Question 1

A convenient parameter for specifying the sensitivity of a radio telescope is its sensitivity in units of K/Jy; that is, the number of Kelvins of antenna temperature T_A produced by an unpolarized point source whose flux density is 1 Jy. 1. What is the effective collecting area A_e of a radio telescope whose sensitivity is 1 K/Jy? 2. The 2.3 GHz feed at Arecibo illuminates an elliptical aperture 225 m by 200 m in size, and the aperture efficiency η_A over this ellipse is $\eta_A \approx 0.70$. What is the sensitivity of this system in K/Jy? 3. The same feed is used with a 1 megawatt transmitter at 2.3 GHz for planetary radar. What is the on-axis power gain G_{max} of this radar system?

Question 2

In class, we showed that a 1-D aperture with constant illumination produces a power pattern described by:

$$P(\theta) \propto \mathrm{sinc}^2(\frac{\theta D}{\lambda}).$$

With D = 10m and $\lambda = 10$ cm, use *numerical techniques*, compute θ and $P(\theta)$ to at least 4 significant figures at the peaks of the first 2 sidelobes. Express them in dB as well as in relative terms (as compared to the main beam). Verify your results analytically.

Question 3

For a circular aperture with uniform illumination, the normalized power pattern $P_n(\theta)$ is known as the *Airy disk* and is described in the textbook by eqn. 6.27 (remember that $u = \sin\theta$ where θ is the angle between the optical axis and the direction in question). 1. As in problem 2, numerically compute θ and $P_n(\theta)$ to at least 4 significant figures for the first 2 sidelobes with D = 100m and λ = 5 cm. 2. Numerically determine the aperture's beam solid angle Ω_A. 3. Compute the main beam solid angle Ω_{MB} and the beam efficiency η_B assuming that the main beam is defined as everything within the first null of eqn. 6.27.

Question 4

You would like to measure the temperature of Mars with an rms uncertainty of about 1% by observing it at $\nu_{RF} \approx 10$GHz. The angular diameter of Mars is θ_M = 18arcsec, and its 10 GHz flux density is 4.2 Jy. You have a radio telescope whose paraboloidal mirror has diameter D = 25m and aperture efficiency η_A = 0.70. The single-channel total-power receiver is connected to a feed sensitive to right circular polarization, and its RF bandwidth is 100MHz. The receiver noise temperature is T_{RX} = 18 K, the atmosphere adds about 3.5 K, the microwave background 3K, and spillover pickup of ground radiation is about 11 K. 1. Show that Mars is a "point source" for your observation; that is, $\theta_M \ll \theta_{FWHM}$, the telescope beamwidth between half-power points. 2. What antenna temperature contribution T_A do you expect from Mars? 3. What is the system noise temperature T_{sys} when the telescope is pointing at Mars? 4. If the receiver has perfect gain stability, how long must you point the telescope at Mars? 5. Estimate the maximum rms receiver gain fluctuation $\Delta G/G$ that you can tolerate during this observation. 6. If the receiver gain stability is not good enough for a total-power observation and you are forced to Dicke switch, how long must you point the telescope at Mars?

Question 5

For pulsar timing observations, astronomers “fold” the data modulo the pulse period. If the number of bins in the pulse profile is such that the pulse fits perfectly into a single bin, then for simple single-peaked pulse profiles, this technique is practically an optimal “matched-filter” for the signal. 1. If we assume a rectangular (i.e. top-hat) pulse shape of width W seconds, measured peak amplitude T_{peak} Kelvin, and spin period P seconds, derive the signalto- noise ratio ($S/N = T_{peak}/\sigma_T$) as a function of the integration time τ , the observed bandwidth $\Delta\nu_{RF}$, W and P. You can assume (as is nearly always true) that $T_{peak} \ll T_{sys}$. 2. For $W \ll P$, how does S/N scale with the pulsar “duty-cycle”, W/P? 3. For a few points of extra credit, explain how this applies to pulsar searches, which are conducted in the frequency domain after a Fourier transform (and where we don’t know W and P a priori).

Question 6

Measurements of many extragalactic sources have shown that they typically have brightness temperatures of $T_B \approx 10^{11}$ K and $S \approx 1$ Jy. Approximately what size telescope is needed to resolve these sources?

Question 7

At the end of section Two-Dimensional Aperture Antennas the notes assert without proof that the wavelength λ_m at which a reflector telescope has maximum gain is given by $\lambda_m = 4\pi\sigma$, where σ is the rms surface error. Derive this equation.