2D modeling and analysis of the impact of structural properties on whisker mechanics

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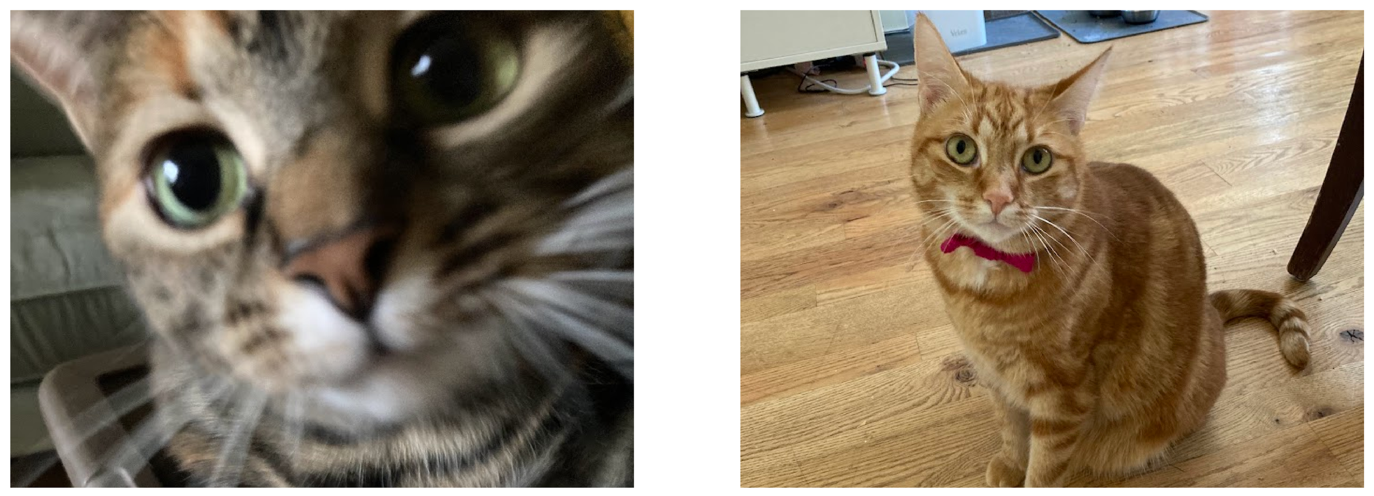
Introduction

- Whiskers or vibrissae are a sensory organ found on many animals including cats, rats, seals, and many others
- Vibrissae are used for texture discrimination, object recognition, positioning, and to gain spatial awareness of the space near the mouth whiskers
- They function by transducing movement to forces at the base
- Vibration does not have a significant effect on sensing
- Vibrissae have many relevant physical properties, including whether they are hollow or filled, the stiffness of the material that composes them and their cores (the medulla, see Figure 6), and their conicity



Images A & B. The author's cats demonstrating their use of whiskers and ability to look dashing in a bowtie

Purpose

To determine what factors have significant impacts on the deflection and deformation of cat whiskers, with focuses on structure, shape, conicity, young's modulus, and cross sectional moment of inertia. Additionally, a goal was to analyze the conditions under which whiskers fail.

Methods

- Whiskers are modeled by linear Cesáro equations (see Equation 1), a type of parameterized curve that gives curvature with respect to arc-distance along the curve
  - See Figure 1 for all the whiskers on a cat charted with this method (data from Luo & Hartman)
- They are defined in terms of their loading, unloaded shape (via the Cesáro equations), and conicity (tip diameter divided by base diameter)

Equations

- The set of differential equations in Equation 3 are taken from Farfán and are the method by which its loaded position and shape is found
- This is formulated as a boundary value problem and can be solved by the implementation of the finite difference methods available in scipy
- This gives the angle of deflection (φ) which can be converted to the derivatives of the x and y locations in Cartesian coordinates via the application of trigonometric functions

Methods of Analysis

- Investigation was mostly achieved via the manipulation of various parameters and analysis of the effects. Conicity and forces were chosen for these examples.

$$\kappa(s) = As + B$$

Equation 1. Cesaro curves relate the curvature (κ) to the distance along the curve (s)

$$f := \frac{FL^2}{EI_b}$$

Equation 2. The relationship between the force parameter f and the force magnitude (F), arc-length (L), young's modulus (E), and moment of inertia at the base (I_b)

Symbols	Meaning
κ	Curvature
φ	Deflection angle
f and β	Tip force parameter and angle
q and α	distributed force parameter and angle
ϑ	Conicity (tip diameter/base diameter)
A, B, and s	The Cesáro variables and distance along curve (as distance/arc length of whisker)

Table 1. A list of all the variables used in Equation 2

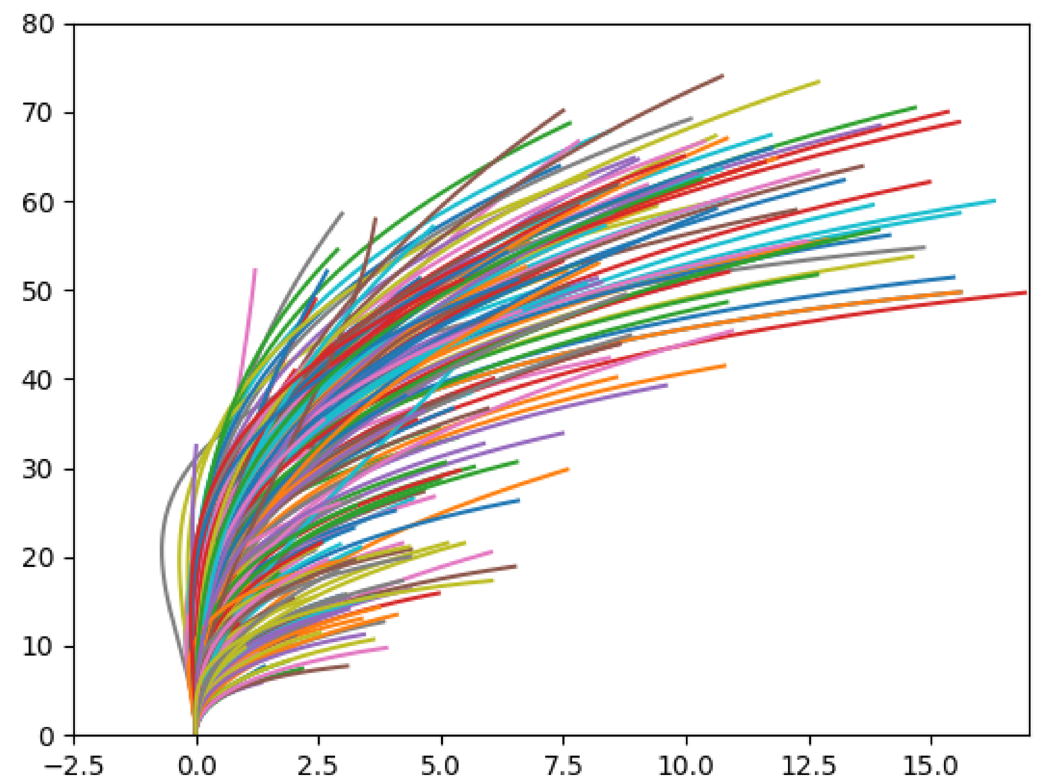


Figure 1. A graphic showing all the whiskers on a cat, aligned with their bases perpendicular to the horizontal

Results

Charts showing how whiskers respond to various types of loading

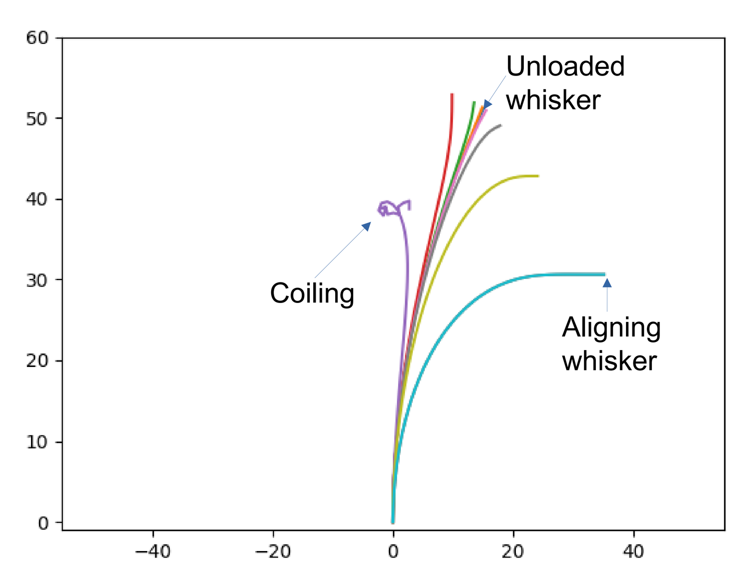


Figure 2. The whisker with point loads at the tip pointing both left and right. Each whisker has a force parameter (see Equation 2) with a magnitude 1 greater than the last. (starting at ±0.001 and ranging to ±1 unit at the extreme left and right)

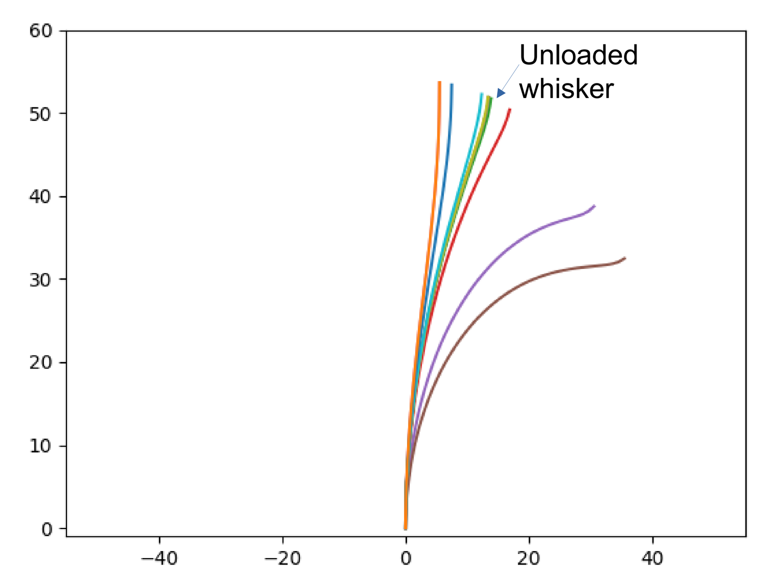


Figure 3. Same as previous but with load distributed across whisker

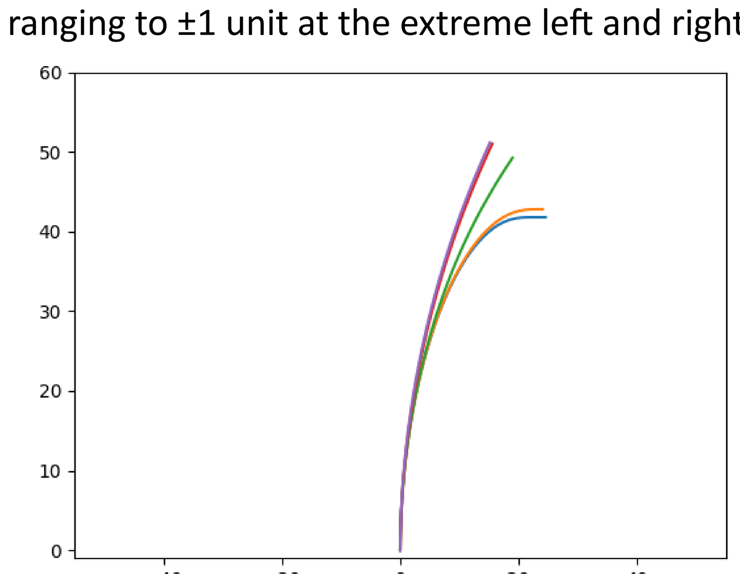


Figure 4. The whisker with a point load at the tip pointing right. Each whisker (right to left) has a conicity with a magnitude 1 greater than the last (starting at 0.0036 and ranging to 360 at the left). Force parameter is 0.01 for all (see Equation 2).

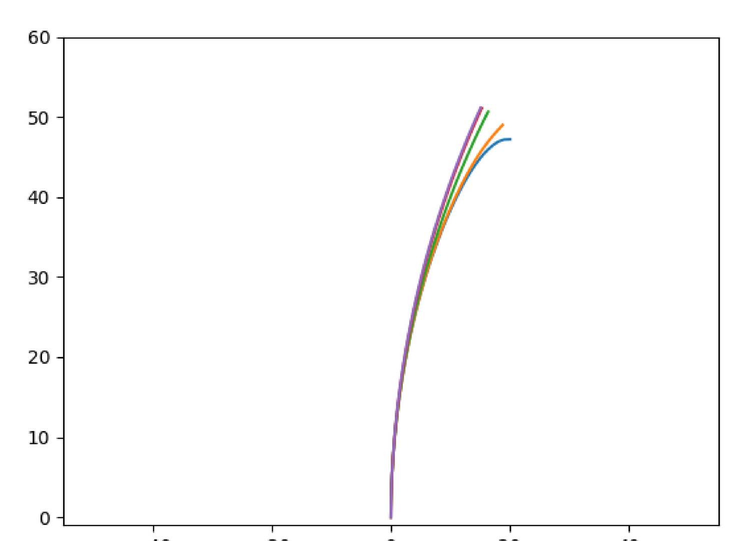


Figure 5. Same as previous but with load distributed across whisker

$$\left.\begin{aligned}
x' &= \cos\varphi \\
y' &= \sin\varphi \\
\varphi' &= \kappa \\
\kappa' &= \frac{f\sin(\varphi+\beta) + q(1-s)\sin(\varphi+\alpha)}{(1+(\vartheta-1)s)^4} - \frac{4(\vartheta-1)}{(1+(\vartheta-1)s)}\left[\varphi' - As - B\right] + A
\end{aligned}\right\}$$

Equation 3. The differential equations used. See Table 1 for an explanation of symbols

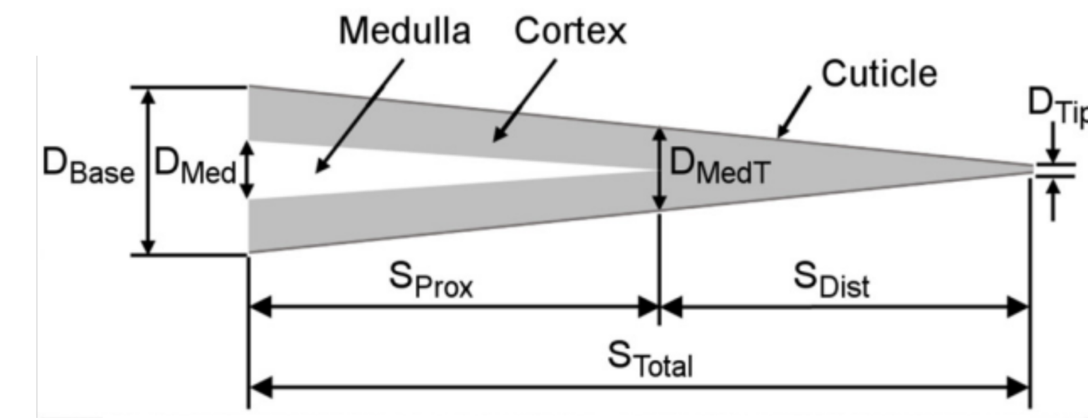


Figure 6. From Belli et. al pg 1808. Structure of a whisker. Medulla as % of length is proportional to arc-length

Discussion

- Whiskers displaced significantly less under equal point loads when the direction was against the intrinsic curvature
- Under point loads in this model, whiskers coiled when they reached a certain point in forces against the curve
  - Not shown in results for clarity, extreme "with the curve" point tip loads result in similar coiling
- "With the curve" forces end up aligning the whisker tip to the curve (see Figure 2)
  - This occurred strongly on tip loads, but also occurred on distributed loads, although the extreme tips did not fully align
- Coiling is an artifact of the model, but should be treated as the whisker failing
- Coiling was not seen under distributed loads
- Conicity had a significant effect on displacement, particularly at the tip and particularly with tip point forces.
- Distributed forces were not strongly affected by conicity outside of extreme cases

Further exploration

- Due to time constraints and issues with initial methodology, young's modulus could not be adequately studied as a paramerer. Initial results seem promising however.
- Similarly, the medulla wasn't able to be adequately studied. Initial results are less promising, however.
- Further work will focus on these areas and on how the whisker's shape may affect it's bending
- Additionally, more focus may be placed on buckling and analyzing the "coiling" behavior

Conclusions

In summary, the purpose and initial goals were only partially achieved. Due to some late but necessary changes in methodology, only some parameters were able to be explored. The explored factors of force magnitude and direction along with conicity provided interesting explorations of how whiskers behave under different loads and respond to parameters, exposing how conicity strongly influences whisker deformation. Further work is needed, but this is a solid start. More emphasis should be placed on buckling (and the coiling behavior) to determine how parameters like shape, curve and thickness might enable greater durability or sensing capabilities.

Figure 7. QR Code links to GitHub with preliminary version of code. Will be updated by week end with fixes to major bugs in graphing system under load, graphing unloaded currently functional.

References

1. Luo, Y., & Hartmann, M. J. (2023). On the intrinsic curvature of animal whiskers. Plos one, 18(1), e0269210. Both Paper and Dataset.
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3. Wolfe, J., Hill, D. N., Pahlavan, S., Drew, P. J., Kleinfeld, D., & Feldman, D. E. (2008). Texture coding in the rat whisker system: slip-stick versus differential resonance. PLoS biology, 6(8), e215.
4. Belli, H. M., Yang, A. E., Bresee, C. S., & Hartmann, M. J. (2017). Variations in vibrissal geometry across the rat mystacial pad: base diameter, medulla, and taper. Journal of Neurophysiology, 117(4), 1807-1820.