BRADLEY University

CS 220 Computer Architecture

HW 03 - Number Systems and Computer Arithmetic 2

Fall 2023

DEPARTMENT OF COMPUTER SCIENCE AND INFORMATION SYSTEMS

PART 0: READING

- **Chapter 11** - Computer Arithmetic | **11.1 - 11.3**

PART 1: COMPUTER ARITHMETIC – MULTIPLICATION AND DIVISION [100 PTS]

QUESTION 1

Given **x = 00101** (multiplicand) and **y = 11010** (multiplier) in **5-bit** two's complement notation (ie., **x = 5**, **y = -6**), compute the product of **p = x * y** with **Booth's algorithm**. **Show All steps** in the table. [20 pts]

M4	M3	M2	M1	M0
0	0	1	0	1

n	A4	A3	A2	A1	A0	Q4	Q3	Q2	Q1	Q0	Q_{-1}
5 initial	0	0	0	0	0	1	1	0	1	0	0
5 shift	0	0	0	0	0	0	1	1	0	1	0
4 sub	1	1	0	1	1	0	1	1	0	1	0
shift	1	1	1	0	1	1	0	1	1	0	1
3 add	0	0	0	1	0	1	0	1	1	0	1
shift	0	0	0	0	1	0	1	0	1	1	0
2 sub	1	1	1	0	0	0	1	0	1	1	0

shift	1	1	1	1	0	0	0	1	0	1	1
1 shift	1	1	1	1	1	0	0	0	1	0	1

- *Note: Add more rows if needed.*

QUESTION 2

Given **x = -5** (multiplicand), **y = 6** (multiplier), compute the product of **p = x * y** with **Booth's algorithm** in 6-bit two's complement notation. **Show All steps** in the table. [20 pts]

M5	M4	M3	M2	M1	M0
1	1	1	0	1	1

n	A5	A4	A3	A2	A1	A0	Q5	Q4	Q3	Q2	Q1	Q0	Q_{-1}
6 initial	0	0	0	0	0	0	0	0	0	1	1	0	0
6 shift	0	0	0	0	0	0	0	0	0	0	1	1	0
5 sub	0	0	0	1	0	1	0	0	0	0	1	1	0
shift	0	0	0	0	1	0	1	0	0	0	0	1	1
4 shift	0	0	0	0	0	1	0	1	0	0	0	0	1
3 add	1	1	1	1	0	0	0	1	0	0	0	0	1
shift	1	1	1	1	1	0	0	0	1	0	0	0	0
2 shift	1	1	1	1	1	1	0	0	0	1	0	0	0
1 shift	1	1	1	1	1	1	1	0	0	0	1	0	0

- *Note: Add more rows if needed.*

QUESTION 3

Use the booth algorithm to multiply **23** (multiplicand) by **29** (multiplier), where each number is presented using **8-bits**. [20 pts]

M7	M6	M5	M4	M3	M2	M1	M0
0	0	0	1	0	1	1	1

n	A7	A6	A5	A4	A3	A2	A1	A0	Q7	Q6	Q5	Q4	Q3	Q2	Q1	Q0	Q_{-1}
8 init	0	0	0	0	0	0	0	0	0	0	0	1	1	1	0	1	0
sub	1	1	1	0	1	0	0	0	0	0	0	1	1	1	0	1	0
shift	1	1	1	1	0	1	0	0	0	0	0	0	1	1	1	0	1
7 add	0	0	0	0	1	0	1	1	0	0	0	0	1	1	1	0	1
shift	0	0	0	0	0	1	0	1	1	0	0	0	0	1	1	1	0
6 sub	1	1	1	0	1	1	1	0	1	1	0	0	0	1	1	1	0
shift	1	1	1	1	0	1	1	1	0	1	1	0	0	0	1	1	1
5 shift	1	1	1	1	1	0	1	1	1	0	1	1	0	0	0	1	1
4 shift	1	1	1	1	1	1	0	1	1	1	0	1	1	0	0	0	1
3	0	0	0	1	0	1	0	0	1	1	0	1	1	0	0	0	1

add																	
shift	0	0	0	0	1	0	1	0	0	1	1	0	1	1	0	0	0
2 shift	0	0	0	0	0	1	0	1	0	0	1	1	0	1	1	0	0
1 shift	0	0	0	0	0	0	1	0	1	0	0	1	1	0	1	1	0

QUESTION 4

Divide **45** by **13** in binary two's complement notation, using **8-bit** words. [20 pts]

- *Note: Use a table similar to the above questions and trace the variations of A and Q registers, and show all steps.*

M7	M6	M5	M4	M3	M2	M1	M0
0	0	0	0	1	1	0	1

n	A7	A6	A5	A4	A3	A2	A1	A0	Q7	Q6	Q5	Q4	Q3	Q2	Q1	Q0
8 init	0	0	0	0	0	0	0	0	0	0	1	0	1	1	0	1
shift	0	0	0	0	0	0	0	0	0	1	0	1	1	0	1	0
sub	1	1	1	1	0	0	1	1	0	1	0	1	1	0	1	0
A<0 Q<0	0	0	0	0	0	0	0	0	0	1	0	1	1	0	1	0
7 shift	0	0	0	0	0	0	0	0	1	0	1	1	0	1	0	0
sub	1	1	1	1	0	0	1	1	1	0	1	1	0	1	0	0
A<0 Q<0	0	0	0	0	0	0	0	0	1	0	1	1	0	1	0	0
6	0	0	0	0	0	0	0	1	0	1	1	0	1	0	0	0

shift																
sub	1	1	1	1	0	1	0	0	0	1	1	0	1	0	0	0
A<0 Q<0	0	0	0	0	0	0	0	1	0	1	1	0	1	0	0	0
5 shift	0	0	0	0	0	0	1	0	1	1	0	1	0	0	0	0
sub	1	1	1	1	0	1	0	1	1	1	0	1	0	0	0	0
A<0 Q<0	0	0	0	0	0	0	1	0	1	1	0	1	0	0	0	0
4 shift	0	0	0	0	0	1	0	1	1	0	1	0	0	0	0	0
sub	1	1	1	1	1	0	0	0	1	0	1	0	0	0	0	0
A<0 Q<0	0	0	0	0	0	1	0	1	1	0	1	0	0	0	0	0
3 shift	0	0	0	0	1	0	1	1	0	1	0	0	0	0	0	0
sub	1	1	1	1	1	1	1	0	0	1	0	0	0	0	0	0
A<0 Q<0	0	0	0	0	1	0	1	1	0	1	0	0	0	0	0	0
2 shift	0	0	0	1	0	1	1	0	1	0	0	0	0	0	0	0
sub	0	0	0	0	1	0	0	1	1	0	0	0	0	0	0	0
A<0 Q>1	0	0	0	0	1	0	0	1	1	0	0	0	0	0	0	1
1 shift	0	0	0	1	0	0	1	1	0	0	0	0	0	0	0	1
sub	0	0	0	0	0	1	1	0	0	0	0	0	0	0	1	0
A<0 Q>1	0	0	0	0	0	1	1	0	0	0	0	0	0	0	1	1

QUESTION 5

Divide **145** by **13** in binary two's complement notation, using **12-bit** words. [20 pts]

- *Note: Use a table similar to the above questions and trace the variations of A and Q registers, and show all steps.*

M11	M10	M9	M8	M7	M6	M5	M4	M3	M2	M1	M0
0	0	0	0	1	1	0	1	0	0	0	0

n	A 11	A 10	A 9	A 8	A 7	A 6	A 5	A 4	A 3	A 2	A 1	A 0	Q 11	Q 10	Q 9	Q 8	Q 7	Q 6	Q 5	Q 4	Q 3	Q 2	Q 1	Q 0
12 initial	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	1	0	0	0	1
shift	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	1	0	0	0	1	0
sub	1	1	1	1	1	1	1	1	0	0	1	1	0	0	0	1	0	0	1	0	0	0	1	0
A<0 add	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	1	0	0	0	1	0
11 shift	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	1	0	0	0	1	0	0
sub	1	1	1	1	1	1	1	1	0	0	1	1	0	0	1	0	0	1	0	0	0	1	0	0
A<0 add	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	1	0	0	0	1	0	0
10 shift	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	1	0	0	0	1	0	0	0
sub	1	1	1	1	1	1	1	1	0	0	1	1	0	1	0	0	1	0	0	0	1	0	0	0
A<0 add	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	1	0	0	0	1	0	0	0
9 shift	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	1	0	0	0	1	0	0	0	0
sub	1	1	1	1	1	1	1	1	0	0	1	1	1	0	0	1	0	0	0	1	0	0	0	0
A<0 add	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	1	0	0	0	1	0	0	0	0

8 shift	0	0	0	0	0	0	0	0	0	0	0	1	0	0	1	0	0	0	1	0	0	0	0	0
sub	1	1	1	1	1	1	1	1	0	1	0	0	0	0	1	0	0	0	1	0	0	0	0	0
A<0 add	0	0	0	0	0	0	0	0	0	0	0	1	0	0	1	0	0	0	1	0	0	0	0	0
7 shift	0	0	0	0	0	0	0	0	0	0	1	0	0	1	0	0	0	1	0	0	0	0	0	0
sub	1	1	1	1	1	1	1	1	0	1	0	1	0	1	0	0	0	1	0	0	0	0	0	0
A<0 add	0	0	0	0	0	0	0	0	0	0	1	0	0	1	0	0	0	1	0	0	0	0	0	0
6 shift	0	0	0	0	0	0	0	0	0	1	0	0	1	0	0	0	1	0	0	0	0	0	0	0
sub	1	1	1	1	1	1	1	1	0	1	1	1	1	0	0	0	1	0	0	0	0	0	0	0
A<0 add	0	0	0	0	0	0	0	0	0	1	0	0	1	0	0	0	1	0	0	0	0	0	0	0
5 shift	0	0	0	0	0	0	0	0	1	0	0	1	0	0	1	0	0	0	0	0	0	0	0	0
sub	1	1	1	1	1	1	1	1	1	1	0	0	0	0	1	0	0	0	0	0	0	0	0	0
A<0 add	0	0	0	0	0	0	0	0	1	0	0	1	0	0	1	0	0	0	0	0	0	0	0	0
4 shift	0	0	0	0	0	0	0	1	0	0	1	0	0	1	0	0	0	0	0	0	0	0	0	0
sub	0	0	0	0	0	0	0	0	0	1	0	1	0	1	0	0	0	0	0	0	0	0	0	0
A<0 Q>1	0	0	0	0	0	0	0	0	0	1	0	1	0	1	0	0	0	0	0	0	0	0	0	1
3 shift	0	0	0	0	0	0	0	0	1	0	1	0	1	0	0	0	0	0	0	0	0	0	1	0
sub	1	1	1	1	1	1	1	1	1	1	0	1	1	0	0	0	0	0	0	0	0	0	1	0
A<0 add	0	0	0	0	0	0	0	0	1	0	1	0	1	0	0	0	0	0	0	0	0	0	1	0
2 shift	0	0	0	0	0	0	0	1	0	1	0	1	0	0	0	0	0	0	0	0	0	1	0	0

sub	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0
A<0 Q>1	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	1	0	1
1 shift	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	1	0	1	0
sub	0	0	0	0	0	0	0	0	0	0	1	1	0	0	0	0	0	0	0	0	1	0	1	0
A<0 Q>1	0	0	0	0	0	0	0	0	0	0	1	1	0	0	0	0	0	0	0	0	1	0	1	1