

Section 2.2: The Limit of a Function

Goal: Explore the idea of a limit from graphical and numerical viewpoints.

Concept of limit

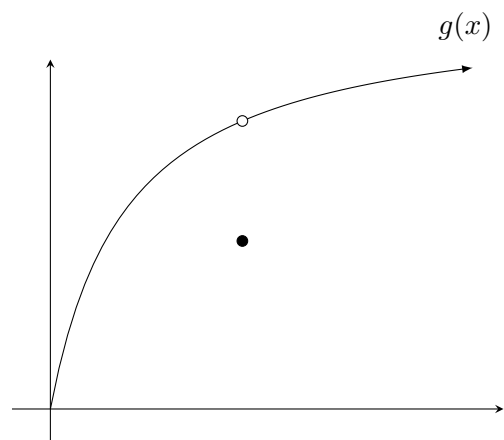
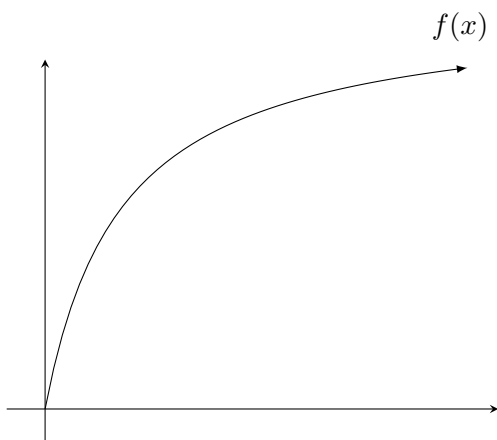
What does $\lim_{x \rightarrow a} f(x) = L$ mean?

Limits 101:

Example 1

Create a table of values and estimate $\lim_{x \rightarrow 3} \frac{x^2 - 9}{x - 3}$.

Graphical viewpoint:



One-sided limits

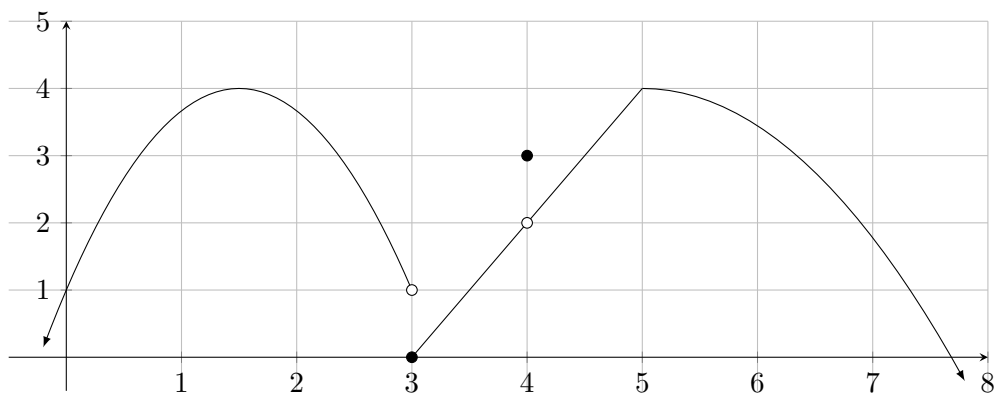
$\lim_{x \rightarrow a^-} f(x) = L$ (said “the limit of $f(x)$ as x approaches a from the left equals L ”) means

$\lim_{x \rightarrow a^+} f(x) = L$ (said “the limit of $f(x)$ as x approaches a from the right equals L ”) means

Note:

Example 2

Use the graph of $y = f(x)$ to fill in the following values.



$$f(3) = \underline{\hspace{2cm}}$$

$$f(4) = \underline{\hspace{2cm}}$$

$$f(5) = \underline{\hspace{2cm}}$$

$$\lim_{x \rightarrow 3^-} f(x) = \underline{\hspace{2cm}}$$

$$\lim_{x \rightarrow 4^-} f(x) = \underline{\hspace{2cm}}$$

$$\lim_{x \rightarrow 5^-} f(x) = \underline{\hspace{2cm}}$$

$$\lim_{x \rightarrow 3^+} f(x) = \underline{\hspace{2cm}}$$

$$\lim_{x \rightarrow 4^+} f(x) = \underline{\hspace{2cm}}$$

$$\lim_{x \rightarrow 5^+} f(x) = \underline{\hspace{2cm}}$$

$$\lim_{x \rightarrow 3} f(x) = \underline{\hspace{2cm}}$$

$$\lim_{x \rightarrow 4} f(x) = \underline{\hspace{2cm}}$$

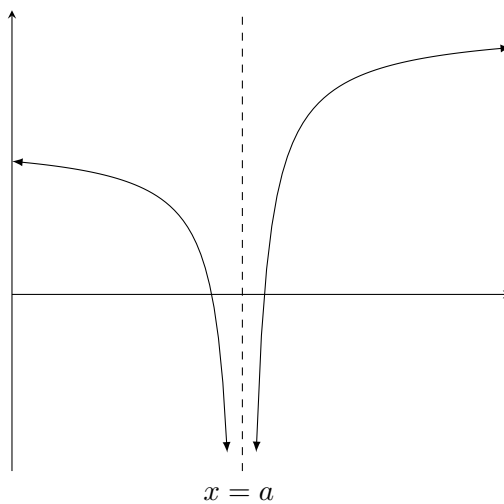
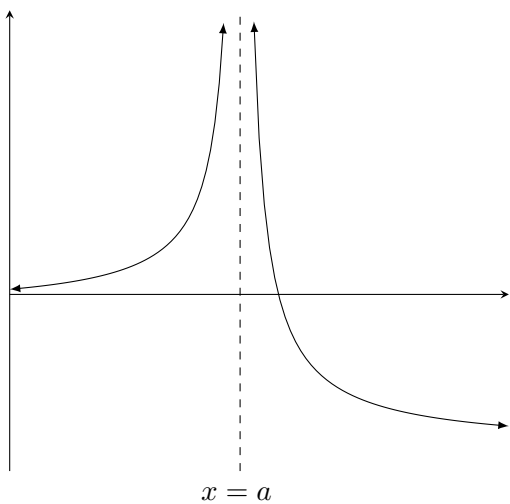
$$\lim_{x \rightarrow 5} f(x) = \underline{\hspace{2cm}}$$

Infinite limits

$\lim_{x \rightarrow a} f(x) = \infty$ (said “the limit of $f(x)$ as x approaches a is positive infinity”) means

$\lim_{x \rightarrow a} f(x) = -\infty$ (said “the limit of $f(x)$ as x approaches a is negative infinity”) means

Graphical viewpoint:



Vertical asymptotes:

Example 3

Determine $\lim_{x \rightarrow 0} \frac{x+3}{x^2}$.

Example 4

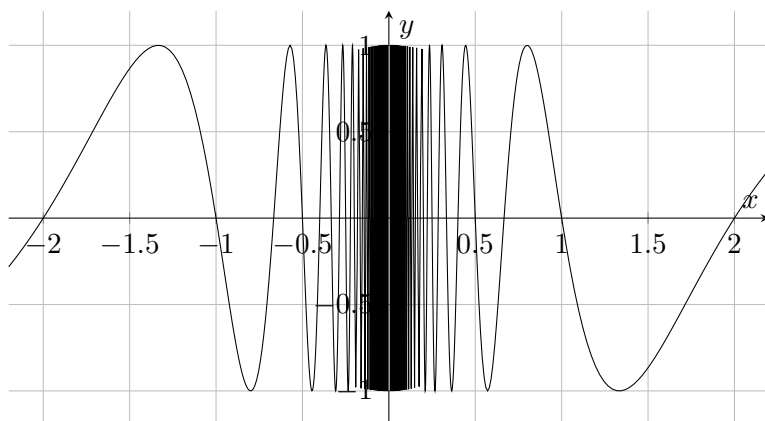
What is $\lim_{x \rightarrow 0^+} \ln(x)$?

Example 5

Determine $\lim_{x \rightarrow \pi^-} \cot(x)$.

Example 6

Below is the graph of $\sin\left(\frac{2\pi}{x}\right)$. What is $\lim_{x \rightarrow 0} \sin\left(\frac{2\pi}{x}\right)$?



Example 7

Sketch a graph of a function f satisfying the following conditions.

(i) $\lim_{x \rightarrow -2} f(x) = -1$

(ii) $f(-2) = 2$

(iii) $\lim_{x \rightarrow 3^-} f(x) = -\infty$

