5.3 Integration Problems and Applications

Basic Rules you'll need to know:

$\int x^n dx = \frac{x^{n+1}}{n+1} + C \;\; for\, n \neq -1$ $\qquad \int \frac{1}{x} dx = \ln|x| + C$ (why the absolute value…let's discuss ☺)

$\int \cos(kx+b)dx = \frac{1}{k}\sin(kx+b) + C$ $\qquad \int \sin(kx+b)dx = -\frac{1}{k}\cos(kx+b) + C$ $\qquad \int e^{kx+b}dx = \frac{1}{k}e^{kx+b} + C$

You try… $\int \sec^2 x\, dx$ $\qquad \int \frac{1}{1+x^2}dx$ $\qquad \int \frac{1}{\sqrt{1-x^2}}dx$

Now let's work these together…

1. $\int \frac{8}{s^4} ds$ $\qquad$ 2. $\int \left(5x^3 - x^{-2} - x^{\frac{-3}{5}}\right)dx$ $\qquad$ 3. $\int \frac{1}{\sqrt{x}} dx$ $\qquad$ 4. $\int (4t-9)^{-3}\, dt$

5. $\int \frac{x^2+2x-3}{x^4} dx$ $\qquad$ 6. $\int \sin 9x\, dx$ $\qquad$ 7. $\int (4\theta + \cos 8\theta)\, d\theta$ $\qquad$ 8. $\int \left(2x + e^{14-2x}\right) dx$

9. $\int \left(x + x^{-1}\right)\left(3x^2 - 5x\right) dx$ $\qquad$ 10. $\int \sec(3x)\tan(3x)\, dx$ $\qquad$ 11. $\int \csc(5z)\cot(5z)\, dz$

12. Solve the differential equation with the given initial condition: $\frac{dy}{dx} = \sec^2 3x,\; y\left(\frac{\pi}{4}\right) = 2$

13. Solve the differential equation with the given initial condition: $f''(t) = t - \cos t,\; f'(0) = 2,\; f(0) = -2$

14. A ball is thrown upward with a speed of 48 ft/s from the edge of a cliff 432 ft above the ground. A second later another ball is thrown upward with a speed of 24 ft/s. Do the balls ever pass each other?

15. A stone was dropped off a cliff and hit the ground with a speed of 120ft/s. What is the height of the cliff?

16. A car is traveling at 50 mi/h when the brakes are fully applied, producing a constant deceleration of 22 ft/s^2. What is the distance traveled before the car comes to a stop?

17. A car braked with a constant deceleration of 16 ft/s^2, produced skid marks measuring 200 ft before coming to a stop. How fast was the car traveling when the brakes were first applied?

5.3

Indefinite Integrals

1. $\int \frac{8}{s^4}\,ds = \int 8s^{-4}\,ds = \frac{8s^{-3}}{-3} + C = \frac{-8}{3s^3} + C$

2. $\int (5x^3 - x^{-2} - x^{-3/5})\,dx = \frac{5x^4}{4} - \frac{x^{-1}}{-1} - \frac{x^{2/5}}{2/5} + C$

$= \frac{5x^4}{4} + \frac{1}{x} - \frac{5x^{2/5}}{2} + C$

3. $\int \frac{1}{\sqrt{x}}\,dx = \int x^{-1/2}\,dx = \frac{x^{\frac{1}{2}}}{\frac{1}{2}} + C = 2\sqrt{x} + C$

4. $\int (4t-9)^{-3}\,dt = \frac{(4t-9)^{-2}}{(-2)(4)} + C = \frac{1}{8(4t-9)^2} + C$

5. $\int \frac{x^2+2x-3}{x^4}\,dx = \int (x^{-2} + 2x^{-3} - 3x^{-4})\,dx$

$= \frac{x^{-1}}{-1} + \frac{2x^{-2}}{-2} - \frac{3x^{-3}}{-3} + C = \frac{-1}{x} - \frac{1}{x^2} + \frac{1}{x^3} + C$

6. $\int \sin 9x\,dx = \frac{-\cos 9x}{9} + C = -\frac{1}{9}\cos 9x + C$

7. $\int (4\theta + \cos 8\theta)\,d\theta = 2\theta^2 + \frac{\sin 8\theta}{8} + C$

8. $\int (2x + e^{14-2x})\,dx = x^2 + \frac{e^{14-2x}}{-2} + C = x^2 - \frac{1}{2}e^{14-2x} + C$

9. $\int (x + x^{-1})(3x^2 - 5x)\,dx = \int (3x^3 - 5x^2 + 3x - 5)\,dx$

$= \frac{3x^4}{4} - \frac{5x^3}{3} + \frac{3x^2}{2} - 5x + C$

10. $\int \sec(3x)\tan(3x)\,dx = \frac{1}{3}\sec 3x + C$

11. $\int \csc(5z)\cot(5z)\,dz = \frac{-1}{5}\csc(5z) + C$

12. $\frac{dy}{dx} = \sec^2 3x \rightarrow y = \frac{1}{3}\tan 3x + C$

$y(\frac{\pi}{4}) = \frac{1}{3}\tan(\frac{3\pi}{4}) + C = 2$

$\frac{1}{3}(-1) + C = 2$

$C = \frac{7}{3}$

so $y = \frac{1}{3}\tan 3x + \frac{7}{3}$

13. $f''(t) = t - \cos t, \quad f'(0) = 2, \quad f(0) = -2$

$f'(t) = \frac{1}{2}t^2 - \sin t + C \rightarrow f'(0) = 0 + C = 2 \rightarrow C = 2$

$f'(t) = \frac{1}{2}t^2 - \sin t + 2$

$f(t) = \frac{1}{6}t^3 + \cos t + 2t + C \rightarrow f(0) = 1 + C = -2 \rightarrow C = -3$

$f(t) = \frac{1}{6}t^3 + \cos t + 2t - 3$

14. Ball 1

$a_1(t) = -32$

$v_1(t) = -32t + C$

$v_1(0) = 48 \rightarrow C = 48$

$v_1(t) = -32t + 48$

$s_1(0)$

$s_1(t) = -16t^2 + 48t + 432$

Ball 2

$a_2(t) = -32$

$v_2(t) = -32t + C$

$v_2(1) = 24 \rightarrow -32 + C = 24$

$C = 56$

$v_2(t) = -32t + 56$

$s_2(t) = -16t^2 + 56t + C$

$s_2(1) = 432 \rightarrow -16 + 56 + C = 432$

$C = 392$

$s_2(t) = -16t^2 + 56t + 392$

$s_1(t) \overset{?}{=} s_2(t)$

$-16t^2 + 48t + 432 = -16 + 56t + 392$

$40 = 8t$

$t = 5$

Balls cross paths 5 seconds after first ball is released.

15. $a(t) = -32$

$v(t) = -32t \rightarrow \quad -32t = -120 \rightarrow t = 3.75s$ (time of impact)

$s(t) = -16t^2 + s_0$

$s(3.75) = -16(3.75)^2 + s_0 \rightarrow -225 + s_0 = 0 \rightarrow \boxed{s_0 = 225 \text{ ft.}}$

16. $a(t) = -22 \text{ ft/s}^2$ $\quad V_0 = 50 \frac{mi}{hr}\left(\frac{hr}{3600s}\right)\left(\frac{5280}{mi}\right) = \frac{220}{3} \text{ ft/s}$

$v(t) = -22t + \frac{220}{3} \rightarrow$ car stops when $v(t) = 0$

$-22t + \frac{220}{3} = 0$

$t = \frac{220}{3(22)} = \frac{10}{3} s$

let $s_0 = 0$

$s(t) = -11t^2 + \frac{220}{3}t$

$s\left(\frac{10}{3}\right) = -11\left(\frac{10}{3}\right)^2 + \frac{220}{3}\left(\frac{10}{3}\right) = \boxed{122.\overline{2} \text{ ft.}}$

17. $a(t) = -16$

$V_0 = ?$

$V(t) = -16t + V_0$

car stops when $v(t) = 0$

$-16t + V_0 = 0$

$V_0 = +16t$

$t = \frac{V_0}{16}$

$s(t) = -8t^2 + V_0 t + 0$

$s\left(\frac{V_0}{16}\right) = 200$

$-8\left(\frac{V_0}{16}\right)^2 + V_0\left(\frac{V_0}{16}\right) = 200$

$\frac{-1}{32}V_0^2 + \frac{1}{16}V_0^2 = 200$

$\frac{1}{32}V_0^2 = 200$

$V_0^2 = 6400$

$V_0 = 80 \text{ ft/s}$

$V_0 = 80 \frac{ft}{s}\left(\frac{1 \text{ mile}}{5280 ft}\right)\left(\frac{3600s}{hr.}\right)$

$= 54.\overline{54} \text{ mph}$