Math 131 Test 3   Name: _____

1. Differentiate and simplify.

   a. (6 points) $y = 7x^3 + x + e^{10}$

   b. (6 points) $s(t) = \sqrt[5]{t} + \dfrac{6}{t^9}$

   c. (6 points) $r(\theta) = 5\csc\theta + \sec\theta$

   d. (6 points) $f(x) = \dfrac{x^2 + 3}{2x^2 - 1}$

e. (6 points) $g(y) = (4y - 3)^5$

2. Differentiate. You don't need to simplify, but put parentheses where they need to be.
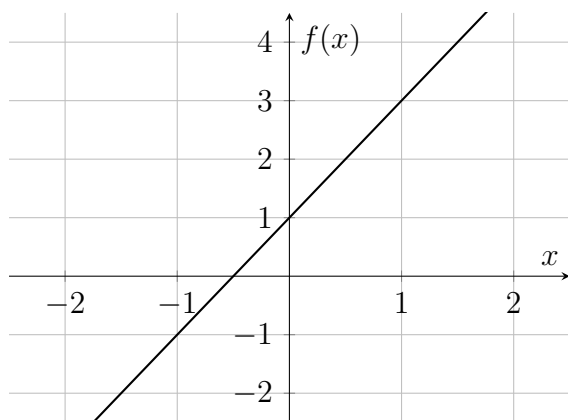
   a. (8 points) $s(t) = t^4 e^t \cot t$

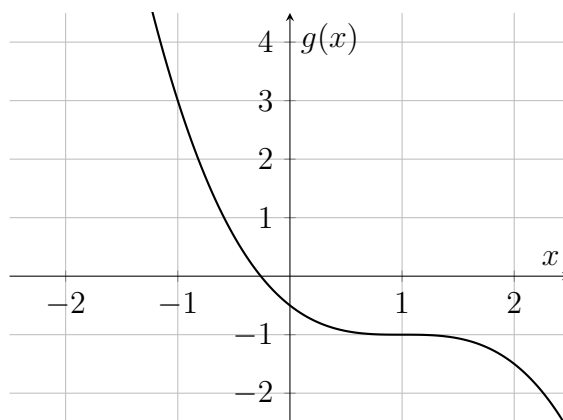   b. (8 points) $y = \dfrac{\tan^4 x}{4x^5 - x^2}$

3. (6 points) Find all x-value(s) where $f(x) = 3x^2 - 12x - 1$ has a horizontal tangent.

4. (6 points) Consider the graphs of $f(x)$ (left graph) and $g(x)$ (right graph). Let $v(x) = f(x)g(x)$ and find $v'(1)$.

(i)

(ii)

5. (7 points) Find the equation of the line tangent to $f(x) = \cos(3x)$ at $x = \dfrac{\pi}{2}$.

6. (5 points) Find the 101st derivative of $g(x) = \sin x$. Show/explain your work clearly.

7. The position of a particle moving back and forth on a horizontal surface is $s(t) = \dfrac{t^3}{3} - 4t$ meters, t in seconds. Let the positive position be distance to the right of the starting position.

    a. (5 points) Find the velocity of the object at $t = 5$. Include units.

    b. (3 points) Which of the following is an interpretation of your answer to a.?

       i. The particle is right of its starting point.

      ii. The particle is left of its starting point.

     iii. The particle is moving to the left.

     iv. The particle is moving to the right.

    c. (5 points) Find the acceleration of the particle when the velocity is 0.

8. (4 points) Given $y = 4xt^4 - t^2x^5 + 4x^2$, find $\dfrac{dy}{dt}$ treating x as constant.

9. (5 points) Let $P(x)$ be the profit of some business selling x units of its product. Suppose it is known that $P'(100) = .013$. One of your friends says, "This business' profit must be decreasing since .013 is such a small number." Is this correct? Explain in one or 2 complete sentence.

10. (8 points) The biomass $B(t)$ of a fish population is the total mass of the members of the population at time t. It is the product of the number of individuals $N(t)$ in the population and the average mass $M(t)$ of a fish at time t. In the case of guppies, breeding occurs continually. Suppose that at time $t = 3$ weeks the population is 500 guppies and is growing at a rate of 20 guppies per week, while the average mass is 2 g and is increasing at a rate of 0.1 g/week. At what rate is the biomass increasing when $t = 3$? You do not need to multiply your answer out.