Rössler Attractor: Chaotic Dynamics

Prof. Ana Isabel C.

Math-Dynamics Lab

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Introduction

The Rössler system is a three-dimensional continuous dynamical system exhibiting chaotic behavior, characterized by dense spiral trajectories around unstable fixed points.

Applications:

- Nonlinear dynamics
- Chaos theory
- Mathematical modeling in physics and biology

Mathematical Definition

Parameters:

- $a = 0.2$: Linear damping coefficient
- $b = 0.2$: Nonlinear interaction strength
- $c = 7.0$: Critical threshold

Differential Equations:

$$\begin{aligned} \dot{x} &= -y - z, \\ \dot{y} &= x + ay, \\ \dot{z} &= b + z(x - c). \end{aligned}$$

Nonlinear terms yield chaotic orbits.

Fixed Points Analysis

Equilibrium points are found by setting $\dot{x} = \dot{y} = \dot{z} = 0$:

$$\begin{cases} -y - z = 0, \\ x + ay = 0, \\ b + z(x - c) = 0. \end{cases}$$

Solving yields two fixed points:

$$\text{Point 1: } (x_1, y_1, z_1) \approx (6.994, -34.971, 34.971),$$
$$\text{Point 2: } (x_2, y_2, z_2) \approx (0.006, -0.029, 0.029).$$

Both points are unstable, driving chaotic dynamics.

Numerical Solution

The nonlinear system requires numerical integration:

- `scipy.integrate.odeint`: Implements Runge-Kutta 4th-order method.
- **Initial Conditions**: $[x_0, y_0, z_0] = [1, 1, 1]$.
- **Time**: $t \in [0, 100]$ with 20,000 points ($\Delta t \approx 0.005$).

$$\mathbf{u}(t) = [x(t), y(t), z(t)], \quad \dot{\mathbf{u}} = \begin{bmatrix} -y - z \\ x + ay \\ b + z(x - c) \end{bmatrix}.$$

Sensitive to initial conditions, capturing chaos.

Visualization

- Implemented in Python using `matplotlib.animation`.
- 3D phase space plot with 20,000 points.
- Orbits form a strange attractor with dense spirals.
- rossler.gif

Conclusion

- The Rössler system illustrates chaotic dynamics in three dimensions.
- Numerical methods enable visualization of complex orbits.
- Useful for education, research, and scientific outreach.

Source code available at: `github.com/IsabelCasPe/Math-Dynamics`