ECE 255 Workbook.

Min terms = 1

Max terms = 0

ECE255 – Introduction to Digital Logic Design
Homework Assignment 2
Due September 98

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NOTE: Some questions require drawing schematics. You can use Logisim Evolution for drawing a schematic and then take a screenshot. Please see the Logisim Tutorial on Canvas as needed.

Let $ABCD_2$ be a 4-bit non-negative integer with corresponding decimal values:

$0000_2 = 0_{10}$
$0001_2 = 1_{10}$
$0010_2 = 2_{10}$
$0011_2 = 3_{10}$
…
$1111_2 = 15_{10}$

The digits *A*, *B*, *C*, and *D* in this 4-bit integer are also to be considered variables. Consider two functions of these 4 variables $f(A, B, C, D)$ and $g(A, B, C, D)$.

1. The function $f(A, B, C, D)$ is defined as:

$$f = \begin{cases} 1 & \text{if the hex value of } ABCD_2 \text{ is less than } A_{16} \\ 0 & \text{otherwise} \end{cases}$$

We have 4^n calculations
outputs true = 1
false = 0

Describe *f* in the following forms listed below:

(a) **Truth Table**
(b) **CSOP** in the concise $\sum m_i$ **notation**
(c) **CPOS** in the concise $\prod M_i$ **notation**

When 0

when 1

CSOP in Σm_i notation

Ⓐ Truth Table 16 bit

input   output.

	A	B	C	D	f	G
m_0	0	0	0	0	1	0
m_1	0	0	0	1	1	0
m_2	0	0	1	0	1	0
m_3	0	0	1	1	1	0
m_4	0	1	0	0	1	0
m_5	0	1	0	1	1	0
m_6	0	1	1	0	1	0
m_7	0	1	1	1	1	0
m_8	1	0	0	0	1	0
m_9	1	0	0	1	1	0
m_{10}	1	0	1	0	0	1
m_{11}	1	0	1	1	0	1
m_{12}	1	1	0	0	0	1
m_{13}	1	1	0	1	0	1
m_{14}	1	1	1	0	0	1
m_{15}	1	1	1	1	0	1

1 if hex of $ABCD_2$ is less than A_{16}, if >, then it's 0

A_{16} is 1010_2 so everything after is 0 but all before are 1.

Ⓑ CPOS $\Sigma \Pi_{m_i}$ notation

$= \Pi(m_0, m_1\ m_2\ m_3\ m_4\ m_5\ m_6\ m_7\ m_8\ m_9)$

Ⓒ CSOP ΣM_i notation

$= \Sigma(m_{10}\ m_{11}\ m_{12}\ m_{13}\ m_{14}\ m_{16})$

2. The function $g(A, B, C, D)$ is defined as:

$$g=\begin{cases}0 & if\ f=1\\ 1 & if\ f=0\end{cases}$$

input / output

	A	B	C	D	f	G
m_0	0	0	0	0	1	0
m_1	0	0	0	1	1	0
m_2	0	0	1	0	1	0
m_3	0	0	1	1	1	0
m_4	0	1	0	0	1	0
m_5	0	1	0	1	1	0
m_6	0	1	1	0	1	0
m_7	0	1	1	1	1	0
m_8	1	0	0	0	1	0
m_9	1	0	0	1	1	0
m_{10}	1	0	1	0	0	1
m_{11}	1	0	1	1	0	1
m_{12}	1	1	0	0	0	1
m_{13}	1	1	0	1	0	1
m_{14}	1	1	1	0	0	1
m_{15}	1	1	1	1	0	1

Describe g in the following forms listed below:

(a) **CSOP** in the concise **$\sum m_i$ notation**
(b) **CPOS** in the concise **$\prod M_i$ notation**

(A) CSOP in Σm_i notation

$\Pi(m_{10} m_{11} m_{12} m_{13} m_{14} m_{15})$

(B) CPOS

$\Sigma(m_0 M_1 m_2 m_3 m_4 m_5 M_6 m_7 m_8 m_9)$

3. Express the function $f(A, B, C, D)$ in the following forms:

$X + X' = 1$

$X \cdot X' = 0$

(a) A **Boolean Algebra expression of the CSOP** function $f(A, B, C, D)$
(b) A **minimized Boolean expression** for $f(A, B, C, D)$
Use the properties and axioms of Boolean Algebra to minimize $f(A, B, C, D)$

input A	B	C	D	output f	G	terms: min term	CSOP
0	0	0	0	1	0	$A\cdot B\cdot C\cdot D\ m_0$	$A\cdot B\cdot C\cdot D\ m_0$
0	0	0	1	1	0	$A\cdot B\cdot C\cdot \bar{D}\ m_1$	$A\cdot B\cdot C\cdot D\ m_1$
0	0	1	0	1	0	$A\cdot B\cdot \bar{C}\cdot D\ m_2$	$A\cdot B\cdot C\cdot D\ m_2$
0	0	1	1	1	0	$A\cdot B\cdot \bar{C}\cdot \bar{D}\ m_3$	$A\cdot B\cdot C\cdot D\ m_3$
0	1	0	0	1	0	$A\cdot \bar{B}\cdot C\cdot D\ m_4$	$A\cdot B\cdot C\cdot D\ m_4$
0	1	0	1	1	0	$A\cdot \bar{B}\cdot C\cdot \bar{D}\ m_5$	$A\cdot B\cdot C\cdot D\ m_5$
0	1	1	0	1	0	$A\cdot \bar{B}\cdot \bar{C}\cdot D\ m_6$	$A\cdot B\cdot C\cdot D\ m_6$
0	1	1	1	1	0	$A\cdot \bar{B}\cdot \bar{C}\cdot \bar{D}\ m_7$	$A\cdot B\cdot C\cdot D\ m_7$
1	0	0	0	1	0	$\bar{A}\cdot B\cdot C\cdot D\ m_8$	$\bar{A}\cdot B\cdot C\cdot D\ m_8$
1	0	0	1	1	0	$\bar{A}\cdot B\cdot C\cdot \bar{D}\ m_9$	$A\cdot B\cdot C\cdot D\ m_9$
1	0	1	0	0	1	$\bar{A}\cdot B\cdot \bar{C}\cdot D\ m_{10}$	
1	0	1	1	0	1	$\bar{A}\cdot B\cdot \bar{C}\cdot \bar{D}\ m_{11}$	
1	1	0	0	0	1	$\bar{A}\cdot \bar{B}\cdot C\cdot D\ m_{12}$	
1	1	0	1	0	1	$\bar{A}\cdot \bar{B}\cdot C\cdot \bar{D}\ m_{13}$	
1	1	1	0	0	1	$\bar{A}\cdot \bar{B}\cdot \bar{C}\cdot D\ m_{14}$	
1	1	1	1			$\bar{A}\cdot \bar{B}\cdot \bar{C}\cdot \bar{D}\ m_{15}$	

what we calculated

can't be a 3 term product.

$f = (\bar{A}\bar{B}\bar{C}\bar{D}) + (\bar{A}\bar{B}\bar{C}D) + (\bar{A}\bar{B}C\bar{D}) + (\bar{A}\bar{B}CD) + (\bar{A}B\bar{C}\bar{D}) + (\bar{A}B\bar{C}D) + (\bar{A}BC\bar{D}) + (\bar{A}BCD) + (A\bar{B}\bar{C}\bar{D}) + (A\bar{B}\bar{C}D)$

$= \bar{A}\bar{B}\bar{C}(\bar{D}+D) + \bar{A}\bar{B}C(\bar{D}+D)$

$= \bar{A}B\bar{C}(\bar{D}+D) + \bar{A}BC(\bar{D}+D)$

$+ A\bar{B}\bar{C}(\bar{D}+D)$

$= \bar{A}\bar{B}\bar{C} + \bar{A}\bar{B}C + \bar{A}B\bar{C} + \bar{A}BC + A\bar{B}\bar{C}$

$= \bar{A}\bar{B}(\bar{C}+C) + \bar{A}B(\bar{C}+C) + A\bar{B}\bar{C}$

$= \bar{A}\bar{B} + \bar{A}B + A\bar{B}\bar{C}$

$= \bar{A}(\bar{B}+B) + A\bar{B}\bar{C}$

$= \bar{A} + A\bar{B}\bar{C}$

$\boxed{= \bar{A} + \bar{B}\bar{C}}$

5 gates 3 not, 1 or, 1 and.

4. Express ***f*(*A*, *B*, *C*, *D*) as a digital logic schematic**, clearly showing AND, NOT, and OR gates used and how they are connected. **Draw by hand or use Logisim to place and connect gates**.

$f(A,B,C,D) = \bar{A} + \bar{B}\bar{C}$

3 not inputs feed

We can try 3 or inputs into inverters to set to 0
and test BC to a NAND gate

Keep one as 1 input
test BC 1,1 into NAND evaluates 0
then feed 1 and 0 into OR to
A BC
get 1+1 = 0