Section 3.4: The Chain Rule

Goal: Differentiate composite functions.

Question: Can we compute the derivative of $y = (x^2 + 3x)^{25}$ or $y = e^{\cos(x)}$?

Note: These are both examples of composite functions.

The **Chain Rule** tells us how to take derivatives of composite functions:

Question revisited

Example 1: Differentiate the following functions:

(a) $y = \sqrt{3x^4 + 5x^2 + 1}$

(b) $y = 5\sec(x^3)$

(c) $y = \sin^9(x)$

The chain rule is a derivative rule

Example 2:

(a) Find the slope of the line tangent to $y = 5e^{\frac{x}{2}}$ at $x = 0$.

(b) An object's position can be modeled $s(t) = \cos(2t + \pi)$ inches, t seconds form now. Find the velocity of the object at $t = \frac{\pi}{4}$.

Example 3: Differentiate the following functions:

(a) $y = \tan(xe^x)$

(b) $y = \sqrt{1 + e^{3x^2}}$

Example 4: The power P (in Watts) in a circuit is $P = Ri^2$ where R is resistance and i is the current.

(a) Suppose $R = 1000\Omega$ and i varies according to the formula $i = \sin(4\pi t)$, where t is measured in minutes. Find $\dfrac{dP}{dt}$ at $t = \frac{1}{3}$.

(b) Explain in words what you computed in (a).