Fast-running model to predict concussion probability using head acceleration or strain implemented and validated

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Introduction

Chronic traumatic encephalopathy, or CTE, is a progressive neurodegenerative condition often caused by repeated forces to the head resulting in high accelerations of the brain [1]. Concern around this issue in the professional football industry has recently grown as over 100 former NFL players have received a postmortem diagnosis of CTE [2]. The brain damage incurred from repetitive impacts and multiple concussions is much greater than that incurred from a single impact [3]; thus, it would be beneficial to have a system to alert players that they may have sustained a single concussion to remove them from gameplay immediately and allow them to receive further advice from a medical professional. This system would require translating kinematic data, such as acceleration, that could be collected from a transducer on the player's helmet into a concussion risk. Data has been collected in a previous study that correlated linear and rotational acceleration data to concussion in football players via video analysis, and a curve-fit of acceleration to concussion probability was determined from this data [4]. A multi-component model has also been developed to translate acceleration data into probability of concussion, which includes a human head finite element analysis model, a micromechanics model, an axon signaling model, a neurologic injury model, and a dose-response model [5]. These models are used in series, with the output of one model used as the input to the next. A simplifying assumption to neglect linear acceleration is used in the model as rotational acceleration oftentimes has a much greater effect on probability of concussion than does linear acceleration [5].

The goal of this study is to validate this model against the curve-fit of experimental data to determine if the model predicts concussion probability based upon kinematic data with sufficient accuracy. This validation will include a Monte Carlo simulation to determine the relative importance of neglecting linear acceleration to error in the model to determine if it is a reasonable assumption to neglect linear acceleration. This will allow the selection of a type of sensor used to predict concussion probability within an alert system to be defended with statistical analysis. For example, if the model and curve fit are shown to predict similar results, a strain gauge may be selected, inputting strain into the axon signaling model to use the multi-component model to predict concussion probability. If, however, linear acceleration is shown to be significant or the models do not produce similar results, accelerometers and gyroscopes would have to be used as the transducer to input data into the experimentally-derived curve fit to yield concussion probability results. Therefore, deliverables for this model include a MATLAB implementation of the multi-component rotational acceleration to concussion probability model, a MATLAB implementation of the acceleration to concussion probability curve-fit, and a Monte Carlo simulation. The Monte Carlo simulation outputs the mean and standard deviation of percent difference between the model and curve-fit as well as a correlation coefficient relating linear acceleration to concussion probability. These metrics aid in validating the model and the assumption of neglecting linear acceleration in determining concussion probability, respectively.

Methods

A. Curve Fit

A curve fit derived from a correlation of accelerations obtained by video analysis to concussion in football players [4] was implemented in MATLAB as in Equation 1, where a is maximum linear acceleration, α is maximum rotational acceleration, and CP is concussion probability.

$$CP = \frac{1}{1 + \exp\left[-(-10.2 + 0.0433a + 0.000873\alpha - 0.000000920a\alpha)\right]} \qquad (\textbf{\textit{Equation}}\ \mathbf{1})$$

B. Model Implementation

A simplified multi-step series model translating accelerations to concussion probability [5] was also implemented in MATLAB. The first component of the model is a tissue response model to obtain strains from accelerations. This component is a simplification of an FEM model intended to increase the speed and simplicity of the model, and was implemented as shown in Equation 2, where ω_p is maximum angular velocity, $\alpha(t)$ is the time-dependent angular acceleration, and ε is axial strain.

$$a\ddot{\varepsilon} + \left(b + c\omega_p\right)\dot{\varepsilon} + d\varepsilon = -\alpha(t) \qquad (\textbf{\textit{Equation}}\ \mathbf{2})$$

The *ode45* function in MATLAB was used to solve the second order differential equation with initial conditions $\varepsilon(0) = 0$ and $\varepsilon'(0) = 0$ (which assumes no axial strain or time rate of change in strain at the time of impact). The parameters a, b, c, and d were given in the paper by Phohomsiri et al. as 3.3, 250, -2.2, and 74800 for x-axis rotation and -3.0, -230, 3.6, and -67320 for y-axis rotation, respectively [5].

The second component of the model is a micromechanics model, which translates the axial strain obtained in the previous step into strain at the Nodes of Ranvier. The micromechanical behavior can be modeled as viscoelastic as shown in Figure 1 [6].

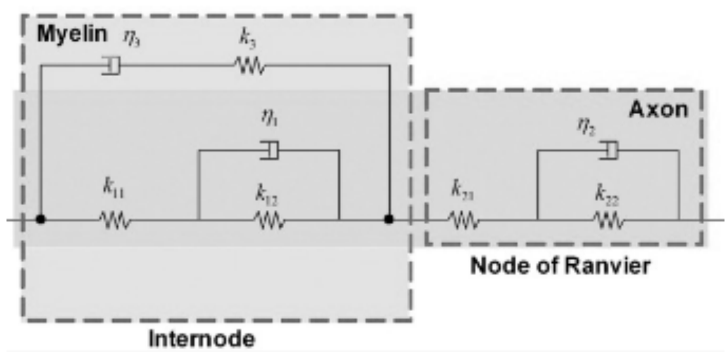


Figure 1: Micromechanical behavior of an axon modeled as a viscoelastic system. Spring and damping constants were determined using the material properties (elastic modulus and viscosity, respectively) of a dorsal root ganglion neuron [6].

Equation 3 was used to determine spring and damping constants, where elastic moduli $E_1 = 19.9$ kPa, $E_2 = 0.42$ kPa, and $E_3 = 50$ kPa, and viscosities $\eta_1 = 2.256$ MPa/s and $\eta_3 = 1$ kPa/s. The internode length used was 125 microns (this length varies between 50 and 200 microns) and the node length was 1 micron. Cross sectional areas of the node, internode, and myelin, respectively, were assumed to be 7.85e-11, 7.85e-11, and 7.54e-11 meters [6].

$$k_{11} = \frac{E_1 A_{\text{internode}}}{L_{\text{internode}}},\quad k_{12} = \frac{E_2 A_{\text{internode}}}{L_{\text{internode}}},\quad \gamma_1 = \frac{\eta_1 A_{\text{internode}}}{L_{\text{internode}}} \qquad \textbf{(\textit{Equation 3a})}$$

$$k_{21} = \frac{E_1 A_{\text{node}}}{L_{\text{node}}},\quad k_{22} = \frac{E_2 A_{\text{node}}}{L_{\text{node}}},\quad \gamma_2 = \frac{\eta_1 A_{\text{node}}}{L_{\text{node}}} \qquad \textbf{(\textit{Equation 3b})}$$

$$k_{31} = \frac{E_3 A_{\text{myelin}}}{L_{\text{internode}}},\quad \gamma_3 = \frac{\eta_3 A_{\text{myelin}}}{L_{\text{internode}}} \qquad \textbf{(\textit{Equation 3c})}$$

A system of equations was derived using the viscoelastic model (derivation shown in Appendix A) to solve for the strain at the output of the Node of Ranvier, using the known axial strain as the input at the internode. Three first order ordinary differential equations were solved using the *ode45* function in MATLAB in order to obtain strains at each of the dampers. The system was then solved for the output strain using the results from the *ode45* solver with the initial conditions that the strain at each damper at time zero was zero.

The factor of reduction in action potential amplitude, ΔA, was then calculated using the maximum strain at the Node of Ranvier found within the impact duration, ε_{NR}, as in Equation 4 [5].

$$\Delta A = \begin{cases} 1.5826\varepsilon_{NR}^2 + 0.2138\varepsilon_{NR}, & \varepsilon_{NR} < 0.73 \\ 1, & \varepsilon_{NR} > 0.73 \end{cases} \qquad (\boldsymbol{Equation\ 4})$$

A dose-response curve determined in previous experiments [5] was then used to predict concussion probability, CP, as in Equation 5 using the reduction in action potential magnitude determined in the previous step.

$$CP = \frac{\exp(x)}{1 + \exp(x)},\qquad x = 7.6182 + 2.4587w,\qquad w = \ln\left(\frac{\Delta A}{1 - \Delta A}\right) \qquad (\boldsymbol{Equation\ 5})$$

C. Data Simulation

Kinematics were derived using a Monte Carlo simulation from means and standard deviations of data obtained in a study that used an in-helmet system with six accelerometers to collect data on linear and rotational accelerations in eight football players who incurred a total of 347 impacts during one game [7]. Maximum linear accelerations were measured at 21.5 ± 19.7g, maximum rotation about the x-axis (backward and downward from the center of mass of the head) 769.9 ± 1082.7 rad/s^2, and maximum rotation about the y-axis (forward from the center of mass of the head) 1382.8 ± 1547.3 rad/s^2 [7]. Rotation about the z-axis is generally considered insignificant for impacts in football studies due to probable angles of impact. Impact duration (duration of positive acceleration) was found to be 6 ± 2 milliseconds [7]. Probability density functions (PDFs) were used in to create a normal distribution for each variable, and a set of 1000 randomly chosen values from each PDF was chosen to represent the variables for the simulated data points (N=1000). An assumption was made that the rotational acceleration about each of the x and y axes can be represented as a scalar multiple of linear acceleration in order to minimize variation in the results and more accurately represent a real impact; thus, the values for angular acceleration were generated based upon scaling by the mean experimentally derived values. A histogram of angular acceleration magnitude used in the study is shown in Figure 2.

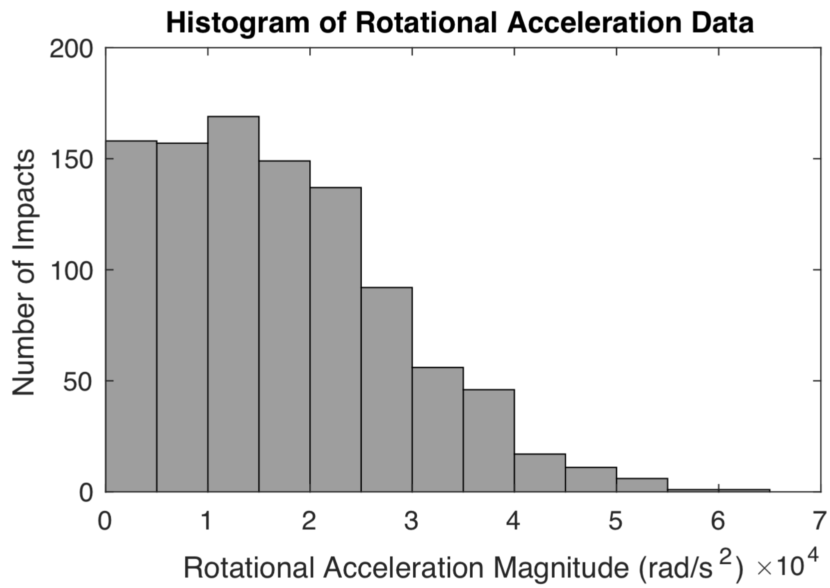


Figure 2: Histogram of angular acceleration magnitudes generated for the study. Data was randomly generated and scaled based on the assumption that rotational acceleration can be estimated as a scalar multiple of linear acceleration in a given impact. Since negative values were produced by the PDF due to high standard deviations, the absolute value of accelerations was taken to produce all positive acceleration values to input to the model, which is reflected in the skewness of the histogram.

Additionally, a head acceleration shape function representative of kinematics from the collected data, shown in Figure 3, was used as a multiplier for the maximum acceleration in order to generate accelerations as a function of time as indicated in the study by Phohomsiri et al. [5]. Although duration of positive acceleration was given as 6 ± 2 milliseconds by the experimental data, a constant duration of 6 milliseconds was used as a control to minimize variability in the results.

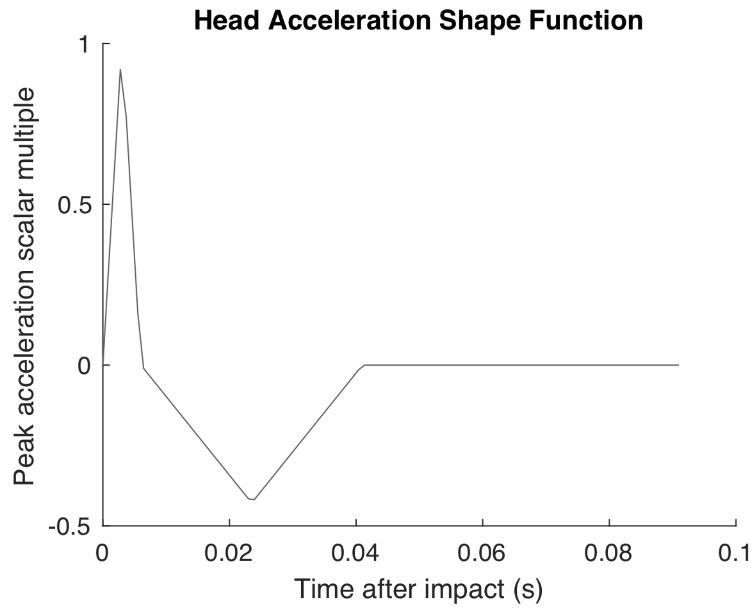


Figure 3: Head acceleration shape function was used as a multiplier for maximum acceleration to generate a time function. The positive impact duration (a constant 6 ms in this simulation) and maximum angular acceleration of an impact were input into the shape function along with a constant negative impact duration of 35 ms and subsequent zero acceleration period of 85 ms in order to generate angular acceleration as a function of time.

D. Model Validation Methods

i. Comparison to Curve Fit

Percent difference in concussion probability was determined in MATLAB for each simulated data point between the curve fit method and model method. Mean and standard deviation of percent difference in concussion probability were calculated for the data set (N = 1000) in order to determine the validity of the model. A paired two-sample t-test was run between the curve fit and model to determine if there was significant difference between the curves. Additionally, a root mean square error analysis was performed between the experimental fit and model curves to determine the degree of similarity between the curves.

ii. Introduction of Noise and Propagation of Error

Error was introduced which simulated data collection from an accelerometer with 95% accuracy. Concussion probabilities for each of 1000 data points using the "noisy" acceleration data were collected and compared against the original model and curve fit outputs. Noise was introduced by scaling acceleration values for each data point with a random number between 0.95 and 1.05, or ± 5%. A root mean square error analysis was performed between the original and noisy data for both the model and curve fit in order to determine the error propagation introduced by noise in acceleration data readings.

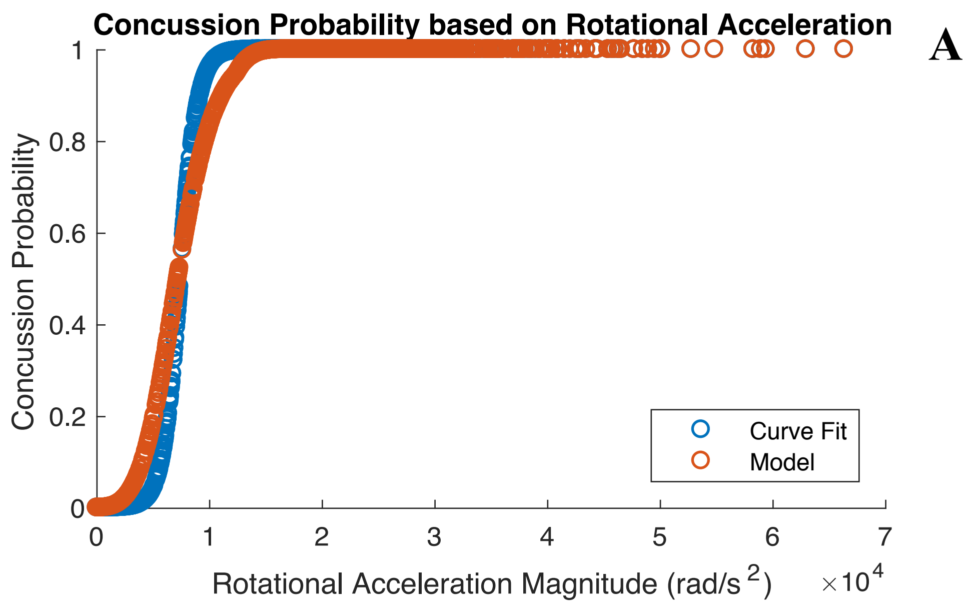
iii. Timing

The runtime both the model and curve fit were determined using the *tic* and *toc* timing functions for 1000 data points. Additionally, a simulation was performed to time the model for 1 – 30 data points, in increments of 1. A linear fit was used to describe run time as a function of the number of data points. This fit was then used to determine maximum sampling frequency for the model to run in real-time by setting the time equal to one second and solving for number of data points.

A link to MATLAB scripts for the curve fit, model, data generation, validation, and analysis is provided in Appendix B.

Results

A plot of concussion probability based on the resultant magnitude of angular acceleration for both the experimental curve fit and model is shown in Figure 4, along with a plot of residuals between the two curves as a function of acceleration. A paired two-sample t-test between the two sets of generated concussion probabilities (via experimental curve fit and model, respectively) resulted in a p-value of 0.88, which fails to reject the null hypothesis that the curves are statistically similar. Additionally, the root mean square error was found to be 0.064 between the two curves, which is 6.4% of the range of possible output values (zero to one).



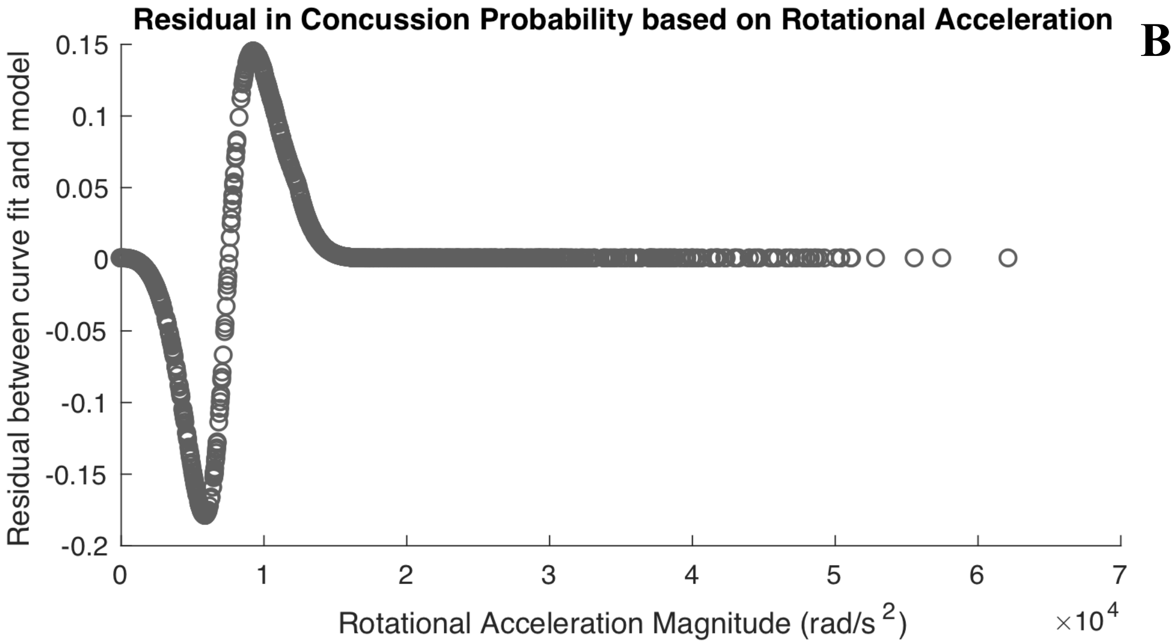


Figure 4: Concussion probability based on rotational acceleration magnitude from generated data for curve fit and model. (A) 1000 samples randomly generated from probability density functions were used to calculate concussion probability based upon a model and curve fit. (B) Residuals from this data plotted as a function of acceleration show a higher degree of correlation between the curves for very high and low accelerations and a varying degree of correlation for accelerations resulting in concussion probabilities in between 0 and 1. Average RMSE (root mean square error) between the two series for this simulation was 0.064, or 6.4% of the possible output range, and maximum RMSE was 0.18, or 18%.

A plot of concussion probability based on the resultant magnitude of angular acceleration for the original model and curve fit results along with the results with ± 5% error introduction in acceleration is shown in Figure 5. RMS error in the result between the original and noisy data for was 0.016, or 1.6% for the curve fit and 0.012, or 1.2% for the model.

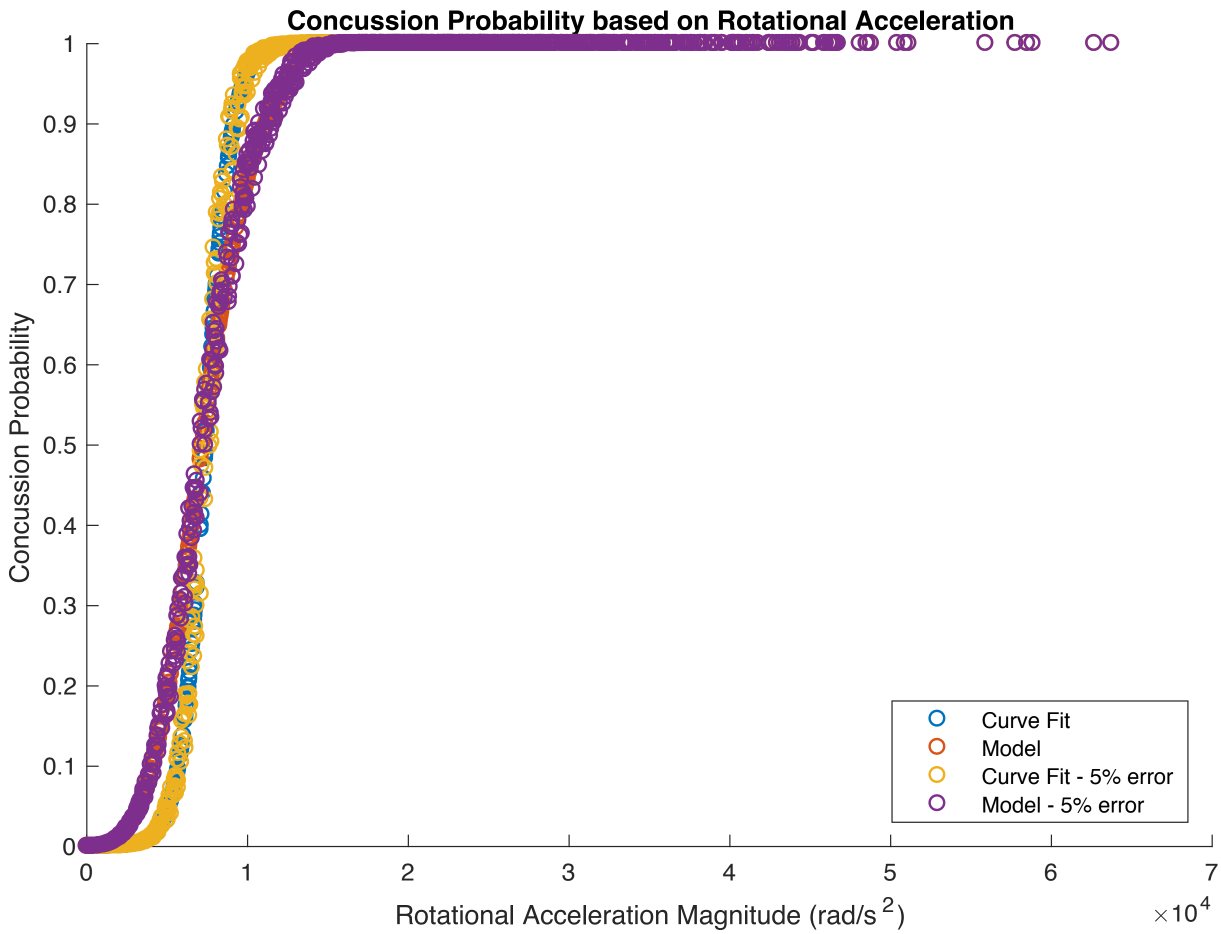


Figure 5: Concussion probability based on rotational acceleration magnitude for curve fit and model with ± 5% acceleration noise introduced. Error was randomly introduced within ± 5% by scaling the accelerations at each data point by a random number between 0.95 and 1.05. RMSE between the original and "noisy" data in the resulting concussion probabilities was 1.6% for the curve fit and 1.2% for the model.

The runtime of the model for 1000 data points was 104 seconds, and for the curve fit was 10.4 milliseconds. Figure 6 shows a plot of runtime based on dataset size for dataset sizes of 1 to 30 in increments of 1 for the model and curve fit. A linear fit was determined as shown in Equation 6 for small data sizes, which was used to calculate a maximum sampling frequency of 7.5 Hz.

$$Model\ Time\ (s) = 0.063 + 0.53(dataset\ size) \qquad (\boldsymbol{Equation\ 6})$$

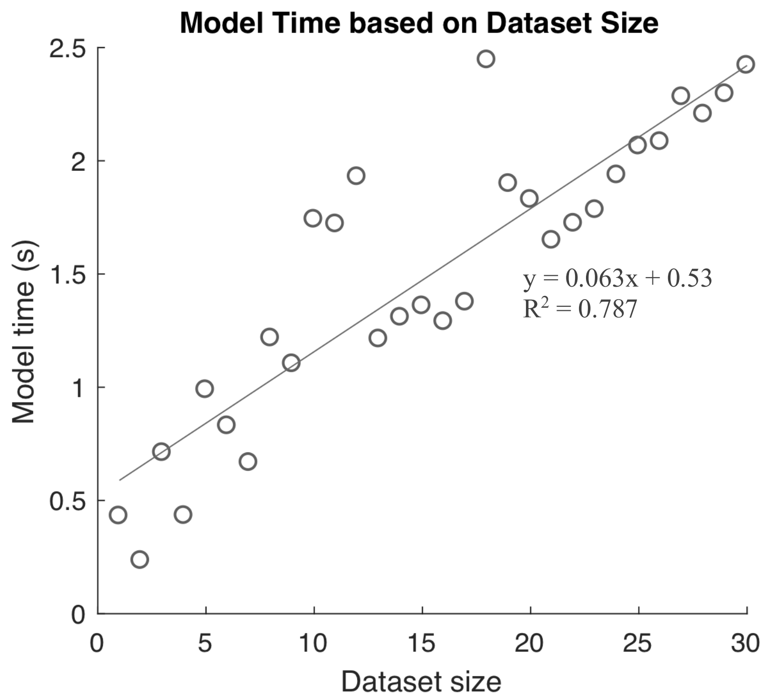


Figure 6: Runtime of model and curve fit for various dataset sizes. The model and curve fit were timed using the *tic* and *toc* functions in MATLAB in order to generate the timing data. Results of timing for dataset sizes of 1 to 30 in increments of 1 were plotted as a function of data size, and *polyfit* and *polyval* were used to determine a linear fit of y = 0.063x + 0.53, with an R^2 value of 0.787.

Discussion

The multi-component model accuracy, error propagation, and run time were tested in this study and compared to those of the experimentally derived curve fit. The curve fit and model were found to produce different curves to correlate acceleration data to correlate acceleration data to probability of concussion as seen in Figure 4. Although the root mean square error (RMSE) between the model and curve fit was relatively low at 6.4% of the possible range of output values, there was a distinct pattern in the error between the models, shown by Figure 4b. This showed that the models were well-correlated for very low or high rotational acceleration magnitudes but varied in degree of correlation up to 15% error between the model and curve fit for rotational acceleration magnitudes around 1e4 ± 1e4 rad/s^2. This correlation may still be sufficient for some applications, however, given that the application for this model would likely be in concussion indication as opposed to concussion diagnosis, which must be performed by a medical professional. If indication is made in 33% increments (concussion probability of 0-33% = low risk, 34-66% = medium risk, 67-100% = high risk), 15% error would be less than half of the increment size.

The simulation performed to show error propagation given noise in acceleration data within 5% of the original acceleration value showed relatively low RMS error values in the resulting concussion probabilities for both the model and curve fit – 1.6% of the possible output range for the curve fit and 1.2% for the model. This shows that the model is slightly less sensitive to noise in data than the curve fit.

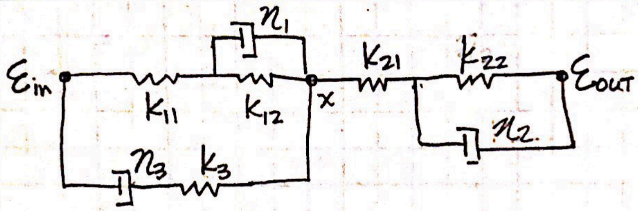
The timing simulation showed that run time for the model was relatively linear (with a linear fit given by Equation 6) and was approximately five orders of magnitude greater than the run time of the curve fit. The increase in computation time between the curve fit and the model is likely due to the mathematical complexity of the model, which requires solving two systems of ordinary differential equations, creating an acceleration time function and performing many simple calculations, while the curve fit only performs a single calculation. The impact of the difference in computation time between the model and curve fit is in the sampling frequency if computations are to be made in real-time. The maximum sampling frequency for the model is 7.5 Hz, while that of the curve fit is orders of magnitude greater. The average duration of a concussive impact is 6 milliseconds [5], so the Nyquist Theorem suggests sampling at half of that period, or 3 milliseconds, in order produce accurate results. This demonstrates that the 7.5 Hz sampling frequency necessary for the model to produce accurate results in real time is insufficient considering the low duration of concussive impacts. The curve fit, however could easily compute results every 3 milliseconds, or at approximately 333 Hz.

An additional consideration in comparing the model and curve fit is the inputs of each function. The inputs to the model are peak rotational acceleration about each axis and the duration of the impact, and the inputs to the curve fit are peak rotational acceleration magnitude and peak linear acceleration magnitude. Additionally, the model could be modified by omitting the first step which converts rotational acceleration to axial strain in order to input a strain time function into the model. These differences in input values could also govern the ability of the model and curve fit to perform for various applications. For example, if pressure sensors were used as opposed to accelerometers to collect data, it would be inappropriate to use the curve fit but would be sensible to use the modified model to calculate concussion probabilities.

Thus, choosing between the model and curve fit in an experimental setting would likely depend on the logistics of the experiment. For wide indication increments, the model would be appropriate but, due to up to 15% error rates between the model and curve fit, the curve fit would be more appropriate for more narrow indication increments. Noisy data may necessitate use of the model, which showed lower error rates given noise in the inputs. Need to make computations in real time would make the curve fit a more appropriate choice due to the high computation time of the model and low duration of concussive impacts. Finally, the inputs available from the experimental data may govern the choice between the model or curve fit.

Appendix

Appendix A: Viscoelastic Micromechanics Model for the Axon



$$\sigma_{in} = \sigma_x = \sigma_{OUT} = \sigma_{\eta 3} + \sigma_{K11}$$
$$= \sigma_{K3} + \sigma_{K11}$$
$$= \sigma_{\eta 3} + \sigma_{K12} + \sigma_{\eta 1}$$
$$= \sigma_{K3} + \sigma_{K12} + \sigma_{\eta 1}$$
$$= \sigma_{K21}$$
$$= \sigma_{K22} + \sigma_{\eta 2}$$

(1) $\varepsilon_{in} + \varepsilon_{K11} + \varepsilon_{K12} + \varepsilon_{K21} + \varepsilon_{K22} = \varepsilon_{OUT}$
(2) $\varepsilon_{\eta 3} + \varepsilon_{K3} = \varepsilon_{K11} + \varepsilon_{K12}$ $+ \varepsilon_{K11}$
(3) $\varepsilon_{K12} = \varepsilon_{\eta 1}$
(4) $\varepsilon_{K22} = \varepsilon_{\eta 2}$

$\sigma = \eta \frac{d\varepsilon}{dt}$ FOR A DAMPER
$\sigma = K\varepsilon$ FOR A SPRING

(5) $K_{21}\varepsilon_{K21} = \eta_3 \frac{d\varepsilon_{\eta 3}}{dt} + K_{11}\varepsilon_{K11}$
(6) $= K_3\varepsilon_{K3} + K_{11}\varepsilon_{K11}$
(7) $= \eta_3 \frac{d\varepsilon_{\eta 3}}{dt} + K_{12}\varepsilon_{K12} + \eta_1 \frac{d\varepsilon_{\eta 1}}{dt}$
(8) $= K_3\varepsilon_{K3} + K_{12}\varepsilon_{K12} + \eta_1 \frac{d\varepsilon_{\eta 1}}{dt}$
(9) $= K_{22}\varepsilon_{K22} + \eta_2 \frac{d\varepsilon_{\eta 2}}{dt}$

SUB DAMPERS INTO SPRINGS IN EQS 5,7-9 USING EQS 2-4, 6

$\varepsilon_{K12} = \varepsilon_{\eta 1}$
$\varepsilon_{K22} = \varepsilon_{\eta 3}$

$\varepsilon_{K3} = \varepsilon_{K11} + \varepsilon_{\eta 1} - \varepsilon_{\eta 3}$
$= K_{21}\varepsilon_{K21} - K_{11}\varepsilon_{K11}$
$\varepsilon_{K11} + \varepsilon_{\eta 1} - \varepsilon_{\eta 3} = K_{21}\varepsilon_{K21} - K_{11}\varepsilon_{K11}$
$K_{21}\varepsilon_{K21} = + \varepsilon_{\eta 1} - \varepsilon_{\eta 3} + (1+K_{11})\varepsilon_{11}$

$\varepsilon_{\eta 1} - \varepsilon_{\eta 3} + \varepsilon_{11} = \eta_3 \frac{d\varepsilon_{\eta 3}}{dt} \longrightarrow \varepsilon_{11} = \eta_3 \frac{d\varepsilon_{\eta 3}}{dt} + \varepsilon_{\eta 3} - \varepsilon_{\eta 1}$

$\varepsilon_{\eta 1} - \varepsilon_{\eta 3} + \varepsilon_{11} = K_3(\varepsilon_{K11} + \varepsilon_{\eta 1} - \varepsilon_{\eta 2})$

$\varepsilon_{\eta 1} - \varepsilon_{\eta 3} + (1+K_{11})\varepsilon_{11} = \eta_3 \frac{d\varepsilon_{\eta 3}}{dt} + K_{12}\varepsilon_{\eta 1} + \eta_1 \frac{d\varepsilon_{\eta 1}}{dt}$

$\varepsilon_{\eta 1} - \varepsilon_{\eta 3} + (1+K_{11})\varepsilon_{11} = K_{22}\varepsilon_{\eta 3} + \eta_2 \frac{d\varepsilon_{\eta 2}}{dt}$

(i) $$\varepsilon_{\eta 1}-\varepsilon_{\eta 3}+(1-k_3)\left(\eta_3\frac{d\varepsilon_{\eta 3}}{dt}+\varepsilon_{\eta 3}-\varepsilon_{\eta 1}\right)=k_3(\varepsilon_{\eta 1}-\varepsilon_{\eta 2})$$

(ii) $$\varepsilon_{\eta 1}-\varepsilon_{\eta 3}+(1+k_{11})\left(\eta_3\frac{d\varepsilon_{\eta 3}}{dt}+\varepsilon_{\eta 3}-\varepsilon_{\eta 1}\right)=\eta_3\frac{d\varepsilon_{\eta 3}}{dt}+k_{12}\varepsilon_{\eta 1}+\eta_1\frac{d\varepsilon_{\eta 1}}{dt}$$

(iii) $$=k_{22}\varepsilon_{\eta 3}+\eta_2\frac{d\varepsilon_{\eta 2}}{dt}$$

$$\frac{d\varepsilon_{\eta 3}}{dt}=\left(\frac{k_3(\varepsilon_{\eta 1}-\varepsilon_{\eta 2})+\varepsilon_{\eta 3}-\varepsilon_{\eta 1}}{1-k_3}+\varepsilon_{\eta 1}-\varepsilon_{\eta 3}\right)\left(\frac{1}{\eta_3}\right)$$

$$\frac{d\varepsilon_{\eta 1}}{dt}=\left(k_{22}\varepsilon_{\eta 3}+\eta_2\frac{d\varepsilon_{\eta 2}}{dt}-\eta_3\frac{d\varepsilon_{\eta 3}}{dt}-k_{12}\varepsilon_{\eta 1}\right)\left(\frac{1}{\eta_1}\right)$$

$$\frac{d\varepsilon_{\eta 2}}{dt}=\left(\varepsilon_{\eta 1}-\varepsilon_{\eta 3}+(1+k_{11})\left(\eta_3\frac{d\varepsilon_{\eta 3}}{dt}+\varepsilon_{\eta 3}-\varepsilon_{\eta 1}\right)-k_{22}\varepsilon_{\eta 3}\right)\left(\frac{1}{\eta_2}\right)$$

TO ODE45

FIND ε_{out} USING OUTPUT

$$\varepsilon_{k11}=\eta_3\frac{d\varepsilon_{\eta 3}}{dt}+\varepsilon_{\eta 3}-\varepsilon_{\eta 1}$$

$$\varepsilon_{k21}=\left(\varepsilon_{\eta 1}-\varepsilon_{\eta 3}+(1+k_{11})\varepsilon_{11}\right)\left(\frac{1}{k_{21}}\right)$$

$$\varepsilon_{k22}=\varepsilon_{\eta 2}$$

$$\varepsilon_{k12}=\varepsilon_{\eta 1}$$

Appendix B: MATLAB code

All source code is available at https://github.com/Isabel-0000/concussion_model

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