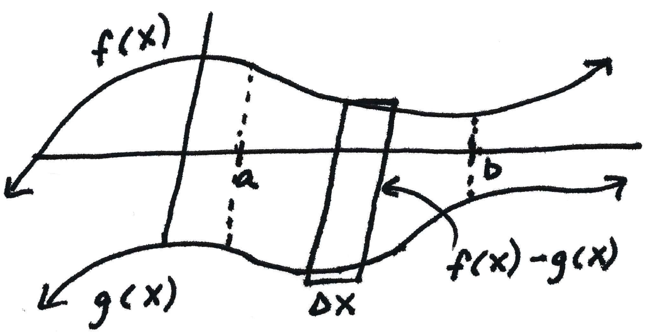
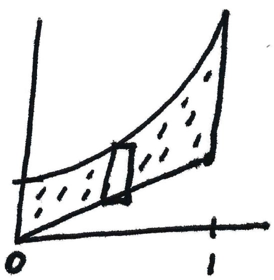
6.1 Area Between Curves



Area between $f(x)$ and $g(x)$ on $x = [a, b]$

$$A = \int_a^b (f(x) - g(x))\, dx$$

1. Find region bounded by $y = e^x$, $y = x$, $x = 0$, and $x = 1$.



$$A = \int_0^1 (e^x - x)\, dx = \left[e^x - \tfrac{1}{2}x^2\right]_0^1 = e - \tfrac{1}{2} - 1 = \boxed{e - \tfrac{3}{2}}$$

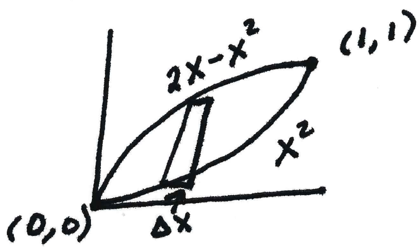
2. Find area of region enclosed by $y = x^2$ and $2x - x^2 = y$

$x^2 = 2x - x^2$

$2x^2 - 2x = 0$

$2x(x-1) = 0$

$(0,0)$, $(1,1)$



$$A = \int_0^1 [(2x - x^2) - x^2]\, dx$$

$$A = \int_0^1 (2x - 2x^2)\, dx$$

$$A = \left[x^2 - \tfrac{2}{3}x^3\right]_0^1 = 1 - \tfrac{2}{3} = \tfrac{1}{3}$$

3. Find area enclosed by $y = x - 1$ and $y^2 = 2x + 6$

$y + 1 = x$

$y^2 - 6 = 2x$

$\tfrac{1}{2}y^2 - 3 = x$

$\tfrac{1}{2}y^2 - 3 = y + 1$

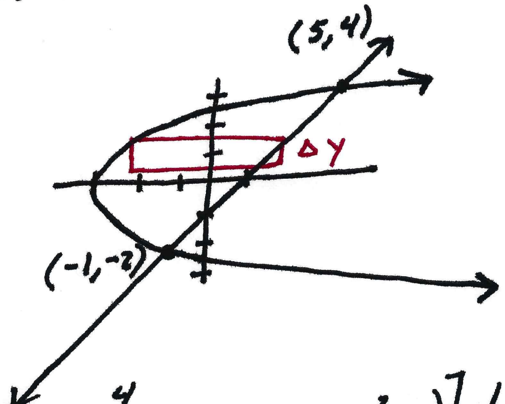
$\tfrac{1}{2}y^2 - y - 4 = 0$

$y^2 - 2y - 8 = 0$

$(y-4)(y+2) = 0$

$y = 4 \quad y = -2$

$(5, 4) \quad (-1, -2)$



$$A = \int_{-2}^{4} \left[(y+1) - \left(\tfrac{1}{2}y^2 - 3\right)\right] dy = \int_{-2}^{4} \left(-\tfrac{1}{2}y^2 + y + 4\right) dy$$

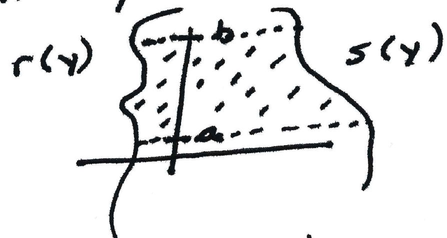
$$A = \left[-\tfrac{1}{6}y^3 + \tfrac{1}{2}y^2 + 4y\right]_{-2}^{4} = \tfrac{40}{3} - \left(\tfrac{-14}{3}\right) = \tfrac{54}{3} = 18$$

4. If $f(x) \geq g(x)$ on $[2,7]$ but $g(x) \geq f(x)$ on $[7,10]$ express area between curves on $[2,10]$

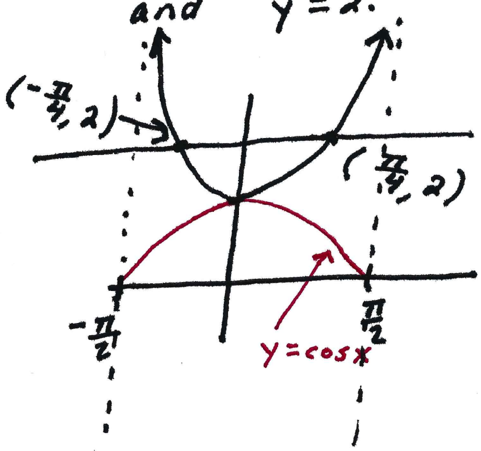
$$A = \int_2^7 (f(x) - g(x))\,dx + \int_7^{10} (g(x) - f(x))\,dx$$

5. If $r(y)$ lies to the left of $s(y)$, then the area between them from $y=a$ to $y=b$ is...

$$A = \int_a^b (s(y) - r(y))\,dy$$



6. Find area of one region bound between $y = \sec^2 x$ and $y = 2$.



$\sec^2 x = 2$

$\sec x = \sqrt{2}$

$\cos x = \frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{2}$

$x = -\frac{\pi}{4}, \frac{\pi}{4}$

$$A = 2\int_0^{\frac{\pi}{4}} (2 - \sec^2 x)\,dx$$

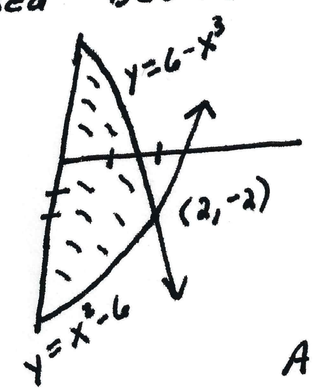
$$A = 2\left[2x - \tan x\right]_0^{\frac{\pi}{4}} = 2\left[\left(\frac{\pi}{2} - 1\right) - 0\right] = \pi - 2$$

7. Find area enclosed between $y = x^2 - 6$, $y = 6 - x^3$, y-axis

$x^2 - 6 = 6 - x^3$

$x^3 + x^2 - 12 = 0$

graphically $\rightarrow x = 2$



$$A = \int_0^2 [(6 - x^3) - (x^2 - 6)]\,dx$$

$$A = \int_0^2 (-x^3 - x^2 + 12)\,dx$$

$$A = \left[\frac{-x^4}{4} - \frac{x^3}{3} + 12x\right]_0^2$$

$$A = \frac{-16}{4} - \frac{8}{3} + 24 = \frac{52}{3}$$