

Name: _____

1. (6 points) Use the limit definition of the derivative to find $f'(x)$ given $f(x) = 5x^2 - 2$.
2. (5 points) The deck of a bridge is suspended 100 feet above a river. If a pebble falls off the side of the bridge, the height, in feet, of the pebble above the water surface after t seconds is given by $s(t) = 100 - 16t^2$. Find the average velocity of the pebble from $t = 0$ to $t = 2$. Include units.

3. Find the limit. Show your work or explain your rationale using calculus.

(a). (5 points) $\lim_{x \rightarrow 0} \frac{42}{x^2}$

(b). (6 points) $\lim_{x \rightarrow 1} \frac{x - 1}{x^2 + x - 2}$

(c). (6 points) $\lim_{x \rightarrow \infty} \frac{x - 4}{x^2 + 6x}$

(d). (5 points) $\lim_{x \rightarrow 0} \frac{\sin x}{x + 2}$

(e). (2 points) $\lim_{x \rightarrow 0^+} \ln(x)$

(f). (5 points) $\lim_{x \rightarrow \infty} \tan^{-1}(e^x)$

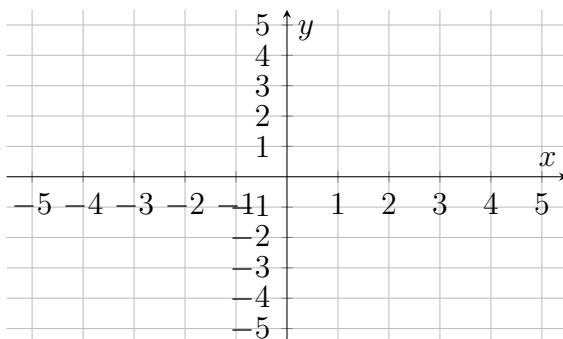
4. (5 points) At what x -value(s) does $f(x) = \frac{x^2 - 7x + 10}{x^2 - 25}$ have a **vertical asymptote** and at what x -value(s) does it have a **hole**? Explain in one or two complete sentences.

5. (5 points) Let $f(x) = \begin{cases} 3x^2 + x - c & \text{if } x \leq 2 \\ cx^2 + 12 & \text{if } x > 2 \end{cases}$. Find the value c so that $\lim_{x \rightarrow 2} f(x)$ exists.

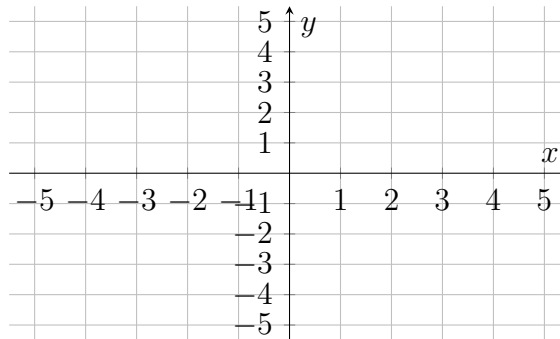
6. (5 points) Sketch one graph of a function $f(x)$ with the following characteristics:

- $f(x)$ has removable discontinuity at $x = -3$
- $\lim_{x \rightarrow 0} f(x) = 2$
- $\lim_{x \rightarrow 3} f(x) = \infty$
- $f(x)$ is continuous on all x -values except $x = -3, 3$

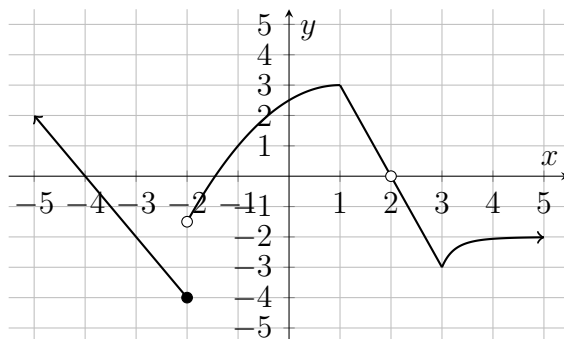
Rough draft (if needed)



Graph to be graded



7. Use the graph of $f(x)$ below to answer the following questions.



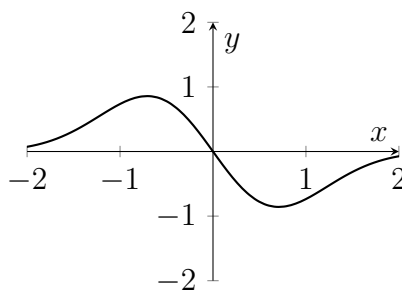
- (2 points) Find $\lim_{x \rightarrow \infty} f(x)$.
- (2 points) Find $\lim_{x \rightarrow -\infty} f(x)$.
- (2 points) Find $\lim_{x \rightarrow -2} f(x)$.
- (2 points) Find $\lim_{x \rightarrow 2} f(x)$.
- (5 points) Which is larger: $f'(-1)$ or $f'(0)$? Explain in one complete sentence.

8. (5 points) On what interval of x -values is $f(x) = \frac{\cos x}{x-3}$ continuous?

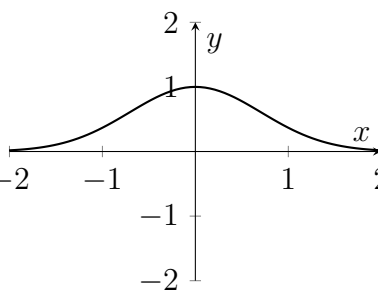
9. (5 points) Use the intermediate value theorem to show there is a solution to $x^2 + \sqrt{x+4} - 10 = 0$ in $[0, 5]$.

10. (6 points) The graphs below are of f , f' , and f'' . Identify each curve. No work/justification needed.

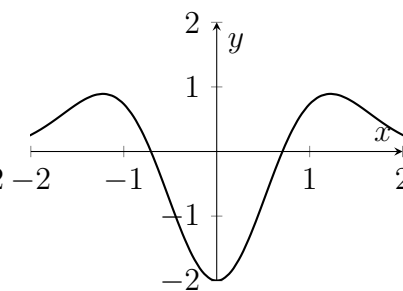
(i)



(ii)



(iii)



11. (5 points) Let $-x^2 \leq f(x) \leq x^2 + 2$. Can the squeeze theorem be used to show that $\lim_{x \rightarrow 0} f(x)$ does not exist? Explain using complete sentences.
12. Let $T(t)$ be the temperature, measured in degrees Fahrenheit, at t hours after 1pm.
- (a). (4 points) Interpret the meaning of $T'(0) = .7$ in the context of this application. Include units.
- (b). (3 points) If $T(0) = 50$ and $T'(0) = .7$, estimate the temperature at 3pm.
13. (4 points) Is it possible that a function can be continuous at $x = 0$ but not differentiable (nondifferentiable) at $x = 0$? Explain. You may sketch a graph as your explanation.