ECE 255
Introduction to Digital Logic Design

$\checkmark\ AB + \bar{A}B$

$\checkmark\ A + \bar{A}B$

$\times\ A\underbrace{(A+B)}_{\text{Not a single variable}} + \bar{A}B$

$\underbrace{\text{Sum}}_{\text{OR}}$ of $\underbrace{\text{Products}}_{\text{AND}}$ & Product of Sums

$(A+B)(C+D)\ \checkmark$

$ABC(D+E)\ \checkmark$

$(A+B)(C+D)+EF\ \times$

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Minterms and Products

- Recall: AND ~~if~~ is a function of variables the result of which is TRUE (logic 1) only when *all* input variables are TRUE (logic 1)
- A ***minterm*** is an AND (a.k.a. "product") in which ***each variable appears once*** in the term, in either complemented or uncomplemented form
- A minterm *m* is 1 for exactly one combination of variables and 0 for all others

Look for $f = 1$

- For a function of *n* variables, how many minterms? 2^n
- Let's write all possible minterms for two variables *A*, *B*

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Minterms and Products – Truth Tables

- Minterms are AND expressions in a function that yield logic 1 (or TRUE) in the output of that function
- Possible minterms are based on any combination of variables (2^n possible minterms for n variable function)

Canonical SOP

given

	A	B	f
$m_0\ \bar{A}\cdot\bar{B}$	0	0	1
$m_1\ \bar{A}\cdot B$	0	1	0
$m_2\ A\cdot\bar{B}$	1	0	1
$m_3\ A\cdot B$	1	1	0

$A=0$ & $B=0$

$A=1$ & $B=0$

$$f(A,B)=\bar{A}\cdot\bar{B}+A\cdot\bar{B}$$

$\bar{A}\cdot\bar{B}=1$ iff $A=0$ & $B=0$

$\bar{A}\cdot B=1$ iff ~~$A=0$~~ & $B=1$

minimal?

$$f(A,B)=(\bar{A}+A)\cdot\bar{B}$$
$$=\bar{B}$$

minimal SOP

Sum of Products Expressions

- A *sum of products (SOP)* expression for a function *f* is an OR of ANDs
  - AND for “products”, OR for “sums”
  - Each AND/product not necessarily a minterm
- A *canonical sum of products (CSOP)* expression is an *OR of minterms* and is unique for a given function
- The $\sum m_i$ notation is convenient, concise notation ($0 \leq i \leq 2^n - 1$ for n variables)

$$f = \underbrace{\bar{A}\bar{B}}_{m_0} + \underbrace{A\bar{B}}_{m_2} \leftarrow \text{CSOP}$$

$$f = \Sigma(m_0 + m_2) \leftarrow$$

$$= \Sigma(0, 2)$$

		A	B	f
m_0	$\bar{A}\bar{B}$	0	0	
m_1	$\bar{A}B$	0	1	
m_2	$A\bar{B}$	1	0	
m_3	AB	1	1	

Maxterms and Sums

- <u>Recall</u>: OR ~~if~~ is a function of variables the result of which is TRUE (logic 1) when *any* input variables are TRUE (logic 1)
- A *<u>Maxterm</u>* is an OR (a.k.a. “sum”) in which *<u>each variable appears once</u>* in the term, in either complemented or uncomplemented form
- A Maxterm *M* is 0 for exactly one combination of variables and 1 for all others

  Look for $f=0$

- For a function of *n* variables, how many Maxterms? 2^n
- Let’s write all possible *Maxterms* for two variables *A*, *B*

$$(\bar{A}+B)(A+B)(\bar{A}+\bar{B}) \leftarrow \text{CPOS} \checkmark$$

$$\underline{A}(A+B) \leftarrow \text{CPOS} \times$$

Product of Sums Expressions

DeMorgan's Laws:

$\overline{X+Y} = \bar{X} \cdot \bar{Y}$ 1

$\overline{XY} = \bar{X} + \bar{Y}$ 2

- A *product of sums (POS)* expression for a function *f* is an AND of ORs
  - AND for "products", OR for "sums"
  - Each OR/sum not necessarily a Maxterm

$\overline{\bar{A}\bar{B}} = \bar{\bar{A}} + \bar{\bar{B}} = \boxed{A+B}$

- A *canonical product of sums (CPOS)* expression is an *AND of Maxterms* and is unique for a given function
- The $\prod M_i$ notation is convenient, concise notation ($0 \le i \le 2^n - 1$ for n variables)

minterm: $\bar{A}\bar{B} = 1$ iff $A=0$ & $B=0$

Maxterm: $A+B=0$ iff $A=0$ & $B=0$

$A+\bar{B}=0$ iff $A=0$ & $B=1$

Maxterm	minterm	A	B	f
M_0 $A+B$	m_0 $\bar{A}\bar{B}$	0	0	1
M_1 $A+\bar{B}$	m_1 $\bar{A}B$	0	1	0
M_2 $\bar{A}+B$	m_2 $A\bar{B}$	1	0	1
M_3 $\bar{A}+\bar{B}$	m_3 AB	1	1	0

CSOP: $f = \bar{A}\bar{B} + A\bar{B} = \Sigma(m_0, m_2)$

CPOS: $f = (A+\bar{B})(\bar{A}+\bar{B}) = \prod(M_1, M$

CPOS and CSOP
Same Function, Different Representations

- Let $f() = \sum m_i = \prod M_j$ be the CSOP and CPOS for function f, respectively
- Here i is the index for a minterm m_i of f if and only if i is <u>not</u> the index of a Maxterm M_i of f

CPOS: $f = (A+\bar{B})(\bar{A}+\bar{B})$

$$= \cancel{A\bar{A}}^{\,0} + A\bar{B} + \bar{A}\bar{B} + \cancel{\bar{B}\bar{B}}\ \bar{B}$$

$$= \bar{B}(\underbrace{\cancel{A+\bar{A}}}_{1}+1)$$

$$= \bar{B}$$

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