Math 141 Test 3

Name: _____

Show all work/rationale. **No notes, internet, calculators, or any other outside resources allowed**.

1. (9 points) Find the x-value(s) where $f(x) = \frac{x^5}{5} - x^4 + 6$ has an inflection point.

2. (7 points) Given $f(x) = xe^x$, $f'(x) = xe^x + e^x$, and $f''(x) = xe^x + 2e^x$, find all x-value(s) where $f(x)$ has a local extrema. For each, define if it is a maximum or minimum.

3. (7 points) Given $f(x) = \dfrac{3x}{x^2+1}$, $f'(x) = \dfrac{3-3x^2}{(x^2+1)^2}$, and $f''(x) = \dfrac{6x^3-18x}{(x^2+1)^3}$, find the y-value(s) of all absolute extrema of $f(x)$ on [0,10]. For each, define whether it is a max or min.

4. (5 points) Suppose $f''(x)$ is continuous on $(-\infty, \infty)$. If $f(2) = 3$, $f'(2) = 0$ and $f''(2) = -3$, which of the following can we tell about $f(x)$ (circle one)?

   (i). $f(x)$ does not have a critical number at $x = 2$.

   (ii). $f(x)$ has a critical number at $x = 2$, but we can't tell if there is a local extrema at $x = 2$.

   (iii). $f(x)$ has a local minimum at $x = 2$.

   (iv). $f(x)$ has a local maximum at $x = 2$.

   Supply rationale for full credit:

5. Find the limit:

(a). (5 points) $\lim_{x\to\infty} \dfrac{7x}{9-5x^6}$

(b). (8 points) $\lim_{x\to 0} \dfrac{e^{3x}-3x-1}{1-\cos(2x)}$

(c). (8 points) $\lim_{x\to 0^+} x^3 \ln x$

6. (8 points) Sketch one graph of a function $f(x)$ with all of the following characteristics:

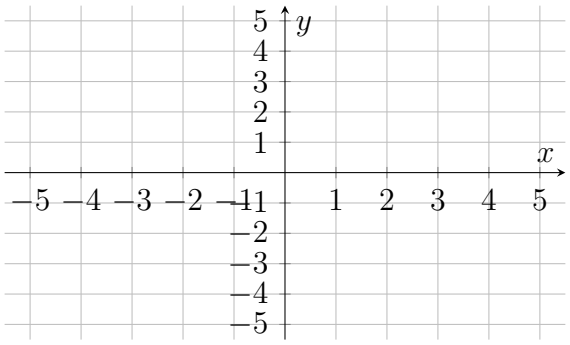
   a. $f'(x) > 0$ on $(0, 3) \cup (3, \infty)$ and $f'(x) < 0$ on $(-\infty, 0)$

   b. $f''(x) > 0$ on $(-3, 3)$ and $f''(x) < 0$ on $(-\infty, -3) \cup (3, \infty)$

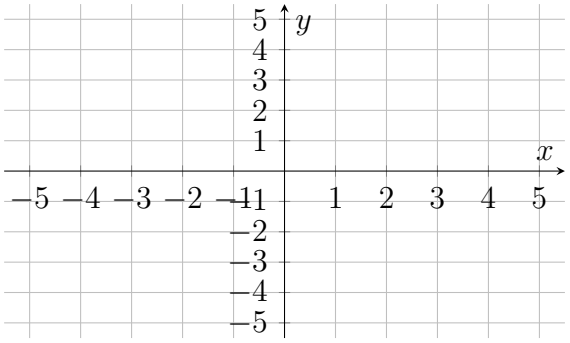
   c. $\lim_{x \to -\infty} f(x) = \infty$ and $\lim_{x \to \infty} f(x) = 0$

   d. $\lim_{x \to 3^-} f(x) = \infty$ and $\lim_{x \to 3^+} f(x) = -\infty$

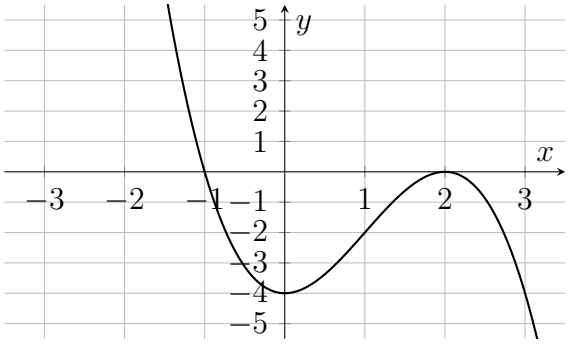
Rough draft (if needed)



Graph to be graded



7. (3 points each) Use the graph of below and answer the following questions. No work required.



   (a). Assume the graph is of $f(x)$. Find the x-value(s) where $f(x)$ has a critical number.

   (b). Assume the graph is of $f(x)$. Find all interval(s) of x-values for which $f(x)$ is concave down.

   (c). Assume the graph is of $f'(x)$ (and the domain of f is all real numbers). Find all interval(s) of x-values where $f(x)$ is increasing.

   (d). Assume the graph is of $f''(x)$ (and the domain of f is all real numbers). Find the x-value(s) where $f(x)$ has an inflection point.

8. Consider $e^{.05}$.

   (a). (6 points) Use linear approximation to approximate $e^{.05}$.

   (b). (4 points) Would you expect this approximation to be an over or under approximation of $e^{.05}$ (I need some work/rationale)?

   (c). (4 points) Use Taylor Polynomials of order 2 to approximate $e^{.05}$.

$$T_2(x) = \frac{1}{2}f''(a)(x-a)^2 + f'(a)(x-a) + f(a)$$

9. (5 points) The water level against a dam was 18 feet deep right after a storm. Four days later, the water level against the same dam was only 6 feet deep. Which of the following 2 conclusions is an example of the Mean Value Theorem (circle one)?

   (i). There had to be a time during those four days that the water level was exactly 10.1234 ft deep.

   (ii). There had to be a time during those four days that the water level water was decreasing at a rate of exactly 3 ft/day.

   Explain your rationale in complete sentences:

10. (12 points) Use optimization to find the dimensions of a rectangular box with a square base that holds a volume of 1000 cm^3 with the least amount of materials (surface area). Remember to justify your answer.