1. $\int x \sqrt[3]{4x^2-5}\, dx = \frac{1}{8}\int \sqrt[3]{4x^2-5}\; 8x\,dx = \frac{1}{8}\int u^{\frac{1}{3}}\,du = \frac{1}{8}\left[\frac{3}{4}u^{\frac{4}{3}}+C\right]$

$= \frac{3\sqrt[3]{(4x^2-5)^4}}{32} + C$

$u = 4x^2-5$

$du = 8x\,dx$

2. $\int (2x(3+x)^7)\,dx = 2\int (u-3)u^7\,du = 2\int (u^8-3u^7)\,du$

$= 2\left[\frac{u^9}{9} - \frac{3u^8}{8} + C\right]$

$u = 3+x \rightarrow u-3 = x$

$du = dx$

$= \frac{2(3+x)^9}{9} - \frac{3(3+x)^8}{4} + C$

3. $\int \frac{-\sec^2 x}{\tan x + 4}\,dx = -\int \frac{1}{u}\,du = -\ln|u| + C = -\ln|\tan x + 4| + C$

$u = \tan x + 4$

$du = \sec^2 x\,dx$

4. $\int_{\frac{\pi}{6}}^{\frac{\pi}{4}} \cos^3 x \sin x\,dx = \int_{\frac{\sqrt{2}}{2}}^{\frac{\sqrt{3}}{2}} u^3\,du = \frac{u^4}{4}\Big|_{\frac{\sqrt{2}}{2}}^{\frac{\sqrt{3}}{2}} = \frac{9}{64} - \frac{4}{64}$

$= \frac{5}{64}$

$u = \cos x$

$du = -\sin x\,dx$

$u(\frac{\pi}{4}) = \frac{\sqrt{2}}{2}$

$u(\frac{\pi}{6}) = \frac{\sqrt{3}}{2}$

5. $\int_{-4}^{2} \frac{t+1}{t+7}\,dt = \int_3^9 \frac{u-6}{u}\,du = \int_3^9 (1-\frac{6}{u})\,du = \left[u - 6\ln|u| + C\right]_3^9$

$= (9-6\ln 9) - (3-6\ln 3)$

$= 6 - 6\ln 9 + 6\ln 3$

$= 6 + 6\ln(\frac{3}{9}) = 6 - 6\ln 3$

$u = t+7 \rightarrow u-6 = t+1$

$du = dt$

$u(-4) = 3$

$u(2) = 9$

6. $\int_1^6 \sqrt{x+3}\,dx = \int_1^6 (x+3)^{\frac{1}{2}}\,dx = \left[\frac{2}{3}(x+3)^{\frac{3}{2}}\right]_1^6 = \frac{2}{3}(27-8)$

$= \frac{38}{3}$

7. $\int 5^{2x-1} e^{2x}\,dx = \frac{1}{5}\int (5e)^{2x}\,dx = \frac{1}{5}\left[\frac{(5e)^{2x}}{2\ln(5e)} + C\right]$

$= \frac{(5e)^{2x}}{10\ln(5e)} + C$

8. $\int \frac{3}{9+4x^2}\,dx = \int \frac{3(\frac{3}{2}du)}{9+9u^2} = \frac{1}{2}\int \frac{du}{1+u^2} = \frac{1}{2}\tan^{-1}u + C$

$= \frac{1}{2}\tan^{-1}\left(\frac{2x}{3}\right) + C$

$4x^2 = 9u^2$

$2x = 3u$

$x = \frac{3}{2}u \rightarrow u = \frac{2x}{3}$

$dx = \frac{3}{2}du$

9. $\int \frac{1}{\sqrt{25-16x^2}}\,dx = \int \frac{\frac{5}{4}du}{\sqrt{25-25u^2}} = \frac{1}{4}\int \frac{du}{\sqrt{1-u^2}} = \frac{1}{4}\sin^{-1}u + C$

$= \frac{1}{4}\sin^{-1}\left(\frac{4x}{5}\right) + C$

$16x^2 = 25u^2$

$4x = 5u$

$x = \frac{5}{4}u \rightarrow u = \frac{4x}{5}$

$dx = \frac{5}{4}du$

10. $\int \frac{dx}{3|x|\sqrt{25x^2-1}} = \frac{1}{3}\int \frac{\frac{1}{5}du}{|\frac{1}{5}u|\sqrt{u^2-1}} = \frac{1}{3}\int \frac{du}{|u|\sqrt{u^2-1}}$

$= \frac{1}{3}\sec^{-1}u + C$

$= \frac{1}{3}\sec^{-1}(5x) + C$

$25x^2 = u^2$

$5x = u$

$x = \frac{1}{5}u \rightarrow u = 5x$

$dx = \frac{1}{5}du$

11. $\int \frac{2-3x}{x^2+16}\,dx = \int \frac{2}{x^2+16}\,dx - 3\int \frac{x}{x^2+16}\,dx$

$x^2 = 16u^2$

$x = 4u \rightarrow u = \frac{x}{4}$

$dx = 4du$

$= \int \frac{2(4du)}{16u^2+16} - \frac{3}{2}\int \frac{2x}{x^2+16}\,dx$

$= \frac{1}{2}\int \frac{du}{u^2+1} - \frac{3}{2}\ln(x^2+16)$

$= \frac{1}{2}\tan^{-1}u - \frac{3}{2}\ln(x^2+16) + C$

$= \frac{1}{2}\tan^{-1}\left(\frac{x}{4}\right) - \frac{3}{2}\ln(x^2+16) + C$

12. $A = \int_0^{\frac{\pi}{2}} (\sin x + 1)\,dx = \left[-\cos x + x\right]_0^{\frac{\pi}{2}} = \frac{\pi}{2} + 1$

13. $y^2 = x + 4$ $\quad 8 - x = 2y^2$ $\quad A = \int_{-2}^{2} \left[(8-2y^2) - (y^2-4)\right] dy$

$x = y^2 - 4$ $\quad x = 8 - 2y^2$

left $\quad$ right $\quad A = \int_{-2}^{2} (12 - 3y^2)\,dy$

$A = \left[12y - y^3\right]_{-2}^{2} = 16 + 16 = 32$

14. $y = \frac{4}{x^3}$, $[4, 9]$

$f_{avg.} = \frac{1}{9-4} \int_4^9 \frac{4}{x^3}\,dx = \frac{1}{5} \int_4^9 4x^{-3}\,dx = \frac{1}{5}\left[\frac{-2}{x^2}\right]_4^9$

$f_{avg.} = \frac{1}{5}\left[\frac{-2}{81} + \frac{1}{8}\right] = \frac{13}{648}$

Find $x \rightarrow \frac{4}{x^3} = \frac{13}{648} \rightarrow 13x^3 = 2592 \rightarrow x = \sqrt[3]{\frac{2592}{13}} \approx 5.842$

15.

$V_o = \pi(\sqrt{x})^2 \Delta x = \pi x \Delta x$

$V = \pi \int_0^4 x\,dx = \pi\left[\frac{x^2}{2}\right]_0^4 = 8\pi$

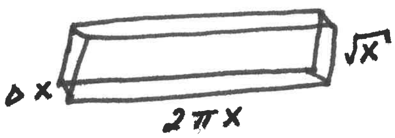
16.

$r_2 = \sqrt{x} + 2$ $\quad V_o = \pi\left[(\sqrt{x}+2)^2 - 2^2\right]\Delta x$

$r_1 = 2$ $\quad V_o = \pi\left[x + 4\sqrt{x}\right]\Delta x$

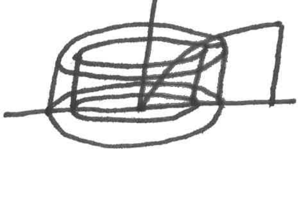
$V = \pi \int_0^4 (x + 4\sqrt{x})\,dx = \pi\left[\frac{x^2}{2} + \frac{8x^{3/2}}{3}\right]_0^4 = \pi\left(8 + \frac{64}{3}\right) = \frac{88\pi}{3}$

17.

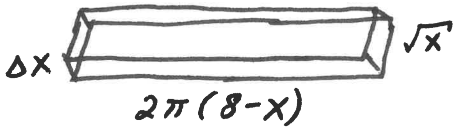


$V_{shell} = 2\pi x \sqrt{x}\,\Delta x = 2\pi x^{3/2} \Delta x$

$V = 2\pi \int_0^4 x^{3/2}\,dx = 2\pi\left[\frac{2}{5}x^{\frac{5}{2}}\right]_0^4 = \frac{128\pi}{5}$



18.



$$V_{shell} = 2\pi(8-x)x^{\frac{1}{2}}\,\Delta x$$
$$= 2\pi\left(8x^{\frac{1}{2}} - x^{3/2}\right)\Delta x$$

$$V = 2\pi\int_0^4 \left(8x^{\frac{1}{2}} - x^{3/2}\right)dx = 2\pi\left[\frac{16}{3}x^{3/2} - \frac{2}{5}x^{\frac{5}{2}}\right]_0^4$$

$$= 2\pi\left(\frac{128}{3} - \frac{64}{5}\right) = \frac{896\pi}{15}$$