Written Assignment 1

Due: Friday 02/02/2024 @ 11:59pm EST

Disclaimer

I encourage you to work together, I am a firm believer that we are at our best (and learn better) when we communicate with our peers. Perspective is incredibly important when it comes to solving problems, and sometimes it takes talking to other humans (or rubber ducks in the case of programmers) to gain a perspective we normally would not be able to achieve on our own. The only thing I ask is that you report who you work with: this is **not** to punish anyone, but instead will help me figure out what topics I need to spend extra time on/who to help. When you turn in your solution (please use some form of typesetting: do **NOT** turn in handwritten solutions), please note who you worked with.

Remember that if you have a partner, you and your partner should submit only **one** submission on gradescope.

Question 1: Shortest Path Composition (25 points)

Consider a graph $G = (V, E, w : E \to \mathbb{R}^{\geq 0})$ where V is a set of vertices, E is a set of (directed) edges, and w is a *weight function* that maps edges to weights where each weight is ≥ 0. Let us define a path p to be a sequence of edges where the destination vertex of one edge is the source vertex of the next edge (if the next edge exists). Let us define the *cost* of an path the traditional way, i.e. the cost of a path p is the sum of the edge weights in p:

$$cost(p) = \sum_{e \in p} w(e)$$

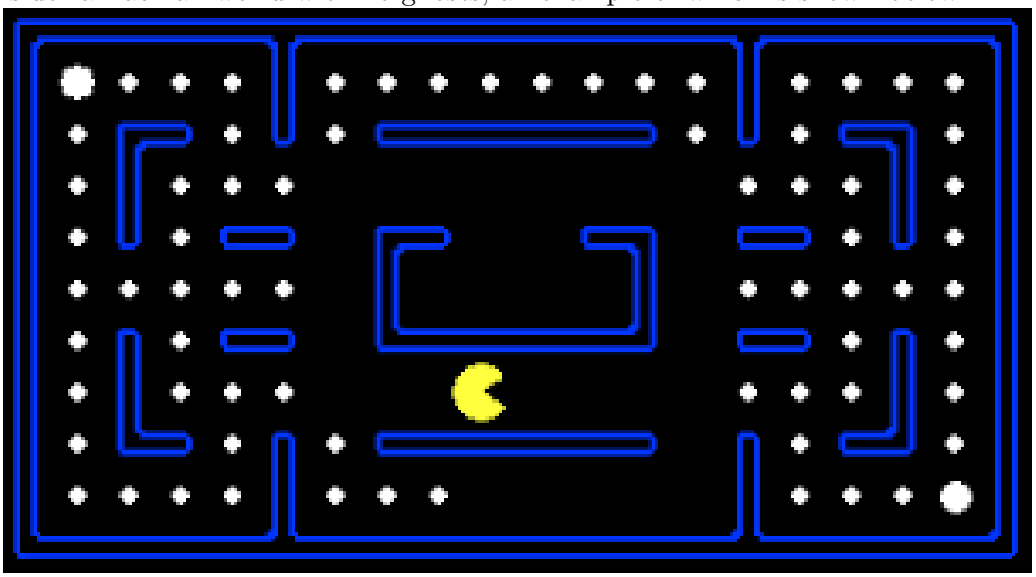
Show that if we know a shortest path $p^* = a \overset{x}{\rightsquigarrow} b$ from vertex a to vertex b has cost x. If we know p^* passes through intermediary vertex c, then let $p_1 = a \overset{y}{\rightsquigarrow} c$, and let $p_2 = c \overset{z}{\rightsquigarrow} b$. Show that if $p^* = p_1 \cup p_2$, then p_1 is a shortest path from a to c, and that p_2 is a shortest path from c to b.

Question 2: Best and Worst Cases for DFS (25 points)

Consider a graph $G = (V, E, w : E \to \mathbb{R}^{\geq 0})$ where V is a set of vertices, E is a set of (directed) edges, and w is a *weight function* that maps edges to weights where each weight is ≥ 0. Let us start a DFS search from vertex a, the goal of which is to find vertex b (where $b \neq a$). In the worst case, where is the goal vertex b in relation to the source vertex a in the DFS expansion. How many vertices and edges must the expansion contain before when we reach b? What about the best case?

Extra Credit: Heuristics for Pacman-NoGhosts (50 points)

Consider a Pacman world with no ghosts, an example of which is shown below:



Here is the description of this search problem:

- **Environment:** A 2-d map of finite size where each square can contain a wall, a food pellet, Pacman, or is unoccupied.
- **Sensors/State:** The state of the world is a structure containing the following fields (in Java-ish syntax):
  - `current_loc: Coordinate`: The (x,y) location of Pacman in the map.
  - `food_remaining_locs: Collection<Coordinate>`: The (x,y) locations of all remaining food pellets in the map.
- **Initial state:** The initial state is an instance of the State with the following field values:
  - `current_loc = (9, 3); // technically any unoccupied square will do`
  - `food_remaining_locs = locations of all food pellets in the map;`
- **Actuators/Actions:** Pacman can move to an adjacent square in a cardinal direction.
- **Transition Model:** Pacman will move to the adjacent square if it is not a wall and if the action will not take Pacman out of bounds of the map. Additionally, if Pacman enters a square that contains a food pellet, Pacman will automatically consume that pellet. When Pacman consumes a pellet, the location of that pellet is removed from the `food_remaining_locs` of that state.
- **Path cost:** Moving to an adjacent square has a cost of 1. Therefore, the cost of a path is the number of edges in the path.
- **Goal Test:** Any state where the number of remaining pellets is zero is a goal state regardless of the (x,y) position of Pacman.
- **Performance Measure:** The number of turns it takes to eat all of the food pellets.

In this problem, we want to minimize the performance metric (i.e. eat all of the pellets in the fewest number of turns). If we cast this problem into a search problem (where states are vertices and actions are edges), then we can use a search algorithm to find the shortest path from our initial state to a goal state. We will use the A^* algorithm equipped with the goal-test function to do so.

In order to use A^*, we need to define a heuristic that estimates the cost of the current state to a goal state. Design a heuristic that is admissible and consistent that will solve this problem. Your heuristic must be non-negative and must return 0 when the current state is a goal state.

Hint: If we are at a square that contains a food pellet, then the smallest cost to a goal state is the number of movements to enter all of the squares that still contain food pellets. So, something we would like to know (and probably cache beforehand) are the distances between pairs of squares that contain food pellets.