1. $\int \tan^{-1} 2x\,dx$

$u = \tan^{-1} 2x$ $\quad dv = dx$

$du = \frac{1}{1+4x^2}\cdot 2dx$ $\quad v = x$

$t = 1+4x^2$

$dt = 8x\,dx$

$$\int \tan^{-1} 2x\,dx = x\tan^{-1}2x - 2\int \frac{x}{1+4x^2}\,dx$$

$$= x\tan^{-1}2x - \frac{1}{4}\int \frac{dt}{t}$$

$$= x\tan^{-1}2x - \frac{1}{4}\ln|t| + C = \boxed{x\tan^{-1}2x - \frac{1}{4}\ln(1+4x^2) + C}$$

2. $\int \frac{\ln(\ln 3x)\ln 3x}{x}\,dx$

$t = \ln 3x$

$dt = \frac{3}{3x}\,dx = \frac{1}{x}\,dx$

$$\int \frac{\ln(\ln 3x)\ln 3x}{x}\,dx = \int t\ln t\,dt$$

$u = \ln t$ $\quad dv = t\,dt$

$du = \frac{1}{t}\,dt$ $\quad v = \frac{1}{2}t^2$

$$\int \frac{\ln(\ln 3x)\ln 3x}{x}\,dx = \frac{1}{2}t^2\ln t - \frac{1}{2}\int t\,dt = \frac{1}{2}t^2\ln t - \frac{1}{4}t^2 + C$$

$$= \boxed{\frac{1}{2}(\ln 3x)^2\ln(\ln 3x) - \frac{1}{4}(\ln 3x)^2 + C}$$

3. $\int e^{2x}\sin 5x\,dx$

$u = \sin 5x$ $\quad dv = e^{2x}dx$

$du = 5\cos 5x\,dx$ $\quad v = \frac{1}{2}e^{2x}$

$$\int e^{2x}\sin 5x\,dx = \frac{1}{2}e^{2x}\sin 5x - \frac{5}{2}\int e^{2x}\cos 5x\,dx$$

$u = \cos 5x$ $\quad dv = e^{2x}dx$

$du = -5\sin 5x\,dx$ $\quad v = \frac{1}{2}e^{2x}$

$$\int e^{2x}\sin 5x\,dx = \frac{1}{2}e^{2x}\sin 5x - \frac{5}{2}\left[\frac{1}{2}e^{2x}\cos 5x + \frac{5}{2}\int e^{2x}\sin 5x\,dx\right]$$

$$\frac{29}{4}\int e^{2x}\sin 5x\,dx = \frac{1}{2}e^{2x}\sin 5x - \frac{5}{4}e^{2x}\cos 5x + C$$

$$\int e^{2x}\sin 5x\,dx = \boxed{\frac{2}{29}e^{2x}\sin 5x - \frac{5}{29}e^{2x}\cos 5x + C}$$

$$= \boxed{\frac{1}{29}e^{2x}(2\sin 5x - 5\cos 5x) + C}$$

4. $\int_1^8 x\ln x\,dx$ $\quad u=\ln x \quad dv = x\,dx$

$du=\frac{1}{x}dx \quad v=\frac{1}{2}x^2$

$$\int_1^8 x\ln x\,dx = \left[\frac{1}{2}x^2\ln x - \frac{1}{2}\int x\,dx\right]_1^8 = \left[\frac{1}{2}x^2\ln x - \frac{1}{4}x^2\right]_1^8$$

$$= (32\ln 8 - 16) - (0-\tfrac{1}{4}) = 32\ln 8 - \frac{63}{4} \approx 50.792$$

5. $$\sin^4 8x = \left(\tfrac{1}{2}(1-\cos 16x)\right)^2 = \tfrac{1}{4}\left[1-2\cos 16x + \cos^2 16x\right]$$

$$= \tfrac{1}{4}\left[1-2\cos 16x + \tfrac{1}{2}(1+\cos 32x)\right]$$

$$= \tfrac{1}{4}\left[\tfrac{3}{2} - 2\cos 16x + \tfrac{1}{2}\cos 32x\right]$$

$$\int \sin^4 8x\,dx = \tfrac{1}{4}\int\left(\tfrac{3}{2}-2\cos 16x + \tfrac{1}{2}\cos 32x\right)dx$$

$$= \tfrac{1}{4}\left[\tfrac{3}{2}x - \tfrac{1}{8}\sin 16x + \tfrac{1}{64}\sin 32x\right] + C$$

$$= \frac{3}{8}x - \frac{1}{32}\sin 16x + \frac{1}{256}\sin 32x + C$$

6. $$\tan^5 x = (\tan^2 x)^2(\tan x) = (\sec^2 x - 1)^2(\tan x) = (\sec^4 x - 2\sec^2 x + 1)(\tan x)$$

so... $$\int \tan^5 x\sec^3 x\,dx = \int(\sec^4 x - 2\sec^2 x + 1)\sec^3 x\tan x\,dx$$

$u=\sec x \rightarrow du = \sec x\tan x\,dx$

$$= \int(u^4-2u^2+1)u^2\,du = \int(u^6-2u^4+u^2)\,du = \frac{u^7}{7} - \frac{2u^5}{5} + \frac{u^3}{3} + C$$

$$= \frac{1}{7}\sec^7 x - \frac{2}{5}\sec^5 x + \frac{1}{3}\sec^3 x + C$$

7. $\displaystyle\int \frac{dx}{\sqrt{16x^2-25}}$

let $4x = 5\sec\theta$

$x = \frac{5}{4}\sec\theta$

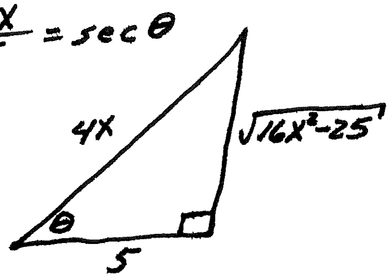
$dx = \frac{5}{4}\sec\theta\tan\theta \, d\theta$

$\sqrt{16x^2-25} = \sqrt{25\sec^2\theta - 25} = \sqrt{25\tan^2\theta} = 5\tan\theta$

$$\int \frac{dx}{\sqrt{16x^2-25}} = \int \frac{\frac{5}{4}\sec\theta\tan\theta\,d\theta}{5\tan\theta} = \frac{1}{4}\int \sec\theta\,d\theta = \frac{1}{4}\ln|\sec\theta + \tan\theta| + C$$

$$= \frac{1}{4}\ln\left|\frac{4x}{5} + \frac{\sqrt{16x^2-25}}{5}\right| + C$$

$\frac{4x}{5} = \sec\theta$



$$= \frac{1}{4}\ln\left|4x + \sqrt{16x^2-25}\right| + C$$

8. $\displaystyle\int \frac{x^2}{\sqrt{36-x^2}}\,dx$

let $x = 6\sin\theta$

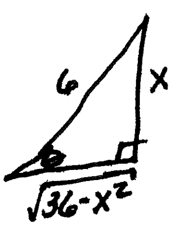
$dx = 6\cos\theta\,d\theta$

$\sqrt{36-x^2} = \sqrt{36-36\sin^2\theta} = \sqrt{36\cos^2\theta} = 6\cos\theta$

$$\int \frac{x^2}{\sqrt{36-x^2}}\,dx = \int \frac{36\sin^2\theta\,(6\cos\theta)}{6\cos\theta}\,d\theta = 36\int \sin^2\theta\,d\theta$$

$$= 18\int(1-\cos 2\theta)\,d\theta = 18\left[\theta - \frac{1}{2}\sin 2\theta\right] + C$$

$$= 18\left[\theta - \sin\theta\cos\theta\right] + C$$



$\frac{x}{6} = \sin\theta$

$$= 18\left[\sin^{-1}\left(\frac{x}{6}\right) - \frac{x}{6}\left(\frac{\sqrt{36-x^2}}{6}\right)\right] + C$$

$$= 18\sin^{-1}\left(\frac{x}{6}\right) - \frac{1}{2}x\sqrt{36-x^2} + C$$

9. $\int \frac{5x-8}{x^2+5x-14}\,dx$

$$\frac{5x-8}{(x+7)(x-2)} = \frac{A}{x+7} + \frac{B}{x-2}$$

$$5x-8 = A(x-2) + B(x+7)$$

$$x=-7 \rightarrow -43 = -9A \rightarrow A = \frac{43}{9}$$

$$x=2 \rightarrow 2 = 9B \rightarrow B = \frac{2}{9}$$

$$\int \frac{5x-8}{x^2+5x-14}\,dx = \frac{43}{9}\int \frac{dx}{x+7} + \frac{2}{9}\int \frac{dx}{x-2}$$

$$= \frac{43}{9}\ln|x+7| + \frac{2}{9}\ln|x-2| + C$$

10. $\int \frac{5}{(x-4)^2(x-1)}\,dx$

$$\frac{5}{(x-4)^2(x-1)} = \frac{A}{x-4} + \frac{B}{(x-4)^2} + \frac{C}{x-1}$$

$$5 = A(x-4)(x-1) + B(x-1) + C(x-4)^2$$

$$x=4 \rightarrow 5 = 3B \rightarrow B = \frac{5}{3}$$

$$x=1 \rightarrow 5 = C(-3)^2 \rightarrow 9C = 5 \rightarrow C = \frac{5}{9}$$

$$x^2 \rightarrow 0 = A + C \rightarrow A = \frac{-5}{9}$$

$$\int \frac{5\,dx}{(x-4)^2(x-1)} = \frac{-5}{9}\int \frac{dx}{x-4} + \frac{5}{3}\int \frac{dx}{(x-4)^2} + \frac{5}{9}\int \frac{dx}{x-1}$$

$$= \frac{-5}{9}\ln|x-4| + \frac{5}{3}\left[\frac{(x-4)^{-1}}{-1}\right] + \frac{5}{9}\ln|x-1| + C$$

$$= \frac{-5}{9}\ln|x-4| - \frac{5}{3(x-4)} + \frac{5}{9}\ln|x-1| + C$$

11.

$$\begin{array}{r} x+1 \\ x^2+1 \enclose{longdiv}{x^3+x^2+4} \\ -(x^3+x) \\ \hline x^2-x+4 \\ -(x^2+1) \\ \hline -x+3 \end{array}$$

$$\int \frac{x^3+x^2+4}{x^2+1}\,dx = \int (x+1)\,dx - \int \frac{x\,dx}{x^2+1} + 3\int \frac{dx}{x^2+1}$$

$$= \frac{1}{2}x^2 + x - \frac{1}{2}\ln|x^2+1| + 3\tan^{-1}x + C$$

12. $\int \sqrt{\frac{1+x}{1-x}}\,dx = \int \frac{\sqrt{1+x}}{\sqrt{1-x}} \cdot \frac{\sqrt{1+x}}{\sqrt{1+x}}\,dx = \int \frac{1+x}{\sqrt{1-x^2}}\,dx$

$= \int \frac{1}{\sqrt{1-x^2}}\,dx + \int \frac{x}{\sqrt{1-x^2}}\,dx$ $\quad u = 1-x^2$, $du = -2x\,dx$

$= \sin^{-1}x - \frac{1}{2}\int u^{-1/2}\,du$

$= \sin^{-1}x - \frac{1}{2}(2u^{1/2}) + C = \sin^{-1}x - \sqrt{1-x^2} + C$

13. $\sin^5 x = (1-\cos^2 x)^2 \sin x = (1 - 2\cos^2 x + \cos^4 x)\sin x$ $\quad u = \cos x$, $du = -\sin x\,dx$

$\int \sin^5 x \cos^4 x\,dx = \int (1 - 2\cos^2 x + \cos^4 x)(\cos^4 x)\sin x\,dx$

$= -\int (u^4 - 2u^6 + u^8)\,du = -\left[\frac{u^5}{5} - \frac{2u^7}{7} + \frac{u^9}{9}\right] + C$

$= -\frac{1}{5}\cos^5 x + \frac{2}{7}\cos^7 x - \frac{1}{9}\cos^9 x + C$

14. $\int x^4 \ln x\,dx$ $\quad u = \ln x$, $du = \frac{1}{x}dx$ $\quad dv = x^4 dx$, $v = \frac{x^5}{5}$

$\int x^4 \ln x\,dx = \frac{1}{5}x^5 \ln x - \frac{1}{5}\int x^4 dx = \frac{1}{5}x^5 \ln x - \frac{1}{25}x^5 + C$

15. $\int_{-\infty}^{0} \frac{dx}{3-4x} = \lim_{a\to-\infty} \int_a^0 (3-4x)^{-1}dx = \lim_{a\to-\infty}\left[-\frac{1}{4}\ln|3-4x|\right]_a^0$

$= -\frac{1}{4}\ln 3 + \lim_{a\to-\infty} \frac{1}{4}\ln|3-4a| = \infty$

16. $\int_0^5 \frac{dx}{\sqrt[3]{5-x}} = \lim_{b\to5^-} \int_0^b (5-x)^{-\frac{1}{3}}dx = \lim_{b\to5^-}\left[-\frac{3}{2}(5-x)^{2/3}\right]_0^b$

$= \lim_{b\to5^-} -\frac{3}{2}(5-b)^{2/3} + \frac{3}{2}(5-0)^{2/3} = \frac{3\sqrt[3]{25}}{2}$

17. $\int_{\frac{6}{\pi}}^{1} \frac{dx}{x\sqrt{x^2-1}} = \lim_{b\to1^-} \int_{\frac{6}{\pi}}^{b} \frac{dx}{x\sqrt{x^2-1}} = \lim_{b\to1^-} \sec^{-1}x \Big|_{6/\pi}^{b}$

$= \lim_{b\to1^-} \sec^{-1}b - \sec^{-1}\left(\frac{6}{\pi}\right) = 0 - \cos^{-1}\left(\frac{\pi}{6}\right)$

$= -\cos^{-1}\left(\frac{\pi}{6}\right)$