Double Pendulum: Chaotic Dynamics

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Introduction

The double pendulum is a classic mechanical system exhibiting chaotic behavior due to its nonlinear dynamics, making it highly sensitive to initial conditions.

Applications:

- Nonlinear dynamics
- Chaos theory
- Mechanical systems analysis

Mathematical Definition

Parameters:

- Lengths: $L_1 = L_2 = 1$ m
- Masses: $m_1 = m_2 = 1$ kg
- Gravity: $g = 9.81$ m/s^2
- State: $\mathbf{s} = [\theta_1, \dot{\theta}_1, \theta_2, \dot{\theta}_2]^T$

Notations:

- $\Delta\theta = \theta_1 - \theta_2$, $c = \cos(\Delta\theta)$, $s = \sin(\Delta\theta)$
- $D = m_1 + m_2 s^2$

Differential Equations:

$$\dot{\theta}_1 = \dot{\theta}_1,$$

$$\ddot{\theta}_1 = \frac{m_2 g \sin(\theta_2) c - m_2 s (L_1 \dot{\theta}_1^2 c + L_2 \dot{\theta}_2^2) - (m_1 + m_2) g \sin(\theta_1)}{L_1 D},$$

$$\dot{\theta}_2 = \dot{\theta}_2,$$

$$\ddot{\theta}_2 = \frac{(m_1 + m_2)(L_1 \dot{\theta}_1^2 s - g \sin(\theta_2) + g \sin(\theta_1) c) + m_2 L_2 \dot{\theta}_2^2 s c}{L_2 D}.$$

Nonlinear terms drive chaotic motion.

Numerical Solution

The nonlinear system is solved numerically:

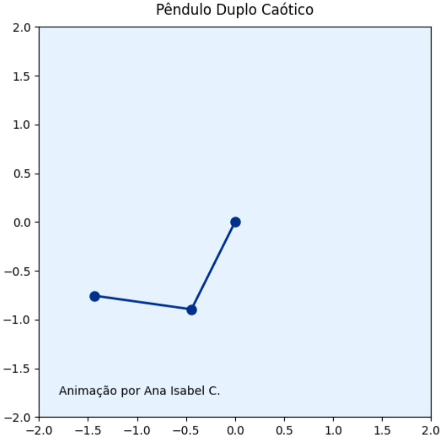
- `scipy.integrate.odeint`: Runge-Kutta 4th-order method.
- **Initial Conditions**: $\mathbf{s}(0) = [\pi/2, 0, \pi, 0]^T$.
- **Time**: $t \in [0, 10]$ s with 1000 points ($\Delta t \approx 0.01$ s).

$$\dot{\mathbf{s}} = \begin{bmatrix} \dot{\theta}_1 \\ \ddot{\theta}_1 \\ \dot{\theta}_2 \\ \ddot{\theta}_2 \end{bmatrix}.$$

Captures sensitivity to initial conditions.

Visualization

- Implemented in Python using `matplotlib.animation`.
- 2D plot of pendulum positions over 1000 time steps.
- Cartesian coordinates: $x_1 = \sin(\theta_1)$, $y_1 = -\cos(\theta_1)$, $x_2 = x_1 + \sin(\theta_2)$, $y_2 = y_1 - \cos(\theta_2)$.



Conclusion

- The double pendulum exemplifies chaotic dynamics in mechanical systems.
- Numerical solutions enable visualization of complex motion.
- Valuable for education, research, and scientific outreach in nonlinear dynamics.

Source code available at: `github.com/IsabelCasPe/Math-Dynamics`