5.4 Notes

Fundamental Theorem of Calculus (Part 1): If $f(x)$ is continuous on $[a,b]$ and has an antiderivative, then

$$\int_a^b f(x)\,dx = F(b) - F(a)$$ where $F(x)$ is the antiderivative

1. $$\int_{-12}^{-4} \frac{dx}{x} = \int_{-12}^{-4} \frac{1}{x}\,dx = \left[\ln|x|\right]_{-12}^{-4} = \ln 4 - \ln 12 = -1.099$$

2. $$\int_0^4 (3x^5 + x^2 - 2x)\,dx = \left[\tfrac{1}{2}x^6 + \tfrac{1}{3}x^3 - x^2\right]_0^4 = \tfrac{1}{2}(4)^6 + \tfrac{1}{3}(4)^3 - 4^2 = 2053.\overline{3}$$

3. $$\int_0^4 \sqrt{y}\,dy = \left[\tfrac{2}{3}y^{3/2}\right]_0^4 = \tfrac{2}{3}(4)^{3/2} = \tfrac{2}{3}(8) = \frac{16}{3}$$

4. $$\int_2^4 \pi^2\,dx = \left[\pi^2 x\right]_2^4 = 4\pi^2 - 2\pi^2 = 2\pi^2$$

5. $$\int_0^{\frac{\pi}{2}} \cos\theta\,d\theta = \sin\theta\,\Big|_0^{\frac{\pi}{2}} = \sin\tfrac{\pi}{2} - \sin 0 = 1$$

6. $$\int_{\frac{\pi}{6}}^{\frac{\pi}{2}} \csc^2 y\,dy = -\cot y\,\Big|_{\frac{\pi}{6}}^{\frac{\pi}{2}} = -\cot\tfrac{\pi}{2} - \left(-\cot\tfrac{\pi}{6}\right) = 0 + \sqrt{3} = \sqrt{3}$$

7. $$\int_0^5 |3-x|\,dx$$ Since $|3-x| = \begin{cases} 3-x, & x \le 3 \\ -(3-x), & x > 3 \end{cases}$

$$\int_0^5 |3-x|\,dx = \int_0^3 (3-x)\,dx + \int_3^5 -(3-x)\,dx$$

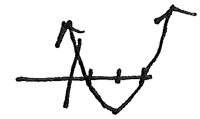
$$= \int_0^3 (3-x)\,dx + \int_5^3 (3-x)\,dx$$

$$= \left[3x - \tfrac{1}{2}x^2\right]_0^3 + \left[3x - \tfrac{1}{2}x^2\right]_5^3$$

$$= \left[\left(9 - \tfrac{9}{2}\right) - 0\right] + \left[\left(9 - \tfrac{9}{2}\right) - \left(15 - \tfrac{25}{2}\right)\right]$$

$$= \tfrac{9}{2} + \tfrac{9}{2} - \tfrac{5}{2} = \frac{13}{2}$$

8. $\int_{-3}^{5} |x^2-4x+3|\,dx$

$x^2-4x+3=(x-3)(x-1) \rightarrow$ 

so $|x^2-4x+3| = \begin{cases} x^2-4x+3, & x=(-\infty,1)\cup(3,\infty) \\ -(x^2-4x+3), & x=(1,3) \end{cases}$

then...

$$\int_{-3}^{5}|x^2-4x+3|\,dx = \int_{-3}^{1}(x^2-4x+3)\,dx - \int_{1}^{3}(x^2-4x+3)\,dx + \int_{3}^{5}(x^2-4x+3)\,dx$$

$$=\left[\tfrac{1}{3}x^3-2x^2+3x\right]_{-3}^{1} - \left[\tfrac{1}{3}x^3-2x^2+3x\right]_{1}^{3} + \left[\tfrac{1}{3}x^3-2x^2+3x\right]_{3}^{5}$$

$$=\left[\left(\tfrac{1}{3}-2+3\right)-(-9-18-9)\right]-\left[(9-18+9)-\left(\tfrac{4}{3}\right)\right]+\left[\left(\tfrac{125}{3}-50+15\right)-0\right]$$

$$=\left[\tfrac{4}{3}+36\right]+\tfrac{4}{3}+\tfrac{20}{3}=\frac{136}{3}=45.\overline{3}$$

9. $\int_{b}^{a} x^4\,dx = \frac{x^5}{5}\Big|_{b}^{a} = \frac{a^5}{5}-\frac{b^5}{5}$

10. $\int_{b}^{b^2}\frac{dx}{x} = \ln|x|\Big|_{b}^{b^2} = \ln b^2 - \ln|b| = \ln\left|\frac{b^2}{b}\right| = \ln|b|$