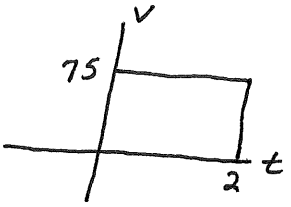
5.6 Notes – Net Change/ Displacement versus total distance traveled

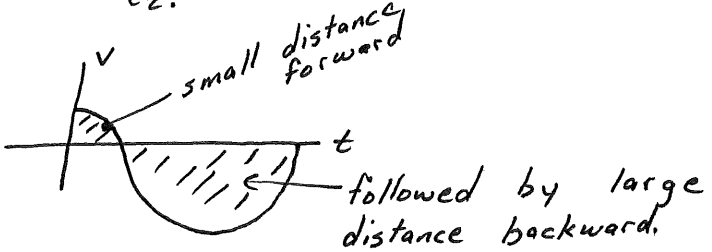
Draw a graph representing the distance traveled if the rate is 75mph and the time is 2 hours. Determine the integral representing this simplistic scenario.



$$\int_0^2 75\,dt = 75t\Big|_0^2 = 75(2) - 75(0) = 150 \text{ miles}$$

In general, we can say displacement is: $\int_{t_1}^{t_2} v(t)\,dt$ = displacement from time t_1 to t_2.

Could displacement be negative? Sure! →



How would we find total distance traveled?

$$\int_{t_1}^{t_2} |v(t)|\,dt$$

Try these problems with me.

1. Find the displacement over the time interval [1, 6] of a helicopter whose (vertical) velocity at time t is $v(t) = .02t^2 + t$ m/s.

$$\text{Displacement} = \int_1^6 (.02t^2 + t)\,dt = \left[\frac{.02t^3}{3} + \frac{t^2}{2}\right]_1^6 = (1.44 + 18) - (.00\bar{6} + .5)$$

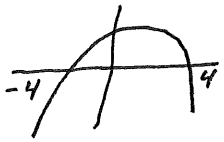
$$= 18.933 \text{ ft.}$$

2. Find the displacement and total distance traveled for a particle in its first 6 seconds of movement if it has a velocity function of $v(t) = 32 - 2t^2$ ft/s. Draw a motion diagram for the particle.

$$\text{Displacement} = \int_0^6 (32 - 2t^2)\,dt = \left[32t - \frac{2t^3}{3}\right]_0^6 = (192 - 144) - 0 = 48 \text{ ft.}$$

Distance → need $32 - 2t^2 \geq 0$

$2(4-t)(4+t) \geq 0$



$$|32 - 2t^2| = \begin{cases} 32 - 4t^2, & t = [0, 4] \\ -(32 - 4t^2), & t > 4 \end{cases}$$

$$\text{Distance} = \int_0^4 (32-2t^2)\,dt + \int_6^4 (32-2t^2)\,dt$$

$$= \left[32t - \tfrac{2}{3}t^3\right]_0^4 + \left[32t - \tfrac{2}{3}t^3\right]_6^4 = \left(\tfrac{256}{3} - 0\right) + \left(\tfrac{256}{3} - 48\right)$$

$$= \frac{368}{3} \approx 122.667 \text{ ft}$$

Motion Diagram