Lecture 5: Application: Gravity with Gravitas

Instructor: Paul Grieco   *Date: September 12/19*

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Up to now, we have focused on a few basic tools of numerical analysis namely methods for solving linear and nonlinear equations. This week, we'll get a chance to see how some of these tools can be used in economic analysis. The paper we will use is:

> Anderson, James and van Wincoop, Eric, Gravity with Gravitas: A Solution to the Border Puzzle, *American Economic Review*, 2003, 93(1): 170-192.

It is available in the lecture notes repository. While it's been around a while, it does a nice job of illustrating the value added of being able to numerically solve systems of equations.

5.1 Motivation: Gravity Equation and the Border Puzzle

A "gravity equation" in trade relates bilateral trade flows to GDP, distance and factors which proxy for trade barriers. A standard gravity equation might look like:

$$\ln x_{ij} = \alpha_1 + \alpha_2 \ln y_i + \alpha_3 \ln y_j + \alpha_4 \ln d_{ij} + \alpha_5 \delta_{ij} + \epsilon_{ij} \tag{5.1}$$

Where,

- x_{ij} is exports from i to j, these may be countries or "regions".

- y_i, y_j are GDP for regions i and j respectively.

- d_{ij} is a measure of distance between i and j.

- δ_{ij} are proxies for trade frictions between i and j.

The intuition for for this regression comes not directly from economic theory, but from the physics law of gravity, hence its name. That said, the regression is extremely good at fitting observed trade patterns.

A paper by McCallum (1995, *AER*), looked at interstate/provincial trade between the United States and Canada. The δ_{ij} was whether the trade had to cross the US-Canada border.

- In McCallums case, this regression fits the data well in the sense that R^2 is around .7 to .8.

- This paper found a substantial "border effect." Trade between Canadian provinces was 22 times higher (2200%) than trade between Canadian provinces and US states after controlling for size and distance.

- This is despite the fact that the US and Canada are in a free trade agreement, share the same language (mostly) and were (at that time, at least) close allies, so where is this big trade barrier coming from?

- The "border puzzle" became one of the great puzzles of international economics, according to Obsfeld and Rogoff (2001).

- Similar studies found similarly high border effects elsewhere (see footnote 3 for a list).

5.2   Explaining the Border Puzzle with Theory

Anderson and van Wincoop offer a succinct explanation for the border puzzle:

1. The empirical model McCallum used is not supported by trade theory.

2. When you work through the theory, you would find that trade flows are determined not by absolute trade costs but by trade costs *relative* to trade costs with all other partners. In other words, outside options matter, and we need a general equilibrium model to account for them.

3. Anderson and van Wincoop work through this theory and find that a consistent gravity equation would include "multilateral resistance" terms that account for the average trade costs between, say, Manitoba, and the rest of the world. This shows that the traditional gravity model equations suffer from endogeneity due to omitted variables.

4. Moreover, the Anderson and van Wincoop model can be used to perform general equilibrium counterfactual analysis to see how flows change when the border is "eliminated" that will account for price changes.

However, Anderson and van Wincoop's model cannot be estimated with OLS.[1]

- Trade costs between New York and Ohio will affect the size of trade between New York and Ontario due to multilateral resistance.

- But trade costs are exactly what we are trying to estimate!

- For a given set of trade cost parameters, they show how to calculate multilateral resistance by solving a system of nonlinear equations.

- Then they can embed this solution into a nonlinear least squares estimation of trade costs.

5.3   Deriving Gravity from a General Equilibrium Model of Trade

Assumptions:

- Each region produces only one good.

- The supply of goods from each region is a fixed endowment.

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[1]There is an "easy" solution, pursued by Helliwell. Since multilateral resitance is due to average trade costs in region i it can be controlled for with region fixed effects. Anderson van Wincoop point out that while this gives consistent estimates of trade costs, it can't be used to do counterfactual analysis since the resistence terms are a function of trade costs, and will change when barriers are removed.

- Goods are differentiated by origin, all goods are substitutable according to a single, constant elasticity of substitution. Consumers in region j maximize their utility:

$$\left(\sum_i \beta_i^{(1-\sigma)/\sigma} c_{ij}^{(\sigma-1)/\sigma} \right)^{\sigma/(\sigma-1)}$$

  subject to the budget constraint:

$$\sum_i p_{ij} c_{ij} = y_j$$

  - β_i is an exogenous preference parameter, σ is the elasticity of substitution.
  - y_j is j's income (exogenous).
  - c_{ij} is units consumption of i's product by j.
  - p_{ij} is the price of a unit of consumption of i's product in country j.

- Prices differ across regions due to trade costs. If p_i is the exporter supply price, then people in region j must pay

$$p_{ij} = p_i t_{ij}$$

  where t_{ij} is the trade cost factor (exogenous). You can think of this as perfect competition in the transport market where the importer pays trade costs to the exporter.

- Therefore, the value of trade flows is $x_{ij} = p_{ij} c_{ij}$ and total income is $y_i = \sum_j x_{ij}$.

Optimal behavior of consumers (taking prices as given) gives rise to nominal demand (in dollars):

$$x_{ij} = \left(\frac{\beta_i p_i t_{ij}}{P_j} \right)^{1-\sigma} y_j$$

Where P_j is just the CES price index:

$$P_j = \left[\sum_i (\beta_i p_i t_{ij})^{1-\sigma} \right]^{1/(1-\sigma)}$$

The key thing to notice is that the price index in country j, and hence flows x_{ij} depends on *all* of j's trading relationships, not just the barriers between i and j.

By imposing the market clearing condition, $y_i = \sum_j x_{ij}$, and assuming that trade costs are symmetric, $t_{ij} = t_{ji}$ it is possible to solve for "scaled" supply prices, $\beta_i p_i$, then demand can be written,

$$x_{ij} = \frac{y_i y_j}{y^W} \left(\frac{t_{ij}}{P_i P_j} \right)^{1-\sigma}$$

where y^W is world income, using the same substitution we get a system of equations for the price indices:

$$P_j^{1-\sigma} = \sum_i P_i^{\sigma-1} \theta_i t_{ij}^{1-\sigma} \tag{5.2}$$

where $\theta_i = y_i/y^W$, and hence, data. Therefore, given trade cost values, this represents a system of equations that can be solved for price indices in every country, which can then be used to compute implied trade flows.

Now parameterize trade costs as

$$t_{ij} = b_{ij} d_{ij}^{\rho}$$

where b_{ij} is a "border effect" that is 1 if the two regions are in the same country and a parameter to be estimated otherwise. Distance between regions is d_{ij} with ρ a distance cost parameter. Taking logs we get the theoretically founded gravity equation:

$$\ln x_{ij} = k + \ln y_i + \ln y_j + (1 - \sigma)\rho \ln d_{ij} + (1 - \sigma) \ln b_{ij} - (1 - \sigma) \ln P_i - (1 - \sigma) P_j$$

The two price indices are the omitted variables from the "traditional" gravity equation, (5.1). We can see from (5.2) that they are clearly correlated with the trade cost variables, and hence introducing endogeneity bias to OLS estimation of (5.1). The other difference is unitary income elasticities, although this can be relaxed by introducing a non-traded good.

5.4   Estimation

Estimate the gravity equation, we'll move the GDP variables to the left hand side and define,

$$\ln z_{ij} \equiv \ln\left(\frac{x_{ij}}{y_i y_j}\right)$$

We'll (conveniently) assume that the z_{ij} we observe is contaminated with measurement error that is uncorrelated with all the explanatory variables. Then the gravity model implies:

$$\ln z_{ij} = \underbrace{k + a_1 \ln d_{ij} + a_1(1 - \delta_{ij}) - \ln P_i^{1-\sigma} - \ln P_j^{1-\sigma}}_{h_{ij}(\cdot)} + \varepsilon_{ij}$$

Where $a_1 = (1 - \sigma)\rho$ and $a_2 = (1 - \sigma)\ln b$. Assume we know σ ("the literature" says it is 5), then we can estimate this using OLS if we just knew P for all regions. Instead, we can solve for P using the N equations:

$$P_j^{1-\sigma} = \sum_i P_i^{\sigma-1} \theta_i e^{a_1 \ln d_{ij} + a_1(1-\delta_{ij})} \tag{5.3}$$

Then plug the result in and compute the implied ε, which is needed for a least squares objective function. We can think of this as a nonlinear least squares problem arising from:

$$\ln z = h(d, \delta, \theta; k, a_1, a_2) + \varepsilon$$

Where this equation is a vector of region pairs and $h(\cdot)$ is the stacked vector of h_{ij} which is of course a function of the full vector of data inputs.

The problem can also be expressed as a constrained optimization:

$$\min_{k,a_1,a_2} \sum_i \sum_{j \neq i} \left[\ln z_{ij} - k - a_1(1 - \delta_{ij}) + \ln P_i^{1-\sigma} + \ln P_j^{1-\sigma}\right]^2 \tag{5.4}$$

subject to (5.3) for all j.

While we could implement this directly. We're instead going to adopt the following algorithm to showcase nonlinear equation solving:

1. Guess a value for $P^{(0)}$, set $t = 0$.

2. While $|P^{(t)} - P^{(t-1)}| > \epsilon$

   (a) Increment t.

   (b) Solve the least squares problem (5.4) for (k, a_1, a_2).

   (c) Given (k, a_1, a_2), solve the nonlinear equations (5.3) for $P^{(t)}$ using Broyden's method.

3. Return estimates (k, a_1, a_2).

This will terminate at a solution to the least squares problem when holding P fixed that satisfies the constraints.

Later we will consider alternative approaches:

- Nested fixed point would explicitly solve for P before computing the objective function. This would directly take into account that $\partial h / \partial a$ includes a $\partial P / \partial a$ term.

- Direct constrained optimization would utilize a Lagrangian.

5.5   Now let's go through the code...

- Some quick examples of data cleaning.

- Makes use of the Comp Econ toolbox of Miranda and Fackler.

- Uses MATLAB optimization toolbox command `lsqnonlin`

- Estimates trade costs for a 70 region model in a few seconds.