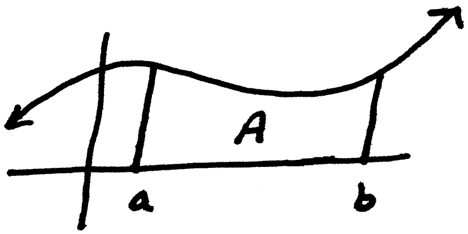
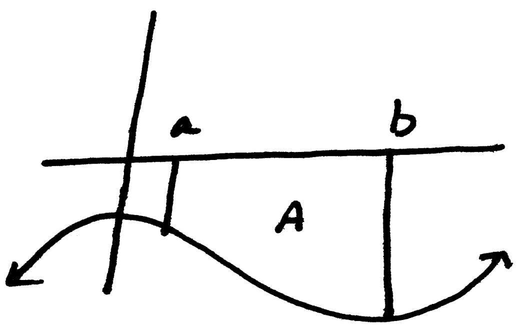
5.2 Notes

$\int_a^b f(x)dx$ represents the signed area between $f(x)$ and the x-axis on $[a,b]$.



$$A = \int_a^b f(x)dx = F(b) - F(a)$$

$$-A = \int_b^a f(x)\,dx = F(a) - F(b)$$

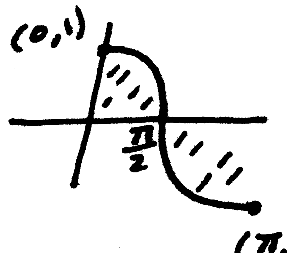


$$-A = \int_a^b f(x)\,dx$$

$$A = \left| \int_a^b f(x)dx \right| = \int_a^b |f(x)|\,dx = \int_b^a f(x)dx$$

Integration Power Rule : $\int x^n dx = \frac{x^{n+1}}{n+1} + C$

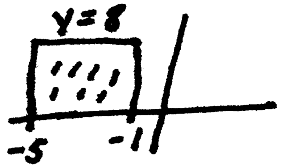
1. $\int_0^{\pi} \cos x\, dx = \sin x \Big|_0^{\pi} = \sin(\pi) - \sin(0) = 0 - 0 = \boxed{0}$

Graphically $\rightarrow$ 

Areas would cancel and give sum of $\boxed{0}$

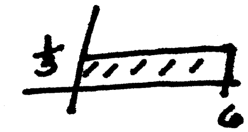
2. $\displaystyle\int_{-5}^{-1} 8\,dx = 8x\Big|_{-5}^{-1} = 8(-1) - 8(-5) = -8 + 40 = 32$

Graphically →



$A = 4(8) = 32$

3. How large could $\displaystyle\int_{0}^{6} f(x)\,dx$ be if $f(x) \le \frac{1}{3}$



$A = \frac{1}{3}(6) = 2$

4. $\displaystyle\int_{-1}^{1} (2x - |x|)\,dx$

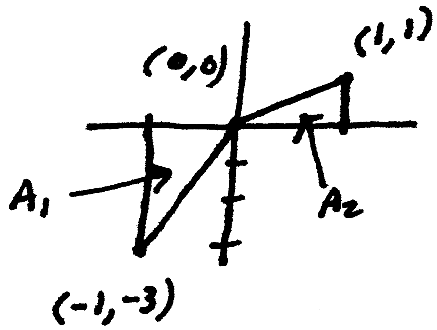
$$|x| = \begin{cases} x, & x \ge 0 \\ -x, & x < 0 \end{cases}$$

so... $\displaystyle\int_{-1}^{1} (2x - |x|)\,dx = \int_{-1}^{0} (2x - (-x))\,dx + \int_{0}^{1} (2x - x)\,dx$

$$= \int_{-1}^{0} 3x\,dx + \int_{0}^{1} x\,dx = \frac{3}{2}x^2\Big|_{-1}^{0} + \frac{x^2}{2}\Big|_{0}^{1}$$

$$= \left[0 - \frac{3}{2}\right] + \left[\frac{1}{2} - 0\right] = -1$$

Graphically...



$A = A_1 + A_2$

$A = \frac{1}{2}(1)(-3) + \frac{1}{2}(1)(1) = \frac{-3}{2} + \frac{1}{2} = -1$

5. Find $\int_2^5 (2x+1)\,dx$ three ways : a.) using $\lim_{n\to\infty} R_n$

b.) using geometry

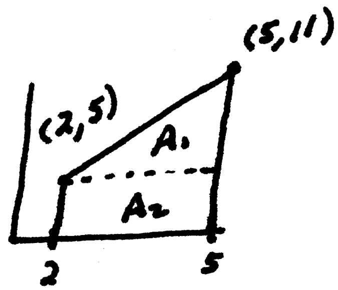
c.) using integration

a.) $\int_2^5 (2x+1)\,dx = \lim_{n\to\infty} \sum_{i=1}^{n} f(a+i\Delta x)\Delta x$

$R_n = \lim_{n\to\infty} \sum f\left(2+\frac{3i}{n}\right)\left(\frac{3}{n}\right)$

$R_n = \lim_{n\to\infty} \frac{3}{n} \sum \left[2\left(2+\frac{3i}{n}\right)+1\right] = \lim_{n\to\infty} \frac{3}{n} \sum \left(5+\frac{6i}{n}\right) = \lim_{n\to\infty} \frac{3}{n}(5n) + \frac{3}{n}\left(\frac{6}{n}\right)\sum i$

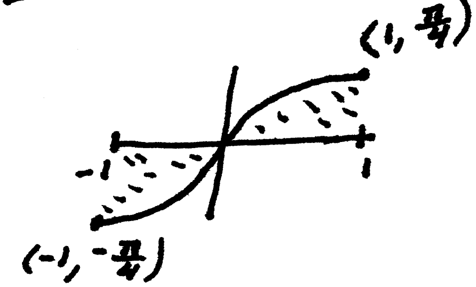
$R_n = \lim_{n\to\infty} 15 + \frac{18}{n^2}\left(\frac{n(n+1)}{2}\right) = \lim_{n\to\infty} 15 + \frac{18n^2+\dots}{2n^2} = 15+9 = 24$

b.) 

$A = A_1 + A_2 = \frac{1}{2}(3)(6) + 3(5) = 9 + 15 = 24$

c.) $\int_2^5 (2x+1)\,dx = \left[x^2+x\right]_2^5 = (5^2+5) - (2^2+2) = 30 - 6 = 24$

6. $\int_{-1}^{1} \tan^{-1} x\,dx$



Areas would cancel

so $\int_{-1}^{1} \tan^{-1} x\,dx = 0$

★ This is true for any $\int_{-a}^{a} (\text{odd function})\,dx = 0$

7. $\int_0^3 (6y^2 + 7y + 1)\,dy = \left[2y^3 + \frac{7}{2}y^2 + y\right]_0^3 = 2(3)^3 + \frac{7}{2}(3)^2 + 3$

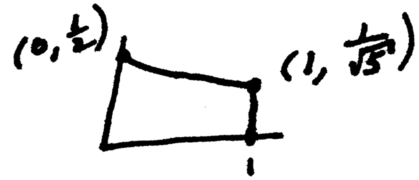
$= 54 + \frac{63}{2} + 3 = \boxed{\frac{177}{2} = 88.5}$

8. $\int_2^0 (x^2 - e^x)\,dx = \frac{x^3}{3} - e^x \Big|_2^0 = (0 - e^0) - \left(\frac{8}{3} - e^2\right) = \boxed{-\frac{11}{3} + e^2 \approx 3.722}$

9. Express as a single integral: $\int_7^3 f(x)dx + \int_3^9 f(x)dx = \boxed{\int_7^9 f(x)\,dx}$

10. Determine upper and lower bounds for $\int_0^1 \frac{1}{\sqrt{x^3+4}}\,dx$

$\frac{1}{\sqrt{x^3+4}}$ decreases on $[0,1]$



L_1 gives upper bound $\rightarrow \int_0^1 \frac{1}{\sqrt{x^3+4}}\,dx \le 1\left(\frac{1}{2}\right) = \frac{1}{2}$

R_1 gives lower bound $\rightarrow \int_0^1 \frac{1}{\sqrt{x^3+4}}\,dx \ge 1\left(\frac{1}{\sqrt{5}}\right) \approx .447$