Math 132 Test 2

Name: _____

Show all work/rationale. **No notes, internet, calculators, or any other outside resources allowed**.

1. Consider $f(x) = x^3 - \frac{3}{2}x^2 - 6x + 1$.

   (a). (7 points) Find all x-value(s) where $f(x)$ has a local extrema using the **first** derivative test. For each, define whether it is a maximum or minimum.

   (b). (5 points) Find all x-value(s) where $f(x)$ has a local extrema using the **second** derivative test. For each, define whether it is a maximum or minimum.

2. (6 points) Explain whether or not $x = 0$ is a critical number of $f(x) = \dfrac{1}{x}$.

3. (10 points) Find the x-value(s) where $f(x) = \dfrac{x^5}{5} + x^4 + 1$ has an inflection point.

4. (8 points) Given $\frac{3x}{x^2+1}$, $f'(x) = \frac{3-3x^2}{(x^2+1)^2}$, and $\frac{6x^3-18x}{(x^2+1)^3}$, find the y-value(s) of all absolute extrema of $f(x)$ on [0,10]. For each, define whether it is a max or min.

5. (10 points) Find all interval(s) of x-values where $f(x) = xe^{2x}$ is increasing and decreasing.

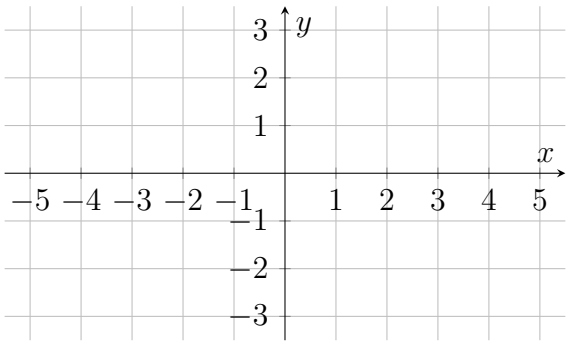
6. (9 points) Sketch one graph of a function $f(x)$ with the following characteristics:

   a. $f(x)$ has a horizontal tangent at $x = -3$.

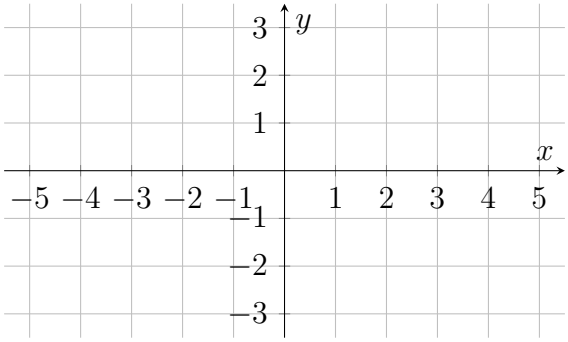
   b. $f(x)$ is decreasing on $(-1, 2)$.

   c. $f(x)$ has an inflection point at $x = 4$.

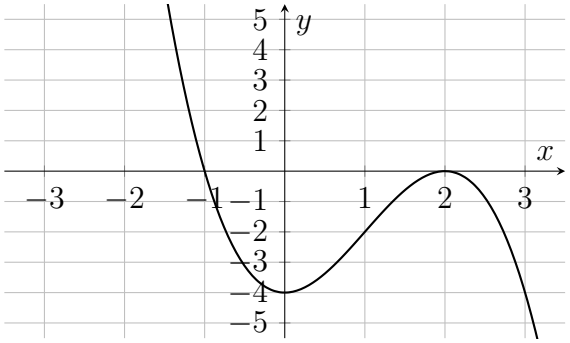
Rough draft (if needed)



Graph to be graded



7. (3 points each) Use the graph of below and answer the following questions. No work required.



(a). Assume the graph is of $f(x)$. Find the x-value(s) of all absolute extrema on $[1, 3]$. For each, define whether it is a max or min.

(b). Assume the graph is of $f(x)$. Find the x-value(s) of all local extrema. For each, define whether it is a max or min.

(c). Assume the graph is of $f'(x)$ (and the domain of f is all real numbers). Find the x-value(s) of all critical numbers of $f(x)$.

(d). Assume the graph is of $f''(x)$ (and the domain of f is all real numbers). Find the interval(s) of x-values where $f(x)$ is concave up.

8. Let $S(t)$ represent the median home price in Knoxville at time t. Home prices are always growing. For small t's, home prices experienced steep increases. As time went on, home prices started to level off. (This scenario is not based off any real data)

   (a). (3 points) Sketch a possible graph of $S(t)$.

   (b). (6 points) Is $S'(t)$ positive, negative, or 0? Is $S''(t)$ positive, negative, or 0? (no explanation needed)

9. Given $f(x) = x^2$ answer the following questions.

   (a). (6 points) Find a value of c that satisfies the conclusion of the Mean Value Theorem over the interval $[-1, 2]$.

   (b). (3 points) Can Rolle's Theorem also be applied on the interval $[-1, 2]$? In one sentence explain why or why not.

10. I am 30 ft from a bottle rocket launching pad. The bottle rocket launches vertically and is attached to a string in my hand.

    (a). (3 points) Without doing any computations (but drawing a picture will be helpful!), should the rate of change of the angle between the string and the horizontal be positive, negative, or zero? Explain your answer.

    (b). (12 points) Find the rate of change of the angle between the string and the horizontal when the rocket is 40 feet in the air and traveling at a rate of 10 ft/s.