the heuristic(s) selected during each stage in the search process.

For a detailed survey on hyper-heuristics and its applications, the interested readers may refer to [4].

4. Multi-start hyper-heuristic approaches for PTP

Depending on the profit values associated with the cities, distance values associated with the edges and the value of α, number, composition and ordering of cities in the optimal tour differ widely from one instance to another. As a hyper-heuristic can quickly adapt according to the characteristics of the instance at hand, we have developed a hyper-heuristic approach for PTP where several low-level heuristics are used. Hereafter, this approach will be referred to as MSHH.

Following subsections describe the main features of our MSHH algorithm for the PTP.

4.1. Solution encoding

We have represented a solution as a linear permutation of the visited cities where depot always occupies the first position. This position of the depot is fixed and can not be altered by any approach. This is done to ensure that the size of the resulting search space (set of all possible solution representations) is same as the size of the solution space (Section 2.2), and hence, there is no redundancy. Please note that a tour is a circular permutation, and, representing a tour as a linear permutation without the restriction on the position of depot results in redundancy as k linear permutations corresponds to a single circular permutation where k is the number of visited cities. A heuristic works in the search space, and the presence of redundant solutions in the search space makes the heuristic to search a space larger than the solution space which can hamper its performance.

4.2. Fitness

Objective function (1) itself is used as the fitness function. Hence, a solution with a higher value of the objective function is considered to be more fit than the solution with a lower value.

4.3. Initialization of solutions by using a construction heuristic

An initial solution S is constructed by following an iterative procedure. This procedure starts with a partial tour containing only depot and randomly determining the number of cities k ($2 \le k \le n$) to be visited. Then during each iteration an unvisited city is selected at random and inserted at the best position in the tour. The best position means the position which yields the highest objective function value. This process is repeated till we have k cities in the tour (including the depot). Please note that inserting a city may decrease the value of the objective function also. However, the process continues till we have k cities in the tour.

4.4. Generation of new solutions by using low-level heuristics

An effective new solution generation procedure should consider all the characteristics of the problem at hand, and, should also maintain a proper balance among the considerations given to different characteristics. The hyper-heuristic generates a new solution S' from the present solution S. The hyper-heuristic is provided with eight low-level heuristics, viz. LH_1, LH_2 LH_3 LH_4LH_5 LH_6 LH_7 and LH_8 [18]. First five heuristics are best improvement heuristics, i.e., they evaluates all possible moves (including retaining the current solution) and choose the best move. As a result, if there is no improving move for any of these heuristics then there is no point in applying that heuristic again till configuration of the tour gets changed by some other heuristic. These five heuristics are used accordingly. Our eight heuristics are described below.

(a) Before (b) After

Fig. 1. Illustration of a swap move.

(a) Before (b) After

Fig. 2. Illustration of an exchange move.

1. *Addition(LH_1):* This heuristic adds an unvisited city at its best possible position in the tour if it leads to largest increase in objective function value. To determine this, all unvisited cities needs to be tried at all positions in the tour. This heuristic is like a constructive operator which tries to increase the set of visited cities. No unvisited city will be added in case no unvisited city exists which can increase the value of the objective function. To find the best increase in objective function value, $(n-k)$ unvisited cities need to be tried in k positions in the tour. Here, k is the length of the current tour. Therefore, this heuristic requires $(k*(n-k))$ operations (maximum number of operations occur when $k=\frac{n}{2}$), which has the worst case time complexity of $\mathcal{O}(n^2)$.
2. *Removal(LH_2):* This heuristic removes a visited city from the tour if it leads to largest increase in objective function value. To determine this, all visited cities needs to be tried for removal. This heuristic is like a destructive operator which tries to decrease the size of the subset to be visited by the salesman. Again no visited city will be removed if no city exists in the tour whose removal leads to the increase in the objective function value. To find the best increase in objective function value, all the $k-1$ visited cities except the depot need to be checked one by one for removal. Therefore, exploring this heuristic requires $(k-1)$ operations which has the worst case time complexity of $\mathcal{O}(n)$.
3. *Swap(LH_3):* This heuristic is designed to deal with the permutation of cities on the salesman tour. This heuristic is a perturbation operator that swaps the positions of two cities in the tour. Among all possible swaps, the swap that yields the largest increase in objective function value is performed. No swap is performed if there exists no move that leads to increase in objective function value. Note that this operator does not change the subset of cities to be visited by the salesman, but the order in which the salesman visits the city. Fig. 1 illustrates a swap move. The red city is swapped with the blue city, and as a result of this swap, the set of visited cities remains the same, the sum of profits in the objective function remains unchanged, but travel distance is reduced, thereby our objective function value is improved. For identifying the large objective value, this heuristic tries to swap each of the $k-1$ visited cities excluding the depot with remaining $k-2$ cities in the tour. Doing so requires $((k-1)*(k-2)/2)$ operations which has the worst case time complexity of $\mathcal{O}(n^2)$.
4. *Exchange(LH_4):* This heuristic is designed to exchange a city between the set of visited cities and the set of unvisited cities. This heuristic is a perturbation operator that exchanges a visited city with an unvisited one. In this exchange, the unvisited city is inserted at its best position in the tour which can be quite different from the position of the visited city that got removed for this exchange. Among all possible exchanges, the exchange that yields the largest increase in objective function value is performed. No exchange is performed if there exists no exchange move that can increase the objective function value. Note that this operator does not change the size of the set of visited cities, but the content of it and order of visited cities. Fig. 2 illustrates an exchange move. The red city is exchanged with a blue city, and as a result, the sum of profit increases or travel distance reduces, thereby our objective function value is improved. This heuristic tries to exchange each of the $(n-k)$ unvisited cities with $k-1$ visited cities excluding the depot in the tour and then finds the best position for the unvisited city to be inserted. Therefore, exploring this heuristic requires $((n-k)*(k-1)*(k-1))$ operations which has the worst case complexity of $\mathcal{O}(n^3)$.
5. *2-Opt(LH_5):* This heuristic performs the standard 2-opt move [11] and tries to improve a solution by removing two non-adjacent edges from the tour and then adding two edges in place of them to get a valid tour. Among all possible such moves, the move that yields the largest increase in objective function value is performed. No move is performed if there exists no such move that can increase the objective function value. This heuristic is a perturbation operator. Note that this operator does not change the set of visited cities, but the order in which cities get visited. Figure 3 shows the illustration of 2-Opt move. Worst case complexity of a 2-opt move described above is $\mathcal{O}(n^2)$.
6. *Relocate (LH_6):* This heuristic removes a visited city (excluding depot) randomly and reinserts it at the best position in the tour. This heuristic is a perturbation operator. Note that this operator does not change the set of visited cities, but the order among them. Fig. 4 shows the illustration of relocate move. The red city is removed from its original position and reinserted at its best position, thereby reducing the travel distance and improving the objective value. Obviously, the worst case complexity of a relocate move is $\mathcal{O}(n)$
7. *Multi-Removal-and-Reinsertion (LH_7):* This heuristic removes each visited city from the tour with probability 0.5. These removed cities are then inserted back into the tour one-by-one in some random order at their best possible positions in the tour. This heuristic is a perturbation operator and deals with the permutation of visited cities.

(a) Before (b) After

Fig. 3. Illustration of a 2-Opt move.

(a) Before (b) After

Fig. 4. Illustration of a relocate move.

(a) Before (b) After

Fig. 5. Illustration of a Multi-Removal-and-Reinsertion move .

Fig. 5 shows the illustration of the multi-removal-and-reinsertion move. The red and blue cities are removed from the tour and reinserted at their best possible positions in the tour. As a result, the set of visited cities stays the same, the amount of profit stays unchanged in the objective function, but travel distance may decrease, thereby improving our objective value. Worst case complexity of this heuristic is $\mathcal{O}(n^2)$ as reinserting all the removed cities back at their best possible positions requires $\mathcal{O}(n^2)$ operations in worst case.

8. *Multi-Removal-and-Addition (LH_8):* This heuristic removes each visited city from the tour with probability 0.4. These removed cities are added to the set of unvisited cities. Then unvisited cities are tried for insertion into the tour in an iterative manner. Initially, all unvisited cities are marked as untried. Then during each iteration, an untried unvisited city is selected randomly and tried for insertion into the tour at the best possible position. If the insertion at the best possible position in the tour improves the objective function, then this unvisited city is inserted, otherwise it is not inserted into the tour and marked as tried. This process continues till no untried unvisited city remains. Note that this heuristic not only changes the content of the set of visited cities, but also ordering among them. Fig. 6 illustrates the multi-removal-and-addition move. Like the previous heuristic, worst case complexity of this heuristic is also $\mathcal{O}(n^2)$ as trying all unvisited cities for insertion at their best possible positions requires $\mathcal{O}(n^2)$ operations in the worst case.

4.5. Acceptance criteria

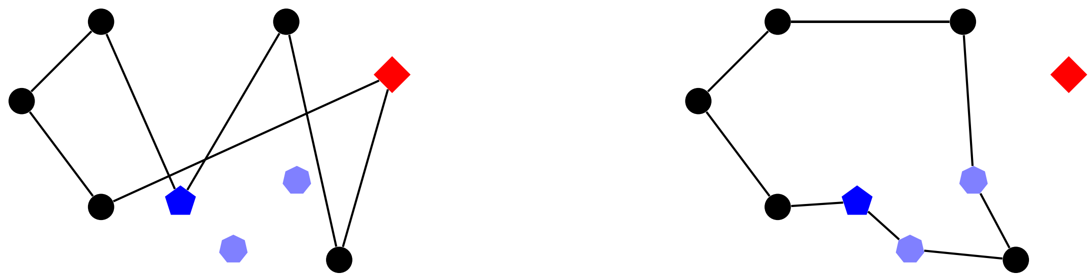
Several acceptance criterias have been proposed in the literature [4]. We have tried AA (all acceptance), OI (only improvement) acceptance criterias with our hyper-heuristic approach. In AA criteria, the new solution returned after applying low level heuristics always replaces the current solution irrespective of its own fitness. On the other hand, in OI criteria, this replacement is done only when new solution is better than the current solution. Between these two criterias, only improvement (OI) criteria yields better quality solutions. Consequently, we will report the results only with OI criteria in this paper.

4.6. Selection mechanism

The selection mechanism decides which low level heuristic to use for generating a new solution from the current solution and contributes significantly in the success of a hyper-heuristic approach. We have used two selection mechanisms, viz. random selection mechanism and greedy selection mechanism, thereby yielding two variants of our hyper-heuristic approach. The variant with random selection mechanism will be referred to as MSHH_RAND, whereas the variant with greedy selection mechanism will be referred to as MSHH_GREEDY. As mentioned in the beginning of Section 4.4, if any of the first 5 heuristics fails to improve a solution, then they will be not be used till the configuration of the solution changes in either of the selection methods. These two selection mechanisms used by us are described below:

4.6.1. Random selection mechanism

The random selection mechanism selects a low-level heuristic randomly and applies it to the solution under consideration to get a new solution. The new solution is accepted / rejected based on the acceptance criteria. The complexity of an iteration of MSHH_RAND is the complexity of the low level heuristic used in that iteration.



(a) Before   (b) After

Fig. 6. Illustration of a Multi-Removal-and-Addition move.

4.6.2. Greedy selection mechanism

Depending on their nature, the low-level heuristics described in Section 4.4 are divided into three groups, viz. permutation, subset alteration, and combination. The permutation group of heuristics does not change the set of visited cities, but changes the ordering among them. Since the set of visited cities remains the same, the sum of profits in the objective function remains unchanged. In order to increase the objective function value, our goal is to decrease the second term (travel cost) of the objective function (Eq. 2) by changing the visiting order of the cities in the tour. This group include swap (LH_3), 2-opt (LH_5), relocate (LH_6) and multiple removal and reinsertion (LH_7). The subset alteration group of heuristics improves the objective value by altering the subset of visited cities by adding or removing a single city. These include addition (LH_1) and removal(LH_2). Note that when a city is added or deleted, except for this city, the visiting order remains the same. The combination group includes exchange(LH_4) and multi removal and addition(LH_8) and changes both the set of visited cities and ordering among them. The greedy selection mechanism (MSHH_GREEDY) selects one low-level heuristic randomly from each group and applies each of the three selected heuristics on the solution at hand to get three new solutions. Among these three new solutions, the best solution is returned. The new best solution is accepted / rejected based on the acceptance criteria. The complexity of an iteration of MSHH_GREEDY is the complexity of the highest complexity low level heuristic among the three low level heuristics used in that iteration. Fig. 7 depicts the greedy selection mechanism.

Algorithm 1: Pseudo-code of MSHH approach for PTP .

Input: Set of parameters for MSHH Algorithm and a PTP instance
Output: Best solution found
for $j \leftarrow 1$ *to* N_{rst} **do**
    $S_j \leftarrow$ Generate_Initial_Solution();
    while *Termination condition not satisfied* **do**
        $L_h \leftarrow$ Selection_Mechanism($LH_1, LH_2, \ldots, LH_8$)
        $S' \leftarrow$ Create_New_Solution(S_j, L_h);
        if *(S' is better than S_j)* **then**
            $S_j \leftarrow S'$;
best $\leftarrow$ best solution among $S_1, S_2, \ldots, S_{N_{rst}}$;
return *best*;

The pseudo-code of our MSHH approach is given in Algorithm 1, where N_{rst} is the number of times the algorithm restarts. *Selection_Mechanism($LH_1, LH_2, \ldots, LH_8$)* is a function that returns the set L_h of low-level heuristics selected as per selection mechanisms described in Section 4.6. L_h contains a single heuristic for random selection mechanism and three heuristics for greedy selection mechanism. *Create_New_Solution(S, L_h)* is a function that produces a new solution by applying the heuristics in L_h on current solution S as per the selection mechanism described in Section 4.6. *Generate_Initial_Solution()* is another function that generates an initial solution as per the procedure described in 4.3.

5. Multi-start iterated local search approach for PTP

We have also developed a multi-start iterated local search (MS-ILS) approach for PTP where variable neighborhood descent (VND) search is utilized as a local search. Iterated local search (ILS) is a metaheuristic that maintains a single solution which is improved in an iterative manner. ILS posses several desirable features [26,27]: simplicity, ease of implementation, robustness, and effectiveness. ILS comprises four principal components, viz. initial solution generation, local search, perturbation procedure & acceptance criterion. ILS starts with generation of an initial solution which is improved through local search. Then this solution becomes the current solution and an iterative process ensues. During each iteration, the current solution is perturbed and local search is applied on the perturbed solution to obtain a new solution. Then depending on the acceptance criteria, this new solution may replace the current solution. Common acceptance criterias are mandatorily replacing the current solution with the new solution, and, replacing only if the new solution is better than the current one. The former criteria yields a first improvement strategy, whereas the latter one results into a random-walk sort of strategy. This iterative process continues till the termination condition is met. Already, ILS has been used to solve numerous combinatorial optimization problems, e.g. [13,31,33,35,39,41], where it has shown its effectiveness in comparison to other state-of-the-art metaheuristic approaches.

Our ILS based approach makes multiple starts and the overall best solution among the best solutions found in each start is returned as the solution found by MS-ILS. Subsequent subsections describes the salient features of MS-ILS.

5.1. Solution encoding, fitness and initial solution generation

Solution encoding, fitness and initial Solution generation is same as describe in Section 4.1, Section 4.2 and Section 4.3 respectively.

5.2. Local search

MS-ILS utilizes variable neighborhood descent (VND) as local search. VND proposed by Mladenović and Hansen [28] is a variant of variable neighborhood search [28] that explores various neighborhood structures in a deterministic fashion. Let S be a solution for a problem under consideration and $f(S)$ its fitness value, and let $\mathcal{N} = \{N_1, N_2, \ldots, N_{r_{\max}}\}$ represents a set of $r_{\max}$ different neighborhood structures. The VND begins taking a solution S as input, and setting the neighborhood indicator variable r to 1, then an iterative process ensues. During each iteration, VND tries to find a solution better than the current solution S in the

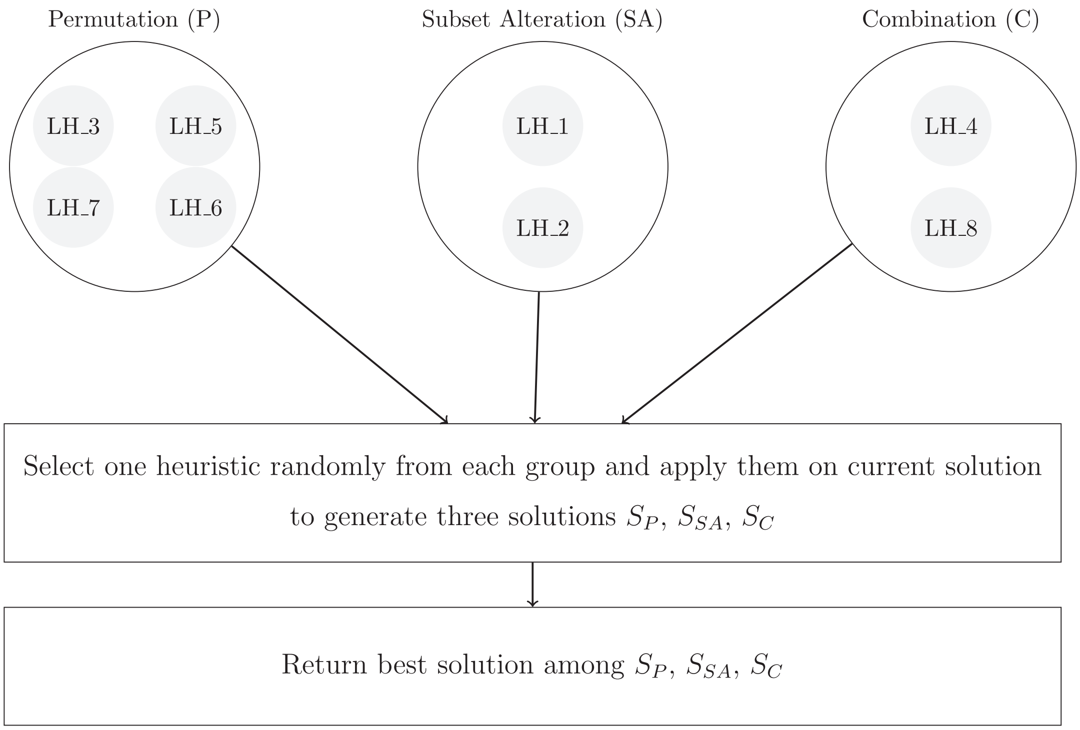


Fig. 7. Greedy selection mechanism .

neighborhood N_r of S. If a better solution is found then the current solution S is replaced with that solution and the VND moves to the first neighborhood structure (i.e., r is reset to 1) in the next iteration, otherwise the VND explores the next neighborhood structure for a better solution in the next iteration. This process is repeated till VND fails to find a better solution in all the $r_{\max}$ neighborhoods. The search for a better solution in each neighborhood can be done in a best improvement or a first improvement manner. VND is frequently used as a local search owing to its ability to explore multiple neighborhood structures in a systematic manner, and, thereby providing a solution of quality higher than that obtained by a method relying on a single neighborhood structure.

The first five heuristics described in Section 4.4 also constitute neighborhoods of their own. We have considered these five neighborhoods in our VND for PTP, and, refer to neighborhoods corresponding to addition, removal, swap, exchange, and 2-opt heuristics by N_1, N_2, N_3, N_4 and N_5 respectively. So $r_{\max} = 5$. All these neighborhoods are explored in first improvement manner, i.e., the search for a better solution is stopped at the moment first solution better than the current solution is found. The pseudo-code of VND is provided in Algorithm 2.

Algorithm 2: Pseudo-code of VND local search for PTP .

Input: A PTP solution S
Output: Improved solution S
$r \leftarrow 1$;
repeat
  $S' \leftarrow$ First-Improvement(S, N_r);
  if *S' is better than S* **then**
    $S \leftarrow S'$;
    $r \leftarrow 1$;
  else
    $r \leftarrow r + 1$;
until $r > 5$;
return S;

5.3. Perturbation procedure and acceptance criteria

The goal of the perturbation mechanism is to escape from the currently local optimal solution. Our perturbation procedure is based on iteratively swapping the positions of two visited cities. During each iteration, two visited cities are selected randomly and their positions in the tour are swapped. This is done 6 times in total.

The current solution is accepted/rejected based on the acceptance criteria described in Section 4.5. The pseudo-code of MS-ILS approach is presented in Algorithm 3, where N_{rst} is the number of times algorithm restarts, *Generate_Initial_Solution()* is a procedure that generates an initial solution as per the procedure described in 4.3. The termination condition is described in Section 7.

Algorithm 3: Pseudo-code of MS-ILS approach for PTP .

Input: Set of parameters for MS-ILS Algorithm and a PTP instance
Output: Best solution found
for $j \leftarrow 1$ *to* N_{rst} **do**
  $S_j \leftarrow$ Generate_Random_Initial_Solution();
  $S_j \leftarrow$ VND(S_j);
  while *Termination condition not satisfied* **do**
    $S' \leftarrow$perturbation(S_j);
    $S'' \leftarrow$ VND(S');
    if *(S'' is better than S_j)* **then**
      $S_j \leftarrow S''$;
best $\leftarrow$ best solution among $S_1, S_2, \ldots, S_{N_{rst}}$;
return *best*;

6. Multi-start general variable neighborhood search for PTP

Variable neighborhood search (VNS) is a metaheuristic for solving combinatorial optimization problems, proposed by Mladenovic and

Hansen [28]. The key idea behind VNS is the systematic search of different neighborhood structures to achieve an optimal or a close-to-optimal solution. The VNS consists of executing alternately, one shake phase (diversification phase) to escape from local optima and one local search phase (intensification phase) to improve the solution together with neighborhood change. General variable neighborhood search (GVNS) [22] is a variant of VNS where the variable neighborhood descent (VND) method is used as the local search. The effectiveness of GVNS has been validated by numerous successful applications, as can be seen in some recent works [29,32,37,40]

Inspired by the success of GVNS in solving combinatorial optimization problems, we have developed a Multi-start General Variable Neighborhood Search (MS-GVNS). The pseudo-code of the proposed MS-GVNS approach is given in Algorithm 5, where N_{rst} is the number of times algorithm restarts. The algorithm comprises four components. First is the generation of the initial solution which is same as in our previous two approaches. Second is shake function for escaping the local maxima traps. Our shake function employs two neighborhoods and generates a random solution S' in one of the two neighborhoods. The two neighborhoods used in shake function consist of neighborhoods implicitly defined by Multi-Removal-and-Reinsertion (LH_7) and Multi-Removal-and-Addition (LH_8) heuristics which are used as first and second neighborhoods respectively. The pseudo-code of shake function is given in Algorithm 4 , where *Generate_Solution(*S*,* LH_i*)* is a function that generates a solution as per heuristic LH_i. The third component is the local search to improve the solution. In our approach, same VND as described in Section 5.2 is used as a local search. The fourth component is the acceptance criteria as described in Section 4.5. MS-GVNS uses OI acceptance criteria.

Algorithm 4: Pseudo-code of shake function .

```
Input: A solution S, neighborhood to be used ℓ
Output: A random solution S′ in chosen neighborhood
if ℓ == 1 then
    S′ ← Generate_Solution(S, LH_7);
else
    S′ ← Generate_Solutin(S, LH_8);
return S′;
```

Algorithm 5: Pseudo-code of MS-GVNS approach for PTP .

```
Input: Set of parameters for MS-GVNS Algorithm and a PTP
       instance
Output: Best solution found
for j ← 1 to N_rst do
    S_j ← Generate_Random_Initial_Solution();
    ℓ ← 1;
    while Termination condition not satisfied do
        S′ ← Shake(S_j, ℓ);
        S″ ← VND(S′);
        if (S″ is better than S_j) then
            S_j ← S″;
            ℓ ← 1;
        else
            ℓ ← 3 − ℓ;
best ← best solution among S_1, S_2, …, S_{N_rst};
return best;
```

Please note that unlike the usual GVNS approaches [23], shake function and VND approach of MS-GVNS use disjoint neighborhoods. This was needed as the neighborhoods of VND were not sufficient to escape from the local maximas while solving PTP, thereby forcing us to use the large neighborhoods implicitly defined by LH_7 and LH_8.

7. Computational results

As our approaches are the first metaheuristic / hyper-heuristic approaches for the PTP, no benchmark instances were available in the literature. Hence, it is inevitable for us to generate new test instances. We generated 77 PTP benchmark instances based on the instances from standard TSPLIB[1]. These instances have cities ranging from 14 to 1379. First city in these instances is always the base city or depot with zero profit. A profit for each city $i \neq 1$ is randomly selected from the interval $[LV_i, UV_i]$, where $LV_i = \lfloor 0.4 \times \delta_i \rfloor$, $UV_i = \lceil 0.4 \times \frac{\delta_i + \Delta_i}{2} \rceil$, $\delta_i = \min_{j=1}^{n} d_{ij}$ and $\Delta_i = \max_{j=1}^{n} d_{ij}$. With these instances, 4 values of α are considered, viz. 0.25, 0.50, 0.75 and 1.0. These instances can be obtained through e-mail from the corresponding author. The MSHH, MS-ILS & MS-GVNS are restarted only 2 times, i.e., N_{rst}=2. We have allowed our approaches to be executed under two different termination criterias with a short and long time to compare our approaches from the perspective of fairness as different approaches may have different covergence behavior.

- **Short time (ST)** - $0.02 \times n$ seconds.
- **Long time (LT)** - $0.2 \times n$ seconds.

where n is number of cities in an instance. Our approaches are multi-start approaches, and hence, every time an approach is executed after a fresh start, it is allowed for a time $\frac{T}{N_{rst}}$, where T is total execution time allowed for the approach. Our approaches are implemented in C and executed on a Linux based 3.10 GHz Core-i5-8600 system with 8 GB RAM. The MSHH, MS-ILS & MS-GVNS have been executed on each test instance twenty times independently, each time with a different random seed. As there are two variants of MSHH, viz. MSHH_RAND and MSHH_GREEDY, we will report the results for each of the two variants. Hence, results are reported for four approaches, viz. MSHH_RAND, MSHH_GREEDY, MS-ILS and MS-GVNS.

Tables 1 to 8 report the results of our four approaches on each of the 77 instances with four different α values and under two different termination criterias. Tables 1, 2, 3, 4 report the results corresponding to values of α equal to 0.25, 0.50, 0.75, 1.0 respectively under ST termination criteria. Tables 5, 6, 7, 8 do the same under LT termination criteria. In all these 8 tables, the first column (named *Instance*) reports the name of the instance with total number of cities at the end. For each of the four approaches, we have reported the best solution, average solution quality and standard deviation of solution values over twenty independent runs under columns named *Best, Average* and *S.D* respectively. The best values are reported in bold font so that they can be identified easily.

Tables 9, 10, 11 and 12 are summary tables. The first two tables present the summary of results in terms of best solution quality under ST and LT termination criterias, whereas the latter two tables does the same in terms of average solution quality. In all these tables, approaches are compared in terms of number of instances out of 77 on which approach on the left performed better ('>'), worse ('<') or same (' = ') in comparison to the approach on the right.

Tables 1, 2, 3, 4, 9 and 11 clearly show that MSHH_RAND and MSHH_GREEDY performed better than MS-ILS and MS-GVNS under ST termination condition on majority of instances in terms of best and average solution quality both. In terms of best and average solution quality, MSHH_RAND performed slightly better than MSHH_GREEDY. MS-GVNS performed better than MS-ILS in terms of both best as well as average solution quality. MSHH_RAND is the best method overall under ST termination critria.

[1] http://elib.zib.de/pub/mp-testdata/tsp/tsplib/tsplib.html

Table 1
Results on PTP instances with $\alpha = 0.25$ under ST termination criteria .

Instance	MSHH_GREEDY			MSHH_RAND			MS-ILS			MS-GVNS		
	Best	Average	S.D	Best	Average	S.D	Best	Average	S.D	Best	Average	S.D
burma14	**799.25**	773.16	5.98	**799.25**	773.16	5.98	**799.25**	**799.25**	**0.00**	**799.25**	**799.25**	**0.00**
ulysses16	**1554.25**	**1554.25**	**0.00**	**1554.25**	**1554.25**	**0.00**	**1554.25**	**1554.25**	**0.00**	**1554.25**	**1554.25**	**0.00**
gr17	**617.00**	**617.00**	**0.00**	**617.00**	**617.00**	**0.00**	**617.00**	**617.00**	**0.00**	**617.00**	**617.00**	**0.00**
gr21	**1110.25**	**1110.25**	**0.00**	**1110.25**	**1110.25**	**0.00**	**1110.25**	**1110.25**	**0.00**	**1110.25**	**1110.25**	**0.00**
ulysses22	**3431.75**	**3431.75**	**0.00**	**3431.75**	**3431.75**	**0.00**	**3431.75**	**3431.75**	**0.00**	**3431.75**	**3431.75**	**0.00**
gr24	**428.50**	**428.50**	**0.00**	**428.50**	**428.50**	**0.00**	**428.50**	**428.50**	**0.00**	**428.50**	**428.50**	**0.00**
fri26	**355.75**	**355.75**	**0.00**	**355.75**	**355.75**	**0.00**	**355.75**	**355.75**	**0.00**	**355.75**	**355.75**	**0.00**
bayg29	**664.50**	664.09	0.09	**664.50**	**664.50**	**0.00**	**664.50**	**664.50**	**0.00**	**664.50**	664.09	0.09
bays29	**710.00**	**710.00**	**0.00**	**710.00**	**710.00**	**0.00**	**710.00**	**710.00**	**0.00**	**710.00**	**710.00**	**0.00**
dantzig42	**353.25**	352.65	0.14	**353.25**	**352.73**	**0.12**	**353.25**	350.15	0.21	**353.25**	352.57	0.15
swiss42	**760.75**	756.35	1.11	**760.75**	757.27	1.32	**760.75**	**760.75**	**0.00**	**760.75**	758.17	1.53
att48	**7549.00**	**7544.48**	1.04	**7549.00**	7542.89	**0.26**	7546.50	7529.30	5.06	**7549.00**	7535.73	1.71
gr48	**3137.25**	3135.62	1.12	**3137.25**	**3135.95**	**0.30**	**3137.25**	3134.95	0.53	**3137.25**	3134.34	0.67
hk48	**9637.75**	**9635.65**	**0.48**	**9637.75**	9633.42	0.99	**9637.75**	9627.52	1.83	**9637.75**	9627.41	2.37
eil51	210.25	209.35	0.31	**210.50**	209.34	0.04	**210.50**	**209.51**	0.05	210.25	209.25	**0.00**
berlin52	**5348.50**	5307.11	9.49	**5348.50**	5040.61	70.63	**5348.50**	**5326.29**	**8.27**	**5348.50**	5290.84	13.23
brazil58	**25068.75**	25008.74	10.90	**25068.75**	24953.16	27.91	**25068.75**	**25067.75**	**4.36**	**25068.75**	24980.94	7.13
st70	**476.50**	**475.40**	**0.02**	**476.50**	474.82	0.15	**476.50**	473.51	0.05	**476.50**	474.96	0.12
eil76	**311.75**	**310.12**	0.03	311.25	309.98	**0.01**	310.75	308.04	0.58	311.25	309.68	0.21
pr76	**98776.25**	**98505.68**	**62.07**	**98776.25**	98394.01	87.69	98739.00	98020.05	164.94	**98776.25**	98435.99	78.06
gr96	68803.25	**68751.00**	16.00	**68853.75**	68660.76	34.36	68673.00	68244.93	62.61	68833.25	68674.12	**1.63**
rat99	**1322.25**	1316.67	0.90	**1322.25**	**1319.50**	**0.23**	1317.25	1306.26	1.49	1321.00	1316.31	1.05
kroA100	**26888.75**	26819.39	15.91	**26888.75**	**26830.76**	**13.30**	26883.00	26699.36	28.08	**26888.75**	26813.26	17.32
kroB100	29835.00	29800.31	**6.61**	**29841.50**	**29806.81**	8.27	29765.50	29573.33	44.09	**29841.50**	29786.66	12.48
kroC100	**32971.75**	32955.39	**0.49**	**32971.75**	**32956.03**	0.64	32953.25	32794.61	33.18	**32971.75**	32947.89	1.23
kroD100	**29250.50**	29194.29	9.19	**29250.50**	**29215.94**	**1.28**	29123.75	28963.08	72.00	29233.50	29153.30	31.10
kroE100	**29286.50**	29260.61	10.69	**29286.50**	**29268.74**	4.07	29196.25	29078.00	69.69	29273.00	29227.34	**3.98**
rd100	**8700.50**	8668.56	**4.57**	**8700.50**	**8673.89**	7.60	**8700.50**	8603.94	14.09	**8700.50**	8663.46	7.39
eil101	**527.00**	523.64	0.78	526.75	**523.81**	0.36	521.25	517.55	0.85	**527.00**	522.90	**0.09**
lin105	24791.50	**24694.71**	7.05	24784.25	24684.38	**3.76**	24735.00	24595.19	17.48	**24811.00**	24680.70	7.85
pr107	**88659.25**	88622.96	8.32	**88659.25**	**88630.90**	1.24	88545.00	88360.80	15.76	**88659.25**	88624.77	**1.15**
gr120	9457.50	**9441.62**	0.43	**9460.00**	8970.21	107.43	9443.75	9373.76	0.51	9453.00	9426.06	**0.16**
pr124	**113631.50**	113589.10	**5.36**	**113631.50**	**113591.26**	6.59	113502.50	113140.64	53.20	**113631.50**	113516.98	6.53
bier127	160073.00	159328.31	84.78	**160094.50**	**159334.70**	**55.28**	159277.50	157739.89	183.51	160076.25	159219.05	148.36
ch130	7898.25	7876.68	**0.71**	**7908.75**	**7884.23**	0.91	7844.25	7803.39	1.69	7898.50	7868.62	1.18
pr136	**139220.50**	**138825.84**	25.38	139218.25	138788.20	**3.66**	138456.25	137650.91	40.74	139135.75	138634.09	115.09
gr137	141204.50	**141079.44**	28.63	**141253.75**	141037.95	20.25	140987.00	140526.55	57.69	141169.25	141038.44	**11.89**
pr144	**139455.75**	**139224.95**	**13.49**	**139455.75**	139190.23	47.95	139342.75	138864.05	28.09	**139455.75**	139135.91	33.30
ch150	**8782.25**	**8754.51**	**0.86**	8765.50	8748.33	1.51	8719.75	8675.40	1.86	8767.50	8739.99	1.61
kroA150	43924.00	43857.29	15.19	**43943.25**	**43868.38**	12.65	43788.50	43554.06	27.06	**43943.25**	43815.88	**4.50**
kroB150	**47407.50**	47308.93	10.31	47386.25	**47314.00**	12.62	47261.75	47056.78	19.33	47385.50	47278.10	**3.64**
pr152	**194344.00**	**194255.77**	**11.59**	**194344.00**	187473.95	1544.27	194165.25	193691.84	64.08	194306.75	194088.25	38.37
u159	**79385.00**	**79193.02**	**25.06**	**79385.00**	79116.43	98.00	79130.50	78644.23	68.89	**79385.00**	79098.40	43.38
si175	4922.25	4894.59	1.99	**4925.75**	**4894.76**	**1.72**	4902.00	4853.90	4.73	4922.00	4887.71	4.92
brg180	169010.00	168963.88	4.90	169032.50	168964.38	9.32	**169035.00**	**169005.25**	**0.06**	168997.50	168949.12	3.24
rat195	3879.25	3870.76	**0.23**	**3881.75**	**3871.62**	0.60	3854.00	3837.64	1.29	3879.25	3865.31	1.08
d198	50741.25	**50705.61**	6.46	**50747.50**	50298.41	97.46	50655.00	50575.46	**4.20**	50741.25	49865.88	194.63
kroA200	59275.25	59089.80	47.27	**59312.75**	**59125.41**	23.42	59018.00	58778.34	**1.45**	59201.00	59055.81	10.34
kroB200	**59996.25**	59855.15	**8.81**	59988.50	**59863.89**	9.55	59699.50	59484.26	62.06	59943.00	59787.78	28.17
gr202	83894.50	83749.60	18.96	**83945.00**	**83765.24**	41.24	83529.00	83245.14	15.69	83793.50	83643.62	**6.57**
ts225	260689.00	**259381.30**	149.68	**261348.25**	259353.17	457.70	259927.25	258944.52	**140.00**	260533.00	259234.34	189.42
tsp225	**7607.75**	**7587.67**	1.80	7604.50	7586.16	4.21	7562.75	7539.56	**1.39**	7588.75	7574.68	3.23
pr226	**305116.25**	**304906.19**	**9.51**	305051.50	304871.03	20.52	304875.00	304633.78	55.34	305047.25	304867.06	28.55
gr229	340068.00	339701.50	**6.82**	**340122.75**	**339761.06**	19.54	339380.50	338275.12	139.74	339982.50	339287.56	61.15
gil262	4665.50	4655.21	1.33	**4668.00**	**4655.81**	**0.59**	4646.25	4631.98	3.27	4661.50	4649.60	1.07
pr264	187450.50	187213.98	80.64	**187500.00**	**187234.69**	**11.11**	186812.75	186586.48	24.60	187338.75	187039.92	31.99
a280	5889.75	5869.25	2.47	**5896.50**	**5874.64**	1.57	5864.25	5836.30	1.42	5882.25	5863.60	**0.71**
pr299	145652.25	**145410.75**	10.95	**145685.75**	145387.64	**0.89**	145131.50	144856.08	38.24	145512.75	145300.64	4.62
lin318	106577.25	**106433.09**	13.74	**106604.75**	101133.61	1223.10	106264.00	106062.93	4.95	106576.75	106353.88	**1.86**
rd400	**38995.00**	38936.47	3.89	38986.50	**38938.22**	**2.47**	38898.50	38809.44	10.28	38949.75	38866.51	6.43
fl417	**85778.50**	**85711.76**	6.25	85759.75	85704.14	4.51	85749.25	85643.93	**4.40**	85747.25	85696.49	9.35
gr431	764113.00	763432.62	**21.94**	**764127.25**	**763505.38**	99.48	762750.50	761901.44	166.40	763951.25	762584.81	232.73
pr439	424397.75	423630.12	62.37	**424405.75**	**423757.81**	114.69	424166.25	422665.25	**12.79**	424157.50	423056.84	24.05
pcb442	**143768.50**	143428.50	**47.43**	143730.50	**143452.39**	75.17	143356.75	142848.89	70.58	143767.50	143164.84	54.28
d493	**160204.75**	**160102.55**	3.68	160188.25	160083.88	16.95	159981.50	159828.06	**2.39**	160116.25	159893.45	5.46
att532	112654.00	112589.90	7.95	**112685.00**	**112600.64**	6.28	112506.25	112377.74	7.23	112531.00	112423.64	**2.67**
ali535	934951.50	**933976.19**	**9.62**	**935137.00**	933964.88	152.25	933586.25	932110.12	81.18	933457.75	932249.62	113.19
si535	19295.75	19263.36	0.89	**19307.25**	**19270.70**	2.23	19254.25	19219.09	1.18	19254.25	19218.20	**0.28**
pa561	6625.00	6609.85	1.57	**6625.75**	**6612.98**	**0.18**	6597.00	6581.79	2.65	6596.75	6585.56	1.33
u574	**138307.75**	138081.38	**8.23**	138289.50	**138104.55**	18.12	138023.25	137739.75	36.53	138061.00	137770.34	18.90
rat575	**21108.50**	**21088.41**	2.79	21107.00	21087.04	3.26	21054.75	21025.87	**0.20**	21061.00	21035.92	1.53
p654	369240.00	369061.97	27.48	**369260.50**	**369073.12**	33.76	369171.25	368928.91	**16.36**	369151.00	368935.81	18.74
d657	**217622.50**	**217456.28**	20.93	217581.25	217429.45	15.72	217143.50	216928.84	19.71	217201.75	216994.44	**12.98**
gr666	**1195527.75**	**1194416.00**	117.23	1194919.00	1194042.88	247.97	1192511.25	1191182.38	**16.94**	1193480.50	1191499.25	189.86
u724	162846.50	162762.77	**1.37**	**162907.50**	**162783.80**	6.49	162649.25	162349.36	4.50	162613.00	162413.64	7.14
rat783	**34040.75**	**34013.54**	5.10	34037.50	34004.56	**3.05**	33962.00	33921.69	3.46	33964.00	33927.39	8.40
nrw1379	280498.75	**280279.66**	47.12	**280591.50**	278560.31	395.73	280211.50	279978.56	37.75	280216.50	280026.56	**18.31**

Table 2
Results on PTP instances with $\alpha = 0.50$ under ST termination criteria .

Instance	MSHH_GREEDY			MSHH_RAND			MS-ILS			MS-GVNS		
	Best	Average	S.D	Best	Average	S.D	Best	Average	S.D	Best	Average	S.D
burma14	**190.50**	186.25	0.98	**190.50**	188.38	0.49	**190.50**	**190.50**	**0.00**	**190.50**	188.38	0.49
ulysses16	**668.00**	577.30	**21.40**	**668.00**	540.08	49.11	**668.00**	**625.80**	32.53	**668.00**	618.55	30.87
gr17	**313.00**	293.05	4.58	**313.00**	285.38	3.70	**313.00**	**309.68**	**0.76**	**313.00**	290.88	5.08
gr21	**456.00**	409.55	10.66	**456.00**	399.23	**1.44**	**456.00**	**439.35**	3.82	**456.00**	426.43	6.78
ulysses22	**2420.50**	2237.10	42.07	**2420.50**	2282.95	178.82	**2420.50**	**2374.65**	**10.52**	**2420.50**	2328.80	21.04
gr24	**156.50**	141.25	3.50	**156.50**	133.00	2.52	**156.50**	**150.75**	1.32	**156.50**	142.62	**0.32**
fri26	**141.00**	136.38	1.06	**141.00**	136.38	1.06	**141.00**	**141.00**	**0.00**	**141.00**	**141.00**	**0.00**
bayg29	**319.50**	**319.50**	**0.00**	**319.50**	**319.50**	**0.00**	**319.50**	**319.50**	**0.00**	**319.50**	**319.50**	**0.00**
bays29	**253.50**	242.55	2.51	**253.50**	240.68	0.73	**253.50**	**253.50**	**0.00**	**253.50**	246.03	1.71
dantzig42	**201.50**	200.80	0.16	**201.50**	200.70	0.18	**201.50**	**201.20**	**0.07**	**201.50**	200.70	0.18
swiss42	**470.50**	464.88	1.29	**470.50**	466.38	2.84	**470.50**	**470.50**	**0.00**	**470.50**	464.88	1.29
att48	**5103.00**	5057.27	10.49	**5103.00**	5046.18	13.04	**5103.00**	5073.88	6.68	**5103.00**	**5077.88**	**5.76**
gr48	**2045.00**	**2045.00**	**0.00**	**2045.00**	2044.55	0.10	**2045.00**	**2045.00**	**0.00**	**2045.00**	2041.85	0.72
hk48	**6850.00**	**6843.62**	**1.46**	**6850.00**	6843.00	8.14	**6850.00**	6824.23	3.05	**6850.00**	6830.38	3.99
eil51	**122.50**	119.90	0.55	**122.50**	120.22	0.52	**122.50**	**120.62**	**0.32**	**122.50**	119.47	0.45
berlin52	**3544.00**	3320.97	26.04	**3544.00**	3153.18	52.15	**3544.00**	**3484.90**	**13.56**	**3544.00**	3293.32	57.51
brazil58	**20881.00**	20877.70	1.77	**20881.00**	20878.25	0.63	20855.50	19503.65	78.27	**20881.00**	**20881.00**	**0.00**
st70	**326.00**	**323.98**	**0.01**	**326.00**	323.95	**0.01**	**326.00**	322.07	0.59	325.50	323.65	0.08
eil76	200.50	198.00	0.34	**202.00**	**198.57**	0.21	199.50	196.00	0.80	201.00	197.95	**0.01**
pr76	**73221.00**	**72450.27**	**55.23**	**73221.00**	72175.38	118.29	73215.00	72160.45	120.34	**73221.00**	72262.52	218.51
gr96	**55594.50**	**55208.68**	88.51	55493.00	55159.85	**20.57**	55530.50	54875.28	145.04	55530.50	55196.50	68.02
rat99	**1038.50**	1032.30	**0.16**	**1038.50**	**1034.22**	0.63	1029.50	1017.65	0.26	1036.50	1029.88	0.66
kroA100	**21815.50**	21606.90	46.71	**21815.50**	**21703.22**	25.76	**21815.50**	21556.95	**24.67**	**21815.50**	21636.38	41.09
kroB100	24417.00	24330.65	24.01	**24499.50**	**24351.72**	12.90	**24499.50**	24120.53	**0.01**	24407.00	24295.83	11.74
kroC100	**27972.00**	27964.85	1.64	**27972.00**	**27972.00**	**0.00**	**27972.00**	27774.80	19.91	**27972.00**	27946.03	46.46
kroD100	**24013.50**	23911.90	**0.25**	**24013.50**	**23931.45**	4.23	23980.00	23703.91	70.64	**24013.50**	23829.78	45.14
kroE100	**24019.00**	23955.53	25.82	**24019.00**	**23975.15**	**6.39**	23959.00	23786.38	12.99	24011.00	23894.28	22.43
rd100	**6767.00**	6710.82	**5.81**	**6767.00**	**6717.98**	21.79	**6767.00**	6664.60	9.77	**6767.00**	6698.27	27.13
eil101	**381.00**	**376.52**	**0.12**	379.00	376.02	**0.12**	375.00	369.35	1.30	379.50	374.30	0.85
lin105	21273.50	21084.22	25.98	**21341.00**	**21100.33**	**3.63**	21330.00	20970.92	29.00	**21341.00**	21098.58	29.27
pr107	**77645.00**	77567.24	17.84	**77645.00**	**77591.35**	**12.31**	77391.00	77077.05	26.14	**77645.00**	77516.15	15.29
gr120	**7829.50**	**7807.40**	5.94	7826.00	7411.38	92.60	7793.50	7717.40	7.82	7823.00	7780.43	**2.51**
pr124	99234.50	**98947.38**	104.93	99234.50	98862.62	85.49	**99389.00**	98768.98	**77.31**	**99389.00**	98929.90	100.92
bier127	133935.50	132457.23	190.12	**134202.00**	**132848.38**	144.50	132656.50	131255.41	**83.60**	133804.00	132283.23	135.88
ch130	6383.50	6327.20	6.03	**6398.50**	**6340.68**	7.04	6335.50	6258.55	**5.27**	6370.00	6301.30	9.13
pr136	116126.00	115464.40	151.78	**116254.50**	**115663.20**	89.08	115112.50	114258.05	**27.89**	116073.00	115255.75	187.49
gr137	**124764.50**	124481.50	7.00	**124764.50**	124455.43	12.98	124302.50	123457.95	122.29	124678.00	**124506.45**	**1.16**
pr144	124708.50	**123863.40**	30.42	**124805.00**	123857.77	**3.73**	124780.00	123669.93	48.88	124717.00	123632.65	59.38
ch150	**7186.50**	**7129.12**	**0.03**	**7186.50**	7121.65	1.69	7118.50	7023.50	1.38	7177.50	7110.35	3.48
kroA150	**37514.50**	37356.78	26.33	**37514.50**	**37384.53**	19.72	37253.50	36941.40	**18.79**	37497.00	37300.53	29.02
kroB150	41024.00	40829.30	14.98	**41059.50**	**40859.97**	23.29	40925.50	40557.30	**2.94**	40967.00	40811.90	24.64
pr152	175929.50	167896.50	1748.61	**176028.50**	164481.95	2578.19	175695.00	**172375.83**	**377.31**	175864.50	167865.92	1787.62
u159	**68816.50**	**68541.25**	62.11	**68816.50**	68440.57	48.96	**68816.50**	67764.93	**4.95**	**68816.50**	68265.45	107.03
si175	**580.50**	565.92	3.34	**580.50**	**568.55**	**1.37**	559.50	518.80	2.57	**580.50**	548.85	2.56
brg180	168470.00	168381.75	10.73	168515.00	168369.25	20.82	**168520.00**	**168457.50**	**0.57**	168445.00	168344.25	5.56
rat195	**3294.00**	3274.68	**0.27**	3293.50	**3276.10**	0.44	3243.00	3217.72	5.00	3289.00	3262.53	1.84
d198	46791.00	46028.00	169.08	**46831.50**	**46047.60**	173.76	46679.00	45814.80	**164.31**	46797.00	45654.35	260.19
kroA200	51913.50	51661.07	**3.46**	**51957.00**	**51689.82**	61.29	51614.50	51223.12	14.60	51939.50	51626.72	57.52
kroB200	**52703.00**	52475.79	23.12	52698.00	**52495.03**	37.74	52196.00	51899.15	28.53	52638.00	52306.36	**22.45**
gr202	74100.50	73847.60	58.02	**74147.00**	**73867.88**	**6.17**	73587.00	73151.43	74.35	73967.50	73724.52	37.96
ts225	228301.50	226662.70	181.31	**229190.50**	**226742.95**	444.39	229178.50	225874.50	654.29	227233.50	226235.59	**109.45**
tsp225	**6637.00**	**6608.68**	0.99	6624.00	6602.77	**0.85**	6560.50	6508.02	5.04	6609.00	6586.10	1.24
pr226	**286018.00**	285504.88	103.95	**286018.00**	**285522.94**	101.87	285855.00	285253.34	**33.19**	285979.50	285444.41	102.11
gr229	308554.50	**308021.97**	47.15	**308682.50**	307765.44	131.90	307333.50	305142.53	101.17	308415.00	307366.44	**39.70**
gil262	**4092.50**	**4067.82**	1.30	4084.50	4066.57	**0.67**	4046.50	4022.97	5.40	4080.50	4057.75	2.47
pr264	**175246.50**	174592.48	**40.72**	175138.00	**174598.33**	111.80	174095.50	173509.09	53.94	175114.50	174238.02	91.08
a280	5228.00	5192.10	**3.01**	**5240.50**	**5203.65**	4.55	5177.00	5132.68	14.26	5215.00	5177.57	10.69
pr299	133451.50	133117.38	20.16	**133607.50**	**133153.08**	**11.11**	132695.00	132169.34	23.89	133285.50	132816.05	36.83
lin318	**96187.00**	**95627.02**	14.23	96033.00	90910.85	1082.30	95323.50	94857.57	36.72	95944.50	95377.85	**10.75**
rd400	35157.00	35026.22	**4.07**	**35193.00**	**35028.30**	19.00	34984.00	34760.75	10.15	35038.50	34907.10	9.73
fl417	**82914.00**	**82775.18**	22.52	82890.00	82760.52	21.46	82857.50	82633.30	**12.34**	82859.00	82692.80	14.18
gr431	**721386.50**	719733.12	310.84	721301.00	**719765.75**	**280.18**	718471.00	717046.06	326.90	720903.00	718490.56	553.45
pr439	396252.50	395124.59	76.15	**396381.00**	**395314.66**	169.61	395564.00	393252.16	**23.32**	395638.50	393897.44	52.78
pcb442	130547.00	**130070.00**	130.08	**130632.00**	130069.43	**87.16**	129563.00	128898.32	124.76	130252.00	129521.90	95.18
d493	151174.50	150972.38	**3.87**	**151176.00**	**150989.98**	33.72	150842.00	150444.50	39.23	151012.00	150588.33	20.80
att532	105620.50	105445.14	18.50	**105651.00**	**105451.77**	30.11	105269.50	105036.75	37.45	105326.50	105112.12	**2.21**
ali535	883615.00	881155.69	120.60	**883866.50**	**881519.38**	154.31	880952.00	877748.00	42.10	881260.50	878271.00	**2.52**
si535	8630.00	**8540.02**	20.53	**8644.50**	8539.02	**17.43**	8572.50	7836.62	119.96	8614.00	8296.90	65.98
pa561	5903.50	5888.29	**0.58**	**5925.50**	**5896.40**	6.68	5868.50	5835.82	5.12	5876.50	5848.88	3.41
u574	**128970.50**	128459.35	77.00	128886.00	**128529.88**	**50.73**	128249.50	127758.49	99.68	128422.00	127883.85	87.79
rat575	**19386.50**	19327.80	6.84	19374.00	**19329.33**	3.14	19252.00	19185.15	3.64	19273.50	19216.21	**1.43**
p654	360498.00	360179.16	50.44	**360626.50**	**360213.59**	70.07	360352.00	359887.94	**22.95**	360335.00	359921.38	25.26
d657	**204921.50**	**204566.73**	26.89	204814.00	204419.91	64.10	203889.00	203471.88	57.61	203987.50	203619.83	**17.36**
gr666	1120318.50	**1118466.00**	219.65	**1120691.50**	1118087.62	**140.60**	1113078.50	1110896.50	171.02	1115702.50	1112253.12	482.96
u724	152022.50	151649.08	24.56	**152141.50**	**151731.91**	47.63	151341.50	150789.27	13.71	151155.00	150942.44	**0.90**
rat783	31735.50	31654.15	**1.69**	**31748.50**	**31669.12**	4.44	31553.00	31496.08	3.31	31576.00	31508.58	15.47
nrw1379	265212.00	**264813.09**	48.43	**265274.50**	264742.22	**1.66**	264687.50	264272.41	108.50	264706.50	264427.78	70.95

Table 3
Results on PTP instances with $\alpha = 0.75$ under ST termination criteria .

Instance	MSHH_GREEDY			MSHH_RAND			MS-ILS			MS-GVNS		
	Best	Average	S.D	Best	Average	S.D	Best	Average	S.D	Best	Average	S.D
burma14	**53.00**	47.62	1.23	**53.00**	**53.00**	**0.00**	**53.00**	-248.39	3.64	**53.00**	**53.00**	**0.00**
ulysses16	**227.50**	224.20	**0.50**	**227.50**	214.44	1.73	**227.50**	-5.70	21.73	**227.50**	**226.40**	1.01
gr17	**158.75**	**158.75**	**0.00**	**158.75**	150.81	1.82	**158.75**	76.50	1.89	**158.75**	**158.75**	**0.00**
gr21	**103.00**	**103.00**	**0.00**	**103.00**	97.85	1.18	**103.00**	63.62	1.00	**103.00**	**103.00**	**0.00**
ulysses22	**1573.50**	1473.75	22.88	**1573.50**	1444.91	84.92	**1573.50**	1534.76	8.89	**1573.50**	**1548.56**	**5.72**
gr24	**0.00**	**-28.43**	0.19	-29.25	-29.25	**0.00**	-29.25	-81.06	3.60	-29.25	-29.25	**0.00**
fri26	**-32.75**	**-33.66**	0.21	**-32.75**	-35.14	0.55	-36.25	-37.39	**0.20**	**-32.75**	-35.45	0.62
bayg29	**46.25**	38.84	1.70	**46.25**	36.74	2.18	**46.25**	**43.25**	**0.69**	**46.25**	36.45	2.25
bays29	**15.75**	14.74	0.23	**15.75**	**15.75**	**0.00**	**15.75**	-12.38	11.96	**15.75**	15.68	0.02
dantzig42	**102.00**	99.21	0.64	**102.00**	95.45	1.50	**102.00**	**102.00**	**0.00**	**102.00**	99.80	0.50
swiss42	**212.25**	200.71	2.65	**212.25**	197.95	3.28	**212.25**	**212.25**	**0.00**	**212.25**	201.20	2.54
att48	**3576.25**	**3576.16**	**0.02**	**3576.25**	3575.99	0.06	**3576.25**	3020.40	12.25	**3576.25**	3575.46	0.22
gr48	**1033.00**	**1033.00**	**0.00**	**1033.00**	988.89	10.12	**1033.00**	**1033.00**	**0.00**	**1033.00**	1016.17	3.86
hk48	**4330.00**	**4327.57**	**0.56**	**4330.00**	4317.88	2.78	**4330.00**	4314.41	2.96	**4330.00**	4266.10	14.66
eil51	**48.00**	**46.94**	0.24	**48.00**	45.95	1.59	**48.00**	46.16	0.42	**48.00**	46.67	**0.10**
berlin52	**2216.50**	2002.46	**6.93**	**2216.50**	1898.17	73.03	**2216.50**	**2083.99**	19.55	**2216.50**	2033.05	42.09
brazil58	**17132.75**	16977.86	35.53	**17132.75**	16295.00	189.61	**17132.75**	**17101.12**	**7.26**	**17132.75**	16902.95	123.01
st70	**200.25**	189.76	2.41	**200.25**	196.12	0.95	200.00	195.55	0.30	**200.25**	**197.99**	**0.06**
eil76	**112.25**	107.71	**0.05**	**112.25**	107.81	0.62	**112.25**	**108.78**	1.32	110.50	105.29	0.51
pr76	**51164.50**	49805.20	19.26	**51164.50**	49512.22	**11.59**	51008.25	**49970.56**	192.47	**51164.50**	49670.64	35.02
gr96	**44280.25**	**43806.62**	**3.64**	44198.00	43722.65	36.91	**44280.25**	43412.47	117.11	**44280.25**	43656.29	124.28
rat99	**768.25**	753.75	3.33	**768.25**	**763.52**	3.39	753.50	745.31	**0.65**	**768.25**	731.60	6.80
kroA100	16988.75	16669.50	73.24	**16996.25**	**16734.34**	**12.60**	16988.75	16733.50	57.98	16988.75	16614.54	40.08
kroB100	19264.25	19063.96	42.09	19264.25	**19075.99**	**7.75**	**19276.75**	19012.97	33.96	19264.25	19041.88	44.48
kroC100	**23097.00**	23070.99	5.97	**23097.00**	23070.99	5.97	**23097.00**	22836.09	28.75	**23097.00**	**23075.35**	**4.97**
kroD100	**19296.00**	19020.30	39.79	**19296.00**	**19123.55**	**39.56**	19094.00	18628.11	58.93	**19296.00**	18930.51	107.31
kroE100	**19265.25**	19021.56	9.99	**19265.25**	**19103.76**	24.43	19219.25	18819.95	147.22	19265.00	18944.81	**5.72**
rd100	4876.50	4816.80	6.93	4876.50	**4835.85**	**1.06**	**4883.50**	4787.44	8.73	4862.00	4788.11	15.57
eil101	254.75	**249.55**	1.79	**255.75**	237.99	2.70	251.75	239.19	**0.04**	253.75	247.82	1.24
lin105	17849.75	**17599.74**	14.46	**18018.50**	17299.15	101.65	17787.25	17495.96	63.20	17864.50	17549.51	**9.35**
pr107	**66664.00**	66540.26	26.27	**66664.00**	**66548.19**	26.57	66394.25	66106.57	36.63	**66664.00**	66490.85	**20.73**
gr120	**6348.75**	**6304.60**	**1.11**	6346.50	6002.21	67.86	6320.00	6178.52	12.55	6343.50	6281.19	6.64
pr124	**85827.75**	84845.35	155.58	**85827.75**	84858.45	152.57	85586.25	84897.65	**16.04**	85586.25	**85444.64**	18.09
bier127	111820.00	110812.57	202.19	111831.50	**110821.85**	**9.50**	111361.00	108915.62	263.97	**111929.00**	110616.11	309.16
ch130	**4990.25**	**4856.94**	6.18	4985.50	4855.51	11.93	4881.75	4783.85	3.87	4957.50	4838.06	**1.59**
pr136	**94567.75**	93635.44	213.89	94559.25	**93804.90**	165.37	92921.50	91540.68	**12.12**	94444.25	93455.51	179.63
gr137	**108622.75**	**108289.14**	**25.49**	**108622.75**	108213.62	42.81	108119.50	106888.34	253.03	108449.75	108025.21	42.97
pr144	109823.75	103944.69	1263.12	109823.75	106970.29	176.77	**110122.75**	**108686.70**	**26.45**	109823.75	107074.38	509.16
ch150	**5768.75**	5620.70	1.96	**5768.75**	**5627.05**	8.79	5604.75	5489.49	11.65	5747.75	5589.15	**0.09**
kroA150	**31422.00**	31157.28	6.19	31415.75	**31160.94**	44.00	30968.00	30558.12	**3.93**	31391.00	31048.97	33.89
kroB150	35105.50	**34873.61**	9.20	**35136.50**	33155.24	385.02	34814.50	34220.56	114.49	35082.50	34812.15	**4.90**
pr152	157797.00	150244.23	1652.37	**157920.50**	147410.66	2217.73	157650.00	**154221.89**	**283.18**	157797.00	150397.02	1617.32
u159	**58604.75**	**58353.77**	42.95	**58604.75**	56221.29	7384.04	58289.25	57314.70	**40.33**	58583.25	57880.03	80.35
si175	**0.00**	-363.14	**24.86**	-123.50	-352.05	36.09	-388.50	-573.40	30.32	**0.00**	**-326.85**	74.98
brg180	**168020.00**	167801.00	16.86	167997.50	167785.77	38.25	168005.00	**167912.00**	**0.69**	167952.50	167763.67	13.46
rat195	2727.25	2708.71	**0.01**	**2732.25**	**2713.49**	4.30	2673.00	2633.38	6.97	2717.75	2688.86	2.49
d198	**42949.25**	42205.00	164.03	42947.75	**42220.49**	153.37	42800.75	41900.15	**145.01**	42933.50	41865.99	224.37
kroA200	**45047.00**	**44635.74**	47.89	45011.75	44594.75	**24.83**	44290.50	43823.26	61.89	44865.75	44421.79	34.57
kroB200	45943.25	45514.12	78.72	**46035.25**	**45527.90**	44.43	45233.25	44561.80	**1.62**	45787.75	45323.54	179.76
gr202	65517.75	65157.29	39.35	**65733.50**	**65304.40**	**14.71**	64843.00	64158.42	157.05	65272.00	64706.40	103.55
ts225	197624.25	195003.70	302.73	**198022.00**	**195214.86**	**269.31**	197264.75	193751.41	426.16	196365.75	194575.31	410.75
tsp225	5701.75	**5664.88**	**0.60**	**5722.75**	5659.34	2.95	5605.75	5514.77	8.02	5688.75	5636.29	4.41
pr226	**267525.75**	262763.44	1035.02	**267525.75**	262871.50	1048.77	267327.50	266158.59	**16.61**	267482.25	**267024.41**	97.12
gr229	**282909.50**	**281534.28**	65.74	282229.25	281301.53	76.63	280151.00	275271.56	72.48	281841.50	280398.53	**59.52**
gil262	**3543.75**	3503.26	**1.43**	3536.00	**3506.15**	6.79	3479.00	3432.09	8.53	3511.25	3482.70	3.62
pr264	**163083.75**	**162252.14**	73.90	163072.25	162162.31	**24.59**	161414.25	160554.05	31.79	162397.50	161708.09	185.27
a280	4586.50	4544.66	**0.84**	**4609.25**	**4546.86**	7.03	4523.00	4461.01	3.21	4581.25	4511.74	6.48
pr299	121640.25	121063.86	**1.87**	**121807.25**	**121101.64**	97.76	120747.50	119700.38	39.83	121425.50	120655.56	96.48
lin318	**85821.50**	**85191.30**	**34.86**	85770.00	80853.70	998.72	84628.25	83965.96	107.36	85387.75	84724.46	45.30
rd400	31470.00	31245.67	4.28	**31489.00**	**31251.13**	**0.72**	31108.50	30825.14	2.38	31292.25	31065.19	29.26
fl417	**80063.50**	**79860.85**	31.74	80045.50	79841.32	32.42	79987.50	79613.84	**13.15**	80003.25	79763.31	36.15
gr431	680625.00	**678387.00**	**374.87**	679776.25	677479.94	526.82	676390.25	673771.31	600.83	**681231.50**	675659.44	1278.32
pr439	368581.00	**364641.56**	1620.03	**369006.25**	363886.75	603.59	366945.50	362469.91	**14.44**	366836.25	362654.69	93.33
pcb442	**117970.00**	116987.41	186.15	117827.50	**117049.91**	**96.22**	116413.25	115354.52	196.16	117446.50	116317.62	122.36
d493	142320.25	**142139.73**	**9.80**	**142435.50**	142068.48	80.29	141814.50	141159.45	32.36	141871.00	141488.02	34.35
att532	98878.75	**98532.57**	14.64	**98911.75**	98530.46	9.86	98494.25	97921.94	77.35	98710.75	98152.93	**4.20**
ali535	**834165.75**	**830389.25**	401.65	832750.25	830287.75	178.20	829426.25	824256.38	**158.15**	830589.00	825091.62	288.35
si535	**2366.25**	**2266.93**	**8.56**	2354.00	2218.60	17.93	2205.75	-1167.35	663.61	2365.25	1192.42	243.77
pa561	**5252.50**	5200.31	**1.36**	5241.00	**5207.01**	6.99	5183.00	5118.85	3.53	5193.50	5147.46	3.15
u574	**119630.75**	**119251.82**	**37.21**	119449.25	119083.02	75.87	118617.50	117937.90	155.91	118933.75	118221.00	161.85
rat575	17642.00	17595.17	9.68	**17692.00**	**17624.08**	11.14	17475.75	17383.47	**4.88**	17543.50	17448.41	20.68
p654	**352027.50**	351438.41	111.46	351980.75	**351465.72**	113.57	351770.25	350927.50	95.78	351759.75	351002.34	**89.17**
d657	**192404.75**	**191849.16**	24.11	192167.50	191688.66	**11.03**	190801.75	190107.02	46.91	190904.00	190397.55	16.34
gr666	**1050738.75**	**1045983.81**	**4.46**	1049212.25	1045528.56	53.67	1040169.50	1033051.56	673.46	1039948.00	1036411.25	383.18
u724	141152.25	140770.23	32.29	**141387.50**	**140864.23**	23.46	140132.25	139284.91	53.59	140188.50	139570.88	**12.25**
rat783	**29499.25**	**29372.74**	2.98	29428.75	29351.56	10.08	29209.50	29097.56	**0.53**	29244.50	29134.39	25.26
nrw1379	250473.25	**249702.80**	280.41	**250499.50**	249650.41	**7.13**	249361.50	248751.81	212.11	249499.75	249061.33	136.63

Table 4
Results on PTP instances with $\alpha = 1.00$ under ST termination criteria .

Instance	MSHH_GREEDY			MSHH_RAND			MS-ILS			MS-GVNS		
	Best	Average	S.D	Best	Average	S.D	Best	Average	S.D	Best	Average	S.D
burma14	**-1.00**	**-33.40**	0.83	-37.00	-37.00	**0.00**	-37.00	-603.20	6.84	-37.00	-37.00	**0.00**
ulysses16	**118.00**	**118.00**	**0.00**	**118.00**	112.05	1.37	**118.00**	-444.35	32.20	**118.00**	**118.00**	**0.00**
gr17	**71.00**	**71.00**	**0.00**	**71.00**	67.45	0.81	**71.00**	-68.50	5.62	**71.00**	**71.00**	**0.00**
gr21	**7.00**	**7.00**	**0.00**	**7.00**	6.65	0.08	**7.00**	**7.00**	**0.00**	**7.00**	**7.00**	**0.00**
ulysses22	**792.00**	694.35	132.22	**792.00**	656.50	**123.54**	**792.00**	**741.15**	188.16	**792.00**	710.35	135.89
gr24	**0.00**	**-81.85**	18.78	**0.00**	-90.80	2.34	-98.00	-188.00	2.29	**0.00**	-88.20	**2.25**
fri26	**0.00**	**-104.60**	1.24	-110.00	-110.00	**0.00**	-114.00	-179.50	2.41	-110.00	-110.00	**0.00**
bayg29	**0.00**	-43.85	3.02	-25.00	-95.05	8.94	**0.00**	-166.00	**1.61**	**0.00**	**-22.80**	12.20
bays29	**0.00**	-44.40	16.88	-55.00	-60.65	**1.30**	**0.00**	-77.75	5.22	**0.00**	**-41.25**	3.15
dantzig42	**36.00**	34.50	0.34	**36.00**	**34.75**	0.29	13.00	13.00	**0.00**	**36.00**	34.00	0.46
swiss42	**70.00**	**65.80**	0.96	**70.00**	63.70	1.45	58.00	58.00	**0.00**	**70.00**	61.90	0.89
att48	**2405.00**	**2405.00**	**0.00**	**2405.00**	**2405.00**	**0.00**	**2405.00**	1319.75	82.99	**2405.00**	**2405.00**	**0.00**
gr48	**297.00**	285.05	2.74	**297.00**	271.20	5.92	**297.00**	186.75	25.29	**297.00**	**287.65**	**2.15**
hk48	**2010.00**	1866.90	44.94	**2010.00**	1748.55	**17.79**	**2010.00**	**1974.85**	63.97	**2010.00**	1865.65	44.66
eil51	**3.00**	**-1.65**	1.23	**3.00**	-2.10	**0.48**	**3.00**	-8.40	2.62	**3.00**	-3.60	0.55
berlin52	1339.00	1244.90	13.33	1339.00	1244.45	13.43	**1361.00**	1052.00	15.83	**1361.00**	**1317.80**	**1.65**
brazil58	**13445.00**	13314.50	26.50	**13445.00**	12733.30	101.33	**13445.00**	**13381.25**	**14.63**	**13445.00**	13343.00	35.10
st70	**95.00**	85.50	2.18	**95.00**	85.35	2.21	**95.00**	87.00	1.38	**95.00**	**90.10**	**1.12**
eil76	**46.00**	**41.95**	**0.93**	**46.00**	40.95	1.16	45.00	33.70	1.31	**46.00**	**41.95**	**0.93**
pr76	**34481.00**	**33409.05**	245.92	**34481.00**	32679.00	460.67	33488.00	30605.70	**142.08**	**34481.00**	32139.90	639.36
gr96	33686.00	**33208.30**	**109.59**	33686.00	33076.30	176.03	**33816.00**	32802.10	175.02	33566.00	32724.60	171.05
rat99	**517.00**	483.95	7.58	**517.00**	486.05	4.81	500.00	480.75	3.15	**517.00**	**507.60**	**1.93**
kroA100	**12511.00**	12205.65	26.46	12481.00	12210.55	248.35	12483.00	**12236.35**	**15.52**	12483.00	12189.70	243.57
kroB100	**14408.00**	13826.73	124.64	**14408.00**	**14042.70**	75.09	14327.00	13908.80	**31.02**	**14408.00**	13951.35	96.04
kroC100	**18420.00**	18315.75	23.92	**18420.00**	18221.85	45.46	18381.00	18154.55	49.91	**18420.00**	**18420.00**	**0.00**
kroD100	**14840.00**	14473.80	84.01	**14840.00**	**14576.10**	**18.61**	14515.00	13893.75	110.29	**14840.00**	14209.80	99.06
kroE100	14662.00	13684.75	1697.62	14662.00	13581.55	1673.94	**14715.00**	**14318.70**	326.39	14662.00	14004.10	**248.94**
rd100	**3261.00**	3122.20	5.46	**3261.00**	**3145.25**	38.83	3254.00	3065.05	**4.60**	**3261.00**	3110.80	17.62
eil101	**147.00**	**138.50**	1.26	**147.00**	134.45	1.50	141.00	123.55	1.94	**147.00**	138.20	**0.41**
lin105	**14813.00**	14175.20	19.78	14605.00	13451.30	173.60	14605.00	**14214.35**	41.44	14605.00	14011.05	**16.30**
pr107	**55854.00**	**55719.40**	30.88	**55854.00**	55696.20	**0.28**	55685.00	55174.70	117.07	55765.00	55659.50	19.16
gr120	**4970.00**	**4895.10**	**6.91**	4968.00	4655.50	52.65	4899.00	4740.95	32.57	4961.00	4854.90	10.35
pr124	**73257.00**	72678.80	132.65	**73257.00**	71875.15	317.02	72484.00	71162.95	63.31	**73257.00**	**73257.00**	**0.00**
bier127	**92376.00**	87299.75	1637.05	92330.00	88406.60	596.85	91747.00	**88896.05**	**197.74**	92016.00	86587.25	2380.70
ch130	**3713.00**	**3457.78**	16.11	**3713.00**	3452.80	10.97	3508.00	3330.80	**2.48**	**3713.00**	3421.10	34.85
pr136	74908.00	73565.15	178.75	**74999.00**	**73598.70**	269.04	73138.00	70555.65	385.50	74908.00	72625.30	**10.16**
gr137	**92739.00**	**92505.75**	20.82	**92739.00**	92442.30	33.43	92099.00	90762.15	297.75	92588.00	92163.50	**15.49**
pr144	**95621.00**	89976.55	1234.82	**95621.00**	92185.60	106.08	95556.00	**93786.10**	**9.43**	**95621.00**	91302.15	260.81
ch150	**4425.00**	**4058.75**	**22.08**	4421.00	3904.90	46.59	4261.00	3999.80	22.99	4424.00	3954.55	34.29
kroA150	**25529.00**	25072.10	**20.44**	25511.00	**25120.30**	59.58	25208.00	24360.20	87.45	25511.00	25032.45	38.87
kroB150	**29541.00**	28431.30	90.78	**29541.00**	27711.50	246.74	29144.00	28279.85	117.89	29525.00	**29063.30**	**24.16**
pr152	139914.00	133209.16	1431.06	139914.00	128522.05	2252.85	139718.00	**136478.50**	**81.56**	**140064.00**	133098.70	1456.40
u159	**48947.00**	**48501.80**	39.73	48924.00	47034.15	6141.03	48370.00	46717.35	**30.82**	48924.00	48131.25	108.69
si175	**0.00**	-480.70	164.56	-180.00	-512.45	**9.28**	**0.00**	**-402.90**	92.43	**0.00**	**-402.90**	92.43
brg180	167420.00	167179.95	**12.17**	167450.00	167175.95	44.52	**167500.00**	**167394.00**	23.86	167333.00	167173.59	18.95
rat195	2204.00	**2179.65**	**1.53**	**2208.00**	2175.75	5.56	2133.00	2072.15	9.14	2182.00	2152.70	1.67
d198	**39158.00**	38462.80	110.85	39134.00	**38476.45**	146.95	38782.00	38073.20	**108.01**	39126.00	38143.95	189.97
kroA200	**38240.00**	36926.85	3509.11	**38240.00**	**37706.50**	74.22	37587.00	36639.00	75.71	38112.00	37619.10	**11.91**
kroB200	39219.00	36769.00	8395.24	**39307.00**	**38759.30**	**10.16**	38092.00	37592.75	19.56	39143.00	38506.78	124.29
gr202	**58013.00**	**57331.50**	123.77	57945.00	54340.90	13000.05	56955.00	55713.35	127.18	57845.00	56891.65	**96.73**
ts225	**167751.00**	**165044.41**	620.94	167041.00	164866.27	353.24	165753.00	161345.42	1011.17	166959.00	163968.09	**17.87**
tsp225	**4834.00**	**4749.80**	12.20	4818.00	4746.40	**0.78**	4717.00	4569.02	14.68	4807.00	4721.12	10.75
pr226	**250804.00**	244825.25	1369.10	**250804.00**	244276.34	1490.20	249863.00	247859.50	**196.04**	250772.00	**248718.59**	457.09
gr229	**257831.00**	256037.75	114.42	257576.00	**256053.88**	**75.05**	253838.00	246106.55	854.70	257406.00	255557.77	424.01
gil262	3020.00	2956.65	5.82	3015.00	**2968.30**	6.13	2952.00	2851.95	8.96	**3027.00**	2938.50	**1.72**
pr264	**150835.00**	149797.75	155.60	150798.00	**150052.00**	**24.55**	148790.00	147829.09	51.83	150590.00	149439.95	80.28
a280	**3982.00**	**3918.40**	11.79	3970.00	3909.50	7.69	3877.00	3777.00	**3.21**	3951.00	3870.25	13.82
pr299	109982.00	109267.55	44.15	**110380.00**	**109469.90**	**11.22**	108522.00	107309.00	76.40	109900.00	108886.50	68.25
lin318	75788.00	69832.35	1179.35	**76358.00**	68699.15	1348.47	74072.00	**71127.65**	**421.29**	75616.00	69306.35	1254.36
rd400	**27899.00**	**27649.53**	23.06	27850.00	27569.70	**2.36**	27555.00	27078.15	6.92	27608.00	27371.67	36.09
fl417	77230.00	**76997.35**	30.59	**77269.00**	76994.95	**25.91**	77151.00	76673.65	27.45	77169.00	76897.15	52.11
gr431	641609.00	**639712.62**	**387.57**	**643147.00**	638731.38	1013.01	638265.00	633487.50	1096.03	640303.00	636372.88	838.54
pr439	**341434.00**	**337741.16**	888.33	341380.00	337551.56	680.09	339084.00	334502.16	**567.31**	338873.00	335007.06	635.70
pcb442	**105677.00**	104419.75	**127.27**	105618.00	**104705.00**	253.05	103661.00	102359.15	301.49	105122.00	103634.00	208.77
d493	133843.00	133362.67	37.09	**133965.00**	**133528.05**	**20.18**	132856.00	132050.30	76.56	133583.00	132609.20	53.41
att532	92543.00	**91802.65**	101.32	**92663.00**	91776.90	**55.04**	91572.00	90850.45	111.62	91728.00	91199.50	63.43
ali535	787508.00	**780898.69**	**55.45**	**787668.00**	771867.25	2457.90	780645.00	772313.00	397.00	781789.00	771466.81	970.25
si535	**-281.00**	**-521.30**	**52.61**	**-281.00**	-532.50	57.70	-915.00	-8377.65	1617.99	**-281.00**	-2847.40	562.85
pa561	**4621.00**	4551.35	**0.08**	4596.00	**4553.60**	2.84	4531.00	4422.50	8.60	4526.00	4470.80	3.85
u574	**110470.00**	109933.60	123.06	110439.00	**109938.75**	101.23	108948.00	107955.75	**19.10**	109512.00	108325.30	37.23
rat575	**16017.00**	15927.60	28.81	**16017.00**	**15945.75**	**2.47**	15745.00	15612.60	5.14	15838.00	15707.55	22.15
p654	343704.00	**342946.59**	148.06	**343900.00**	342917.94	171.16	343508.00	342189.84	165.21	343508.00	342369.31	**141.71**
d657	179959.00	179247.80	**12.57**	**180285.00**	**179256.92**	52.52	177688.00	176715.42	107.50	177886.00	177247.05	33.71
gr666	**982548.00**	**976301.62**	478.18	982492.00	975850.19	1523.73	968744.00	956562.31	**234.62**	971578.00	963594.31	563.74
u724	**130994.00**	130028.80	**44.00**	130577.00	**130066.20**	81.86	129207.00	127804.30	139.19	129138.00	128123.57	48.96
rat783	**27248.00**	**27108.28**	12.55	27203.00	27103.05	**5.49**	26887.00	26720.50	7.69	26910.00	26777.50	27.87
nrw1379	235540.00	**234854.75**	488.14	**235834.00**	234761.09	**22.46**	234256.00	233359.41	344.22	234634.00	233908.09	210.17

Table 5
Results on PTP instances with $\alpha = 0.25$ under LT termination criteria .

Instance	MSHH_GREEDY			MSHH_RAND			MS-ILS			MS-GVNS		
	Best	Average	S.D	Best	Average	S.D	Best	Average	S.D	Best	Average	S.D
burma14	**799.25**	773.16	5.98	**799.25**	773.16	5.98	**799.25**	**799.25**	**0.00**	**799.25**	**799.25**	**0.00**
ulysses16	**1554.25**	**1554.25**	**0.00**	**1554.25**	**1554.25**	**0.00**	**1554.25**	**1554.25**	**0.00**	**1554.25**	**1554.25**	**0.00**
gr17	**617.00**	**617.00**	**0.00**	**617.00**	**617.00**	**0.00**	**617.00**	**617.00**	**0.00**	**617.00**	**617.00**	**0.00**
gr21	**1110.25**	**1110.25**	**0.00**	**1110.25**	**1110.25**	**0.00**	**1110.25**	**1110.25**	**0.00**	**1110.25**	**1110.25**	**0.00**
ulysses22	**3431.75**	**3431.75**	**0.00**	**3431.75**	**3431.75**	**0.00**	**3431.75**	**3431.75**	**0.00**	**3431.75**	**3431.75**	**0.00**
gr24	**428.50**	**428.50**	**0.00**	**428.50**	**428.50**	**0.00**	**428.50**	**428.50**	**0.00**	**428.50**	**428.50**	**0.00**
fri26	**355.75**	**355.75**	**0.00**	**355.75**	**355.75**	**0.00**	**355.75**	**355.75**	**0.00**	**355.75**	**355.75**	**0.00**
bayg29	**664.50**	**664.50**	**0.00**	**664.50**	**664.50**	**0.00**	**664.50**	**664.50**	**0.00**	**664.50**	664.09	0.09
bays29	**710.00**	**710.00**	**0.00**	**710.00**	**710.00**	**0.00**	**710.00**	**710.00**	**0.00**	**710.00**	**710.00**	**0.00**
dantzig42	**353.25**	**353.25**	**0.00**	**353.25**	**353.25**	**0.00**	**353.25**	350.25	**0.00**	**353.25**	353.02	0.05
swiss42	**760.75**	758.52	1.61	**760.75**	757.98	1.49	**760.75**	**760.75**	**0.00**	**760.75**	758.17	1.53
att48	**7549.00**	**7547.80**	**0.28**	**7549.00**	7547.15	0.72	**7549.00**	7547.66	0.31	**7549.00**	7545.61	0.37
gr48	**3137.25**	**3137.25**	**0.00**	**3137.25**	**3137.25**	**0.00**	**3137.25**	**3137.25**	**0.00**	**3137.25**	**3137.25**	**0.00**
hk48	**9637.75**	**9637.75**	**0.00**	**9637.75**	**9637.75**	**0.00**	**9637.75**	9637.19	0.13	**9637.75**	9633.23	1.04
eil51	210.25	209.64	0.03	**210.50**	209.57	0.02	**210.50**	**210.28**	0.05	210.25	209.51	**0.00**
berlin52	**5348.50**	5323.88	5.65	**5348.50**	5050.39	68.39	**5348.50**	**5348.50**	**0.00**	**5348.50**	5311.69	8.45
brazil58	**25068.75**	25011.44	10.28	**25068.75**	24969.58	19.88	**25068.75**	**25068.75**	**0.00**	**25068.75**	24986.61	18.84
st70	**476.50**	**475.57**	0.02	**476.50**	475.05	0.10	**476.50**	475.56	**0.01**	**476.50**	475.20	0.07
eil76	311.75	**310.79**	0.18	**312.25**	310.71	**0.16**	311.50	310.21	**0.16**	311.25	310.39	0.26
pr76	**98776.25**	98558.99	49.84	**98776.25**	98425.89	80.38	**98776.25**	**98605.14**	**39.26**	**98776.25**	98467.86	70.75
gr96	**68853.75**	**68777.32**	22.04	**68853.75**	68747.15	14.60	**68853.75**	68699.73	14.00	**68853.75**	68721.14	**9.15**
rat99	**1322.25**	1319.29	0.30	**1322.25**	**1320.60**	0.31	1318.75	1314.46	**0.24**	**1322.25**	1318.60	1.34
kroA100	**26888.75**	26848.81	9.16	**26888.75**	**26852.49**	8.32	26883.00	26823.10	**2.84**	**26888.75**	26822.28	15.25
kroB100	29835.00	29807.76	8.32	**29864.75**	**29812.53**	3.68	29825.75	29713.36	22.28	29841.50	29803.60	**1.23**
kroC100	**32971.75**	32970.11	0.38	**32971.75**	**32970.82**	**0.21**	32953.25	32897.04	12.90	**32971.75**	32955.57	0.53
kroD100	**29250.50**	29219.10	7.20	**29250.50**	**29239.88**	2.44	29233.50	29118.11	16.49	**29250.50**	29212.78	**2.00**
kroE100	29286.50	29273.56	**0.13**	29286.50	**29279.83**	1.53	29269.50	29185.91	0.82	**29296.00**	29268.70	6.26
rd100	**8700.50**	8681.51	1.60	**8700.50**	**8687.84**	**0.15**	**8700.50**	8658.27	12.28	**8700.50**	8678.46	0.79
eil101	**527.00**	**524.61**	0.48	**527.00**	524.41	0.21	524.25	521.92	0.25	**527.00**	524.14	**0.08**
lin105	**24816.50**	**24716.10**	2.43	**24816.50**	24710.24	**0.40**	24790.25	24697.44	7.96	**24816.50**	24708.14	12.59
pr107	**88659.25**	88635.25	5.51	**88659.25**	**88642.00**	3.79	88648.25	88543.44	1.13	**88659.25**	88630.18	**1.07**
gr120	**9460.00**	**9446.01**	0.92	**9460.00**	8976.69	106.98	9445.25	9402.30	**0.70**	**9460.00**	9438.70	0.76
pr124	**113631.50**	**113631.50**	**0.00**	**113631.50**	**113631.50**	**0.00**	113520.50	113281.23	85.45	**113631.50**	113630.35	2.37
bier127	160073.00	159401.23	101.51	**160166.75**	**159524.09**	**55.33**	159810.50	158979.84	454.72	160117.25	159465.08	149.62
ch130	**7909.25**	7886.19	**1.48**	**7909.25**	**7893.51**	3.04	7889.00	7839.86	6.34	7908.75	7883.01	2.12
pr136	**139222.75**	138913.88	**7.43**	**139222.75**	**138975.20**	44.57	138846.25	138086.69	15.70	139167.25	138838.05	68.30
gr137	141210.75	**141159.30**	10.37	**141253.75**	141145.89	3.58	141178.75	140858.62	73.44	141183.00	141138.05	**2.70**
pr144	**139455.75**	**139374.80**	5.61	**139455.75**	139317.16	18.84	139342.75	138998.94	**2.85**	**139455.75**	139297.77	3.22
ch150	8783.50	**8765.03**	2.10	**8786.00**	8756.40	2.62	8743.75	8702.08	**0.99**	8782.25	8757.12	1.41
kroA150	43937.25	43875.29	11.06	**43943.25**	**43878.35**	**10.36**	43865.25	43671.43	15.62	**43943.25**	43862.16	11.49
kroB150	**47408.50**	**47357.88**	**3.07**	**47408.50**	47356.90	7.61	47300.75	47166.15	16.72	47405.75	47339.36	17.29
pr152	**194393.50**	**194298.97**	21.50	**194393.50**	194287.61	18.90	194248.00	193878.16	77.29	**194393.50**	194229.66	**5.60**
u159	**79385.00**	**79332.94**	11.94	**79385.00**	79239.01	108.00	79299.25	78862.85	19.59	**79385.00**	79280.88	**4.90**
si175	**4929.50**	**4903.94**	**0.16**	4925.75	4902.04	3.55	4902.75	4860.12	3.30	4922.75	4898.45	1.16
brg180	169010.00	168970.25	1.09	169032.50	168966.25	8.89	**169057.50**	**169047.19**	**0.50**	168997.50	168949.12	3.24
rat195	**3894.50**	**3879.22**	0.46	3886.50	3878.20	0.18	3861.50	3847.84	**0.09**	3884.50	3873.84	0.90
d198	50747.00	50715.62	6.34	50747.50	**50722.66**	**3.92**	50693.25	50593.01	7.74	**50748.25**	50298.62	101.60
kroA200	59288.00	59146.78	39.58	**59312.75**	**59162.35**	20.62	59066.25	58828.90	10.64	59271.75	59127.55	**2.36**
kroB200	**60004.75**	59899.29	8.65	59988.50	**59904.76**	**7.34**	59699.50	59517.04	62.81	60002.00	59866.75	41.75
gr202	83941.50	83794.44	8.67	**83960.00**	**83805.12**	35.53	83616.50	83304.14	18.27	83937.25	83775.93	**3.06**
ts225	260868.00	259886.66	**64.20**	**261428.00**	**260045.75**	317.11	261092.75	259650.02	330.98	261348.25	259591.81	402.95
tsp225	**7612.50**	**7595.85**	**0.44**	7606.25	7593.66	2.49	7582.00	7545.16	1.93	7600.25	7587.74	1.38
pr226	**305138.75**	**305017.34**	**7.72**	305121.00	304970.66	21.13	304918.00	304710.59	37.72	305110.75	304946.00	19.62
gr229	340160.75	339967.88	44.83	**340208.00**	**339983.06**	**6.18**	339443.25	338430.31	105.80	340148.00	339749.78	19.32
gil262	4668.75	**4659.86**	1.35	**4671.25**	4659.30	**0.30**	4660.25	4640.31	1.76	4664.75	4656.06	1.13
pr264	187511.25	**187355.94**	20.43	**187528.25**	187351.77	**8.08**	186847.25	186643.25	46.80	**187528.25**	187318.69	12.23
a280	5891.50	5878.48	1.04	**5899.25**	**5882.21**	1.33	5871.50	5842.00	0.75	5894.25	5874.54	**0.70**
pr299	145717.00	145512.14	**3.88**	**145734.50**	**145524.16**	20.95	145263.50	144894.92	30.99	145625.00	145443.84	8.24
lin318	106652.50	**106529.80**	**6.41**	**106659.00**	101198.95	1225.26	106359.00	106130.64	10.58	106651.50	106486.89	19.98
rd400	39017.75	38978.57	**1.88**	**39025.75**	**38978.66**	3.29	38931.50	38824.24	6.89	39003.25	38947.30	2.25
fl417	**85784.75**	**85727.60**	7.82	85777.50	85720.61	6.45	85751.50	85652.31	**6.32**	85778.50	85721.35	11.32
gr431	764399.00	764079.44	19.92	**764483.25**	**764130.44**	**1.79**	763008.75	762225.56	106.33	764368.25	763598.00	176.70
pr439	424665.50	424143.84	**25.14**	**424859.50**	**424228.09**	72.64	424211.25	422713.16	83.93	424729.50	423978.78	87.91
pcb442	143892.75	143641.59	**33.40**	**144011.75**	**143668.23**	56.95	143356.75	142899.38	79.29	143949.50	143526.58	95.57
d493	**160309.50**	**160212.02**	8.38	160298.50	160205.19	**5.26**	160031.75	159875.30	8.44	160258.50	160161.75	22.20
att532	112770.50	**112695.60**	1.01	**112828.75**	112693.23	9.34	112527.25	112418.85	6.06	112698.00	112625.43	**0.76**
ali535	**935778.25**	934817.25	**1.67**	935541.00	**934861.56**	20.18	934256.50	932797.31	222.95	935321.25	934430.12	67.59
si535	**19334.25**	**19307.93**	1.28	19331.25	19305.92	**0.67**	19313.75	19267.14	4.96	19315.25	19291.55	7.24
pa561	**6636.75**	6623.47	1.20	6633.25	**6624.38**	1.06	6607.75	6597.81	2.54	6622.75	6615.34	**0.67**
u574	**138400.00**	138235.91	14.19	138393.25	**138253.42**	3.97	138114.25	137876.59	22.81	138313.50	138133.92	**2.83**
rat575	21143.75	**21119.11**	**0.60**	**21145.00**	21116.09	3.02	21069.00	21042.09	3.35	21124.50	21095.12	2.09
p654	369327.00	369126.09	33.41	**369329.75**	**369135.34**	33.53	369212.25	368998.88	**25.21**	369318.75	369108.88	31.17
d657	**217858.25**	**217660.00**	16.69	217800.50	217655.80	**6.36**	217365.25	217172.52	39.45	217633.00	217489.16	28.24
gr666	1196577.25	**1195743.62**	33.69	**1196662.25**	1195415.38	248.48	1194774.75	1192471.38	**23.48**	1195663.00	1194674.62	106.89
u724	**163120.25**	162980.05	**0.79**	163110.50	**163005.59**	11.91	162761.75	162549.48	3.21	163045.00	162814.02	27.88
rat783	**34077.25**	34051.29	2.17	34073.75	**34053.80**	3.28	33979.75	33959.21	**0.85**	34031.00	34008.50	3.96
nrw1379	280784.50	**280670.91**	11.62	**280785.25**	280621.00	**11.01**	280381.75	280124.09	38.56	280330.00	280150.09	17.40

Table 6
Results on PTP instances with $\alpha = 0.50$ under LT termination criteria .

Instance	MSHH_GREEDY			MSHH_RAND			MS-ILS			MS-GVNS		
	Best	Average	S.D	Best	Average	S.D	Best	Average	S.D	Best	Average	S.D
burma14	**190.50**	186.25	0.98	**190.50**	188.38	0.49	**190.50**	**190.50**	**0.00**	**190.50**	188.38	0.49
ulysses16	**668.00**	577.30	**21.40**	**668.00**	540.08	49.11	**668.00**	**625.80**	32.53	**668.00**	618.55	30.87
gr17	**313.00**	293.05	4.58	**313.00**	285.38	3.70	**313.00**	**309.68**	**0.76**	**313.00**	290.88	5.08
gr21	**456.00**	409.55	10.66	**456.00**	399.23	**1.44**	**456.00**	**441.88**	3.24	**456.00**	426.43	6.78
ulysses22	**2420.50**	2237.10	42.07	**2420.50**	2282.95	178.82	**2420.50**	**2374.65**	**10.52**	**2420.50**	2328.80	21.04
gr24	**156.50**	141.25	3.50	**156.50**	133.00	2.52	**156.50**	**150.75**	1.32	**156.50**	142.62	**0.32**
fri26	**141.00**	136.38	1.06	**141.00**	136.38	1.06	**141.00**	**141.00**	**0.00**	**141.00**	**141.00**	**0.00**
bayg29	**319.50**	**319.50**	**0.00**	**319.50**	**319.50**	**0.00**	**319.50**	**319.50**	**0.00**	**319.50**	**319.50**	**0.00**
bays29	**253.50**	242.55	2.51	**253.50**	240.68	0.73	**253.50**	**253.50**	**0.00**	**253.50**	246.03	1.71
dantzig42	**201.50**	200.80	0.16	**201.50**	200.70	0.18	**201.50**	**201.30**	**0.05**	**201.50**	200.70	0.18
swiss42	**470.50**	466.38	0.95	**470.50**	466.38	2.84	**470.50**	**470.50**	**0.00**	**470.50**	464.88	1.29
att48	**5103.00**	5064.23	8.90	**5103.00**	5051.30	11.86	**5103.00**	**5102.25**	**0.17**	**5103.00**	5087.45	3.57
gr48	**2045.00**	**2045.00**	**0.00**	**2045.00**	**2045.00**	**0.00**	**2045.00**	**2045.00**	**0.00**	**2045.00**	**2045.00**	**0.00**
hk48	**6850.00**	6843.62	**1.46**	**6850.00**	6843.62	8.29	**6850.00**	**6844.38**	1.58	**6850.00**	6841.50	1.95
eil51	**122.50**	121.00	**0.00**	**122.50**	121.10	0.32	**122.50**	**121.97**	0.12	**122.50**	120.10	0.60
berlin52	**3544.00**	3332.70	23.35	**3544.00**	3178.22	83.91	**3544.00**	**3544.00**	**0.00**	**3544.00**	3327.82	49.59
brazil58	**20881.00**	20877.70	1.77	**20881.00**	20878.25	0.63	**20881.00**	19517.67	75.63	**20881.00**	**20881.00**	**0.00**
st70	**326.00**	324.12	0.03	**326.00**	323.95	**0.01**	**326.00**	**325.18**	0.19	**326.00**	324.02	**0.01**
eil76	**202.00**	199.25	0.29	**202.00**	**199.82**	0.07	**202.00**	199.57	0.25	**202.00**	199.43	**0.02**
pr76	**73221.00**	72515.59	**40.24**	**73221.00**	72357.95	76.41	**73221.00**	**72886.80**	46.30	**73221.00**	72586.93	144.09
gr96	**55594.50**	**55456.80**	**31.59**	**55594.50**	55390.72	32.40	**55594.50**	55346.10	56.99	55530.50	55330.22	37.34
rat99	**1038.50**	**1036.08**	0.71	**1038.50**	1035.60	0.94	1037.00	1032.72	**0.17**	**1038.50**	1033.88	0.54
kroA100	**21815.50**	21632.42	42.00	**21815.50**	**21710.47**	24.09	**21815.50**	21699.03	**7.35**	**21815.50**	21637.88	40.75
kroB100	24499.50	24348.08	28.01	**24542.50**	**24377.30**	18.08	24537.50	24335.78	33.44	**24542.50**	24344.22	**4.88**
kroC100	**27972.00**	27964.85	1.64	**27972.00**	**27972.00**	**0.00**	**27972.00**	27933.58	7.59	**27972.00**	27964.85	14.76
kroD100	**24013.50**	23952.05	**8.96**	**24013.50**	**23987.03**	16.98	**24013.50**	23867.10	62.77	**24013.50**	23943.40	16.08
kroE100	24027.00	23988.20	33.31	24027.00	**23996.60**	**3.30**	**24045.50**	23903.90	13.97	24027.00	23954.95	36.35
rd100	**6767.00**	6738.82	**0.50**	**6767.00**	**6738.95**	6.44	**6767.00**	6731.30	1.88	**6767.00**	6724.82	3.37
eil101	**381.00**	**377.12**	**0.26**	379.50	**377.12**	0.37	379.00	374.95	0.93	379.50	376.20	0.41
lin105	**21341.00**	21164.42	**8.24**	**21341.00**	21193.03	14.80	**21341.00**	**21198.72**	47.77	**21341.00**	21142.60	34.66
pr107	**77645.00**	**77604.50**	9.29	**77645.00**	**77604.50**	9.29	77555.50	77432.60	38.22	**77645.00**	77575.38	**0.49**
gr120	**7831.50**	**7817.20**	1.45	**7831.50**	7425.18	91.73	7825.50	7782.35	9.90	7830.00	7807.85	**1.34**
pr124	99234.50	99006.00	**10.21**	99234.50	98973.10	17.76	**99389.00**	**99016.15**	32.15	**99389.00**	98978.43	18.91
bier127	134133.00	132903.08	158.20	**134202.00**	**133093.67**	**32.31**	133578.50	132642.73	105.81	134105.50	132713.38	81.36
ch130	6416.50	6360.65	**1.64**	6398.50	**6364.35**	6.39	6414.50	6338.95	5.84	**6417.50**	6337.60	2.39
pr136	116126.00	115757.54	84.53	**116324.00**	**115787.27**	**61.65**	116027.00	115216.18	63.05	116201.50	115758.12	84.40
gr137	**124764.50**	124603.50	20.99	**124764.50**	124523.12	**2.55**	124764.00	124371.88	53.02	**124764.50**	**124613.95**	23.39
pr144	**124821.50**	**124197.23**	46.16	**124821.50**	124145.38	69.71	124788.00	123869.18	**3.17**	**124821.50**	124093.90	22.46
ch150	7187.50	**7141.30**	2.80	**7192.00**	7134.40	4.38	7173.50	7128.25	**1.09**	7181.50	7125.30	1.42
kroA150	37552.00	37428.82	28.26	**37564.00**	**37433.82**	8.41	37428.50	37261.88	25.44	37514.50	37406.12	**4.79**
kroB150	**41059.50**	40933.38	6.45	**41059.50**	**40960.05**	**0.33**	40983.50	40819.95	22.82	41024.00	40904.78	4.30
pr152	**176028.50**	168090.80	1736.03	**176028.50**	164598.12	2551.53	175763.50	**172743.80**	**567.51**	**176028.50**	168034.38	1748.98
u159	68816.50	**68688.38**	29.39	68816.50	68597.80	**12.89**	**68879.00**	68211.62	104.01	68816.50	68512.50	56.90
si175	580.50	572.08	1.93	580.50	**572.45**	**0.47**	**582.00**	571.40	2.43	580.50	566.70	1.79
brg180	168470.00	168397.50	**0.57**	168515.00	168373.75	19.79	**168570.00**	**168546.50**	1.49	168445.00	168344.25	5.56
rat195	**3304.00**	**3287.82**	**1.34**	3301.50	3286.51	1.95	3287.50	3258.68	2.33	3293.00	3279.07	1.39
d198	46834.00	46058.54	174.58	**46842.00**	**46072.40**	169.56	46732.50	45930.20	**157.79**	46831.50	45695.50	260.50
kroA200	51990.50	51773.75	**10.61**	**52055.50**	**51802.84**	57.96	51904.00	51622.21	12.45	51943.00	51752.72	13.14
kroB200	**52768.50**	52535.12	**36.74**	52763.50	**52572.38**	36.79	52474.00	52291.85	41.83	52710.00	52502.35	67.41
gr202	**74250.50**	**74008.55**	54.70	74236.50	74005.52	36.83	73825.50	73400.82	65.08	74187.00	73935.71	**15.32**
ts225	228952.00	227066.12	**100.23**	229190.50	227059.88	371.68	**229870.00**	**227624.70**	428.85	228108.00	226751.75	311.15
tsp225	**6647.50**	**6621.62**	**0.26**	6643.50	6618.23	1.31	6598.50	6548.93	2.66	6636.00	6611.40	2.20
pr226	**286018.00**	**285604.34**	88.59	**286018.00**	285546.16	101.94	285945.00	285577.19	**28.86**	**286018.00**	285513.78	88.26
gr229	308984.50	**308420.97**	80.30	**309293.00**	308310.59	**24.23**	307936.50	306320.41	125.17	309161.50	307921.41	28.77
gil262	4097.50	**4077.00**	2.75	4093.50	4074.72	**0.85**	4068.50	4043.07	4.46	**4106.50**	4073.10	2.50
pr264	**175246.50**	**174916.41**	**5.60**	175225.50	174855.91	33.47	174451.00	173878.34	21.07	175196.50	174773.33	31.24
a280	5241.50	5216.07	5.18	**5249.50**	**5216.38**	**2.78**	5210.50	5169.05	3.22	5229.00	5199.48	10.09
pr299	133733.00	133342.12	**1.86**	**133826.00**	**133388.84**	8.18	133101.50	132667.41	25.60	133470.00	133143.88	26.24
lin318	**96251.50**	**95792.10**	45.52	96156.00	91078.05	1061.15	95726.50	95217.23	47.32	96171.00	95740.65	**0.31**
rd400	35221.50	**35124.40**	11.33	**35223.50**	35112.35	2.49	35039.50	34832.35	23.25	35184.50	35057.07	**0.59**
fl417	**82925.50**	**82800.93**	22.47	82898.00	82794.68	**20.46**	82857.50	82683.02	21.11	82895.00	82781.68	23.56
gr431	722085.50	**720981.12**	**36.21**	**722105.00**	720801.75	298.99	719523.00	717839.44	325.79	721647.00	720100.50	354.79
pr439	**397292.00**	**396306.06**	88.66	397082.00	396086.72	228.33	395864.00	393909.16	269.64	396872.50	395621.84	**51.09**
pcb442	**131107.00**	130447.50	124.23	130997.50	**130519.05**	103.82	129763.50	129185.60	**70.45**	130688.50	130153.15	80.10
d493	151352.50	151205.92	**17.34**	**151420.50**	**151230.75**	30.45	150842.00	150544.75	37.68	151349.50	151105.41	39.14
att532	105757.00	**105659.02**	38.09	**105814.50**	105631.40	38.79	105531.50	105173.18	**12.35**	105743.50	105519.23	26.68
ali535	884922.00	882798.69	128.78	**885087.50**	**883161.81**	75.44	882206.00	879372.69	264.10	884509.00	882221.19	**66.46**
si535	8654.00	8566.98	17.67	**8667.00**	**8575.05**	20.18	8631.50	8565.40	**15.16**	8638.50	8540.65	15.57
pa561	**5934.50**	5913.93	0.44	5933.00	**5915.48**	3.56	5885.00	5862.15	**0.31**	5917.50	5898.05	4.46
u574	**129111.50**	128776.52	72.26	129101.50	**128802.93**	**14.47**	128589.50	128022.73	40.90	128863.50	128573.30	48.34
rat575	19422.00	19376.42	**3.42**	**19429.00**	**19382.17**	10.74	19277.50	19221.28	4.65	19386.50	19340.03	5.05
p654	**360714.50**	360285.38	66.56	360679.50	**360305.41**	63.23	360522.00	360131.47	**15.38**	360627.50	360269.91	79.97
d657	205263.50	204961.55	**52.52**	**205328.50**	**204974.88**	77.92	204447.00	204029.50	95.78	205089.00	204676.73	94.58
gr666	1122409.00	**1120429.62**	76.42	**1123125.50**	1120384.00	**38.99**	1118234.50	1114486.62	508.77	1120483.50	1118093.25	378.93
u724	152307.50	152064.08	27.17	**152370.50**	**152114.80**	**1.90**	151476.50	151183.91	10.64	152109.00	151764.52	26.04
rat783	**31803.50**	31732.97	9.18	31800.00	**31757.00**	**0.11**	31602.00	31566.10	0.94	31723.00	31675.35	1.07
nrw1379	**265995.00**	**265675.72**	44.10	265929.50	265629.09	**26.02**	264989.50	264531.88	109.17	264955.50	264672.41	75.11

Table 7
Results on PTP instances with $\alpha = 0.75$ under LT termination criteria .

Instance	MSHH_GREEDY			MSHH_RAND			MS-ILS			MS-GVNS		
	Best	Average	S.D	Best	Average	S.D	Best	Average	S.D	Best	Average	S.D
burma14	**53.00**	47.62	1.23	**53.00**	**53.00**	**0.00**	**53.00**	-248.39	3.64	**53.00**	**53.00**	**0.00**
ulysses16	**227.50**	224.20	**0.50**	**227.50**	214.44	1.73	**227.50**	-5.70	21.73	**227.50**	**226.40**	1.01
gr17	**158.75**	**158.75**	**0.00**	**158.75**	150.81	1.82	**158.75**	76.50	1.89	**158.75**	**158.75**	**0.00**
gr21	**103.00**	**103.00**	**0.00**	**103.00**	97.85	1.18	**103.00**	63.62	1.00	**103.00**	**103.00**	**0.00**
ulysses22	**1573.50**	1473.75	22.88	**1573.50**	1444.91	84.92	**1573.50**	1536.49	8.49	**1573.50**	**1548.56**	**5.72**
gr24	**0.00**	**-28.43**	0.19	-29.25	-29.25	**0.00**	-29.25	-81.06	3.60	-29.25	-29.25	**0.00**
fri26	**-32.75**	**-33.49**	**0.17**	**-32.75**	-34.96	0.51	-36.25	-37.33	0.19	**-32.75**	-34.23	0.34
bayg29	**46.25**	38.84	1.70	**46.25**	36.74	2.18	**46.25**	**43.25**	**0.69**	**46.25**	36.45	2.25
bays29	**15.75**	14.74	0.23	**15.75**	**15.75**	**0.00**	**15.75**	8.32	1.39	**15.75**	15.68	0.02
dantzig42	**102.00**	99.21	0.64	**102.00**	95.45	1.50	**102.00**	**102.00**	**0.00**	**102.00**	99.80	0.50
swiss42	**212.25**	200.71	2.65	**212.25**	197.95	3.28	**212.25**	**212.25**	**0.00**	**212.25**	201.20	2.54
att48	**3576.25**	**3576.25**	**0.00**	**3576.25**	**3576.25**	**0.00**	**3576.25**	3088.85	27.95	**3576.25**	**3576.25**	**0.00**
gr48	**1033.00**	**1033.00**	**0.00**	**1033.00**	988.89	10.12	**1033.00**	**1033.00**	**0.00**	**1033.00**	1019.33	3.14
hk48	**4330.00**	**4330.00**	**0.00**	**4330.00**	**4330.00**	**0.00**	**4330.00**	**4330.00**	**0.00**	**4330.00**	**4330.00**	**0.00**
eil51	**48.00**	46.94	0.24	**48.00**	45.95	1.59	**48.00**	**47.77**	**0.05**	**48.00**	46.67	0.10
berlin52	**2216.50**	2012.81	**9.31**	**2216.50**	1915.28	69.11	**2216.50**	**2097.43**	22.64	**2216.50**	2036.19	41.37
brazil58	**17132.75**	16977.86	35.53	**17132.75**	16333.30	180.83	**17132.75**	**17132.75**	**0.00**	**17132.75**	16979.55	140.59
st70	**200.25**	189.96	2.36	**200.25**	197.91	0.54	200.00	197.64	0.38	**200.25**	**198.79**	**0.12**
eil76	**112.25**	108.47	0.22	**112.25**	109.31	0.27	**112.25**	**111.89**	**0.15**	**112.25**	110.46	0.68
pr76	**51164.50**	49995.61	62.94	**51164.50**	49815.62	58.01	**51164.50**	**50706.84**	**23.55**	**51164.50**	49965.46	102.65
gr96	**44280.25**	44045.40	58.42	**44280.25**	43965.40	**18.89**	**44280.25**	**44071.96**	47.78	**44280.25**	43934.88	60.36
rat99	**768.25**	**766.08**	**0.50**	**768.25**	765.42	0.84	767.25	759.35	1.58	**768.25**	764.62	0.72
kroA100	16988.75	16719.99	61.66	**16996.25**	16773.99	49.27	**16996.25**	**16966.25**	**6.88**	16988.75	16736.19	57.94
kroB100	19264.25	19087.46	19.03	19302.75	19129.56	18.74	**19398.00**	**19260.85**	**15.00**	19264.25	19081.66	35.35
kroC100	**23097.00**	23070.99	5.97	**23097.00**	23078.25	4.30	**23097.00**	23056.99	16.29	**23097.00**	**23097.00**	**0.00**
kroD100	**19296.00**	19120.59	40.24	**19296.00**	**19225.97**	**16.06**	19207.75	18978.59	46.48	**19296.00**	19062.14	137.51
kroE100	19265.25	**19185.38**	**18.32**	19265.25	19174.58	20.80	**19340.00**	19132.56	47.59	**19340.00**	19129.83	29.12
rd100	4876.50	4843.07	**2.97**	**4883.50**	4849.46	4.18	**4883.50**	**4852.12**	5.42	4876.50	4840.09	3.65
eil101	254.75	250.97	1.83	**255.75**	238.88	2.49	255.25	249.24	**0.00**	254.50	**251.40**	0.60
lin105	**18030.75**	17716.00	6.02	**18030.75**	17398.15	78.94	17966.75	**17828.51**	41.64	18018.50	17686.60	**5.47**
pr107	**66664.00**	**66571.45**	0.64	**66664.00**	66554.90	25.03	66620.75	66397.09	**0.27**	**66664.00**	66531.61	30.08
gr120	**6361.50**	**6327.69**	**0.50**	6354.00	6016.06	75.64	6338.75	6294.80	2.88	6357.75	6316.21	10.89
pr124	**85827.75**	84877.14	148.29	**85827.75**	**85551.26**	6.37	85727.25	85249.62	**2.78**	85586.25	85536.05	2.88
bier127	111827.50	111184.39	39.03	111873.75	**111212.82**	70.85	**112566.50**	110860.54	**18.48**	112085.50	111171.84	237.81
ch130	**4990.25**	4882.71	6.78	**4990.25**	4883.69	**6.55**	**4990.25**	**4898.98**	9.46	4990.00	4876.91	7.02
pr136	**94567.75**	94113.98	104.10	**94567.75**	**94116.74**	103.47	94403.50	93621.11	**79.41**	**94567.75**	94023.80	86.13
gr137	**108622.75**	108339.55	13.93	**108622.75**	**108450.90**	11.62	**108622.75**	108248.74	22.14	**108622.75**	108352.20	**11.02**
pr144	109823.75	104247.21	1193.72	109823.75	107302.16	252.91	**110147.50**	**109090.98**	**66.30**	110027.50	107301.86	492.93
ch150	**5768.75**	**5662.06**	**0.70**	**5768.75**	5654.91	15.18	5738.75	5630.73	3.39	**5768.75**	5616.50	6.37
kroA150	**31435.00**	**31285.04**	26.20	**31435.00**	31276.72	20.14	31391.50	31065.94	16.30	31429.50	31192.79	**4.20**
kroB150	35105.50	**34935.40**	23.38	**35136.50**	33210.36	372.37	35007.00	34745.72	**12.91**	35105.50	34920.57	19.98
pr152	157909.50	150658.59	1557.31	**157920.50**	147519.45	2280.00	157776.25	**154715.05**	**587.69**	157797.00	150536.80	1585.25
u159	58604.75	**58454.21**	**25.76**	58604.75	56508.16	7449.85	**58789.50**	58188.60	55.95	58604.75	58198.79	78.80
si175	**0.00**	-363.14	**24.86**	-123.50	-352.05	36.09	-388.50	-573.40	30.32	**0.00**	**-326.85**	74.98
brg180	168020.00	167816.00	**2.06**	167997.50	167792.91	36.61	**168080.00**	**168050.38**	3.53	167952.50	167763.67	13.46
rat195	**2743.00**	2721.36	2.15	2736.50	**2727.29**	1.65	2714.00	2686.64	**1.58**	**2743.00**	2711.28	4.35
d198	42950.75	42246.86	154.94	**42965.75**	**42269.79**	159.66	42845.00	42129.51	**149.52**	42949.25	41940.11	215.51
kroA200	45176.50	**44801.89**	**27.67**	**45244.25**	44760.11	30.25	44779.75	44435.69	35.63	45041.75	44660.69	31.33
kroB200	**46035.25**	**45634.81**	77.44	**46035.25**	45619.72	**53.33**	45915.50	45267.72	71.05	45915.25	45506.54	117.30
gr202	65706.50	65397.19	**41.08**	**65796.75**	**65458.30**	77.65	65404.00	64896.84	65.59	65620.00	65099.50	78.35
ts225	197748.00	195513.34	512.67	198022.00	195631.64	**219.23**	**200390.50**	**197180.73**	336.67	197647.50	194984.06	367.80
tsp225	5738.50	**5692.00**	**2.01**	**5742.00**	5683.85	3.69	5644.25	5575.49	6.14	5728.50	5668.56	2.17
pr226	**267525.75**	262826.28	1020.61	**267525.75**	262899.75	1051.82	267407.50	266910.47	**30.52**	**267525.75**	**267136.16**	74.01
gr229	**282909.50**	281863.97	80.92	282832.00	**282085.38**	217.92	281691.75	278301.28	**20.37**	282656.25	281483.91	60.93
gil262	3545.50	3516.55	4.98	**3558.50**	**3518.54**	4.58	3524.25	3470.34	0.15	3540.00	3507.78	**0.05**
pr264	**163083.75**	**162614.30**	**2.23**	**163083.75**	162454.62	2.95	162083.50	161274.02	18.64	162916.50	162414.75	108.92
a280	4612.75	**4568.95**	4.92	**4628.50**	4565.34	6.12	4574.75	4528.09	**2.49**	4597.50	4545.23	11.75
pr299	121904.50	121352.77	14.69	**121983.25**	**121445.19**	**1.05**	121302.75	120286.34	84.04	121628.25	121145.00	60.80
lin318	**85986.00**	**85501.05**	46.85	85937.00	81205.12	1009.23	85395.25	84543.68	55.33	85884.00	85296.98	**11.71**
rd400	**31567.50**	31394.31	6.32	31566.50	**31394.89**	**1.29**	31207.00	30957.75	22.31	31416.75	31291.60	5.60
fl417	**80090.25**	**79895.98**	32.00	80064.50	79874.93	27.51	79987.50	79718.19	**18.11**	80044.75	79873.94	40.25
gr431	**682348.75**	**680116.56**	**73.51**	681723.00	679484.31	422.23	677035.50	674389.38	466.55	681693.00	678748.44	675.53
pr439	369492.50	**365896.91**	1629.92	**369888.75**	365416.56	456.58	368805.50	365268.38	798.91	369403.00	365485.72	**160.31**
pcb442	118341.50	**117730.68**	222.17	118354.25	117607.64	**131.72**	117081.50	115933.51	262.63	**118462.50**	117308.62	218.89
d493	142664.00	142379.02	6.25	142676.75	**142390.16**	7.93	141906.25	141393.41	**6.00**	**142694.00**	142133.75	60.68
att532	99211.25	98762.89	46.32	**99236.00**	**98863.27**	37.97	98582.75	98090.06	49.20	98992.25	98671.29	**3.38**
ali535	836263.25	832573.50	471.90	**836455.00**	**833043.56**	**124.65**	831800.25	826991.75	465.43	835029.75	831599.00	229.81
si535	2376.25	**2285.56**	8.76	2375.50	2232.00	19.16	2340.25	2159.99	4.13	**2386.00**	2281.12	**2.78**
pa561	5266.25	5234.00	1.15	**5275.75**	**5244.71**	0.97	5221.75	5161.04	1.37	5267.00	5214.46	**0.58**
u574	**119877.75**	**119570.80**	44.32	119868.00	119526.05	**18.40**	119014.50	118373.15	122.19	119730.50	119260.71	27.31
rat575	17725.50	17666.24	22.02	**17751.00**	**17704.19**	10.25	17516.25	17454.51	**7.23**	17720.25	17636.15	10.36
p654	352135.50	351596.38	96.61	**352171.25**	**351620.44**	91.61	351938.25	351339.81	**81.72**	352079.25	351555.38	112.67
d657	**192947.75**	192412.58	**1.74**	192915.75	**192451.02**	10.15	191714.25	191022.23	32.98	192639.75	192013.80	90.38
gr666	**1053116.50**	1048788.25	397.88	1051588.00	**1049363.12**	**83.35**	1044371.25	1040897.44	1336.11	1048930.00	1046009.62	1031.31
u724	141708.75	141338.83	92.82	**141728.25**	**141347.31**	**22.81**	140388.00	140000.11	41.79	141498.75	140897.12	32.15
rat783	29544.75	29487.75	5.79	**29551.25**	**29493.19**	7.18	29290.50	29225.20	**2.57**	29459.00	29395.50	12.90
nrw1379	**251325.50**	250903.84	49.12	251221.25	**250922.77**	**44.91**	249747.50	249174.11	209.42	249831.00	249429.70	123.99

Table 8
Results on PTP instances with $\alpha = 1.00$ under LT termination criteria .

Instance	MSHH_GREEDY			MSHH_RAND			MS-ILS			MS-GVNS		
	Best	Average	S.D	Best	Average	S.D	Best	Average	S.D	Best	Average	S.D
burma14	**-1.00**	**-33.40**	0.83	-37.00	-37.00	**0.00**	-37.00	-603.20	6.84	-37.00	-37.00	**0.00**
ulysses16	**118.00**	**118.00**	**0.00**	**118.00**	112.05	1.37	**118.00**	-444.35	32.20	**118.00**	**118.00**	**0.00**
gr17	**71.00**	**71.00**	**0.00**	**71.00**	67.45	0.81	**71.00**	-38.25	1.32	**71.00**	**71.00**	**0.00**
gr21	**7.00**	**7.00**	**0.00**	**7.00**	6.65	0.08	**7.00**	**7.00**	**0.00**	**7.00**	**7.00**	**0.00**
ulysses22	**792.00**	695.40	132.46	**792.00**	656.85	**123.62**	**792.00**	**741.15**	188.16	**792.00**	712.80	136.46
gr24	**0.00**	**-81.85**	18.78	**0.00**	-90.80	2.34	-98.00	-188.00	2.29	**0.00**	-88.20	**2.25**
fri26	**0.00**	**-104.60**	1.24	-110.00	-110.00	**0.00**	-114.00	-179.50	2.41	-110.00	-110.00	**0.00**
bayg29	**0.00**	-43.85	3.02	-25.00	-95.05	8.94	**0.00**	-164.35	**1.98**	**0.00**	**-22.80**	12.20
bays29	**0.00**	-44.40	16.88	-55.00	-60.65	**1.30**	**0.00**	-77.75	5.22	**0.00**	**-41.25**	3.15
dantzig42	**36.00**	**36.00**	**0.00**	**36.00**	**36.00**	**0.00**	13.00	13.00	**0.00**	**36.00**	**36.00**	**0.00**
swiss42	**70.00**	**65.80**	0.96	**70.00**	63.70	1.45	58.00	58.00	**0.00**	**70.00**	63.70	1.45
att48	**2405.00**	**2405.00**	**0.00**	**2405.00**	**2405.00**	**0.00**	**2405.00**	1898.55	215.78	**2405.00**	**2405.00**	**0.00**
gr48	**297.00**	**291.00**	**1.38**	**297.00**	271.20	5.92	**297.00**	186.75	25.29	**297.00**	287.65	2.15
hk48	**2010.00**	1876.90	41.50	**2010.00**	1762.30	**15.21**	**2010.00**	**1978.60**	64.83	**2010.00**	1873.15	40.64
eil51	**3.00**	-0.85	0.88	**3.00**	-1.40	**0.32**	**3.00**	**1.05**	0.45	**3.00**	-1.10	1.12
berlin52	1339.00	1245.25	13.25	1339.00	1246.25	13.02	**1361.00**	1052.00	15.83	**1361.00**	**1317.80**	**1.65**
brazil58	**13445.00**	13314.50	26.50	**13445.00**	12733.30	101.33	**13445.00**	**13381.25**	**14.63**	**13445.00**	13343.00	35.10
st70	**95.00**	85.50	2.18	**95.00**	85.50	2.18	**95.00**	**95.00**	**0.00**	**95.00**	91.00	0.92
eil76	**46.00**	41.95	**0.93**	**46.00**	40.95	1.16	**46.00**	36.95	2.05	**46.00**	**41.95**	**0.93**
pr76	**34481.00**	**34290.50**	**43.70**	**34481.00**	33909.50	742.96	34289.00	31668.85	74.07	**34481.00**	33975.35	758.07
gr96	33686.00	33462.20	51.34	33686.00	33320.00	202.12	**33816.00**	**33613.90**	**28.70**	**33816.00**	33246.95	59.66
rat99	**517.00**	486.50	7.00	**517.00**	489.55	6.30	**517.00**	507.65	**0.15**	**517.00**	**513.45**	0.81
kroA100	12511.00	12215.05	31.19	12481.00	12275.65	35.41	**12513.00**	**12488.20**	**19.78**	12483.00	12290.50	32.00
kroB100	14408.00	13839.90	121.61	14408.00	14059.70	71.19	**14463.00**	**14312.50**	**20.07**	14408.00	14049.40	73.55
kroC100	**18420.00**	18404.00	3.67	**18420.00**	**18420.00**	**0.00**	**18420.00**	**18420.00**	**0.00**	**18420.00**	**18420.00**	**0.00**
kroD100	**14840.00**	14623.05	49.77	**14840.00**	**14777.80**	**14.27**	14674.00	14558.10	63.34	**14840.00**	14533.25	70.37
kroE100	14662.00	13886.70	1285.81	14662.00	13830.40	1731.03	**14762.00**	**14631.90**	**4.38**	14662.00	14123.30	276.29
rd100	**3261.00**	3153.80	**1.79**	**3261.00**	**3180.40**	46.89	**3261.00**	3171.80	20.46	**3261.00**	3179.05	6.87
eil101	**147.00**	**142.40**	0.37	**147.00**	135.50	1.26	146.00	138.20	1.79	**147.00**	141.90	**0.02**
lin105	**14825.00**	14367.75	**12.33**	**14825.00**	13666.05	124.33	**14825.00**	**14685.30**	29.30	**14825.00**	14077.48	53.12
pr107	**55854.00**	**55729.05**	28.67	**55854.00**	55705.10	34.16	**55854.00**	55588.75	**22.08**	**55854.00**	55695.35	27.38
gr120	4970.00	**4928.60**	2.43	4978.00	4679.40	51.53	**4980.00**	4907.95	3.45	4970.00	4914.70	**0.76**
pr124	**73257.00**	72678.80	132.65	**73257.00**	72165.80	250.34	72522.00	71788.10	168.37	**73257.00**	**73257.00**	**0.00**
bier127	92376.00	87711.30	1712.66	92748.00	88765.85	186.48	**93029.00**	**91685.10**	**131.94**	92202.00	87757.45	1895.77
ch130	**3713.00**	3502.75	**5.79**	**3713.00**	3523.90	13.74	3669.00	**3529.30**	14.15	**3713.00**	3499.85	16.78
pr136	**74999.00**	73855.55	245.37	**74999.00**	**73955.00**	303.75	73955.00	72929.00	**32.58**	**74999.00**	73766.30	224.90
gr137	**92757.00**	**92646.35**	**9.26**	**92757.00**	92507.50	22.60	**92757.00**	92355.65	21.26	92739.00	92570.55	24.90
pr144	95621.00	90116.85	1202.63	95621.00	92455.50	666.11	**95622.00**	**94605.70**	**8.42**	95621.00	91415.20	282.82
ch150	**4425.00**	4168.95	**0.24**	**4425.00**	3975.75	35.62	**4425.00**	**4250.70**	15.21	**4425.00**	4095.65	7.19
kroA150	**25641.00**	25364.85	41.95	**25641.00**	**25407.10**	**13.05**	25394.00	25142.30	52.61	25585.00	25314.25	73.47
kroB150	**29541.00**	29227.80	91.95	**29541.00**	27843.25	338.79	29493.00	28897.40	**13.63**	**29541.00**	**29246.35**	66.15
pr152	140002.00	133416.09	1383.58	140002.00	128730.95	2444.21	140051.00	**136950.66**	**36.33**	**140064.00**	133290.30	1412.44
u159	**48947.00**	**48733.65**	**27.38**	48924.00	47106.05	6157.53	48753.00	48394.95	45.67	48924.00	48345.15	116.51
si175	**0.00**	-480.70	164.56	-180.00	-512.45	**9.28**	**0.00**	**-402.90**	92.43	**0.00**	**-402.90**	92.43
brg180	167420.00	167203.95	11.25	167450.00	167184.45	42.57	**167580.00**	**167550.00**	**2.29**	167333.00	167173.59	18.95
rat195	2207.00	2192.35	**0.77**	**2219.00**	**2192.45**	6.09	2176.00	2141.70	7.87	2205.00	2177.22	3.16
d198	**39159.00**	**38546.10**	**107.80**	**39159.00**	38536.75	133.12	38990.00	38310.55	110.68	39151.00	38220.00	184.45
kroA200	**38256.00**	37814.75	75.19	**38256.00**	**37828.20**	71.30	38192.00	37440.00	172.52	38138.00	37761.65	**6.80**
kroB200	39556.00	36934.30	8433.16	**39572.00**	**38930.35**	48.26	39126.00	38538.80	**12.80**	39538.00	38878.60	88.00
gr202	58111.00	**57534.15**	128.44	**58134.00**	54491.95	13034.70	57868.00	57041.10	**92.20**	57979.00	57286.15	150.30
ts225	167751.00	165548.25	505.35	**169267.00**	165911.91	347.59	169072.00	**166784.00**	403.77	167485.00	164571.41	**84.06**
tsp225	4841.00	**4778.80**	10.14	4841.00	4769.52	**0.12**	4730.00	4662.55	6.07	**4851.00**	4758.75	10.84
pr226	**250804.00**	244842.50	1367.66	**250804.00**	244305.50	1490.86	249863.00	249041.95	**3.43**	**250804.00**	**250399.91**	84.45
gr229	258132.00	256839.70	93.21	**258181.00**	**256870.09**	**2.55**	255453.00	250994.84	109.63	257606.00	256545.16	243.38
gil262	3030.00	2982.85	7.38	**3031.00**	**2990.30**	5.90	2978.00	2909.30	1.31	3028.00	2964.05	**0.45**
pr264	**151197.00**	**150502.66**	**4.21**	151084.00	150441.75	56.95	149888.00	148918.20	16.93	151043.00	150223.55	8.84
a280	3990.00	**3938.15**	12.19	3985.00	3936.00	10.55	3933.00	3888.30	**0.39**	**3994.00**	3915.90	6.63
pr299	110286.00	109783.40	39.37	**110694.00**	**109889.55**	38.42	109190.00	108129.05	100.50	110024.00	109387.05	**33.02**
lin318	75975.00	70142.35	1219.49	**76613.00**	68941.05	1412.96	74639.00	**73872.30**	**57.88**	76147.00	69813.60	1160.02
rd400	**27980.00**	**27798.50**	**15.26**	27944.00	27763.15	40.80	27592.00	27232.00	29.37	27887.00	27651.25	30.23
fl417	77287.00	77059.35	27.84	**77301.00**	**77062.70**	23.56	77151.00	76840.65	22.63	77245.00	77025.15	**19.99**
gr431	644187.00	**641853.62**	**409.37**	**644612.00**	641741.12	498.04	638592.00	634848.38	826.49	643162.00	640346.00	646.03
pr439	341944.00	**339232.09**	596.27	**342899.00**	338670.34	690.00	339553.00	336678.84	**67.94**	341681.00	338079.69	84.72
pcb442	**106671.00**	**105405.35**	**34.10**	106540.00	105318.25	241.86	104551.00	103182.20	173.94	106417.00	104892.95	216.56
d493	134258.00	133802.41	**10.19**	**134269.00**	**133904.91**	13.33	132933.00	132399.09	73.62	134115.00	133538.05	21.55
att532	92683.00	**92173.70**	**26.54**	**92845.00**	92151.75	73.59	91850.00	91178.80	46.39	92536.00	91956.45	44.63
ali535	789454.00	**784244.38**	**121.05**	789810.00	774576.19	2515.96	784525.00	777408.88	770.58	**789827.00**	782967.06	797.44
si535	**-281.00**	**-396.60**	24.00	**-281.00**	-453.85	39.65	-915.00	-1164.00	**6.19**	**-281.00**	-439.10	10.35
pa561	4637.00	4586.40	6.29	**4640.00**	**4605.70**	2.91	4532.00	4482.20	**0.73**	4603.00	4551.35	6.11
u574	**111169.00**	110481.75	**2.70**	110907.00	**110510.60**	64.79	109683.00	108692.25	149.87	110575.00	110048.60	62.95
rat575	16099.00	16019.90	22.23	**16107.00**	**16039.95**	14.69	15819.00	15730.50	5.85	16069.00	15957.60	**1.06**
p654	343959.00	343172.31	125.65	**343964.00**	**343179.66**	145.99	343629.00	342973.44	**100.84**	343855.00	343104.44	110.70
d657	180715.00	179945.41	**18.45**	**181219.00**	**180000.50**	71.00	179015.00	178084.50	47.83	180049.00	179364.25	127.96
gr666	**987134.00**	980503.38	332.80	984982.00	**980853.31**	947.19	974771.00	969938.88	1562.07	982484.00	976371.12	**24.12**
u724	131335.00	130742.30	**5.90**	**131471.00**	**130832.55**	67.55	129517.00	128864.95	77.55	131192.00	130021.75	51.45
rat783	27333.00	27241.80	4.40	**27347.00**	**27264.85**	6.16	26999.00	26916.60	7.94	27224.00	27134.00	**1.84**
nrw1379	**237029.00**	**236468.95**	125.94	236780.00	236381.00	**36.94**	234797.00	233941.25	349.92	235089.00	234439.20	201.93

Table 9
Best solution quality: Performance comparison summary under ST termination criteria .

	α=0.25			α=0.50			α=0.75			α=1			
	>	<	=	>	<	=	>	<	=	>	<	=	
MSSH_GREEDY	18	**30**	29	14	**30**	33	**25**	20	32	**28**	14	35	MSSH_RAND
	58	2	17	**50**	6	21	**55**	3	19	**60**	4	13	MS-ILS
	42	6	29	**46**	6	25	**46**	3	28	**44**	3	30	MS-GVNS
MSSH_RAND	**30**	18	29	**30**	14	33	20	**25**	32	14	**28**	35	MSSH_GREEDY
	58	1	18	**52**	3	22	**54**	5	18	**57**	8	12	MS-ILS
	45	4	28	**48**	3	26	**47**	5	25	**38**	8	31	MS-GVNS
MS-ILS	2	**58**	17	6	**50**	21	3	**55**	19	4	**60**	13	MSSH_GREEDY
	1	**58**	18	3	**52**	22	5	**54**	18	8	**57**	12	MSSH_RAND
	8	**51**	18	7	**48**	22	9	**48**	20	6	**53**	18	MS-GVNS
MS-GVNS	6	**42**	29	6	**46**	25	3	**46**	28	3	**44**	30	MSSH_GREEDY
	4	**45**	28	3	**48**	26	5	**47**	25	8	**38**	31	MSSH_RAND
	51	8	18	**48**	7	22	**48**	9	20	**53**	6	18	MS-ILS

Table 10
Best solution quality: Performance comparison summary under LT termination criteria .

	α=0.25			α=0.50			α=0.75			α=1			
	>	<	=	>	<	=	>	<	=	>	<	=	
MSSH_GREEDY	14	**27**	36	17	**22**	38	15	**25**	37	14	**23**	40	MSSH_RAND
	54	3	20	**44**	7	26	**46**	10	21	**45**	11	21	MS-ILS
	34	10	33	**37**	6	**34**	35	7	35	**34**	7	36	MS-GVNS
MSSH_RAND	**27**	14	36	**22**	17	38	**25**	15	37	**23**	14	40	MSSH_GREEDY
	55	1	21	**44**	7	26	**46**	7	24	**45**	13	19	MS-ILS
	39	4	34	**37**	4	36	**38**	9	30	**32**	10	35	MS-GVNS
MS-ILS	3	**54**	20	7	**44**	26	10	**46**	21	11	**45**	21	MSSH_GREEDY
	1	**55**	21	7	**44**	26	7	**46**	24	13	**45**	19	MSSH_RAND
	4	**53**	20	7	**44**	26	12	**43**	22	10	**44**	23	MS-GVNS
MS-GVNS	10	**34**	33	6	**37**	34	7	**35**	35	7	**34**	36	MSSH_GREEDY
	4	**39**	34	4	**37**	36	9	**38**	30	10	**32**	35	MSSH_RAND
	53	4	20	**44**	7	26	**43**	12	22	**44**	10	23	MS-ILS

Table 11
Average solution quality: Performance comparison summary under ST termination criteria .

	α=0.25			α=0.50			α=0.75			α=1			
	>	<	=	>	<	=	>	<	=	>	<	=	
MSSH_GREEDY	31	**38**	8	32	**43**	2	**43**	33	1	**52**	24	1	MSSH_RAND
	63	7	7	**60**	15	2	**61**	15	1	**60**	16	1	MS-ILS
	66	3	8	**63**	12	2	**61**	14	2	**54**	18	5	MS-GVNS
MSSH_RAND	**38**	31	8	**43**	32	2	33	**43**	1	24	**52**	1	MSSH_GREEDY
	58	11	8	**58**	18	1	**56**	21	0	**57**	20	0	MS-ILS
	58	12	7	**57**	17	3	**51**	24	2	**46**	28	3	MS-GVNS
MS-ILS	7	**63**	7	15	**60**	2	15	**61**	1	16	**60**	1	MSSH_GREEDY
	11	**58**	8	18	**58**	1	21	**56**	0	20	**57**	0	MSSH_RAND
	10	**59**	8	16	**59**	2	15	**62**	0	14	**61**	2	MS-GVNS
MS-GVNS	3	**33**	8	12	**63**	2	14	**61**	2	18	**54**	5	MSSH_GREEDY
	12	**58**	7	17	**57**	3	24	**51**	2	28	**46**	3	MSSH_RAND
	59	10	8	**59**	16	2	**62**	15	0	**61**	14	2	MS-ILS

Table 12
Average solution quality: Performance comparison summary under LT termination criteria .

	α=0.25			α=0.50			α=0.75			α=1			
	>	<	=	>	<	=	>	<	=	>	<	=	
MSSH_GREEDY	**34**	30	13	**36**	34	7	**39**	36	2	**45**	29	3	MSSH_RAND
	61	7	9	**52**	23	2	**53**	22	2	**52**	24	1	MS-ILS
	57	11	9	**51**	24	2	**50**	26	1	**48**	28	1	MS-GVNS
MSSH_RAND	30	**34**	13	34	**36**	7	36	**39**	2	29	**45**	3	MSSH_GREEDY
	57	11	9	**51**	24	2	**50**	26	1	**48**	28	1	MS-ILS
	58	11	8	**56**	17	4	**47**	26	4	**38**	33	6	MS-GVNS
MS-ILS	7	**61**	9	23	**52**	2	22	**53**	2	24	**52**	1	MSSH_GREEDY
	11	**57**	9	24	**51**	2	26	**50**	1	28	**48**	1	MSSH_RAND
	13	**55**	9	30	**44**	3	22	**54**	1	21	**53**	3	MS-GVNS
MS-GVNS	11	**57**	9	24	**51**	2	26	**50**	1	28	**48**	1	MSSH_GREEDY
	11	**58**	8	17	**56**	4	26	**47**	4	33	**38**	6	MSSH_RAND
	55	13	9	44	30	3	**54**	22	1	**53**	21	3	MS-ILS

Table 13
Wilcoxon signed-rank test between different approaches under ST termination criteria.

α value	Total Instances	NWT	Compared Approaches	R+	R-	Z	Z_{cri}	P-value	Significant
0.25	77	69	MSHH_GREEDY vs MSHH_RAND	1337	1078	-0.774	-1.64	0.2194	no
	77	70	MSHH_GREEDY vs MS-ILS	2408	77	-6.821	-1.64	0	yes
	77	69	MSHH_GREEDY vs MS-GVNS	2369	46	-6.945	-1.64	0	yes
	77	69	MSHH_RAND vs MS-ILS	2024	391	-4.882	-1.64	0	yes
	77	70	MSHH_RAND vs MS-GVNS	2022	463	-4.562	-1.64	0	yes
	77	69	MS-ILS vs MS-GVNS	178	2237	-6.155	-1.64	0	yes
0.5	77	75	MSHH_GREEDY vs MSHH_RAND	1216	1634	-1.104	-1.64	0.1349	no
	77	75	MSHH_GREEDY vs MS-ILS	2588	262	-6.141	-1.64	0	yes
	77	75	MSHH_GREEDY vs MS-GVNS	2589	261	-6.147	-1.64	0	yes
	77	76	MSHH_RAND vs MS-ILS	2507	419	-5.405	-1.64	0	yes
	77	74	MSHH_RAND vs MS-GVNS	2222	553	-4.496	-1.64	0	yes
	77	75	MS-ILS vs MS-GVNS	354	2496	-5.655	-1.64	0	yes
0.75	77	76	MSHH_GREEDY vs MSHH_RAND	1805	1121	-1.771	-1.64	0.0383	yes
	77	76	MSHH_GREEDY vs MS-ILS	2528	398	-5.514	-1.64	0	yes
	77	75	MSHH_GREEDY vs MS-GVNS	2455	395	-5.439	-1.64	0	yes
	77	77	MSHH_RAND vs MS-ILS	2306	697	-4.085	-1.64	0	yes
	77	75	MSHH_RAND vs MS-GVNS	2020	830	-3.142	-1.64	0.0008	yes
	77	76	MS-ILS vs MS-GVNS	404	2599	-5.573	-1.64	0	yes
1	77	76	MSHH_GREEDY vs MSHH_RAND	1971	955	-2.63	-1.64	0.0043	yes
	77	76	MSHH_GREEDY vs MS-ILS	2528	398	-5.514	-1.64	0	yes
	77	72	MSHH_GREEDY vs MS-GVNS	2023	605	-3.979	-1.64	0	yes
	77	77	MSHH_RAND vs MS-ILS	2262	741	-3.861	-1.64	0.0001	yes
	77	74	MSHH_RAND vs MS-GVNS	1851	294	-2.497	-1.64	0.0063	yes
	77	75	MS-ILS vs MS-GVNS	523	2327	-4.763	-1.64	0	yes

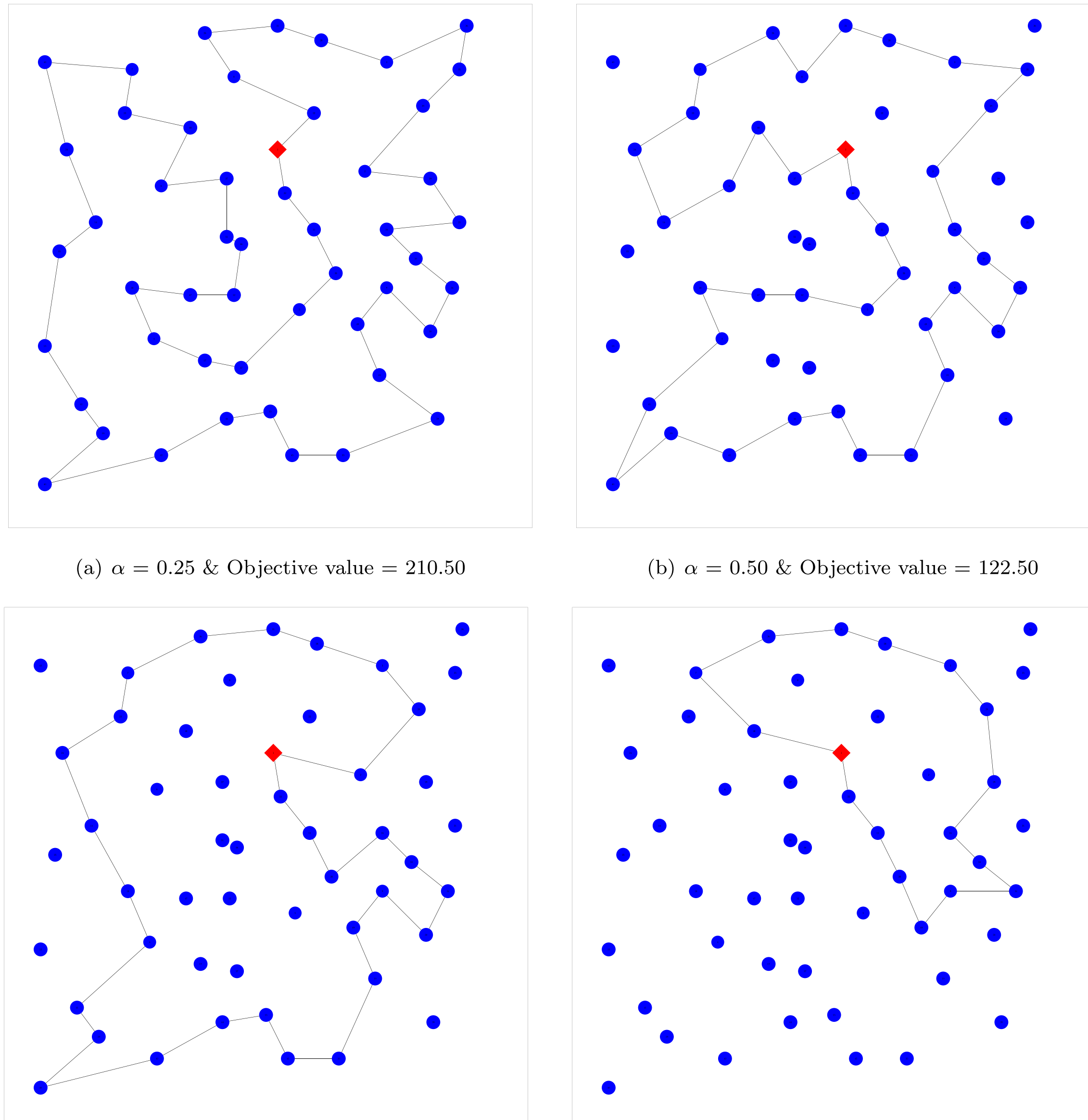
Table 14
Wilcoxon signed-rank test between different approaches under LT termination criteria.

α value	Total Instances	NWT	Compared Approaches	R+	R-	Z	Z_{cri}	P-value	Significant
0.25	77	64	MSHH_GREEDY vs MSHH_RAND	1038	1038	-0.013	-1.64	0.4947	no
	77	68	MSHH_GREEDY vs MS-ILS	2230	116	-6.459	-1.64	0	yes
	77	69	MSHH_GREEDY vs MS-GVNS	2325	90	-6.681	-1.64	0	yes
	77	68	MSHH_RAND vs MS-ILS	2057.5	288.5	-5.405	-1.64	0	yes
	77	70	MSHH_RAND vs MS-GVNS	2038	377	-4.966	-1.64	0	yes
	77	70	MS-ILS vs MS-GVNS	247	2099	-5.658	-1.64	0	yes
0.5	77	70	MSHH_GREEDY vs MSHH_RAND	1401	1084	-0.928	-1.64	0.1768	no
	77	75	MSHH_GREEDY vs MS-ILS	2280	570	-4.515	-1.64	0	yes
	77	74	MSHH_GREEDY vs MS-GVNS	2444	331	-5.692	-1.64	0	yes
	77	75	MSHH_RAND vs MS-ILS	2161	689	-3.886	-1.64	0	yes
	77	73	MSHH_RAND vs MS-GVNS	2108	593	-4.164	-1.64	0	yes
	77	74	MS-ILS vs MS-GVNS	728	2047	-3.553	-1.64	0.0002	yes
0.75	77	75	MSHH_GREEDY vs MSHH_RAND	1536	1314	-0.586	-1.64	0.2789	no
	77	75	MSHH_GREEDY vs MS-ILS	2162	688	-3.892	-1.64	0	yes
	77	72	MSHH_GREEDY vs MS-GVNS	2220	481	-4.78	-1.64	0	yes
	77	76	MSHH_RAND vs MS-ILS	1973	953	-2.64	-1.64	0	yes
	77	73	MSHH_RAND vs MS-GVNS	1776	925	-2.339	-1.64	0.0097	yes
	77	76	MS-ILS vs MS-GVNS	690	2236	-4.002	-1.64	0	yes
1	77	74	MSHH_GREEDY vs MSHH_RAND	1737.5	1037.5	-1.886	-1.64	0.0297	yes
	77	76	MSHH_GREEDY vs MS-ILS	2085	841	-3.22	-1.64	0.0006	yes
	77	71	MSHH_GREEDY vs MS-GVNS	1918	638	-3.667	-1.64	0.0001	yes
	77	76	MSHH_RAND vs MS-ILS	1818	1108	-1.838	-1.64	0.033	yes
	77	73	MSHH_RAND vs MS-GVNS	1459	1097	-1.037	-1.64	0.1498	no
	77	74	MS-ILS vs MS-GVNS	744	2031	-3.467	-1.64	0.0003	yes

Similar conclusions can be drawn for LT termination criteria also. Tables 5, 6, 7, 8, 10 and 12 clearly demonstrate that MSHH_RAND and MSHH_GREEDY performed better than MS-ILS and MS-GVNS under LT termination condition on the majority of instances in terms of best and average solution quality both. Further, MSSH_RAND performed better than MSHH_GREEDY in terms of average solution quality. However, in terms of best solution quality, MSHH_RAND performed only slightly better than MSHH_GREEDY. MS-GVNS outperformed MS-ILS in terms of both best as well as average solution quality. MSHH_RAND is again the best method overall under ST termination critria.

We have used the Wilcoxon signed-rank test [16,30,42] to check whether there are significant differences among the performances of our four approaches. To carry out this test, we set the significance level to 5% (i.e. p-value ≤ 0.05) and made use of the calculator available online[2]. Tables 13 and 14 report the results of the Wilcoxon signed-rank test corresponding to values of α equal to 0.25, 0.50, 0.75, 1.0 under ST and LT termination criteria respectively. In these tables, the column name NWT reports the number of instances without a tie. The column name R^+ reports the sum of ranks for the instances in which the first method outperforms the second method in the row and R^- reports the sum of ranks for the opposite. In all the cases, the number of instances without tie exceeds thirty (NWT > 30), so we used the test statistic Z. According to the Wilcoxon signed-rank test, Z value is compared with the critical value Z_{cri}. If Z value is not exceeding Z_{cri} ($Z \leq Z_{cri}$), then there is a significant difference between two methods, else there is no significant difference. The column named P-value provides the corre-

[2] https://mathcracker.com/wilcoxon-signed-ranks.php



(a) $\alpha = 0.25$ & Objective value = 210.50

(b) $\alpha = 0.50$ & Objective value = 122.50

(c) $\alpha = 0.75$ & Objective value = 48.0

(d) $\alpha = 1$ & Objective value = 3.0

Fig. 8. Best solution found by MSHH_RAND on instance eil51 for different α values under LT termination criteria.

sponding p-values for direct comparison with the significance level. If the p-value is less than or equal to significance level then there is a significant difference between two methods. The conclusions that can be derived from these two tables are almost the same. In both ST & LT termination conditions, there are significant differences between performance of different approaches in all cases except for performance difference between MSHH_RAND and MS-GVNS in one case ($alpha = 1.0$ under LT termination criteria) and MSHH_RAND and MSHH_GREEDYin 5 cases ($\alpha = 0.25, 0.5$ under ST termination criteria and $\alpha = 0.25, 0.5, 0.75$ under LT termination criteria).

To understand the variation in the composition of the tour with values of α, we have taken instance eil51 and plotted the best solution obtained through MSHH_RAND under LT termination criteria. Figs. 8(a), 8(b), 8(c) and 8(d) show these solutions. In these figures, a red coloured square denotes the depot and remaining cities are shown as blue circles. These figures show that the composition of the tour changes significantly with the α values and with increase in the value of α, the objective value decreases.

To assess the convergence behavior of the proposed approaches, we have plotted the convergence behavior on instance gr431 for four α values under LT termination condition. Figs. 9(a), 9(b), 9(c) and 9(d) show these convergence behaviors. We can observe that the MSHH_GREEDY converges faster than MSHH_RAND, MS-GVNS and MS-ILS. The convergence of MS-GVNS is faster than MSHH_RAND and MS-ILS.

MSHH_GREEDY uses one heuristic from each of the three groups. The subset alteration group includes two heuristics, viz. addition (LH_1) and removal(LH_2). The permutation group includes 4 heuristics, viz. swap (LH_3), 2-opt (LH_5), relocate (LH_6) and multiple removal and reinsertion (LH_7). The combination group also includes two heuristics, viz. exchange (LH_4) and multi removal and addition (LH_8). Therefore, MSHH_GREEDY approach can use 16 ($4 \times 2 \times 2$) different combinations of three heuristics. We have analyzed the results produced by each of these 16 combinations on 4 instances of different sizes, viz. *rat195, tsp225, fl417* and *ali535* under LT termination criteria for different values of α. For this analysis, we have used following nomenclature for

Table 15
Best solution found by MSSH with different greedy selection combinations on 4 different instances.

Name	alpha=0.25				alpha=0.50				alpha=0.75				alpha=1.0			
	B1	B2	B3	B4	B1	B2	B3	B4	B1	B2	B3	B4	B1	B2	B3	B4
MSSH-134	3842.25	7559.25	85715.75	933132.25	3221.00	6548.00	82793.00	880510.00	2621.50	5603.25	79875.50	829083.00	2071.00	4681.00	77014.00	780502.00
MSSH-154	3845.25	7563.75	85732.75	933623.75	3225.50	6558.50	82822.50	881493.00	2633.00	5612.00	79931.50	831034.50	2096.00	4714.00	77075.00	783233.00
MSSH-164	3861.00	7582.25	85738.50	934321.25	3247.00	6580.00	82844.50	882538.50	2657.50	5623.50	79966.00	831653.25	2137.00	4719.00	77122.00	783801.00
MSSH-174	3890.00	7609.75	85777.50	935396.75	3300.50	6644.50	82925.00	884894.00	2717.75	5737.25	80076.25	835324.00	2206.00	4833.00	77284.00	788862.00
MSSH-138	3883.50	7608.25	85764.00	935437.50	3303.00	6643.00	82894.50	884362.00	2741.25	5721.00	80044.50	834863.75	2206.00	**4841.00**	77276.00	789122.00
MSSH-158	3887.50	7609.50	85768.25	935557.75	3299.50	6641.50	82894.50	884624.00	2738.75	5727.50	80057.50	835517.75	2205.00	4839.00	77286.00	788392.00
MSSH-168	3883.25	7607.75	85783.00	935728.25	3303.50	6639.00	82922.50	884738.00	2742.20	5723.50	80089.50	835059.50	**2207.00**	4837.00	77286.50	788951.00
MSSH-178	3891.75	7603.75	85776.25	935573.25	3301.50	6643.50	82922.00	884919.50	2740.25	5730.75	80084.00	835933.00	2202.00	4840.00	77298.00	788958.00
MSSH-234	3829.25	7509.00	80345.75	930933.25	3209.00	6488.50	77404.50	873975.00	2631.00	5462.25	74499.50	819180.25	2073.00	4502.00	71809.00	766458.00
MSSH-254	3835.75	7516.00	80444.75	932477.25	3220.00	6502.50	77647.50	876216.50	2668.50	5483.25	74779.00	822552.75	2111.00	4552.00	72322.00	773368.00
MSSH-264	3846.50	7525.00	80519.50	931545.75	3241.50	6504.50	77646.50	874351.50	2680.00	5530.50	74650.25	820230.50	2142.00	4599.00	72228.00	768435.00
MSSH-274	3857.50	7560.25	80319.00	934096.00	3257.50	6557.00	77673.00	878302.50	2701.75	5579.75	74387.00	825794.00	2121.00	4627.00	71302.00	776114.00
MSSH-238	3891.75	7607.75	85779.75	935437.50	3301.50	6632.00	82906.50	884079.00	2731.25	5726.25	80085.75	835208.00	2206.00	4833.00	77283.00	788680.00
MSSH-258	3886.25	7610.75	85767.00	935765.25	3302.50	6647.00	82898.00	884407.00	2742.00	5728.25	80086.25	835045.75	2206.00	4831.00	77283.00	789431.00
MSSH-268	3890.25	7609.50	85781.00	935770.25	3300.00	6635.50	82921.00	884914.50	2741.75	5726.75	80089.00	835863.00	2205.00	4827.00	77212.00	789224.00
MSSH-278	3889.00	7603.50	85784.50	935294.25	3302.50	6639.50	82924.50	884750.50	2740.50	5729.75	80049.00	835902.75	2205.00	4839.00	77286.00	789099.00
MSHH_GREEDY	**3894.50**	**7612.50**	**85784.75**	**935778.25**	**3304.00**	**6647.50**	**82925.50**	**884922.00**	**2743.00**	**5738.50**	**80090.25**	**836263.25**	**2207.00**	**4841.00**	**77287.00**	**789454.00**

Table 16
Average solution quality obtained by MSSH with different greedy selection combinations on 4 different instances.

Name	alpha=0.25				alpha=0.50				alpha=0.75				alpha=1.0			
	A1	A2	A3	A4	A1	A2	A3	A4	A1	A2	A3	A4	A1	A2	A3	A4
MSSH-134	3825.84	7529.18	85614.73	931576.72	3194.65	6480.80	82584.80	876867.47	2580.62	5459.16	79532.91	821233.91	1989.90	4460.70	76538.80	758835.00
MSSH-154	3831.40	7538.44	85644.66	932673.72	3205.40	6496.55	82654.25	878843.95	2594.75	5489.38	79636.14	824314.16	2006.95	4501.90	76674.90	762426.75
MSSH-164	3843.51	7555.84	85643.55	932686.38	3222.72	6523.65	82640.98	878858.07	2618.64	5528.32	79618.59	823970.41	2035.80	4545.40	76643.20	762608.05
MSSH-174	3875.30	7592.52	85722.16	934717.01	3278.32	6617.32	82779.93	**883133.72**	2701.49	5683.74	79813.50	830175.75	2159.10	4731.15	76878.75	771464.90
MSSH-138	3876.24	7588.39	85716.60	934404.51	3280.50	6607.20	82773.52	882083.07	2714.72	5676.18	79866.90	831778.75	2186.15	4766.95	77037.20	783034.25
MSSH-158	3875.32	7592.41	85725.95	934677.68	3282.28	6610.55	82792.25	882496.82	2717.61	5677.74	79880.50	832415.21	2185.60	4766.35	77043.30	783833.50
MSSH-168	3876.22	7594.00	85726.34	934802.81	3285.90	6617.70	82800.23	882994.45	**2722.38**	5683.00	79898.85	832366.49	2188.30	4771.65	77048.90	782503.10
MSSH-178	3876.86	7594.36	85724.01	**934819.49**	3287.57	6619.07	82785.30	882930.65	2722.31	5685.31	79881.69	832205.69	2188.25	4775.90	77036.20	783588.05
MSSH-234	3104.36	6380.46	60913.39	629962.50	2580.68	5452.57	58311.03	588958.88	2083.29	4566.75	55728.70	550627.01	1614.00	3743.45	53217.40	512822.95
MSSH-254	3123.86	6412.96	61013.69	632920.31	2604.30	5497.52	58426.45	593141.78	2111.28	4621.34	55889.16	555755.29	1635.30	3782.20	53465.15	518896.30
MSSH-264	3134.54	6433.76	61025.46	632618.18	2624.72	5528.25	58458.10	592023.88	2130.81	4665.96	56003.86	554629.84	1651.85	3829.35	53508.70	517521.40
MSSH-274	3146.15	6461.56	60979.61	633661.56	2612.82	5511.65	58262.93	592733.97	2079.89	4568.36	55689.20	552109.41	1570.45	3639.40	52823.50	511323.85
MSSH-238	3875.50	7591.64	85717.44	934490.59	3279.22	6607.02	82772.65	882015.82	2714.03	5672.14	79861.60	831838.64	2186.80	4756.50	77030.40	783397.35
MSSH-258	3874.05	7591.43	85723.12	934677.38	3281.90	6616.23	82786.57	882285.97	2716.82	5678.54	79890.52	831946.32	2191.85	4762.05	77050.95	783910.40
MSSH-268	3878.50	7594.94	85727.35	934806.18	3286.57	6612.68	82800.52	883070.05	2720.15	5684.21	79886.46	832517.36	2185.65	4770.30	77052.80	782529.50
MSSH-278	3877.96	7594.79	85724.44	934790.69	3286.38	6618.88	82782.93	882857.38	2721.79	5689.89	79875.71	832547.20	2189.30	4768.85	77054.60	784064.70
MSHH_GREEDY	**3879.22**	**7595.85**	**85727.60**	934817.28	**3287.82**	**6621.62**	**82800.93**	882798.68	2721.36	**5692.00**	**79895.98**	**832573.53**	**2192.35**	**4778.8**	**77059.35**	**784244.35**

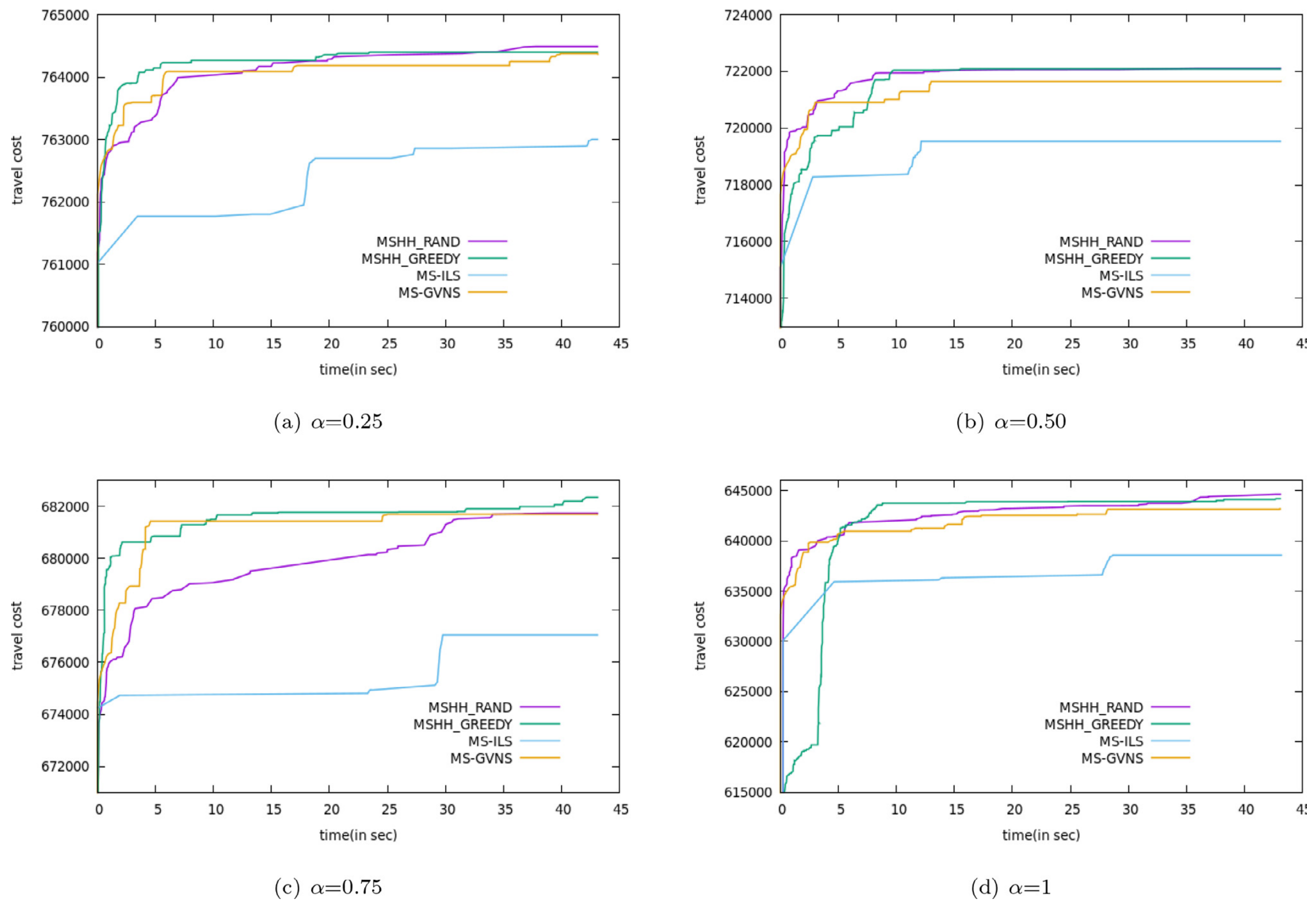


Fig. 9. Convergence behavior on instance gr431 under LT termination condition.

each of these 16 combinations:

$$\begin{aligned}
&\text{MSHH-134} \longleftarrow \text{LH_1},\ \text{LH_3},\ \text{and LH_4}\\
&\text{MSHH-154} \longleftarrow \text{LH_1},\ \text{LH_5},\ \text{and LH_4}\\
&\text{MSHH-164} \longleftarrow \text{LH_1},\ \text{LH_6},\ \text{and LH_4}\\
&\qquad\qquad\vdots\\
&\text{MSHH-268} \longleftarrow \text{LH_2},\ \text{LH_6},\ \text{and LH_8}\\
&\text{MSHH-278} \longleftarrow \text{LH_2},\ \text{LH_7},\ \text{and LH_8}
\end{aligned}$$

Table 15 and Table 16 present the results of best and average solution qualities of the above 16 combinations and MSHH_GREEDY on 4 instances. In these two tables, the first column denotes the name of combination. In Table 15, columns B1, B2, B3, and B4 present the best solution quality of rat195, tsp225, fl417 and ali535 instance respectively for different values of α. Likewise, columns A1, A2, A3, and A4 in Table 16 present the average solution quality. In these two tables, best values among 16 combinations and MSHH_GREEDY are represented in bold font. From these two tables, we can conclude that the combinations MSHH-168, MSHH-178 and MSHH-278 provide the best results and contribute significantly to the success of MSHH_GREEDY though none of these combinations alone provided as good or better results as obtained by MSHH_GREEDY in most cases, thereby demonstrating the need of different heuristics used by us.

8. Conclusions

In this paper, a variant of the traveling salesman problem (TSP) called the profitable tour problem (PTP) is studied. The objective of the PTP is to maximize the total profit collected from visited cities minus total travel cost. As our contribution, we have developed a mathematical model, generated new benchmark instances and proposed four multi-start approaches, viz. MSHH_RAND, MSHH_GREEDY, MS-ILS and MS-GVNS to solve the PTP. The first two approaches are based on hyper-heuristic and other two approaches are metaheuristic approaches. To evaluate the performance of proposed approaches, test instances with various sizes are generated for the PTP from instances available in TSPLIB. Computational results on these test instances show that MSHH_RAND and MSHH_GREEDY performed better than MS-ILS and MS-GVNS in terms of best as well as average solution quality. MS-GVNS performed better than MS-ILS in terms of best as well as average solution quality. MSHH_RAND performed the best among all the four approaches.

Similar approaches can be developed for related TSP variants and other related permutation based problems. Since our approaches are the first heuristic approaches for PTP, these approaches will be used as baselining approaches for evaluating the performance of future approaches for this problem. As a future work, we intend to study multiple salesmen variant of PTP. Another possible future work is to formulate and study the PTP as a bi-objective problem where one objective is to maximize the total profit and the other objective is to minimize the total travel cost. The concept of dividing low-level heuristics into different groups as per their functionality and using only one heuristic from each group under greedy selection mechanism saves the computation time. Similar groupings can be tried when developing hyper-heuristic approaches for other problems. Our GVNS approach utilize different neighborhoods for shake function and VND which deviates from the usual practice of using the same neighborhoods for both. This is done as the neighborhoods used in VND are not able to produce sufficient shaking to escape the local maxima. Similar disjoint neighborhoods may be tried for other problems whenever it is observed that neighborhoods used in VND are incapable of producing sufficient shaking. Another possible future work is to enhance the performance of our proposed approaches by hybridizing them with machine learning based methods.

Declaration of Competing Interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

CRediT authorship contribution statement

Kasi Viswanath Dasari: Methodology, Software, Formal analysis, Investigation, Writing - original draft, Visualization. **Venkatesh Pandiri:** Methodology, Validation, Formal analysis, Visualization. **Alok Singh:** Supervision, Conceptualization, Validation, Formal analysis, Writing - review & editing, Resources, Project administration, Funding acquisition.

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References

[1] E. Balas, The prize collecting traveling salesman problem, Networks 19 (6) (1989) 621–636.
[2] M.E. Bruni, L. Brusco, G. Ielpa, P. Beraldi, The risk-averse profitable tour problem, in: ICORES, 2019, pp. 459–466.
[3] T. Bulhões, M.H. Ha, R. Martinelli, T. Vidal, The vehicle routing problem with service level constraints, Eur J Oper Res 265 (2) (2018) 544–558.
[4] E.K. Burke, M. Gendreau, M. Hyde, G. Kendall, G. Ochoa, E. Özcan, R. Qu, Hyper-heuristics: a survey of the state of the art, Journal of the Operational Research Society 64 (12) (2013) 1695–1724.
[5] K. Chakhlevitch, P. Cowling, Hyperheuristics: Recent Developments, in: Adaptive and multilevel metaheuristics, Springer, 2008, pp. 3–29.
[6] H. Chentli, R. Ouafi, W.R. Cherif-Khettaf, Behaviour of a hybrid ILS heuristic on the capacitated profitable tour problem, in: ICORES, 2018, pp. 115–123.
[7] H. Chentli, R. Ouafi, W.R. Cherif-Khettaf, Impact of iterated local search heuristic hybridization on vehicle routing problems: application to the capacitated profitable tour problem, in: International Conference on Operations Research and Enterprise Systems, Springer, 2018, pp. 80–101.
[8] H. Chentli, R. Ouafi, W.R. Cherif-Khettaf, A selective adaptive large neighborhood search heuristic for the profitable tour problem with simultaneous pickup and delivery services, RAIRO-Operations Research 52 (4–5) (2018) 1295–1328.
[9] D.L. Cortés-Murcia, H.M. Afsar, C. Prodhon, A branch-and-price algorithm for the electric capacitated profitable tour problem with mandatory stops, IFAC-Papers On-Line 52 (13) (2019) 1572–1577.
[10] P. Cowling, G. Kendall, E. Soubeiga, A hyperheuristic approach to scheduling a sales summit, in: International Conference on the Practice and Theory of Automated Timetabling, Springer, 2000, pp. 176–190.
[11] G.A. Croes, A method for solving traveling salesman problems, Oper Res 6 (1958) 791–812.
[12] W.B. Crowston, F. Glover, J.D. Trawick, et al., Probabilistic and parametric learning combinations of local job shop scheduling rules, Technical Report, Research Memorandum, No. 117, GSIA, Carnegie Mellon University, Pittsburgh, 1963.
[13] D.P. Cuervo, P. Goos, K. Sörensen, E. Arráiz, An iterated local search algorithm for the vehicle routing problem with backhauls, Eur J Oper Res 237 (2) (2014) 454–464.
[14] M. Dell'Amico, F. Maffioli, P. Värbrand, On prize-collecting tours and the asymmetric travelling salesman problem, International Transactions in Operational Research 2 (3) (1995) 297–308.
[15] J. Denzinger, M. Fuchs, M. Fuchs, High performance ATP systems by combining several AI methods, in: Proceedings of the 15th International Joint Conference on Artifical Intelligence - Volume 1, in: IJCAI'97, Morgan Kaufmann Publishers Inc., San Francisco, CA, USA, 1997, pp. 102–107.
[16] J. Derrac, S. Garca, D. Molina, F. Herrera, A practical tutorial on the use of nonparametric statistical tests as a methodology for comparing evolutionary and swarm intelligence algorithms, Swarm Evol Comput 1 (1) (2011) 3–18.
[17] D. Feillet, P. Dejax, M. Gendreau, The profitable arc tour problem: solution with a branch-and-price algorithm, Transportation Science 39 (4) (2005) 539–552.
[18] D. Feillet, P. Dejax, M. Gendreau, Traveling salesman problems with profits, Transportation science 39 (2) (2005) 188–205.
[19] H. Fisher, Probabilistic learning combinations of local job-shop scheduling rules, Industrial scheduling (1963) 225–251.
[20] M. Gansterer, M. Küçüktepe, R.F. Hartl, The multi-vehicle profitable pickup and delivery problem, OR Spectrum 39 (1) (2017) 303–319.
[21] M.R. Garey, D.S. Johnson, Computers and intractability: A guide to the theory of NP-completeness, W. H. Freeman, San Francisco, 1979.
[22] P. Hansen, N. Mladenović, J.A.M. Pérez, Variable neighbourhood search: methods and applications, Ann Oper Res 175 (1) (2010) 367–407.
[23] P. Hansen, N. Mladenović, R. Todosijević, S. Hanafi, Variable neighborhood search: basics and variants, EURO Journal on Computational Optimization 5 (2017) 423–454.
[24] Y. Hu, Y. Yao, W.S. Lee, A reinforcement learning approach for optimizing multiple traveling salesman problems over graphs, Knowledge Based Systems 204 (2020) 106244.
[25] G. Lera-Romero, J.J. Miranda-Bront, A branch and cut algorithm for the time-dependent profitable tour problem with resource constraints, Eur J Oper Res 289 (3) (2021) 879–896.
[26] H.R. Lourenço, O.C. Martin, T. Stutzle, Iterated local search, International Series in Operations Research and Management Science (2003) 321–354.
[27] H.R. Lourenço, O.C. Martin, T. Stützle, Iterated Local Search: Framework and Applications, in: Handbook of Metaheuristics, Springer, 2010, pp. 363–397.
[28] N. Mladenović, P. Hansen, Variable neighborhood search, Computers & operations research 24 (11) (1997) 1097–1100.
[29] N. Mladenović, R. Todosijević, D. Urošević, et al., An efficient general variable neighborhood search for large travelling salesman problem with time windows, Yugoslav Journal of Operations Research 23 (1) (2013) 19–30.
[30] A.W. Mohamed, A.A. Hadi, K.M. Jambi, Novel mutation strategy for enhancing SHADE and LSHADE algorithms for global numerical optimization, Swarm Evol Comput 50 (2019) 100455.
[31] P.H.V. Penna, A. Subramanian, L.S. Ochi, An iterated local search heuristic for the heterogeneous fleet vehicle routing problem, Journal of Heuristics 19 (2) (2013) 201–232.
[32] A. Sifaleras, I. Konstantaras, General variable neighborhood search for the multi--product dynamic lot sizing problem in closed-loop supply chain, Electronic Notes in Discrete Mathematics 47 (2015) 69–76.
[33] M.M. Silva, A. Subramanian, L.S. Ochi, An iterated local search heuristic for the split delivery vehicle routing problem, Computers & Operations Research 53 (2015) 234–249.
[34] C.-H. Song, K. Lee, W.D. Lee, Extended simulated annealing for augmented TSP and multi-salesmen TSP, in: Neural Networks, 2003. Proceedings of the International Joint Conference on, volume 3, IEEE, 2003, pp. 2340–2343.
[35] A. Subramanian, M. Battarra, An iterated local search algorithm for the travelling salesman problem with pickups and deliveries, Journal of the Operational Research Society 64 (3) (2013) 402–409.
[36] P. Sun, L.P. Veelenturf, S. Dabia, T. Van Woensel, The time-dependent capacitated profitable tour problem with time windows and precedence constraints, Eur J Oper Res 264 (3) (2018) 1058–1073.
[37] R. Todosijević, A. Mjirda, M. Mladenović, S. Hanafi, B. Gendron, A general variable neighborhood search variants for the travelling salesman problem with draft limits, Optimization Letters 11 (6) (2017) 1047–1056.
[38] T. Tsiligirides, Heuristic methods applied to orienteering, Journal of the Operational Research Society 35 (9) (1984) 797–809.
[39] P. Venkatesh, A. Singh, R. Mallipeddi, A Multi-start Iterated Local Search Algorithm for the Maximum Scatter Traveling Salesman Problem, in: IEEE Congress on Evolutionary Computation, CEC 2019, Wellington, New Zealand, June 10–13, 2019, IEEE, 2019, pp. 1390–1397.
[40] P. Venkatesh, G. Srivastava, A. Singh, A general variable neighborhood search algorithm for the k-traveling salesman problem, Procedia Comput Sci 143 (2018) 189–196.
[41] P. Venkatesh, G. Srivastava, A. Singh, A Multi-start Iterated Local Search Algorithm with Variable Degree of Perturbation for the Covering Salesman Problem, in: Harmony Search and Nature Inspired Optimization Algorithms, Springer, 2019, pp. 279–292.
[42] F. Wilcoxon, S.K. Katti, R.A. Wilcox, Critical values and probability levels for the wilcoxon rank sum test and the wilcoxon signed rank test, Selected tables in mathematical statistics 1 (1970) 171–259.
[43] Y. Wu, W. Song, Z. Cao, J. Zhang, A. Lim, Learning improvement heuristics for solving routing problems, arXiv preprint arXiv:1912.05784 (2019).
[44] L. Xin, W. Song, Z. Cao, J. Zhang, Multi-decoder attention model with embedding glimpse for solving vehicle routing problems, arXiv preprint arXiv:2012.10638 (2020).
[45] L. Xin, W. Song, Z. Cao, J. Zhang, Step-wise deep learning models for solving routing problems, IEEE Trans. Ind. Inf. (2020). Early access
[46] M. Zhang, J. Wang, H. Liu, The probabilistic profitable tour problem, International Journal of Enterprise Information Systems (IJEIS) 13 (3) (2017) 51–64.