Hénon Map: Fractal

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Introduction

The Hénon map, introduced by Michel Hénon in 1976, is a discrete dynamical system that exhibits fractal chaos. This 3D visualization uses the variation of x as the third axis.

Applications:

- Chaos theory
- Fractal geometry
- Dynamical systems

Mathematical Definition

Parameters:

- $a = 1.4$: Nonlinearity parameter
- $b = 0.3$: Dissipation parameter

Equations:

$$x_{n+1} = 1 - ax_n^2 + y_n,$$
$$y_{n+1} = bx_n.$$

The term bx_n introduces dissipation, causing contraction in phase space. Nonlinear term $-ax_n^2$ drives fractal chaos.

Numerical Simulation

- `numpy`: Iterates trajectories. **Initial Conditions**: Two nearby trajectories: $(x_0, y_0) = (0.1, 0.1)$ and $(0.101, 0.1)$.
- **Iterations**: 10,000 per trajectory.
- **3D Visualization**: Variation $\Delta x = x_{n+1} - x_n$ as the third axis.

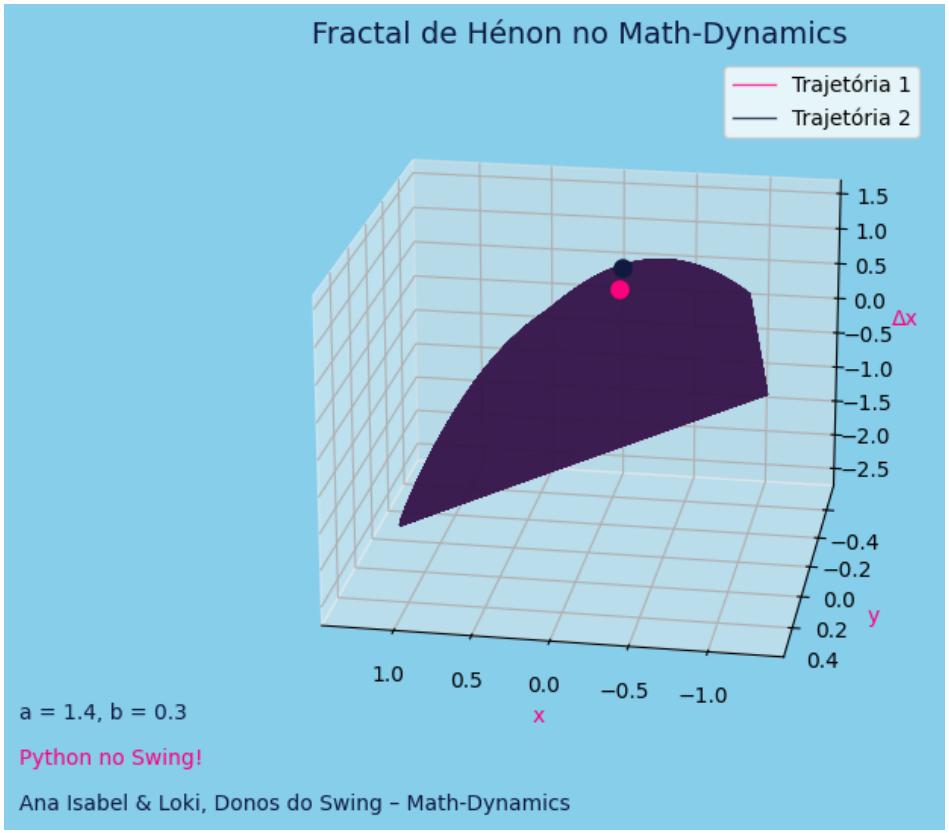
$$\mathbf{z}_{n+1} = \begin{bmatrix} x_{n+1} \\ y_{n+1} \\ \Delta x \end{bmatrix}.$$

Shows sensitivity to initial conditions.

Visualization

- Python with `matplotlib`.
- 3D plot of trajectories: x_n vs. y_n vs. Δx.

Below: Dynamic visualization of fractal trajectories in 3D.



Conclusion

- The Hénon map showcases fractal chaos in 3D.
- Two close initial conditions reveal sensitive dependence.
- A captivating tool for exploring chaotic behavior visually.

Source code: `https://github.com/IsabelCasPe/Math-Dynamics`