Markov Chains: Foundations

Part 1 - C_Markov Course

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July 2025

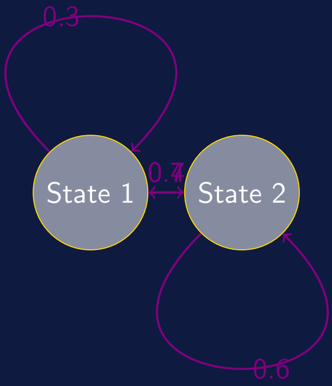
- Markov Chains model systems with probabilistic transitions.
- Key property: Future states depend only on the current state (Markov property).
- Applications: Stochastic processes, finance, and network analysis.

Objective

Explore foundational concepts over 1 month, based on *Markov Chains and Mixing Times* (Levin et al., 2009).

Transition Matrices

- Represent transitions between states in a matrix P.
- Example: $P = \begin{bmatrix} 0.7 & 0.3 \\ 0.4 & 0.6 \end{bmatrix}$.
- Rows sum to 1 (stochastic matrix).



Classification of States

- **Transient**: Can leave and never return.
- **Recurrent**: Eventually returns with probability 1.
- **Absorbing**: Once entered, cannot leave.

Example

In a random walk, states with no return path are transient.

Steady-State Behavior

- Long-term distribution: $\pi P = \pi$, where π is the stationary vector.
- Convergence depends on the chain's structure.
- Example: For $P = \begin{bmatrix} 0.7 & 0.3 \\ 0.4 & 0.6 \end{bmatrix}$, solve $\pi_1 = 0.571, \pi_2 = 0.429$.

Example: Transition Matrix

- Matrix $P = \begin{bmatrix} 0.7 & 0.3 \\ 0.4 & 0.6 \end{bmatrix}$ models transitions.
- After 2 steps: $P^2 = \begin{bmatrix} 0.61 & 0.39 \\ 0.46 & 0.54 \end{bmatrix}$.

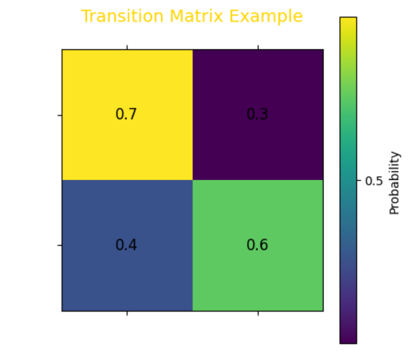


Figure: Visualization of Transition Matrix

Example: Classification of States

- Matrix: $P = \begin{bmatrix} 0.5 & 0.5 & 0.0 \\ 0.2 & 0.7 & 0.1 \\ 0.0 & 0.0 & 1.0 \end{bmatrix}$.
- State 0: Transient (can leave).
- State 2: Absorbing (stays forever).

Example: Steady-State Behavior

- Stationary distribution: $\pi = [0.571, 0.429]$ for $P = \begin{bmatrix} 0.7 & 0.3 \\ 0.4 & 0.6 \end{bmatrix}$.
- Solved using eigenvalue method.

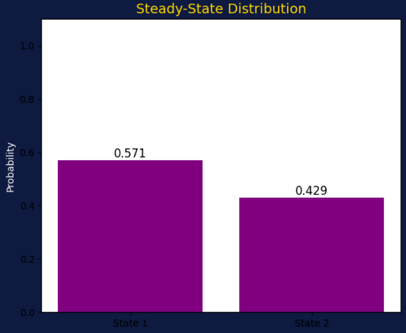


Figure: Steady-State Distribution

References

- LEVIN, D. A., PERES, Y., WILMER, E. L. – *Markov Chains and Mixing Times*, American Mathematical Society, 2009.

- Additional resources to be added as the course progresses.