Min terms = 1

Max terms = 0

ECE255 – Introduction to Digital Logic Design
Homework Assignment 2
Due September 18

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NOTE: Some questions require drawing schematics. You can use Logisim Evolution for drawing a schematic and then take a screenshot. Please see the Logisim Tutorial on Canvas as needed.

Let $ABCD_2$ be a 4-bit non-negative integer with corresponding decimal values:

$0000_2 = 0_{10}$
$0001_2 = 1_{10}$
$0010_2 = 2_{10}$
$0011_2 = 3_{10}$
…
$1111_2 = 15_{10}$

The digits *A*, *B*, *C*, and *D* in this 4-bit integer are also to be considered variables. Consider two functions of these 4 variables $f(A, B, C, D)$ and $g(A, B, C, D)$.

1. The function $f(A, B, C, D)$ is defined as:

$$f = \begin{cases} 1 & \text{if the hex value of } ABCD_2 \text{ is less than } A_{16} \\ 0 & \text{otherwise} \end{cases}$$

We have 4^n calculations

outputs true = 1

false = 0

Describe *f* in the following forms listed below:

(a) **Truth Table** → when 0
(b) **CSOP** in the concise $\sum m_i$ **notation**
(c) **CPOS** in the concise $\prod M_i$ **notation** → when 1

CSOP in $\sum m_i$ notation

(A) Truth Table 16 bit

input				output	
A	B	C	D	f	G
m_0 0	0	0	0	1	0
m_1 0	0	0	1	1	0
m_2 0	0	1	0	1	0
m_3 0	0	1	1	1	0
m_4 0	1	0	0	1	0
m_5 0	1	0	1	1	0
m_6 0	1	1	0	1	0
m_7 0	1	1	1	1	0
m_8 1	0	0	0	1	0
m_9 1	0	0	1	1	0
m_{10} 1	0	1	0	0	1
m_{11} 1	0	1	1	0	1
m_{12} 1	1	0	0	0	1
m_{13} 1	1	0	1	0	1
m_{14} 1	1	1	0	0	1
m_{15} 1	1	1	1	0	1

1 if hex of $ABCD_2$ is less than A_{16}, if >, then it's 0

A_{16} is 1010_2 so everything after is 0 but all before are 1.

(B) CPOS $\Sigma\Pi_{M_i}$ notation

$= \Pi(m_0, m_1, m_2, m_3, m_4, m_5, m_6, m_7, m_8, m_9)$

(C) CSOP Σ_{M_i} notation

$= \Sigma(m_{10}, m_{11}, m_{12}, m_{13}, m_{14}, m_{15})$

2. The function $g(A, B, C, D)$ is defined as:

$$g=\begin{cases}0 & \text{if } f=1\\ 1 & \text{if } f=0\end{cases}$$

Describe g in the following forms listed below:

(a) **CSOP** in the concise $\sum m_i$ **notation**
(b) **CPOS** in the concise $\prod M_i$ **notation**

input / output.

	A	B	C	D	f	g
m_0	0	0	0	0	1	0
m_1	0	0	0	1	1	0
m_2	0	0	1	0	1	0
m_3	0	0	1	1	1	0
m_4	0	1	0	0	1	0
m_5	0	1	0	1	1	0
m_6	0	1	1	0	1	0
m_7	0	1	1	1	1	0
m_8	1	0	0	0	1	0
m_9	1	0	0	1	1	0
m_{10}	1	0	1	0	0	1
m_{11}	1	0	1	1	0	1
m_{12}	1	1	0	0	0	1
m_{13}	1	1	0	1	0	1
m_{14}	1	1	1	0	0	1
m_{15}	1	1	1	1	0	1

(A) CSOP in $\sum m_i$ notation

$\prod (m_{10} m_{11} m_{12} m_{13} m_{14} m_{15})$

(B) CPOS $\sum (m_0 m_1 m_2 m_3 m_4 m_5 m_6 m_7 m_8 m_9)$

3. Express the function $f(A, B, C, D)$ in the following forms:

$X + X' = 1$

$X \cdot X' = 0$

(a) A **Boolean Algebra expression of the CSOP** function $f(A, B, C, D)$
(b) A **minimized Boolean expression** for $f(A, B, C, D)$
Use the properties and axioms of Boolean Algebra to minimize $f(A, B, C, D)$

input / output. terms

A	B	C	D	f	g	min term
0	0	0	0	1	0	$A\cdot B\cdot C\cdot D$ m_0
0	0	0	1	1	0	$A\cdot B\cdot C\cdot \bar{D}$ m_1
0	0	1	0	1	0	$A\cdot B\cdot \bar{C}\cdot D$ m_2
0	0	1	1	1	0	$A\cdot B\cdot \bar{C}\cdot \bar{D}$ m_3
0	1	0	0	1	0	$A\cdot \bar{B}\cdot C\cdot D$ m_4
0	1	0	1	1	0	$A\cdot \bar{B}\cdot C\cdot \bar{D}$ m_5
0	1	1	0	1	0	$A\cdot \bar{B}\cdot \bar{C}\cdot D$ m_6
0	1	1	1	1	0	$A\cdot \bar{B}\cdot \bar{C}\cdot \bar{D}$ m_7
1	0	0	0	1	0	$\bar{A}\cdot B\cdot C\cdot D$ m_8
1	0	0	1	1	0	$\bar{A}\cdot B\cdot C\cdot \bar{D}$ m_9
1	0	1	0	0	1	$\bar{A}\cdot B\cdot \bar{C}\cdot D$ m_{10}
1	0	1	1	0	1	$\bar{A}\cdot B\cdot \bar{C}\cdot \bar{D}$ m_{11}
1	1	0	0	0	1	$\bar{A}\cdot \bar{B}\cdot C\cdot D$ m_{12}
1	1	0	1	0	1	$\bar{A}\cdot \bar{B}\cdot C\cdot \bar{D}$ m_{13}
1	1	1	0	0	1	$\bar{A}\cdot \bar{B}\cdot \bar{C}\cdot D$ m_{14}
1	1	1	1			$\bar{A}\cdot \bar{B}\cdot \bar{C}\cdot \bar{D}$ m_{15}

↳ what we calculate

CSOP →

$A\cdot B\cdot C\cdot D$ m_0
$A\cdot B\cdot C\cdot \bar{D}$ m_1
$A\cdot B\cdot \bar{C}\cdot D$ m_2
$A\cdot B\cdot \bar{C}\cdot \bar{D}$ m_3
$A\cdot \bar{B}\cdot C\cdot D$ m_4
$A\cdot \bar{B}\cdot C\cdot \bar{D}$ m_5
$A\cdot \bar{B}\cdot \bar{C}\cdot D$ m_6
$A\cdot \bar{B}\cdot \bar{C}\cdot \bar{D}$ m_7
$\bar{A}\cdot B\cdot C\cdot D$ m_8
$\bar{A}\cdot B\cdot C\cdot \bar{D}$ m_9

can't be a 3 term product.

$f = (\bar{A}\bar{B}\bar{C}\bar{D}) + (\bar{A}\bar{B}\bar{C}D) + (\bar{A}\bar{B}C\bar{D}) + (\bar{A}\bar{B}CD)$
$+ (\bar{A}B\bar{C}\bar{D}) + (\bar{A}B\bar{C}D) + (\bar{A}BC\bar{D}) + (\bar{A}BCD)$
$+ (A\bar{B}\bar{C}\bar{D}) + (A\bar{B}\bar{C}D)$
$= \bar{A}\bar{B}\bar{C}(\bar{D}+D) + \bar{A}\bar{B}C(\bar{D}+D)$

$= \bar{A}B\bar{C}(\bar{D}+D) + \bar{A}BC(\bar{D}+D)$
$+ A\bar{B}\bar{C}(\bar{D}+D)$
$= \bar{A}\bar{B}\bar{C} + \bar{A}\bar{B}C + \bar{A}B\bar{C} + \bar{A}BC + A\bar{B}\bar{C}$
$= \bar{A}\bar{B}(\bar{C}+C) + \bar{A}BC(\bar{C}+C) + A\bar{B}\bar{C}$
$= \bar{A}\bar{B} + \bar{A}\bar{B} + A\bar{B}\bar{C}$
$= \bar{A}(\bar{B}+B) + A\bar{B}\bar{C}$
$= \bar{A} + A\bar{B}\bar{C}$ $\boxed{= \bar{A} + \bar{B}\bar{C}}$

5 gates — 2 not, 1 or, 1 and.

4. Express ***f(A, B, C, D)*** **as a digital logic schematic**, clearly showing AND, NOT, and OR gates used and how they are connected. **Draw by hand or use Logisim to place and connect gates.**

$f(A,B,C,D) = \bar{A} + \bar{B}\bar{C}$ 3 not inputs feed

(NOT A) OR (NOT B AND NOT C)

invert A, B, C and put $\bar{B}\bar{C}$ into a NAND Gate better doing A OR B

NOT A => A(1)

B NAND C = 0