Review

8/28/2023

Counting in binary & hex

3-bit

Binary: 000, 001, 010, 011, 100, 101, 110, 111 $(2^3 = 8)$ # of bits

Convert to decimal: 0 1 2 3 4 5 6 7

→ Converting binary to decimal

$b_7 b_6 b_5 b_4 b_3 b_2 b_1 b_0$ | 8-bit binary number

→ Decimal equivalent:

bit value, bit position, base_2

$$D = b_7 \cdot 2^7 + b_6 \cdot 2^6 + b_5 \cdot 2^5 + b_4 \cdot 2^4 + b_3 \cdot 2^3 + b_2 \cdot 2^2 + b_1 \cdot 2^1 + b_0 \cdot 2^0$$

<u>Example</u>: $B = (1101)_2$

base_10 → $D = 1 \cdot 2^3 + 1 \cdot 2^2 + \cancel{0 \cdot 2^1} + 1 \cdot 2^0$

$= 2^3 + 2^2 + 2^0 = (13)_{10}$

Ex: $B = (1001\ 0110)_2$

$$D = 1\cdot 2^7 + \cancel{0\cdot 2^6} + \cancel{0\cdot 2^5} + 1\cdot 2^4 + \cancel{0\cdot 2^3} + 1\cdot 2^2 + 1\cdot 2^1 + \cancel{0\cdot 2^0}$$

$$= 2^7 + 2^4 + 2^2 + 2^1$$

$$= 128 + 16 + 4 + 2 = (150)_{10}$$

What about hex?

- for binary to hex, look at $\underbrace{\text{4-bit nibbles}}_{\text{8-bit: bytes}}$ → one hex symbol

binary	hex	decimal
0000	0	0
0001	1	1
0010	2	2
0011	3	3
0100	4	4
⋮	⋮	⋮
1100	C	12
1101	D	13
1110	E	14
1111	F	15

2^4

For binary to hex conversion, it is easier to arrange the binary number in groups of 4 bits

Ex: $\underbrace{0100}_{4}\ \underbrace{0110}_{6}$

Ex:

$$B = (\underbrace{1001}\ \underbrace{0110})_2$$

base-16 ⟶ $H = (\ 9 \quad 6\ )_{16}$

$\therefore\ (1001\ 0110)_2 \equiv (96)_{16}$

↑ equivalent to

Ex: $B = 1101\ 0111$

Conv. to Hex:

LSD: $(0111)_2 = (7)_{10} = (7)_{16}$

MSD: $(1101)_2 = (13)_{10} = (D)_{16}$

$\therefore\ (1101\ 0111)_2 \equiv (D7)_{16}$

MSD LSD ← big endian

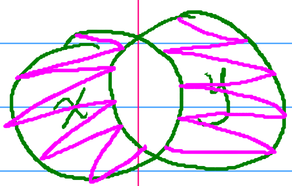
Another Common Operator: XOR

exclusive-OR $\longrightarrow$ symbol $\oplus$

Output is '1' if either $x=1$ & $y=0$ or $x=0$ & $y=1$

TT:

x	y	$f(x) = x \oplus y$
0	0	0
0	1	1
1	0	1
1	1	0



$$f(x,y) = x \oplus y$$
$$= x\bar{y} + \bar{x}y$$

(AND, OR, NOT) is Boolean Complete

can be used to implement any circuits such as XOR

NOR

NAND

DeMorgan's Laws via TTs

A	B	$A \cdot B$	$A+B$	$\bar{A}$	$\bar{B}$	$\overline{(A+B)}$	$\bar{A} \cdot \bar{B}$	$\overline{(A \cdot B)}$	$\bar{A}+\bar{B}$
0	0	0	0	1	1	1	1	1	1
0	1	0	1	1	0	0	0	1	1
1	0	0	1	0	1	0	0	1	1
1	1	1	1	0	0	0	0	0	0

DeMorgan's Laws: $\overline{(A+B)} = \bar{A} \cdot \bar{B}$

$\overline{(A \cdot B)} = \bar{A} + \bar{B}$