Julia Set: Dynamic Fractal Visualization

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Introduction

The Julia set is a fractal defined in the complex plane, arising from the iterative dynamics of a quadratic polynomial. Its intricate boundaries exhibit self-similarity and chaotic behavior.

Applications:

- Complex dynamics
- Fractal geometry
- Visualization of chaotic systems

Mathematical Definition

Parameters:

- Domain: $z = x + iy$, $x, y \in [-2, 2]$.
- Resolution: 800×800 pixels.
- Iterations: 100.
- Dynamic constant: $c(t) = -0.8 + 0.156i + 0.1\left(1 + \frac{t}{2\pi}\right)\sin(t)(1 + i)$, $t \in [0, 2\pi]$.

Iteration:

$$z_{n+1} = z_n^2 + c(t), \quad z_0 = x + iy.$$

The Julia set consists of points where $|z_n|$ remains bounded as $n \to \infty$.
Dynamic $c(t)$ yields evolving fractal boundaries.

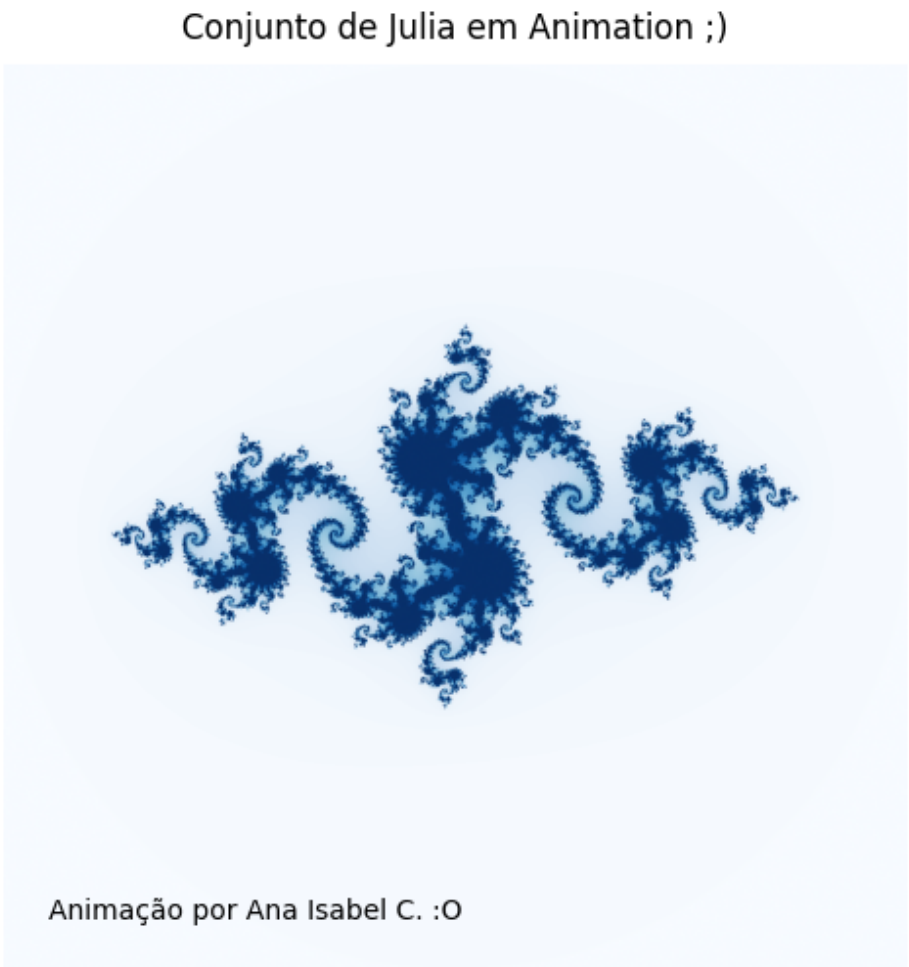
Numerical Implementation

- **Algorithm**: For each pixel z_0, iterate $z_{n+1} = z_n^2 + c(t)$ up to 100 times, tracking divergence ($|z_n| \geq 2$).
- **Output**: Normalized iteration count visualizes escape rates.
- **Animation**: 100 frames, with $c(t)$ varying over $t \in [0, 2\pi]$.
- **Tools**: Python with NumPy and `matplotlib.animation`.

$$\text{Output}(z_0) = \frac{\text{Number of iterations before } |z_n| \geq 2}{\text{Max iterations}}.$$

Visualization

- Plotted as a 2D heatmap with 800×800 resolution.
- Color scheme emphasizes fractal boundaries.
- Animation captures the evolution of the Julia set as $c(t)$ oscillates.



Conclusion

- The Julia set animation showcases the interplay of chaos and fractal geometry.
- Combines mathematical rigor with computational visualization.
- Valuable for education, research, and scientific outreach in complex dynamics.

Source code available at: `github.com/IsabelCasPe/Math-Dynamics`