Numerical Methods Class Test

Test 1: Friday 4/11/16, 16:00

Do **not** turn over this paper to look at the questions until the start of the class test at 16:00.

Download the template python file `class_test_1.py` from Blackboard (under `Assignments >> Class Test`).

For each question you should complete the function associated with the question so that it returns the required output, or plots a figure to the screen. Your function should return precisely the output stated in the question; excessive output may be penalized.

You may do your working in the command window, but only the script uploaded will be marked.

Once you have completed the test you should upload the completed `class_test_1.py` file to Blackboard. Go to `Assignments >> Class Test` and click on `View/Complete Assignment`. Upload your solution using the `Attach local file` field. **Ensure that you click the SUBMIT button once you are happy**. If you do not click the SUBMIT button we will be unable to read your file, and hence unable to mark it!

In completing the test you may use, and are expected to use, the help system and online resources. You are not allowed to communicate with others.

1. Enter the matrix
$$A = \begin{pmatrix} 1 & 3 \\ 4 & 5 \end{pmatrix}$$
and cube it.

   Output: *Only* the A^3.

2. Create a vector $\mathbf{v}$ containing 40 points linearly spaced between 1 and 2. Compute the dot product of the vector with itself.

   Output: *Only* $\mathbf{v} \cdot \mathbf{v}$.

3. Using the vector $\mathbf{v}$ from question 2, produce a vector $\mathbf{w}$ with 40 points containing
$$w(v) = v^2 \cos(\pi v), \quad v \in [1, 2].$$
Sum the *odd* elements of the vector; that is, sum every other element starting from the first.

   Output: *Only* $\sum_{n \text{ odd}} w_n$.

4. Solve the linear system $A\mathbf{z} = \mathbf{b}$ where
$$A = \begin{pmatrix} 8 & 2 & 4 \\ 2 & -12 & 6 \\ 4 & 6 & 1 \end{pmatrix}, \quad \mathbf{b} = \begin{pmatrix} 4 \\ 9 \\ 2 \end{pmatrix}.$$
Output: *Only* the solution $\mathbf{z}$.

5. Find the eigenvalues of the matrix $A - 2I$, where A is as given in question 4 and I is the identity matrix, outputting the minimum and maximum absolute magnitude.

   Output: $|\lambda_1|$ and $|\lambda_3|$ where λ_i are the eigenvalues of $A - 2I$ and $|\lambda_1| < |\lambda_2| < |\lambda_3|$.

6. Using just the `numpy` command `tril`, return the matrix containing those entries of A *strictly* below the diagonal (and zero elsewhere).

   Output: The matrix formed from the *strictly* lower triangular parts of A.

7. In Figure 1 plot
$$y(x) = \log(x) \sin(2\pi x)$$
using a spacing $h = 0.01$ in the interval $1 \leq x \leq 4$. Add axis labels (`'x'` and the function definition) and a title (`'Figure for question 7'`).

   Output: *Only* Figure 1 as described.

8. In Figure 2 show a surface plot of
$$z(x, y) = e^{-x^2} \cos(2\pi y^2).$$
The range should be $x \in [0, 2]$, $y \in [0, 1]$ with an equal spacing of $h = 0.05$ in both directions. Use the default viewing angle. Use an array stride of 1 in each direction. Use the `hot` colormap. Labels and titles are not required.

   Output: *Only* Figure 2 as described.

9. Using the `scipy newton` routine, find the s closest to zero such that
$$s + s^2 = \cos^3(s)$$
to the default tolerance.

   Output: *Only* the root s.

10. If the vector sequence $\mathbf{y}_n = (u_n, v_n)^T$ with initial data $\mathbf{y}_1 = (0.25, 0.75)^T$, is constructed from the map
$$\mathbf{y}_{n+1} = \begin{pmatrix} \cos(v_n) \\ \sin(u_n) \end{pmatrix},$$
find the 5^{th} and 50^{th} members of the sequence.

    Output: *Only* the vectors $\mathbf{y}_5$ and $\mathbf{y}_{50}$.