Name: Isaac Hong Zhang Jie
Student No.: 2023403322

Homework 1

Monday, 25 September 2023 10:22 AM

2) $P(D^+) = 10^{-6}$ $S_n = 1.0$ $S_p = 0.9999$ $S_n = \frac{TP}{TP+FN}$ $S_p = \frac{TN}{TN+FP}$

$$P(D^+|T^+) = \frac{P(T^+|D^+)P(D^+)}{P(T^+)}$$

$$= \left(\frac{TP}{TP+FN}\right)\left(P(T^+|D^+)P(D^+) + P(T^+|D^-)P(D^-)\right)^{-1} P(D^+)$$

$$= S_n\left(S_n \cdot P(D^+) + \frac{FP}{TN+FP}(1-P(D^+))\right)^{-1} P(D^+)$$

$$= S_n\left(S_n \cdot P(D^+) + \alpha(1-P(D^+))\right)^{-1} P(D^+)$$

$$= S_n\left(S_n \cdot P(D^+) + (1-S_p)(1-P(D^+))\right)^{-1} P(D^+)$$

$$= \frac{10^{-6}}{10^{-6} + (1-0.9999)(1-10^{-6})}$$

$$\approx 0.0099010 \quad (5 s.f.)$$

$$\approx 0.00990 \quad (3 s.f.)$$

Given that the probability of actually having the disease when the test is positive is 0.00990 which is equivalent to 0.99%, I might decide not to take the test so that I do not scare myself unnecessarily in the event I test positive since it is so unlikely to get the disease though the results may be positive.

b) If the disease is now infectious, its prevalence may increase and may no longer be 10^{-6}.

Assuming that $P(D^+)$ is now 10^{-3} instead,

$$P(D^+|T^+) = \frac{10^{-3}}{10^{-3}+(1-0.9999)(1-10^{-3})}$$

$$= 0.90917 \quad (5\text{s.f.})$$

$$\approx 0.909 \quad (3\text{s.f.})$$

Now, the probability of being infected with the disease when the test is positive is a lot higher

at 0.909 (90.1%). Hence, if the prevalence of the disease increases due to it being infectious,

I would consider taking it as it seems to become more useful.

If the prevalence of the disease did not increase, I might only consider taking the test if I come into contact with someone who tested positive

Therefore, the probability we are interested in would be:

$P(D^+ | T^+, C)$ where D^+ means the disease is present,
T^+ means that the test is positive
C means that the user has been in close contact with someone who tested positive

[redacted]

$$P(D^+ \mid T^+, C) = \frac{P(T^+, C \mid D^+)\,P(D^+)}{P(T^+, C)}$$

Assuming that both tests (T^+ and C) are conducted independently.

$$= \frac{P(T^+ \mid D^+)P(D^+)P(C \mid D^+)}{P(T^+, C \mid D^+)P(D^+) + P(T^+, C \mid D^-)P(D^-)}$$

$$= \frac{S_n \cdot P(D^+)P(C \mid D^+)}{S_n \cdot P(D^+)P(C \mid D^+) + (1 - S_p)P(D^-)P(C \mid D^-)}$$

Assuming that the close contact took the same test for the disease with $S_n = 100\%$ and $S_p = 99.9\%$,

$$P(D^+ \mid T^+, C) = \frac{S_n^2 P(D^+)}{S_n^2 P(D^+) + (1 - S_p)^2 P(D^-)}$$

$$= \frac{10^{-6}}{10^{-6} + (1 - 0.9999)^2 (1 - 10^{-6})}$$

$$= 0.9901 \quad (5 \text{ s.f.})$$

$$\approx 0.99 \quad (3 \text{ s.f.})$$

Given that the probability of actually having the disease now that the taker has both been tested positive and been in close contact with an individual who tested positive is about 0.99 (99%), I would take the test when I come into close contact with someone who tested positive, as the test now seems a lot more accurate.

However, since it is also possible for me to unaware of coming into contact with someone who tested positive (when there is no contact tracing), I think I would just still do the test to play safe.

If I were to be positive, I would ask my family and friends to take the test too if they ever came into close contact with me. As a precautionary measure, I would also quarantine myself for an extended period of time to be sure I am not infected (when I do not develop any symptoms) or till I recover from the disease.