Attività 1 - Regressione Lineare

es 1: φ_1 : popolazione [in 10'000 unità] } Dati , $N = 97$ città
Y : profitto [in 10'000 \$]

modello lineare

$$Y(i) = \vartheta_0 + \vartheta_1 \varphi_1 + \varepsilon$$

Funzione di costo

$$J(\bar{\vartheta}) = \frac{1}{N} \Sigma \left(Y(i) - \vartheta_0 - \vartheta_1 \varphi_1 \right)^2$$

$$J(\vartheta_0, \vartheta_1) = \frac{1}{N} \Sigma \left(Y(i) - \vartheta_0 - \vartheta_1 \cdot \varphi(i) \right)^2$$

Gradiente

$$\nabla J(\vartheta_0, \vartheta_1) = \left[\frac{dJ(\vartheta_0,\vartheta_1)}{d\vartheta_0} \quad \frac{dJ(\vartheta_0,\vartheta_1)}{d\vartheta_1} \right]^T$$

$$\frac{dJ(\vartheta_0,\vartheta_1)}{d\vartheta_0} = \frac{2}{N} \Sigma \left(Y(i) - \vartheta_0 - \vartheta_1 \cdot \varphi_1(i) \right) \cdot (-1) = -\frac{2}{N} \Sigma \left(Y(i) - X\bar{\vartheta} \right)$$

$$\frac{dJ(\vartheta_0,\vartheta_1)}{d\vartheta_1} = \frac{2}{N} \Sigma \left(Y(i) - \vartheta_0 - \vartheta_1 \cdot \varphi_1(i) \right) \cdot \left(-\varphi(i) \right) = -\frac{2}{N} X(:,2)^T (Y - X\bar{\vartheta})$$

con $X = \begin{bmatrix} 1 & \varphi_1(1) \\ \vdots & \vdots \\ 1 & \varphi_1(N) \end{bmatrix}$ e $\bar{\vartheta} = [\vartheta_0 \ \vartheta_1]$ e $Y = \begin{bmatrix} Y_1 \\ \vdots \\ Y_N \end{bmatrix}$

stima

$$\hat{\vartheta}^{(k+1)} = \hat{\vartheta}^{(k)} - \alpha \cdot \nabla J(\bar{\vartheta}) \big|_{\vartheta = \hat{\vartheta}^{(k)}}$$

con α = learning rate

Gradient descent ... in MatLab

? quanto vale la stima del vettore $\hat{\vartheta}$ $\longrightarrow$ $\hat{\vartheta}_0 = -3{,}87$, $\hat{\vartheta}_1 = 1,$

? qual è il profitto predetto per population = 35 $\longrightarrow \hat{Y}(\varphi_1 = 35) = +2912{,}76$ \$

Compute cost (Funzione di costo)

$$J(\bar{\vartheta}) = \frac{1}{N} \Sigma \left(Y(i) - \vartheta_0 - \vartheta_1 \cdot \varphi(i) \right)^2 = \frac{1}{N} (Y - X\bar{\vartheta})^T (Y - X\bar{\vartheta})$$

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Running Gradient Descent ...
Theta found by gradient descent: -3.878138 1.191261
For population = 35,000, we predict a profit of 2912.764904
Visualizing J(theta_0, theta_1) ...
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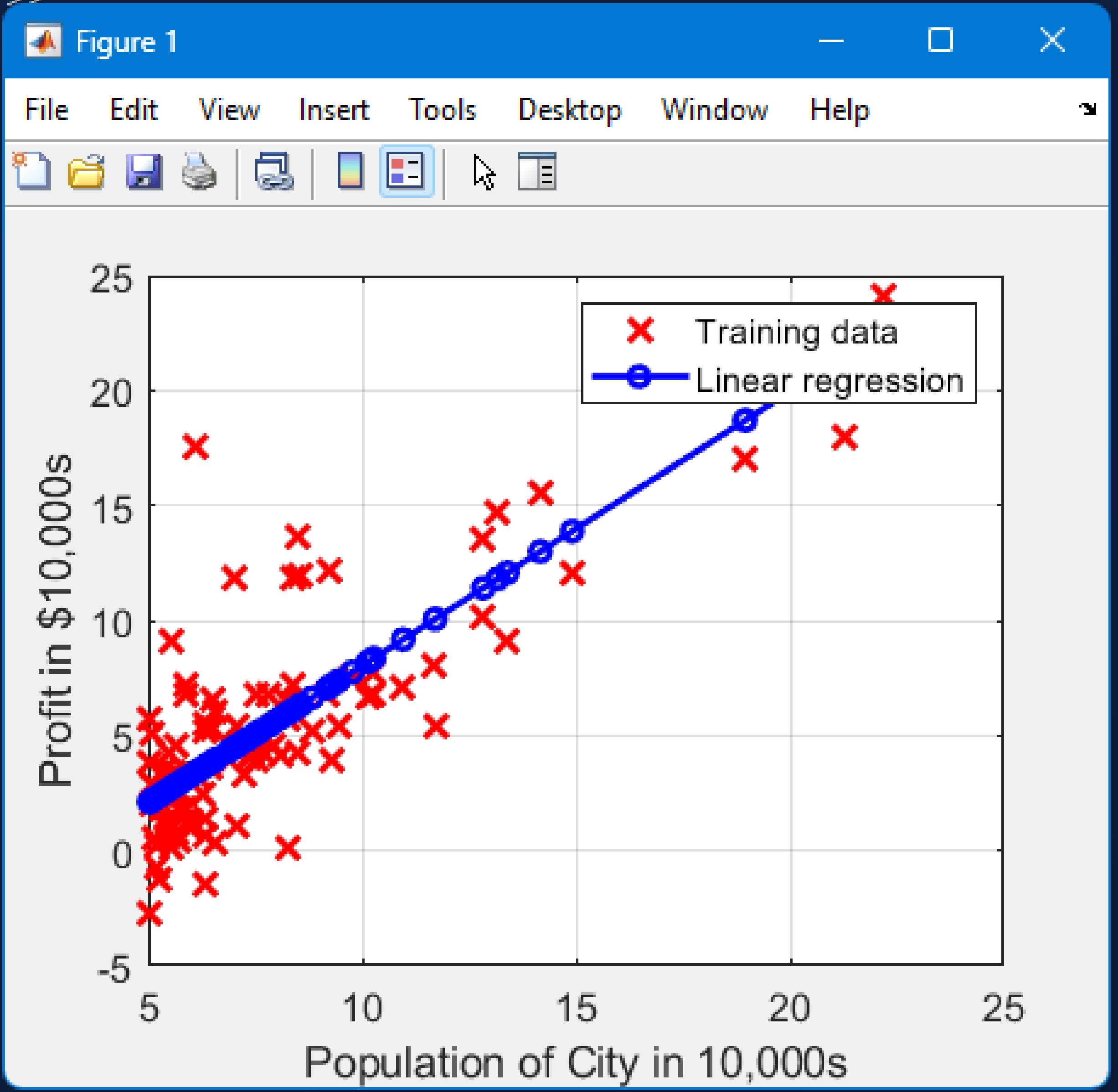

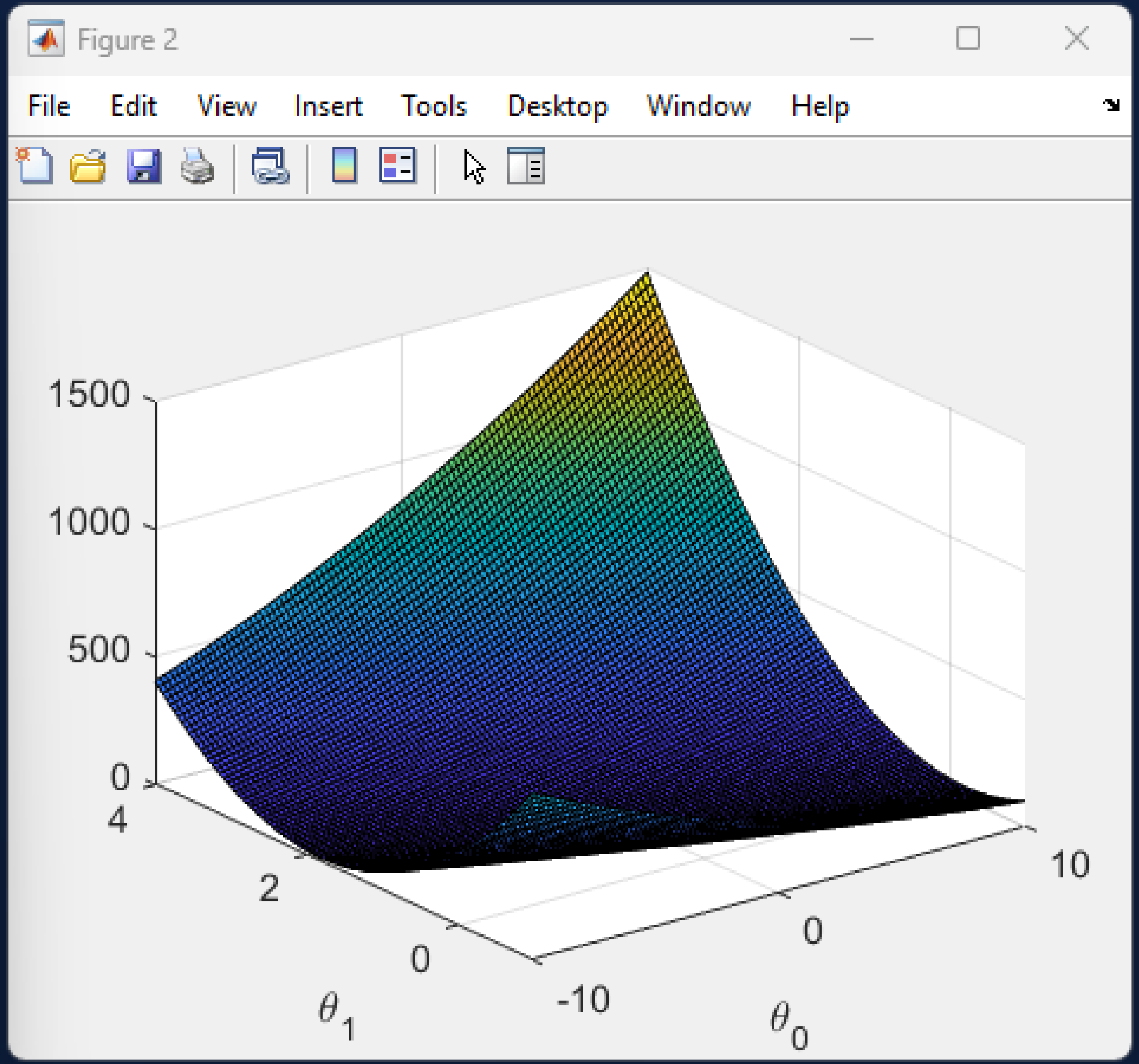

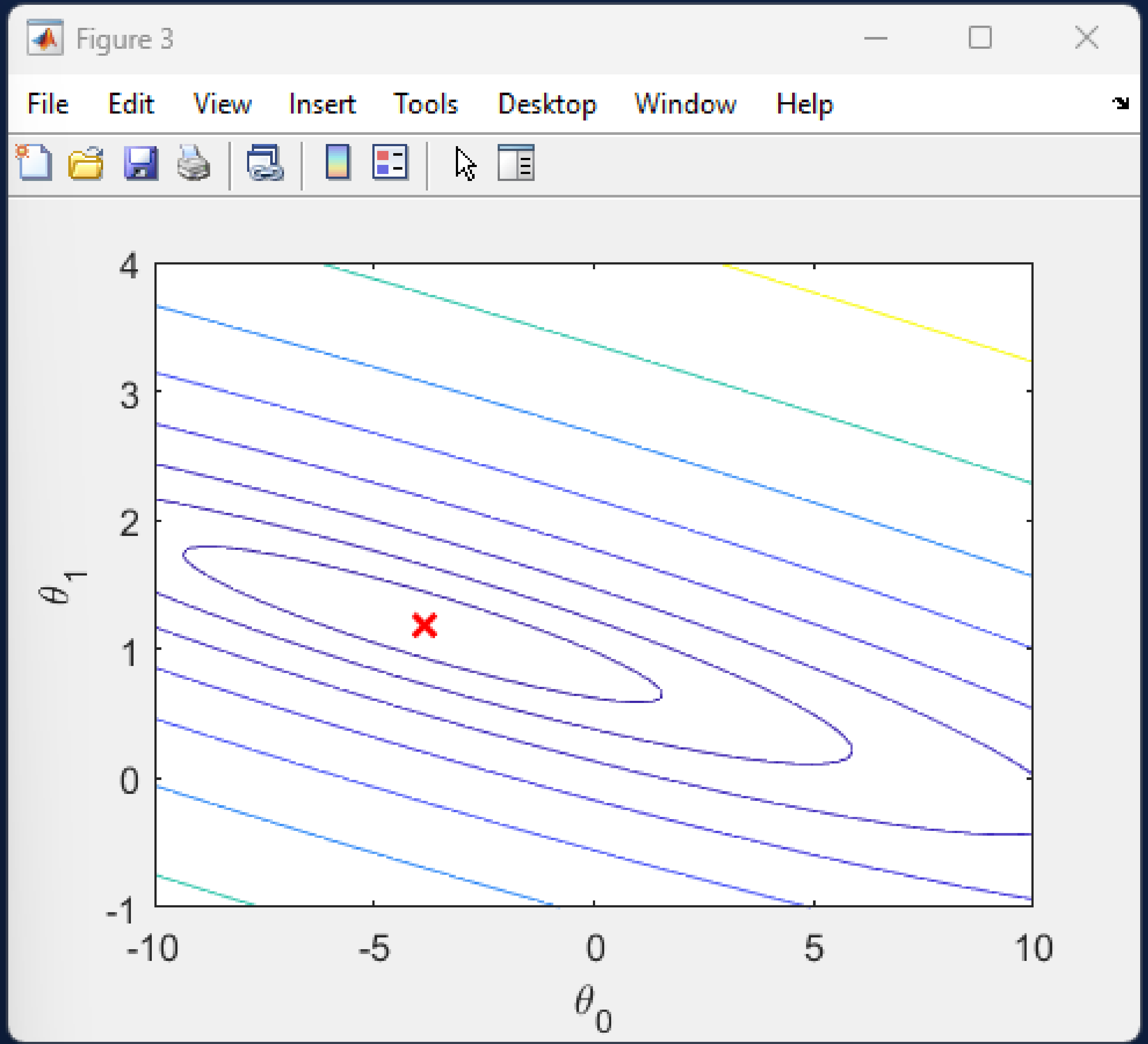

es 2 : φ_1 : Area [Feet^2]

φ_2: Numero di camere da letto

y : prezzo [\$]

modello lineare

$$y(i) = \theta_0 + \theta_1 \varphi_1 + \theta_2 \varphi_2 + \varepsilon = \bar{\varphi}^T(i)\theta + \varepsilon(i)$$

con $\bar{\varphi}(i) = [1 \quad \varphi_1(i) \quad \varphi_2(i)]^T$, $X = \begin{bmatrix} \bar{\varphi}^T(1) \\ \bar{\varphi}^T(2) \\ \bar{\varphi}^T(N) \end{bmatrix}$, $\theta = \begin{bmatrix} \theta_0 \\ \theta_1 \\ \theta_2 \end{bmatrix}$, $Y = \begin{bmatrix} y(1) \\ y(2) \\ \vdots \\ y(47) \end{bmatrix}$ ε_N

Normal equation

pseudo inversa

$$\hat{\theta} = (X^T X)^{-1} X^T Y \xrightarrow{\text{MATLAB}} \text{theta\_hat} = \text{pinv}(X' * X) * X' * y$$

Gradient Descent :

Features normalize

$$\varphi_j(i) = \frac{\varphi_j(i) - \hat{\mu}_j}{\sqrt{\hat{\sigma}_j^2}} \quad j = 1, \dots, d-1$$

$$\hat{\mu}_j = \frac{1}{N} \sum_{i=1}^{N} \varphi_j(i) \quad j = 1, \dots, d-1$$

$$\sigma_j^2 = \frac{1}{N} \sum_{i=1}^{N} (x_j(i) - \mu_j)^2 \quad j = 1, \dots, d-1$$

Gradiente

$$\nabla J(\bar{\theta}) = -\frac{2}{N} \cdot X^T \cdot (Y - X\bar{\theta})$$

stima

$$\hat{\theta}^{(k+1)} = \theta^{(k)} - \alpha \nabla J(\bar{\theta}) \big|_{\theta = \theta^{(k)}}$$

Compute cost

$$J(\theta) = \frac{1}{N} (Y - X\bar{\theta})^T (Y - X\bar{\theta})$$

? Quanto vale la stima del vettore $\hat{\theta}$ $\rightarrow$ $\hat{\theta}_0 = 340398{,}69$ $\hat{\theta}_1 = 109855{,}30$ $\hat{\theta}_2 = -5873{,}74$

? Qual è il profitto predetto per una casa con area di 1650 sq-ft e 3 camer

ricordarsi di normalizzare i dati $\rightarrow$ ([1650 3] - μ) ./ sigma

$\hookrightarrow \hat{y} = \$\,293236{,}30$

$\rightarrow$ From normal equations : $\hat{\theta}_0 = 89597{,}91$ $\hat{\theta}_1 = 139{,}21$ $\hat{\theta}_2 = -8738{,}02$

$\rightarrow$ From normal equations : $\hat{y} = \$\,293081{,}46$ ✓

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First 10 examples from the dataset:
 x = [2104 3], y = 399900
 x = [1600 3], y = 329900
 x = [2400 3], y = 369000
 x = [1416 2], y = 232000
 x = [3000 4], y = 539900
 x = [1985 4], y = 299900
 x = [1534 3], y = 314900
 x = [1427 3], y = 198999
 x = [1380 3], y = 212000
 x = [1494 3], y = 242500
Theta computed from the normal equations:
 89597.909543
 139.210674
 -8738.019112

Predicted price of a 1650 sq-ft, 3 br house (using normal equations):
 $293081.464335



Normalizing Features ...
Running gradient descent ...
Theta computed from gradient descent:
 340398.694491
 109855.300268
 -5873.743442

Predicted price of a 1650 sq-ft, 3 br house (using gradient descent):
 $293236.305203
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