★ A sequence is a function whose domain is a subset of integers. 0, 1, 1, 2, 3, 5, 8, 13, 21, 34, 55, etc

Fibonacci sequence: $A_n = A_{n-1} + A_{n-2}$

1. $a_n = \frac{1}{2n-1}$ for $n = 1, 2, 3, \ldots$ write out first 3 terms

$b_n = a_{n+1} \rightarrow b_1 = a_2 = \frac{1}{3},\ b_2 = a_3 = \frac{1}{5},\ b_3 = a_4 = \frac{1}{7}$

$c_n = a_{n+3} \rightarrow c_1 = a_4 = \frac{1}{7},\ c_2 = a_5 = \frac{1}{9},\ c_3 = a_6 = \frac{1}{11}$

$d_n = a_n^2 \rightarrow d_1 = (a_1)^2 = 1,\ d_2 = a_2^2 = \frac{1}{9},\ d_3 = a_3^2 = \frac{1}{25}$

$e_n = 2a_n - a_{n+1} \Rightarrow e_1 = 2(1) - \frac{1}{3} = \frac{5}{3},\ e_2 = 2(\frac{1}{3}) - \frac{1}{5} = \frac{7}{15},\ e_3 = 2(\frac{1}{5}) - \frac{1}{7} = \frac{9}{35}$

2. $b_n = \frac{(2n-1)!}{n!}$ $\quad b_1 = \frac{1!}{1!} = 1,\ b_2 = \frac{3!}{2!} = 3,\ b_3 = \frac{5!}{3!} = 20 \quad b_4 = \frac{7!}{4!} = 210$

$b_n = 1, 3, 20, 210, \ldots$

★ We say a sequence converges to L if $\lim_{n\to\infty} a_n = L$.
Otherwise the sequence is divergent.

3. $a_n = 20 - \frac{4}{n^2}$ $\quad \lim_{n\to\infty} 20 - \frac{4}{n^2} = 20$

4. $a_n = \frac{4+n-3n^2}{4n^2+1} \rightarrow \lim_{n\to\infty} \frac{4+n-3n^2}{4n^2+1} = \lim_{n\to\infty} \frac{\frac{4}{n^2} + \frac{1}{n} - 3}{4 + \frac{1}{n^2}} = \frac{-3}{4}$

★ Geometric sequence $a_n = c\,r^n$

we know $\lim_{n\to\infty} r^n = \begin{cases} 0 & 0<r<1 \\ 1 & r=1 \\ \text{diverges to } \infty & r>1 \end{cases}$

5. $Z_n = (\frac{1}{3})^n$ $\quad \lim_{n\to\infty} (\frac{1}{3})^n = 0$

6. $a_n = \frac{\sqrt{n}}{\sqrt{n}+4}$ $\quad \lim_{n\to\infty} \frac{1}{1+\frac{4}{\sqrt{n}}} = 1$

7. $d_n = \ln(n^2+4) - \ln(n^2-1)$ $\quad \lim_{n\to\infty} \ln\left(\frac{n^2+4}{n^2-1}\right) = \ln(1) = 0$

8. $c_n = \dfrac{n}{n+n^{1/n}}$ $\quad \lim\limits_{n\to\infty} \dfrac{1}{1+\dfrac{n^{1/n}}{n}} = \lim\limits_{n\to\infty} \dfrac{1}{1+n^{\frac{1-n}{n}}}$

$= \dfrac{1}{1+0} = 1$

9. $b_n = \dfrac{(-1)^n n^3 + \overbrace{2^{-n}}^{(\frac{1}{2})^n}}{3n^3 + \underbrace{4^{-n}}_{(\frac{1}{4})^n}}$ limit goes to either $-\frac{1}{3}$ or $\frac{1}{3}$ so b_n diverges.

10. $d_n = n^2\left(\sqrt[3]{n^3-1} - n\right)$

$\lim\limits_{n\to\infty} \dfrac{\sqrt[3]{n^3-1} - n}{n^{-2}} \overset{H}{=} \lim\limits_{n\to\infty} \dfrac{\frac{1}{3}(n^3-1)^{-2/3}(3n^2) - 1}{-2n^{-3}} = \lim\limits_{n\to\infty} \dfrac{n^5 - n^3(n^3-1)^{2/3}}{-2(n^3-1)^{2/3}}$

too messy
try alg. first

$d_n = n^3\left(\sqrt[3]{1-n^{-3}} - 1\right)$

$\lim\limits_{n\to\infty} \dfrac{(1-n^{-3})^{1/3} - 1}{n^{-3}} \overset{H}{=} \lim\limits_{n\to\infty} \dfrac{\frac{1}{3}(1-n^{-3})^{-2/3}(\cancel{3n^{-4}})}{-\cancel{3n^{-4}}} = \lim\limits_{n\to\infty} \dfrac{-1}{3(1-n^{-3})^{2/3}}$

$= -\dfrac{1}{3}$