5.5 Notes

The Area Function starting at $x=a$: $A(x) = \int_a^x f(t)\,dt$

Please note $A(a) = 0$

1. Find $A(3)$ if $A(x) = \int_3^x f(t)\,dt$

$$A(3) = \int_3^3 f(t)\,dt = F(3) - F(3) = 0$$

Fundamental Theorem Calculus (Part 2): $A'(x) = f(x)$

If $A(x) = \int_a^x f(t)\,d(t)$, then $A'(x) = f(x)$

so $\frac{d}{dx} \int_a^x f(t)\,dt = f(x)$

why? $\rightarrow$ $A(x) = \int_a^x f(t)\,dt = F(x) - F(a)$

then $A'(x) = F'(x) - F'(a) = f(x) - 0 = f(x)$.

2. Find $A'(x)$ if $A(x) = \int_5^x (t^2 - 5)\,dt = \left[\frac{1}{3}t^3 - 5t\right]_5^x$

$$A(x) = \left[\frac{1}{3}x^3 - 5x\right] - \left[\frac{125}{3} - 25\right]$$

$$A'(x) = x^2 - 5$$

3. $H(u) = \int_2^u \sqrt{t^4 - t}\,dt$, Find $H(2)$, $H'(2)$

$H(2) = \int_2^2 \sqrt{t^4 - t}\,dt = 0$, $H'(2) = \sqrt{2^4 - 2} = \sqrt{14}$

4. Find $F(x)$ if $f(x) = \frac{1}{2}x^5$ and $F(5) = 0$

$$F(x) = \int_5^x \frac{1}{2}t^5\,dt = \frac{1}{12}t^6 \Big|_5^x = \frac{x^6}{12} - \frac{5^6}{12}$$

5. $\int_{\frac{\pi}{4}}^x \cos u\,du = \sin u \Big|_{\frac{\pi}{4}}^x = \sin x - \sin\frac{\pi}{4} = \sin x - \frac{\sqrt{2}}{2}$

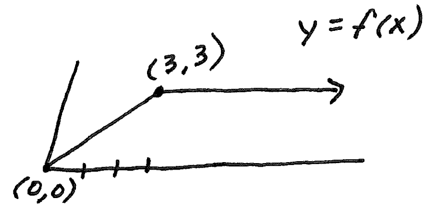
6. $\displaystyle\int_2^{\sqrt{x}} \frac{dt}{t} = \ln|t|\Big|_2^{\sqrt{x}} = \ln\sqrt{x} - \ln 2 = \frac{1}{2}\ln x - \ln 2$

7. Define $F(x)$ in integral form if $f(x) = e^{-x^2}$ and $F(4) = 0$

$$F(x) = \int_4^x e^{-t^2}\,dt$$

8. $\displaystyle\frac{d}{dx}\int_1^x \sin(t^2)\,dt = \sin x^2$

9. Find $A(x)$ if



$$f(x) = \begin{cases} x, & 0 \le x \le 3 \\ 3, & x > 3 \end{cases} \qquad F(x) = A(x) = \int_0^x t\,dt = \frac{1}{2}t^2\Big|_0^x = \frac{1}{2}x^2$$

We know an Area of $A(3) = \frac{1}{2}(3)^2 = \frac{9}{2}$ is accumulated by the point when $x = 3$

when $x > 3 \rightarrow F(x) = \int 3\,dx = 3x + C$

but we need $F(3) = \frac{9}{2} = 3(3) + C \rightarrow C = -\frac{9}{2}$

so $F(x) = A(x) = \begin{cases} \frac{1}{2}x^2, & 0 \le x \le 3 \\ 3x - \frac{9}{2}, & x > 3 \end{cases}$

Chain Rule $\rightarrow$ If $G(x) = A(g(x))$, then $G'(x) = A'(g(x)) \cdot g'(x)$

10. $\displaystyle\frac{d}{dx}\int_1^{\frac{1}{x}} \sin(t^2)\,dt = \sin\left(\frac{1}{x}\right)^2 \cdot \frac{d}{dx}\left(\frac{1}{x}\right) = \frac{-\sin\left(\frac{1}{x}\right)^2}{x^2}$

11. $\displaystyle\frac{d}{dx}\int_{x^3}^0 \sin^2 t\,dt = -\sin^2(x^3)\cdot 3x^2 = -3x^2\sin^2(x^3)$

12. $\displaystyle\frac{d}{dx}\int_{x^2}^{x^4} \sqrt{t}\,dt = \sqrt{x^4}\,(4x^3) - \sqrt{x^2}\,(2x) = 4x^5 - 2x^2$

13. $\displaystyle\frac{d}{du}\int_{-u}^{3u+9} \sqrt{x^2+1}\,dx = \sqrt{(3u+9)^2+1}\,(3) - \sqrt{(-u)^2+1}\,(-1)$

$$= 3\sqrt{9u^2+54u+82} + \sqrt{u^2+1}$$