Department of Mathematics – University of Tennessee

Math 131 Calculus 1A

Test 2

Name: _____

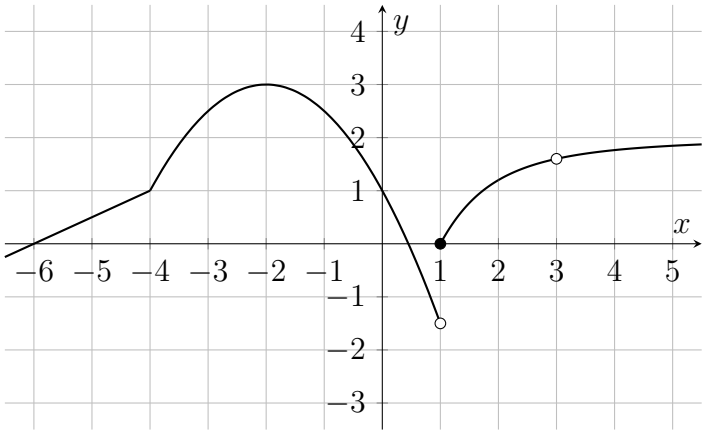
Time allowed: 50 minutes

Instructions:

- Calculators are not allowed.
- All electronic devices must be put away.
- Answers with insufficient or incorrect working will not receive full credit.
- Simplify answers whenever possible.

Page	Points	Score
2	10	
3	10	
4	10	
5	10	
6	10	
Total:	50	

1. Use the graph of $y = f(x)$ below to answer the following questions.



(a) (1 point) At which x-value(s) does f have a jump discontinuity?

(b) (2 points) Write an equation for the horizontal asymptote to $y = f(x)$.

(c) (1 point) At which x-value(s) is f continuous but not differentiable?

(d) (2 points) On what open interval(s) is $f'(x) < 0$?

(e) (2 points) Estimate the value of $f'(-5)$.

(f) (2 points) Is the value of $f'(2)$ or $f'(4)$ larger? Explain in a complete sentence.

2. (4 points) Find the value(s) of c for which the function $f(x)$ defined below is continuous at $x = -2$.

$$f(x) = \begin{cases} e^{x+2} & \text{if } x \neq -2 \\ c & \text{if } x = -2 \end{cases}$$

Justify your answer using the definition of continuity.

3. (6 points) Consider the function $g(x) = 7 - 2x - x^3$.

   (a) Find the values of $\lim_{x \to -\infty} g(x)$ and $\lim_{x \to \infty} g(x)$. *No justification is required.*

   (b) Explain why the equation $g(x) = 0$ has a solution in the interval $(1, 2)$. *Hint: use the Intermediate Value Theorem.*

4. (6 points) Evaluate each limit.

   (a) $\lim_{t\to\infty} \dfrac{1-4t^3}{3+5t^3}$

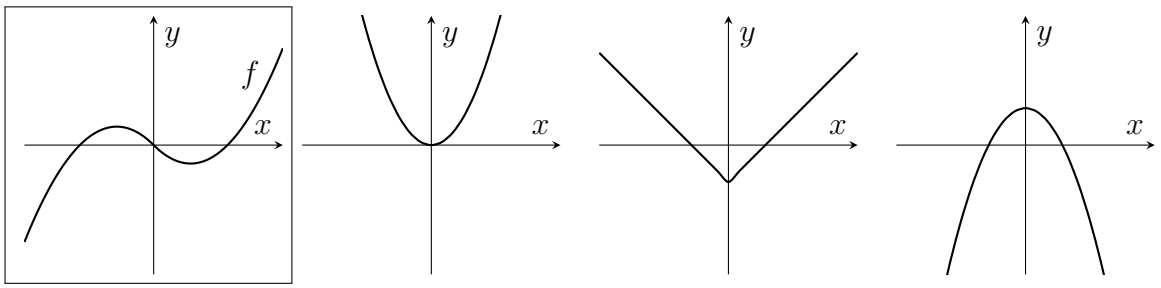
   (b) $\lim_{x\to\infty} \ln\left(\dfrac{1}{x}\right)$

5. (4 points) Find all of the horizontal and vertical asymptotes of the function

$$f(x) = \frac{x^2+5x+6}{x^3-4x^2}.$$

Show any limit calculations.

6. (1 point) The graph of a function f is given in the box on the left below. Circle the graph that shows the derivative f'.



7. (3 points) Suppose that the function $k(x)$ satisfies $k(3) = 5$ and $k'(3) = -4$. Find an equation of the tangent line to $y = k(x)$ at the point $(3, 5)$.

8. (6 points) The function $H(t)$ represents the height of a tree in meters, where t is measured in years. Suppose that $H(4) = 6.2$ and $H'(4) = 1.1$.

   (a) What is the meaning of $H(4) = 6.2$?

   (b) What is the meaning of $H'(4) = 1.1$?

   (c) Estimate $H(6)$.

9. (5 points) Use the limit definition of the derivative to compute the derivative of the function $f(x) = x^2 - 2x$.

10. (5 points) Sketch a graph of a function g that satisfies all of the following conditions.

- g is continuous except at $x = 1$
- g has an infinite discontinuity at $x = 1$
- g is not differentiable at $x = -2$
- $\lim_{x \to \infty} g(x) = \infty$