Math 132 Test 3

Name: _____

Show all work/rationale. **No notes, internet, calculators, or any other outside resources allowed**.

1. Find the limit.

   (a). (6 points) $\lim_{x\to 0} \dfrac{1-\cos(4x)}{\tan x}$

   (b). (5 points) $\lim_{x\to 4^-} \dfrac{x+1}{4-x}$

   (c). (7 points) $\lim_{x\to \infty} \dfrac{x^2+3}{e^{5x}}$

(d). (8 points) $\lim_{x \to 0^+} x^3 \ln x$

2. (8 points) Sketch one graph of a function $f(x)$ with all of the following characteristics:

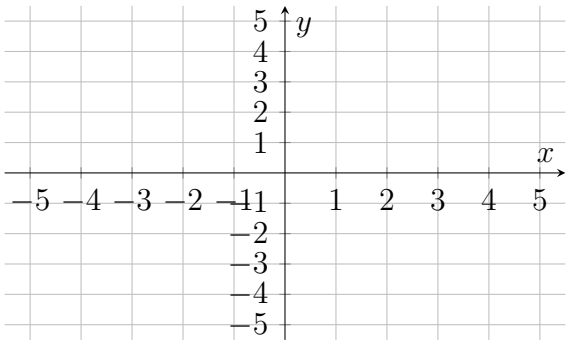
   a. $f'(x) > 0$ on $(0, 3) \cup (3, \infty)$ and $f'(x) < 0$ on $(-\infty, 0)$

   b. $f''(x) > 0$ on $(-3, 3)$ and $f''(x) < 0$ on $(-\infty, -3) \cup (3, \infty)$

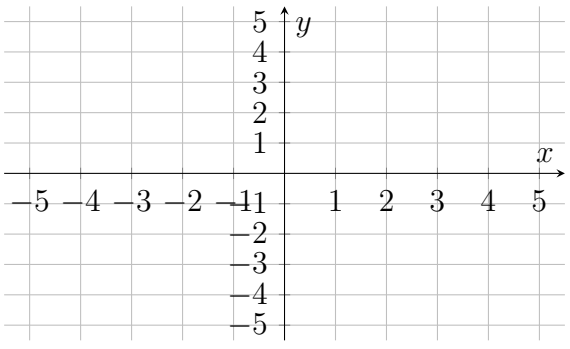
   c. $\lim_{x \to -\infty} f(x) = \infty$ and $\lim_{x \to \infty} f(x) = 0$

   d. $\lim_{x \to 3^-} f(x) = \infty$ and $\lim_{x \to 3^+} f(x) = -\infty$

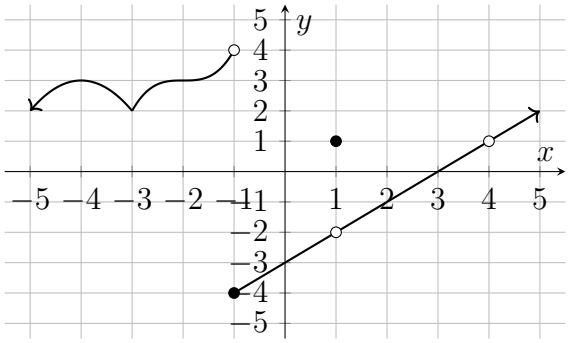
Rough draft (if needed)



Graph to be graded



3. (3 points each) Use the graph below of $f(x)$ to answer the following questions. No work required.



(a). At what x-value(s) is $f(x)$ discontinuous?

(b). Find $\lim_{x \to -3} f(x)$.

(c). Find $\lim_{x \to 1} f(x)$.

(d). Find $\lim_{x \to 4} f(x)$.

(e). Find all x-value(s) a where $\lim_{x \to a^+} f(x)$ and $\lim_{x \to a^-} f(x)$ both exist, but $\lim_{x \to a} f(x)$ does not exist.

4. (6 points) On what interval(s) of x-values is $f(x) = \dfrac{x-5}{x^2-2x-8}$ continuous?

5. Consider $f(x) = \dfrac{2x-3}{x+2}$

(a). (3 points) Find all vertical asymptote(s). Ensure you have some work/rationale for credit.

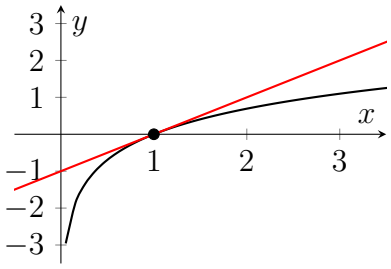
(b). (5 points) Find all horizontal asymptote(s). Ensure you take a limit affiliated with horizontal asymptotes for credit.

6. (8 points) Use the Taylor Polynomial of order 2 to approximate $e^{.05}$.

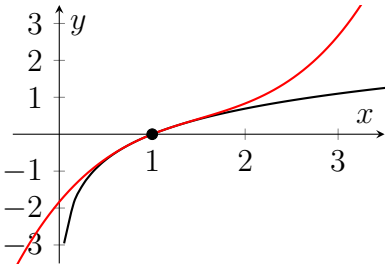
$$T_2(x) = \frac{1}{2}f''(a)(x-a)^2 + f'(a)(x-a) + f(a)$$

7. The following graphs show three different approximations to $f(x) = \ln x$ at $x = 1$.

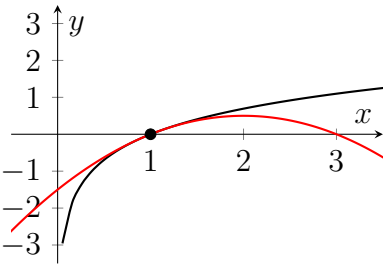
(i)



(ii)



(iii)



(a). (2 pt) Which graph is of $f(x)$ and linear approximation?

(b). (2 pt) Which graph is of $f(x)$ and its 3rd degree Taylor approximation $T_3(x)$?

(c). (4 pts) Explain your rationale to (a) and (b) in one complete sentence.

(d). (2 points) Which of the following do you expect to give us the best approximation for ln(1.1)? (circle one)

(a) Linear approximation

(b) Approximation using the second-order Taylor polynomial

(c) Approximation using the third-order Taylor polynomial

8. (5 points) The loggerhead sea turtles nests in Florida among other places. Let's assume the population of loggerhead sea turtles off the coast of Florida is currently 10,000 and is increasing at a rate of 500 turtles/year. Use linear approximation to approximate the population of loggerhead sea turtles off the coast of Florida in 3 years.

9. (14 points) Use optimization to find the dimensions of a rectangular box with a square base that holds a volume of 1000 cm^3 with the least amount of materials (surface area). Remember to justify your answer.