10.4 Notes

Absolute Convergence: $\sum a_n$ absolutely converges if $\sum |a_n|$ converges.

Conditional Convergence: If $\sum a_n$ converges but $\sum |a_n|$ diverges, then $\sum a_n$ is conditionally convergent.

Leibniz Test (also called Alternating Series Test): $\sum (-1)^n a_n$ converges if $\lim_{n\to\infty} a_n = 0$.

1. Determine convergence of $\sum_{n=1}^{\infty} (-1)^{n-1} \frac{1}{n^{2/3}}$

Alternating Series so... $\rightarrow$ $\lim_{n\to\infty} \frac{1}{n^{2/3}} = 0$ $\therefore \sum_{n=1}^{\infty} (-1)^{n-1} \frac{1}{n^{2/3}}$ converges

However, $\sum_{n=1}^{\infty} \left| (-1)^{n-1} \frac{1}{n^{2/3}} \right| = \sum_{n=1}^{\infty} \frac{1}{n^{2/3}}$ is a divergent p-series ($p = \frac{2}{3} \leq 1$)

$\therefore \sum_{n=1}^{\infty} (-1)^{n-1} \frac{1}{n^{2/3}}$ converges conditionally.

2. Determine convergence of $\sum_{n=1}^{\infty} \frac{(-1)^n n^4}{n^3+1}$

Alternating series $\longrightarrow$ $\lim_{n\to\infty} \frac{n^4}{n^3+1} = \infty \neq 0$

Since $\lim_{n\to\infty} \frac{n^4}{n^3+1} \neq 0$, $\sum_{n=1}^{\infty} \frac{(-1)^n n^4}{n^3+1}$ diverges by the Alternating Series Test.

(could also use Divergence Test)

3. Determine convergence of $\sum_{n=1}^{\infty} \frac{\sin\left(\frac{\pi n}{4}\right)}{n^2}$

Consider $\sum |a_n|$. $\left| \frac{\sin\left(\frac{\pi}{4}\right)}{n^2} \right| \leq \frac{1}{n^2}$

We know $\sum \frac{1}{n^2}$ is a convergent p-series $(p = 2 > 1)$.

Since $\left| \frac{\sin\left(\frac{\pi}{4}\right)}{n^2} \right| \leq \frac{1}{n^2}$, $\sum_{n=1}^{\infty} \left| \frac{\sin\left(\frac{\pi}{4}\right)}{n^2} \right|$ converges by the comparison test.

This shows that $\sum_{n=1}^{\infty} \frac{\sin\left(\frac{\pi}{4}\right)}{n^2}$ converges absolutely.