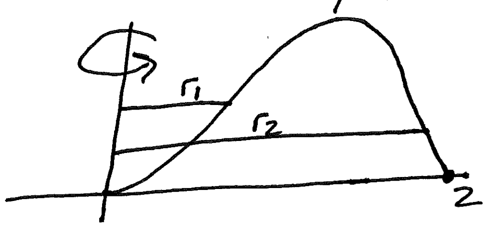
Think of why the slicing method of 6.3 would fail in the following problem.

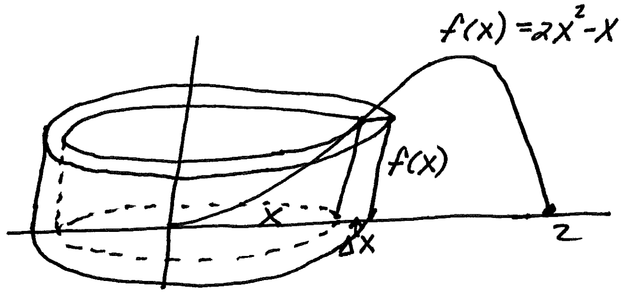
1. Rotate region bounded by $y = 2x^2 - x^3$ and the x-axis about the y-axis. Find resulting solid

Slicing gives...

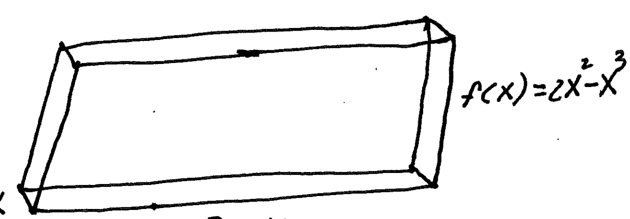


$y = 2x^2 - x^3 \rightarrow$ can't solve for $x = f(y)$ $\therefore$ can't use slicing (washer) method.

So... We use cylindrical shells



pull apart any arbitrary shell and...



$2\pi x$

$C = 2\pi r$

$r = x$ (in this case)

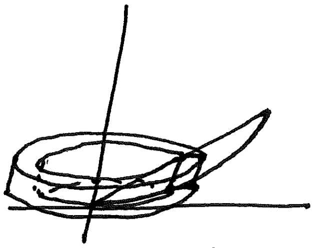
$$V_{shell} = 2\pi x (2x^2 - x^3)\,\Delta x$$

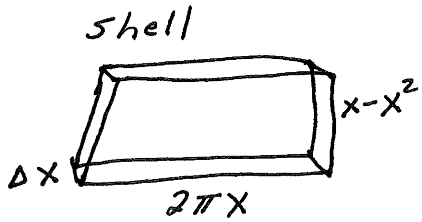
$$V = 2\pi \int_0^2 x(2x^2 - x^3)\,dx = 2\pi \int_0^2 (2x^3 - x^4)\,dx = 2\pi \left[\frac{x^4}{2} - \frac{x^5}{5} \right]_0^2$$

$$V = 2\pi \left(8 - \frac{32}{5}\right) = 2\pi \left(\frac{8}{5}\right) = \frac{16\pi}{5}$$

2. Rotate region enclosed by $y=x$ and $y=x^2$ about y-axis.

$$V_{shell} = 2\pi x(x-x^2)\Delta x$$

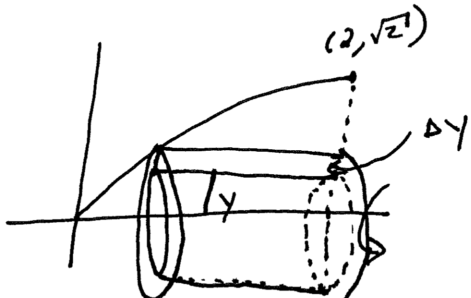


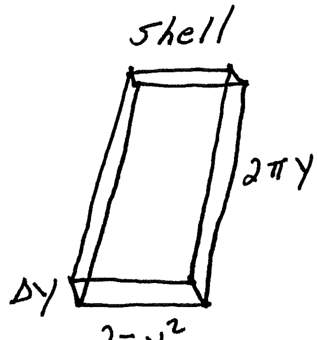


$$V = 2\pi\int_0^1 (x^2-x^3)\,dx = 2\pi\left[\frac{x^3}{3}-\frac{x^4}{4}\right]_0^1 = 2\pi\left(\frac{1}{3}-\frac{1}{4}\right) = \frac{\pi}{6}$$

3. $y=\sqrt{x}$, $x=[0,2]$ about x-axis

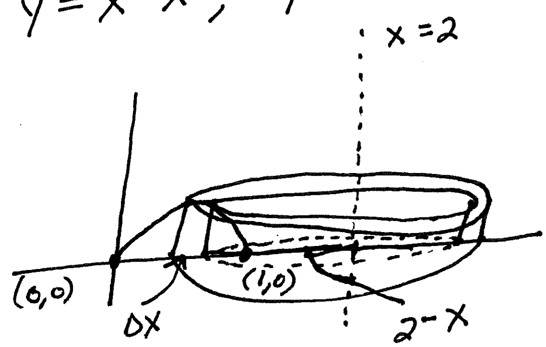
$$V_{shell} = 2\pi y(2-y^2)\Delta y = 2\pi(2y-y^3)\Delta y$$

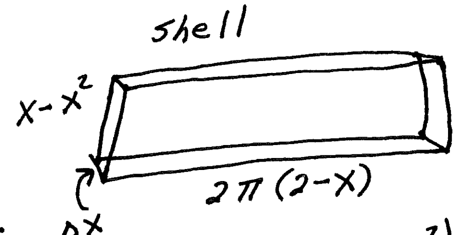




$$V = 2\pi\int_0^{\sqrt{2}} (2y-y^3)\,dy = 2\pi\left[y^2-\frac{y^4}{4}\right]_0^{\sqrt{2}} = 2\pi(2-1) = 2\pi$$

4. $y=x-x^2$, $y=0$ about $x=2$

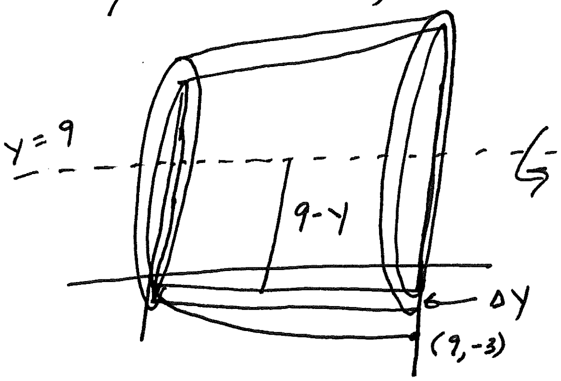


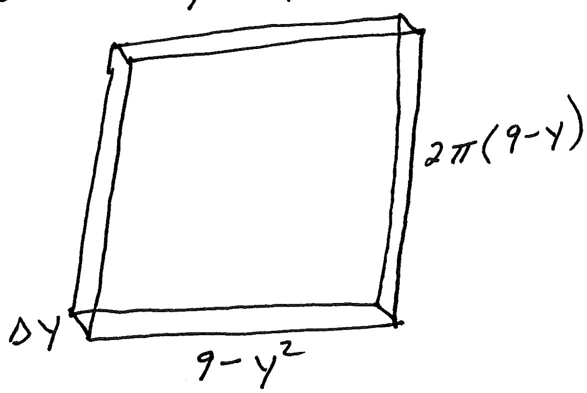


$$V_{shell} = 2\pi(2-x)(x-x^2)\Delta x = 2\pi(2x-2x^2-x^2+x^3)\Delta x$$

$$V = 2\pi\int_0^1 (2x-3x^2+x^3)\,dx = 2\pi\left[x^2-x^3+\frac{x^4}{4}\right]_0^1 = 2\pi\left(\frac{1}{4}\right) = \frac{\pi}{2}$$

5. $y = -\sqrt{x}$, $x = 9$ about $y = 9$





$$V_{shell} = 2\pi(9-y)(9-y^2)\,\Delta y = 2\pi\left(81-9y^2-9y+y^3\right)\Delta y$$

$$V = 2\pi\int_{-3}^{0}\left(81-9y^2-9y+y^3\right)dy = 2\pi\left[81y-3y^3-\frac{9y^2}{2}+\frac{y^4}{4}\right]_{-3}^{0}$$

$$V = 2\pi\left[0-\left(-243+81-\frac{81}{2}+\frac{81}{4}\right)\right] = 2\pi\left(\frac{729}{4}\right) = \frac{729\pi}{2}$$