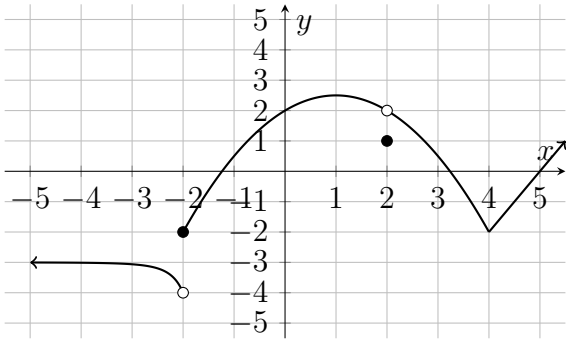
Math 131 Test 2

Name: _____

Show all work/rationale. **No notes, internet, calculators, or any other outside resources allowed**.

1. (8 points) Given $f(x) = 4x^2 + 7$, find $f'(x)$ using the limit definition of the derivative.

2. Given the graph of $f(x)$ below answer the following questions:



   a. (3 points) At what x-value(s) a does $\lim_{x \to a} f(x)$ exist, but $f(x)$ is not continuous at a?

   b. (3 points) At what x-value(s) is $f(x)$ not continuous?

   c. (3 points) At what x-value(s) does $f(x)$ have a horizontal tangent line?

   d. (3 points) Find $f'(4.5)$.

3. Find the limit. Show work or explain your rationale.

a. (4 points) $\lim_{x\to\infty} 3x^4 - 5x^7$

b. (6 points) $\lim_{x\to-\infty} \dfrac{2x^5 - 7}{3 - 5x}$

c. (5 points) $\lim_{x\to 1} 5e^{\frac{x-1}{x^2+10}}$

d. (4 points) $\lim_{x\to\infty} \dfrac{7}{e^x}$

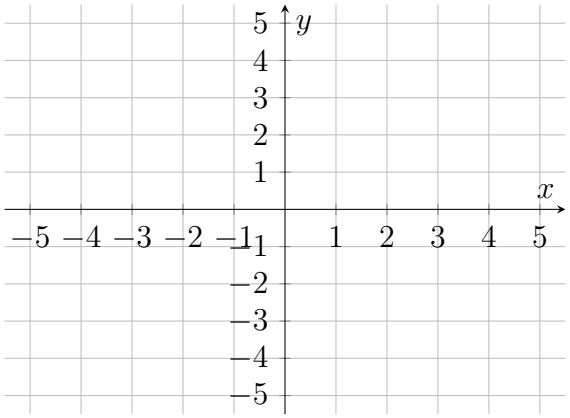
4. (8 points) Sketch one graph of a function $f(x)$ with the following characteristics:
(2 grids are given in case you want a rough draft. If you graph in both, ensure it's clear which is your final version)

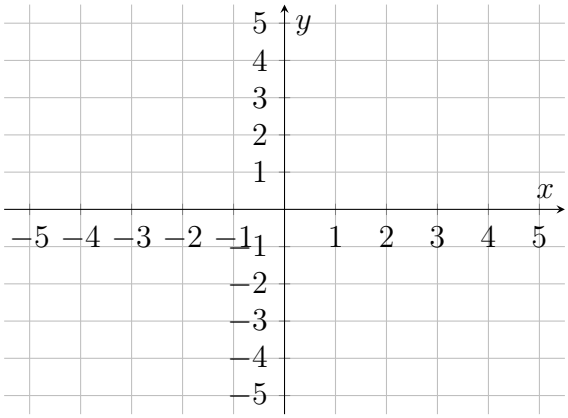
   a. $f(x)$ is continuous everywhere except at $x = -3$

   b. $\lim_{x\to-\infty} f(x) = \infty$ and $\lim_{x\to\infty} f(x) = -\infty$

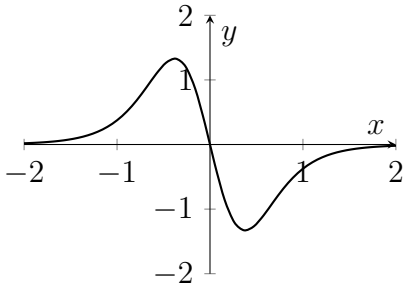
   c. $f(x)$ has removable discontinuity at $x = -3$

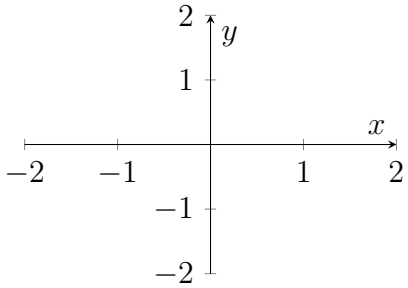
   d. $f(x)$ is continuous yet nondifferentiable at $x = 0$





5. (5 points) Given the graph of $f(x)$ below, sketch a curve for $f'(x)$.





6. (8 points) Find all horizontal asymptotes of $f(x) = \dfrac{3x^2 - 4x}{7 - x^2}$ by finding a limit associated with horizontal asymptotes.

7. (5 points) At what interval(s) of x-values is $f(x) = \sqrt{7 - 5x}$ continuous?

8. (6 points) Is $f(x) = \begin{cases} (4+x)^{3/2} & \text{if } x > 0 \\ \ln(1+x) & \text{if } x \leq 0 \end{cases}$ continuous at $x = 0$? Use the formal definition of continuity to determine. Use proper notation.

9. (5 points) Let $f'(x) = 4x^5 - x + 3$ and $f(1) = 10$. Find an equation of the line tangent to $f(x)$ at $x = 1$.

10. (4 points) Assume that

$$\lim_{x\to\infty} f(x) = L \text{ and } \lim_{x\to L} g(x) = \infty.$$

Which of the following statements are correct? Circle all that apply.

a. $x = L$ is a vertical asymptote of g.

b. $y = L$ is a horizontal asymptote of g.

c. $x = L$ is a vertical asymptote of f.

d. $y = L$ is a horizontal asymptote of f.

11. (5 points) A team of ornithologists (bird scientists) are studying the development of Sandhill cranes. The ornithologists study a newly hatched Sandhill crane chick, who has an initial height 3 inches. After one month, the ornithologists re-measure the bird, and now the same chick is 25 inches tall. The scientists ask you to decide if this chick was ever exactly 20 inches tall. Respond either "yes" or "no", and support your argument by naming and using the theorem we covered affiliated with this story.

12. A rechargeable battery is plugged into a charger. Suppose $C(t)$ is the percentage of full capacity that the battery reaches at time t (in hours).

    a. (5 points) Interpret the meaning of $C'(3) = 10$ in context of this application.

    b. (3 points) If $C(3) = 50$, use $C'(3) = 10$ to estimate $C(5)$.

    c. (2 points) Interpret the meaning of $C(5)$ using your answer to b.

13. (5 points) Suppose that $w(t)$ gives the weight (in pounds) of a goldendoodle puppy t months after it is born. What is larger: $w'(1)$ or $w'(30)$? Explain in a couple complete sentences.