

ECE 255

Introduction to Digital Logic Design

$\checkmark AB + \bar{A}B$
 $\checkmark A + \bar{A}B$
 $\times A(\underline{A+B})\bar{A}B$

Not a
single variable

Sum of Products & Product of Sums

OR

AND

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Outline

- Review of number conversion
- Boolean expressions in VHDL
- Sum of Products (SOP) Expressions
- Canonical Sum of Products (CSOP) – Formulating Boolean Functions
- Canonical Product of Sums
- Logic Gates and Schematics

Converting ^{hex}Base-16 to ^{decimal}Base-10

- Integer b serves as the base of a number system (e.g. $b=16$ for base-16)
- Converting base- b number to decimal
 - Each base- b digit is represented first as its decimal equivalent (base-16 digit A represented as 10 for conversion, digit E represented as 14)
 - Each digit for each position p is multiplied by b^p (A, represented by 10, in position 2 is multiplied by 16^2 or 256)
 - Results for all positions are summed together to reveal the decimal number
- Example: Convert $(2AF)_{16}$ to decimal (base-10)

$$\begin{aligned}d_0 &= \overset{\text{hex}}{F} = 15 \\d_1 &= A = 10 \\d_2 &= 2 = 2\end{aligned}$$

$$\begin{aligned}D &= 2 \cdot 16^2 + 10 \cdot 16^1 + 15 \cdot 16^0 \\&= \sum_p d_p \cdot b^p \\&= 512 + 160 + 15 \\&= (687)_{10}\end{aligned}$$

Converting Base-16 to Base-10

- Example: Convert $(F0D)_{16}$ to decimal (base-10)

$$\begin{aligned} D &= 15 \cdot 16^2 + 0 \cdot 16^1 + 13 \cdot 16^0 \\ &= 3840 + 13 \\ &= (3853)_{10} \end{aligned}$$

Boolean Functions in VHDL

Very High Speed Integrated Circuit

FPGA

- VHDL = VHSIC Hardware Description Language
- We'll introduce you to VHDL along the way and your labs will use VHDL
- For basic logic operators in VHDL:
 - Keyword “AND” represents the AND operator – type it out, not \cdot
 - Keyword “OR” represents the OR operator – type it out, not $+$
 - Keyword “NOT” for complement or NOT operator – type it out, not $'$ or $-$
 - Keyword “XOR” for the exclusive-OR operator
- The VHDL assignment operator is \leq used to assign right side to left side
- Examples of use:

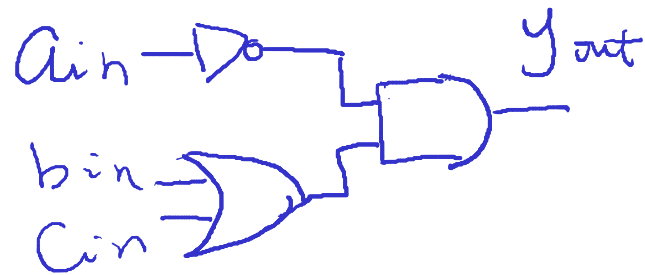
concurrent $\left\{ \begin{array}{ll} y \leq a \text{ AND } b; & \text{-- implements function } y(a,b) = a \cdot b \\ y \leq a \text{ OR } b; & \text{-- implements function } y(a,b) = a + b \\ y \leq \text{NOT}(a); & \text{-- implements function } y(a,b) = a' = \bar{a} \end{array} \right.$

Boolean Functions in VHDL – Notes & Examples

VHDL $\leftarrow y_{out} \leftarrow (\text{NOT } a_{in}) \text{ AND } (b_{in} \text{ OR } c_{in})$

Logic exp. $\leftarrow y_{out} = \bar{a}_{in} \cdot (b_{in} + c_{in})$

\leftarrow assign — value of right drives y_{out} as output



; — end expression

-- Comments

Minterms and Products

- Recall: AND ^{is} a function of variables the result of which is TRUE (logic 1) only when *all* input variables are TRUE (logic 1)
- A minterm is an AND (a.k.a. “product”) in which each variable appears once in the term, in either complemented or uncomplemented form
- A minterm m is 1 for exactly one combination of variables and 0 for all others
- For a function of n variables, how many minterms?
- Let's write all possible minterms for two variables A, B

Minterms and Products – Truth Tables

- Minterms are AND expressions in a function that yield logic 1 (or TRUE) in the output of that function
- Possible minterms are based on any combination of variables
(2^n possible minterms for n variable function)

← given

	A	B	f
$m_0 \bar{A} \cdot \bar{B}$	0	0	1
$m_1 \bar{A} \cdot B$	0	1	0
$m_2 A \cdot \bar{B}$	1	0	1
$m_3 A \cdot B$	1	1	0

$\rightarrow A=0 \ \& \ B=0$
 $\rightarrow A=1 \ \& \ B=0$

← Canonical SOP

$$f(A, B) = \bar{A} \cdot \bar{B} + A \cdot \bar{B}$$

$$\bar{A} \cdot \bar{B} = 1 \quad \text{iff} \quad A=0 \ \& \ B=0$$

$$\bar{A} \cdot B = 1 \quad \text{iff} \quad A=0 \ \& \ B=1$$

→ minimal?

$$\begin{aligned}
 f(A, B) &= (\bar{A} + A) \cdot \bar{B} \\
 &= \bar{B} \quad \leftarrow \text{minimal SOP}
 \end{aligned}$$