1. $\int -3x\sqrt{3x^2+8}\,dx = \frac{-1}{2}\int \underline{-3x}\sqrt{3x^2+8}\,\underline{dx(-2)} = \frac{-1}{2}\int \sqrt{u}\,du$

$u = 3x^2+8$

$du = 6x\,dx$

$= \frac{-1}{2}\left[\frac{2}{3}u^{3/2} + C\right]$

$= \frac{-\sqrt{(3x^2+8)^3}}{3} + C$

2. $\int (5x(4-x)^9)\,dx = -5\int x(4-x)^9(-dx) = -5\int (4-u)u^9\,du$

$u = 4-x \rightarrow u-4 = -x$

$du = -dx \quad 4-u = x$

$= -5\int (4u^9 - u^{10})\,du$

$= -5\left[\frac{4u^{10}}{10} - \frac{u^{11}}{11} + C\right]$

$= -2(4-x)^{10} + \frac{5(4-x)^{11}}{11} + C$

3. $\int \frac{\csc^2 x}{\cot x - 7}\,dx = -\int \frac{-\csc^2 x}{\cot x - 7}\,dx = -\int \frac{1}{u}\,du = -\ln|u| + C$

$= -\ln|\cot x - 7| + C$

$u = \cot x - 7$

$du = -\csc^2 x\,dx$

4. $\int_{\pi/4}^{\pi/3} \cos x \sin^4 x\,dx = \int_{\frac{\sqrt{2}}{2}}^{\frac{\sqrt{3}}{2}} u^4\,du = \frac{u^5}{5}\Big|_{\frac{\sqrt{2}}{2}}^{\frac{\sqrt{3}}{2}} = \frac{9\sqrt{3}}{160} - \frac{4\sqrt{2}}{160}$

$\approx .0621$

$u = \sin x \quad u\left(\frac{\pi}{4}\right) = \frac{\sqrt{2}}{2}$

$du = \cos x\,dx \quad u\left(\frac{\pi}{3}\right) = \frac{\sqrt{3}}{2}$

5. $\int_{-3}^{3} \frac{t-2}{t+5}\,dt = \int_2^8 \frac{u-7}{u}\,du = \int_2^8 \left(1 - \frac{7}{u}\right)du = \left[u - 7\ln u\right]_2^8$

$= (8 - 7\ln 8) - (2 - 7\ln 2)$

$= 6 - 7\ln 4 \approx -3.704$

$u = t+5 \quad u(-3) = 2$

$du = dt \quad u(3) = 8$

$t = u-5$

$t-2 = u-7$

6. $\int_0^{20} (4x+1)^{\frac{1}{4}}\,dx = \frac{4(4x+1)^{\frac{5}{4}}}{4\cdot 5}\Big|_0^{20} = \frac{1}{5}\left(81^{\frac{5}{4}} - 1^{\frac{5}{4}}\right)$

$= \frac{1}{5}(243-1) = \frac{242}{5}$

7. $\int 7^{\pi x}\pi^{\pi x}\,dx = \int (7\pi)^{\pi x}\,dx = \frac{(7\pi)^{\pi x}}{\pi\ln(7\pi)} + C$

8. $\int \frac{1}{16+x^2}\,dx = \int \frac{4\,du}{16+16u^2} = \frac{1}{4}\int \frac{1}{1+u^2}\,du = \frac{1}{4}\tan^{-1}u + C$

$= \frac{1}{4}\tan^{-1}\left(\frac{x}{4}\right) + C$

$x^2 = 16u^2$

$x = 4u \rightarrow u = \frac{x}{4}$

$dx = 4du$

9. $\int \frac{dx}{\sqrt{15-9x^2}} = \int \frac{\sqrt{\frac{5}{3}}\,du}{\sqrt{15-15u^2}} = \sqrt{\frac{5}{3\cdot 15}}\int \frac{du}{\sqrt{1-u^2}}$

$= \frac{1}{3}\sin^{-1}u + C$

$= \frac{1}{3}\sin^{-1}\left(\frac{\sqrt{3}\,x}{\sqrt{5}}\right) + C$

$9x^2 = 15u^2$

$x^2 = \frac{5}{3}u^2$

$x = \sqrt{\frac{5}{3}}\,u \rightarrow \frac{\sqrt{3}\,x}{\sqrt{5}}$

$dx = \sqrt{\frac{5}{3}}\,du$

10. $\int_{\frac{1}{4}}^{\frac{1}{2}} \frac{5\,dx}{3x\sqrt{16x^2-1}} = \frac{5}{3}\int_1^2 \frac{\frac{1}{4}\,du}{\frac{1}{4}u\sqrt{u^2-1}} = \frac{5}{3}\int_1^2 \frac{du}{u\sqrt{u^2-1}}$

$= \frac{5}{3}\left[\sec^{-1}u\right]_1^2$

$= \frac{5}{3}\left[\cos^{-1}\frac{1}{2} - \cos^{-1}1\right]$

$= \frac{5}{3}\left(\frac{\pi}{3} - 0\right) = \frac{5\pi}{9}$

$16x^2 = u^2$ $\quad u = 4x$

$4x = u$ $\quad u\left(\frac{1}{4}\right) = 1$

$x = \frac{1}{4}u$ $\quad u\left(\frac{1}{2}\right) = 2$

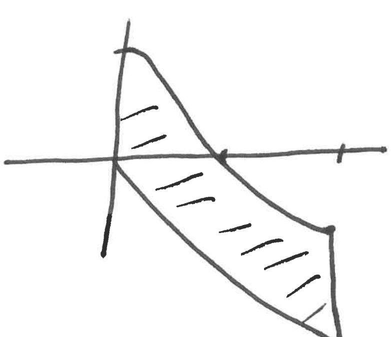
$dx = \frac{1}{4}du$

11. $\int \frac{(17x+2)\,dx}{x^2+4} = \frac{17}{2}\int \frac{2x\,dx}{x^2+4} + \int \frac{2\,dx}{x^2+4}$ $\leftarrow$ $x^2 = 4u^2$, $x = 2u \rightarrow u = \frac{x}{2}$, $dx = 2du$

$= \frac{17}{2}\ln|x^2+4| + \int \frac{4\,du}{4u^2+4}$

$= \frac{17}{2}\ln|x^2+4| + \tan^{-1}\left(\frac{x}{2}\right) + C$

12.



$A = \int_0^{\pi} (\cos x - (-x))\,dx = \left[\sin x + \frac{x^2}{2}\right]_0^{\pi}$

$A = \frac{\pi^2}{2}$

13. $y^2 = x + 6$ $\qquad$ $2 - x = y^2$ $\qquad$ intersections $\rightarrow$ $y^2 - 6 = 2 - y^2$

$x = y^2 - 6$ $\qquad$ $2 - y^2 = x$ $\qquad$ $2y^2 = 8$

left $\qquad$ right $\qquad$ $y^2 = 4$

$y = \pm 2$

$$A = \int_{-2}^{2} \left[(2-y^2) - (y^2-6)\right] dy$$

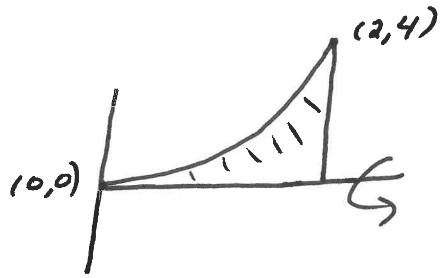
$$A = \int_{-2}^{2} (8 - 2y^2)\, dy = \left[8y - \frac{2}{3}y^3\right]_{-2}^{2} = \frac{32}{3} - \left(\frac{-32}{3}\right) = \frac{64}{3}$$

14. $y = \frac{6}{x^2}$ $\qquad$ $f_{avg.} = \frac{1}{10-9} \int_{9}^{10} \frac{6}{x^2}\, dx = \frac{-6}{x}\Big|_{9}^{10} = \frac{-6}{10} + \frac{6}{9}$

$f_{avg} = \frac{1}{15}$

$$\frac{6}{x^2} = \frac{1}{15} \rightarrow x^2 = 90 \rightarrow x = \sqrt{90} = 3\sqrt{10} \approx 9.487$$

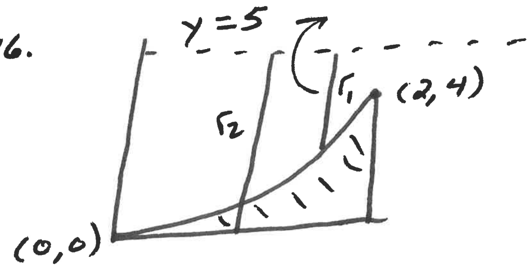
15.

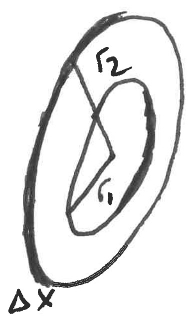


$$V_0 = \pi (x^2)^2 \Delta x = \pi x^4 \Delta x$$

$$V = \pi \int_0^2 x^4 dx = \pi \left[\frac{x^5}{5}\right]_0^2 = \frac{32\pi}{5}$$

16.





$$V_0 = \pi \left[5^2 - (5 - x^2)^2\right] \Delta x$$

$$V_0 = \pi \left[25 - (25 - 10x^2 + x^4)\right] \Delta x$$

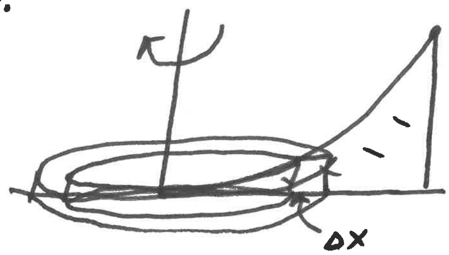
$$V_0 = \pi (10x^2 - x^4) \Delta x$$

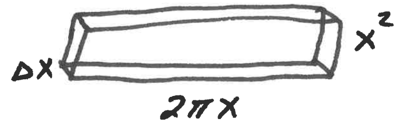
$r_2 = 5$

$r_1 = 5 - x^2$

$$V = \pi \int_0^2 (10x^2 - x^4)\, dx = \pi \left[\frac{10x^3}{3} - \frac{x^5}{5}\right]_0^2 = \frac{304\pi}{15}$$

17.

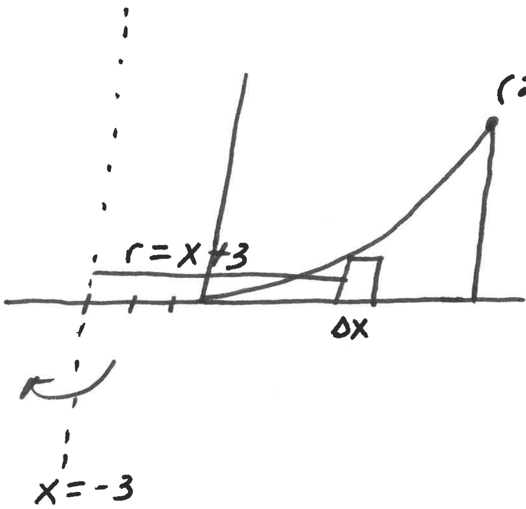


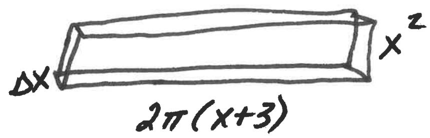


$$V_{shell} = 2\pi x(x^2)\Delta x = 2\pi x^3 \Delta x$$

$$V = 2\pi \int_0^2 x^3 dx = 2\pi \left[\frac{x^4}{4}\right]_0^2 = 8\pi$$

18.





$$V_{shell} = 2\pi(x+3)(x^2)\,\Delta x$$
$$= 2\pi(x^3+3x^2)\,\Delta x$$

$$V = 2\pi \int_0^2 (x^3+3x^2)dx = 2\pi \left[\frac{x^4}{4} + x^3\right]_0^2 = 24\pi$$