Section 2.3: Calculating Limits Using Limit Laws

Goal: Develop tools for computing limits without relying on tables and graphs.

Limit laws

- $\lim_{x \to a} k =$

- $\lim_{x \to a} x =$

Assume that $\lim_{x \to a} f(x)$ and $\lim_{x \to a} g(x)$ exist. Then we have the following:

- $\lim_{x \to a} f(x) \pm g(x) =$

- $\lim_{x \to a} kf(x) =$

- $\lim_{x \to a} f(x)g(x) =$

- Assuming $\lim_{x \to a} g(x) \neq 0$, then $\lim_{x \to a} \dfrac{f(x)}{g(x)} =$

- Let n be a positive integer. Then $\lim_{x \to a} (f(x))^n =$

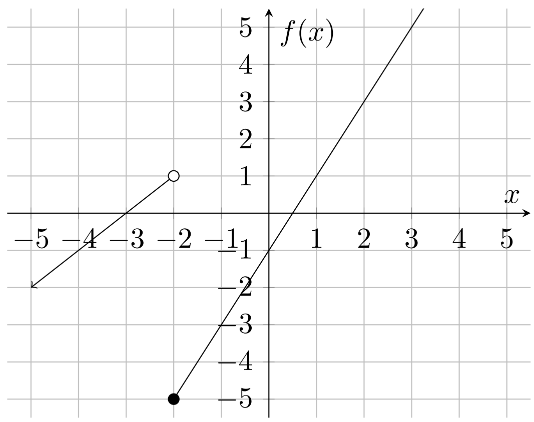
- Let n be a positive integer. Then $\lim_{x \to a} \sqrt[n]{f(x)} =$

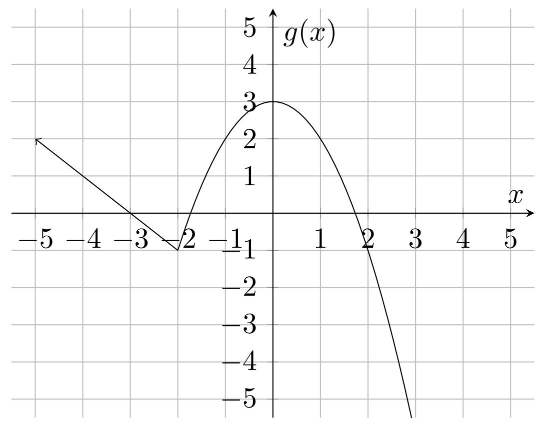
Example 1

Find the limit using limit laws: $\lim_{x \to 4} \frac{x^2 - 9}{x - 3}$

Example 2

Use the following graphs to find the limit: $\lim_{x \to 2} xf(x) - g(x)$





Direct Substitution Property and Limits of Piecewise Functions

Example 3

Determine $\lim_{x \to 3} g(x)$, where $g(x) = \begin{cases} \sqrt{7 - x} & \text{if } x \leq 3 \\ x - 2 & \text{if } x > 3 \end{cases}$.

Limits in the "0/0" form:

Example 4

Find the following limits:

a. $\lim\limits_{x \to 3} \dfrac{x^2 - 9}{x - 3}$

b. $\lim\limits_{x \to 1} \dfrac{3x^2 + 4x - 7}{x^2 - x}$

Example 5

Find the following limits:

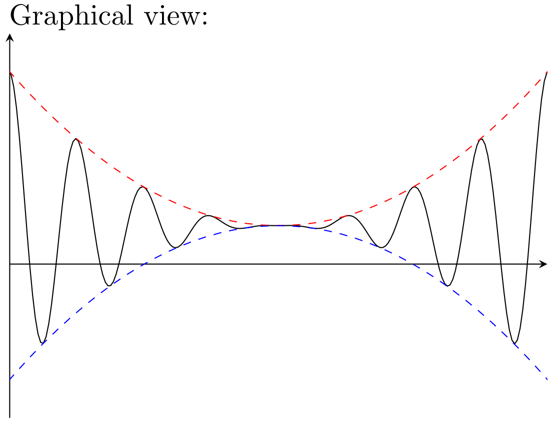
(a) $\lim_{x \to 0} \frac{x}{\sqrt{4+x}-2}$

(b) $\lim_{x \to 5} \frac{\frac{1}{5}-\frac{1}{x}}{x-5}$

Squeeze Theorem

Statement:

Graphical view:



Example 6

Determine $\lim_{x \to 1} g(x)$ assuming $g(x)$ is a function such that $2x - 1 \leq g(x) \leq x^2$.

Example 7

Let $h(x)$ be a function such that $1 - x^2 \leq h(x) \leq x + 3$. Does the squeeze theorem show that $\lim_{x \to 0} h(x)$ does not exist? Explain your answer using complete sentences.

Example 8

Determine $\lim_{x \to 0} x^2 \sin\left(\frac{3\pi}{x}\right)$.