Math 1920 Final Exam Practice Name:

Work all the following problems in order on other paper. Show all steps. Circle answer.

Test 1

1. $\int (7-x^2)(3x+8x^3)\,dx$

2. $f''(x)=5x-8,\ f'(2)=0,\ f(3)=3.\ Find\ f'(x)\ and\ f(x).$

3. $\int_0^{10} |3x-9|\,dx$

Test 2

4. $\int 3x\sqrt{5-x}\,dx$

5. $\int \frac{\sec^2 x}{\tan x+5}\,dx$

6. $\int \frac{5\,dx}{\sqrt{16-81x^2}}$

7. $\int \frac{3x-8}{x^2+8}\,dx$

Test 3

8. $\int \frac{\ln x}{x^2}\,dx$

9. $\int \sin^3 x\,dx$

10. $\int \frac{dx}{(x^2+5)^{3/2}}$

11. $\int \frac{x^2+2}{(x-1)(x-4)(x+2)}\,dx$

12. $\int x^9 \ln x\,dx$

Final Exam Practice

1. $$\int (7-x^2)(3x+8x^3)\,dx = \int (21x + 56x^3 - 3x^3 - 8x^5)\,dx = \int (-8x^5 + 53x^3 + 21x)\,dx$$

$$= \frac{-8x^6}{6} + \frac{53x^4}{4} + \frac{21x^2}{2} + C = \boxed{-\frac{4}{3}x^6 + \frac{53}{4}x^4 + \frac{21}{2}x^2 + C}$$

2. $f''(x) = 5x - 8$, $f'(2) = 0$, $f(3) = 3$

$$f'(x) = \frac{5}{2}x^2 - 8x + C \rightarrow f'(2) = \frac{5}{2}(2)^2 - 8(2) + C = 10 - 16 + C = -6 + C = 0 \rightarrow C = 6$$

$$\therefore \boxed{f'(x) = \frac{5}{2}x^2 - 8x + 6}$$

$$f(x) = \frac{5}{6}x^3 - 4x^2 + 6x + C \rightarrow f(3) = \frac{5}{6}(3)^3 - 4(3)^2 + 6(3) + C = \frac{9}{2} + C = 3 \rightarrow C = \frac{6}{2} - \frac{9}{2} = \frac{-3}{2}$$

$$\therefore \boxed{f(x) = \frac{5}{6}x^3 - 4x^2 + 6x - \frac{3}{2}}$$

3. $$\int_0^{10} |3x-9|\,dx$$

$$3x - 9 = 0$$
$$3x = 9$$
$$x = 3$$

$$\rightarrow |3x-9| = \begin{cases} -(3x-9), & x < 3 \\ 3x-9, & x \geq 3 \end{cases}$$

$$I = -\int_0^3 (3x-9)\,dx + \int_3^{10} (3x-9)\,dx = -\left[\frac{3x^2}{2} - 9x\right]_0^3 + \left[\frac{3x^2}{2} - 9x\right]_3^{10}$$

$$I = -(-13.5 - 0) + (60 + 13.5) = 87$$

4. $$-\int 3x\sqrt{5-x}\,dx\,(-1) = 3\int -x\sqrt{5-x}\,(-dx) = 3\int (u-5)\,u^{1/2}\,du = 3\int (u^{3/2} - 5u^{1/2})\,du$$

$u = 5 - x \rightarrow u - 5 = -x$

$du = -dx$

$$I = 3\left[\frac{2}{5}u^{\frac{5}{2}} - \frac{10}{3}u^{3/2} + C\right]$$

$$= \boxed{\frac{6\sqrt{(5-x)^5}}{5} - 10\sqrt{(5-x)^3} + C}$$

5. $\int \frac{\sec^2 x}{\tan x + 5}\,dx = \int \frac{1}{u}\,du = \ln|u| + C = \boxed{\ln|\tan x + 5| + C}$

$u = \tan x + 5$

$du = \sec^2 x\,dx$

6. $\int \frac{5\,dx}{\sqrt{16 - 81x^2}} = \int \frac{5\left(\frac{4}{9}du\right)}{\sqrt{16 - 16u^2}} = \frac{5(4)}{9(4)} \int \frac{du}{\sqrt{1-u^2}} = \frac{5}{9}\sin^{-1}(u) + C$

$= \boxed{\frac{5}{9}\sin^{-1}\left(\frac{9x}{4}\right) + C}$

$81x^2 = 16u^2$

$x^2 = \frac{16}{81}u^2$

$x = \frac{4}{9}u \rightarrow u = \frac{9x}{4}$

$dx = \frac{4}{9}du$

7. $\int \frac{3x - 8}{x^2 + 8}\,dx = \frac{3}{2}\int \frac{2x\,dx}{x^2+8} - 8\int \frac{1}{x^2+8}\,dx = \frac{3}{2}\ln|x^2+8| - 8\int \frac{\sqrt{8}\,du}{8u^2+8}$

$x^2 = 8u^2$

$x = \sqrt{8}u \rightarrow u = \frac{x}{\sqrt{8}}$

$dx = \sqrt{8}\,du$

$I = \frac{3}{2}\ln(x^2+8) - \sqrt{8}\int \frac{du}{u^2+1} = \frac{3}{2}\ln(x^2+8) - \sqrt{8}\tan^{-1}(u) + C = \boxed{\frac{3}{2}\ln(x^2+8) - \sqrt{8}\tan^{-1}\left(\frac{x}{\sqrt{8}}\right) + C}$

8. $\int \frac{\ln x}{x^2}\,dx$ $\quad u = \ln x \quad du = \frac{1}{x}dx \quad dv = \frac{1}{x^2}dx = x^{-2}dx \quad v = \frac{x^{-1}}{-1} = \frac{-1}{x}$

$$I = \frac{-\ln x}{x} + \int \frac{1}{x^2}dx = \frac{-\ln x}{x} - \frac{1}{x} + C$$

9. $\int \sin^3 x\,dx = -\int (1-\cos^2 x)\sin x\,dx(-1) = -\int (1-u^2)\,du = -\left[u - \frac{u^3}{3} + C\right]$

$u = \cos x$

$du = -\sin x\,dx$

$$I = -\cos x + \frac{1}{3}\cos^3 x + C$$

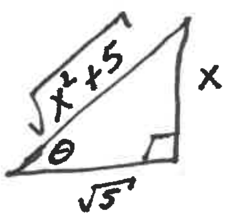
10. $\int \frac{dx}{(x^2+5)^{3/2}}$ $\quad$ let $x = \sqrt{5}\tan\theta \quad dx = \sqrt{5}\sec^2\theta\,d\theta$

$(x^2+5)^{3/2} = (5\tan^2\theta + 5)^{3/2} = \left[5(\tan^2\theta+1)\right]^{3/2} = (5\sec^2\theta)^{3/2} = 5\sqrt{5}\sec^3\theta$

$$I = \int \frac{\sqrt{5}\sec^2\theta\,d\theta}{5\sqrt{5}\sec^3\theta} = \frac{1}{5}\int \frac{1}{\sec\theta}\,d\theta = \frac{1}{5}\int \cos\theta\,d\theta = \frac{1}{5}\sin\theta + C$$

$$I = \frac{1}{5}\left(\frac{x}{\sqrt{x^2+5}}\right) + C$$

$\frac{x}{\sqrt{5}} = \tan\theta \rightarrow$



11. $\int \frac{x^2+2}{(x-1)(x-4)(x+2)}\,dx$

$$\frac{x^2+2}{(x-1)(x-4)(x+2)} = \frac{A}{x-1} + \frac{B}{x-4} + \frac{C}{x+2}$$

$$x^2+2 = A(x-4)(x+2) + B(x-1)(x+2) + C(x-1)(x-4)$$

$x=1 \rightarrow \quad 3 = A(-3)(3) \rightarrow -9A=3 \rightarrow A = \frac{-1}{3}$

$x=4 \rightarrow \quad 18 = B(3)(6) \rightarrow 18B=18 \rightarrow B=1$

$x=-2 \rightarrow \quad 6 = C(-3)(-6) \rightarrow 18C=6 \rightarrow C=\frac{1}{3}$

$$I = \frac{-1}{3}\int\frac{dx}{x-1} + \int\frac{dx}{x-4} + \frac{1}{3}\int\frac{dx}{x+2} = \frac{-1}{3}\ln|x-1| + \ln|x-4| + \frac{1}{3}\ln|x+2| + C$$

12. $\int x^9 \ln x\,dx$

$u = \ln x \qquad dv = x^9\,dx$

$du = \frac{1}{x}\,dx \qquad v = \frac{1}{10}x^{10}$

$$I = \frac{1}{10}x^{10}\ln x - \frac{1}{10}\int x^9\,dx = \frac{1}{10}x^{10}\ln x - \frac{1}{100}x^{10} + C$$