Math 141 Test 2

Name: _____

1. (6 points each) Differentiate and simplify.

(a). $y = \dfrac{4}{x^3} + \dfrac{x^2}{7}$

(b). $F(t) = \dfrac{t^2 - 3}{2t^2 + 3}$

(c). $r(\theta) = 4\csc\theta + \cot\theta$

(d). $f(x) = \arctan(5x)$

2. (8 points each) Differentiate. You don't need to simplify.

   (a). $f(x) = e^{4x} \tan(x^2)$
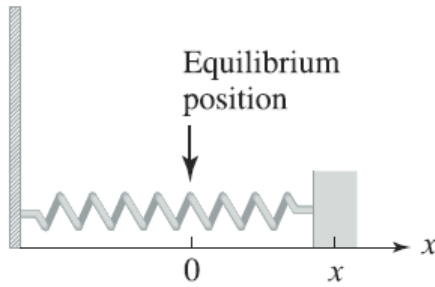
   (b). $y = \sec(\ln(x^2 + 1)$

3. Consider $f(x) = x^3 - 12x + 1$

   (a). (5 points) Find all x-value(s) where $f(x)$ has a tangent line with a slope of 15.

   (b). (3 points) Find the equation of the line tangent to $f(x)$ at $x = 1$.

4. A mass on a spring vibrates horizontally on a smooth, level surface as shown in the figure. Its equation of motion is $x(t) = 5 \sin t$, where t is measured in seconds and x in cm.


Equilibrium position

0   x

x

(a). (4 points) Find the acceleration at time t.

(b). (4 points) In what direction is the mass moving at $t = \frac{\pi}{3}$?

5. (4 points) Given $y = t^4 x + x^2 t^5$, find $\frac{dy}{dt}$ treating x as constant.

6. (8 points) Use logarithmic differentiation to find $\frac{dy}{dx}$ or y' given $y = (x)^{5x}$. Write your final answer only in terms of x.
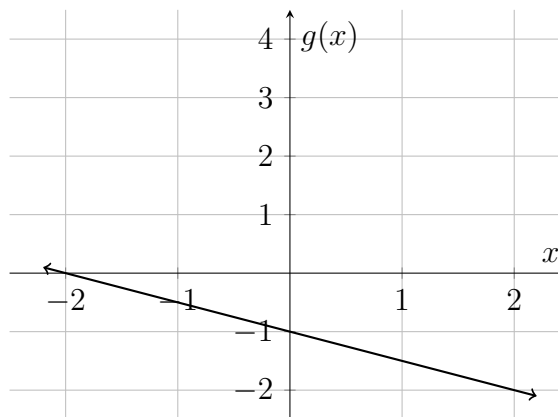
7. (8 points) Use implicit differentiation to find $\frac{dy}{dx}$ or y' given $xy^5 = 3y + 4e^x$.

8. (5 points) Consider the graphs of $f(x)$ (left graph) and $g(x)$ (right graph). Let
$v(x) = \dfrac{f(x)}{g(x)}$ and find $v'(0)$.

(i)
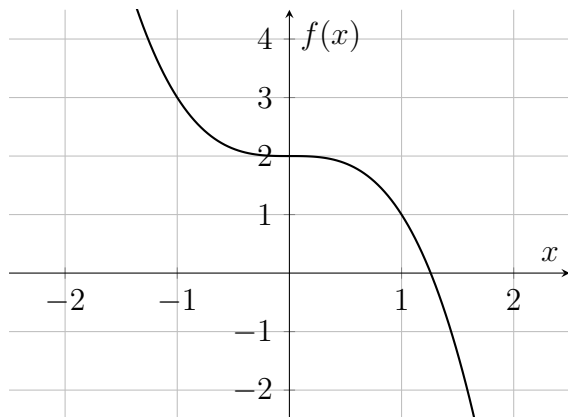
(ii)

9. The ideal gas law relates the pressure P (in Pascals), volume V (in cubic meters), and temperature T (in degrees Kelvin) to the number of moles of a gas n (in moles).

(a). (2 points) What are the units for $\frac{dP}{dV}$?

(b). (5 points) Interpret the meaning of $\frac{dP}{dV} < 0$ in the context of this application in one or 2 complete sentences. (Note: You don't need the formula nor any prior knowledge about the ideal gas law to answer this question)

10. (12 points) I am 30 ft from a bottle rocket launching pad. The bottle rocket launches vertically and is attached to a string in my hand. Find the rate of change of the angle between the string and the horizontal when the rocket is 40 feet in the air and traveling at a rate of 10 ft/s.