# Risk measures, systemic risk and default cascades

Ana Isabel Castillo Pereda
Department of Applied Mathematics
University of São Paulo

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## Content of this talk

- We will discuss various types of financial risks and the risk measures proposed in the literature to evaluate these risks.
- We will present DebtRank as method to evaluate systemic risk We briefly comment on the situation in Brazil.
- For most of the talk we will concentrate on random financial networks, which we introduce next. We will present systemic default cascades for the various risks.
- We will discuss how an alteration of the evaluation of systemic risk and the inclusion of the shadow banking system will affect the default cascades.

# Part I: Risk measures

## (Intrinsic) Risk Measures

- Monetary risk measures quantify the minimal amount of external capital that must be added to make an investment position acceptable. They are given by maps $\rho_{\mathcal{A},r}$ from a function space $\mathcal{X} \subseteq \mathbb{R}^{\Omega}$ to $\mathbb{R}$ of the form

$$\rho_{\mathcal{A},r}(X_T) = \inf\left\{ m \in \mathbb{R} \mid X_T + mr\mathbb{1}_{\Omega} \in \mathcal{A} \right\} . \qquad (1)$$

  They measure the risk of a financial position $X_T \in \mathcal{X}$ with respect to certain acceptability criteria and a risk-free investment. The latter are specified as a subset $\mathcal{A} \subset \mathcal{X}$, the acceptance set, and the risk-free return rate $r > 0$, respectively.

More generally, one may replace $r\mathbb{1}_\Omega$ in Equation (1) by the random return $S_T/S_0$ of an eligible traded assets $S = (S_0, S_T)$ with initial unitary price $S_0 \in \mathbb{R}^+$ and payoff $S_T\colon \Omega \to \mathbb{R}_0^+$. This yields

$$\rho_{\mathcal{A},S}(X_T) = \inf\left\{ m \in \mathbb{R} \mid X_T + m\frac{S_T}{S_0} \in \mathcal{A} \right\} .$$

The application of this approach requires to raise the monetary amount $\rho_{\mathcal{A},S}(X_T)$ and carry it in the eligible asset $S$. However, the possible acquisition of additional capital is not completely accounted for by monetary risk measures.

- Intrinsic risk measure (Farkas & Smirnow) were introduced to identify the smallest percentage of the currently held financial position which has to be sold and reinvested in an eligible asset such that the resulting position becomes acceptable. For a single financial position $X = (X_0, X_T)$ and a single eligible asset $S = (S_0, S_T)$, the intrinsic risk measure is given by

$$R_{\mathcal{A},S}(X) = \inf\left\{\lambda \in [0,1] \mid (1-\lambda)X_T + \lambda X_0 \frac{S_T}{S_0} \in \mathcal{A}\right\} .$$

Both the original risk measures and the intrinsic risk measure are based on acceptance sets. We now specify their basic properties:

## Acceptance sets

A subset $\mathcal{A} \subset \mathcal{X}$ is called an acceptance set if it satisfies

- non-triviality: $\mathcal{A} \neq \emptyset$ and $\mathcal{A} \neq \mathcal{X}$, and
- monotonicity: $X_T \in \mathcal{A}$, $Y_T \in \mathcal{X}$, and $Y_T \geq X_T$ imply $Y_T \in \mathcal{A}$.

An element $X_T \in \mathcal{A}$ is called acceptable. Similarly, we say $X_T \notin \mathcal{A}$ is unacceptable. An acceptance set $\mathcal{A} \subset \mathcal{X}$ is called

- a cone or conic if $X_T \in \mathcal{A}$ implies $\lambda X_T \in \mathcal{A}$ for all $\lambda > 0$;
- convex if $X_T, Y_T \in \mathcal{A}$ implies $\lambda X_T + (1-\lambda) Y_T \in \mathcal{A}$ for all $\lambda \in [0,1]$,
- closed if $\mathcal{A} = \overline{\mathcal{A}}$.

The basic assumptions on acceptance sets and risk measures give rise to number of interesting properties, for which we refer to the literature. Instead of discussing these properties, we will now briefly discuss the various risks which have to be taken into account when selecting a particular acceptance set or when setting up bank regulations (as in the Basel III agreement).

### Literature on risk measures:

- Artzner, Delbaen, Eber & Heath, Coherent measures of risk, Mathematical Finance, 9(3):203–228, 1999.
- Föllmer & Schied, Stochastic Finance, De Gruyter, 2016.
- Farkas & Smirnow, Intrinsic risk measure, 2017.

## Risk (according to Duffie and Singleton)

- market risk: unexpected changes in market prices;
- credit risk: unexpected changes in credit quality, in particular if a counterparty defaults on one of their contractual obligations;
- liquidity risk: the costs of adjusting financial positions may increase substantially;
- operational risk: fraud, errors or other operational failures may lead to loss in value;
- systemic risk: risk of market wide illiquidity or chain reaction defaults, with significant impact on the macroeconomy.

To the extent that the first four risk categories are focussed on individual institutions, they are not deemed to be systemic risk.

## Systemic Risk

Systemic risk evaluates the risk that default cascades will affect the viability of the currency, the money supply, the supply of credit, major market indices, the interest rates, and ultimately the production economy and level of employment.

## Shadow Banking Institutions

In addition to traditional retail and investment banks, financial networks host many shadow banking institutions, including hedge funds, pension and investment funds, savings and credit institutions, and so on. These may all be subject to divergent regulations and may very well cause systemic risks.

# Part II: DebtRank

DebtRank evaluates the additional stress caused by some initial shock using a dynamical system. It maintains two dynamic variables for each bank $i \in \mathcal{B}$:

- $h_i(t) \in [0, 1]$ is the stress level of $i$. When $h_i(t) = 0$, $i$ is undistressed. In contrast, when $h_i(t) = 1$, $i$ is on default. In-between values lead to partial stress of $i$.
- $s_i(t) \in \{U, D, I\}$ is a categorical variable and denotes the state of $i$. $U$, $D$, and $I$ stand for undistressed, distressed, and inactive, respectively.

The update rules of the dynamical system are:

$$h_i(t) = \min\left\{1, h_i(t-1) + \sum_{j \in D(t)} V_{ij}\, h_j(t-1)\right\}, \qquad (2)$$

and

$$s_i(t) = \begin{cases} D & \text{if } h(t) > 0 \text{ and } s(t-1) \neq I \ , \\ I & \text{if } s_i(t-1) = D \ , \\ s_i(t-1) \ , & \text{otherwise.} \end{cases} \tag{3}$$

in which $t \geq 0$ and $D(t) = \left\{ u \in \mathcal{B} \mid s_u(t-1) = D \right\}$. The summation in (2) includes only banks which were distressed in the previous iteration. Once distressed, they become inactive in the next iteration due to (3).

The matrix

$$V_{ij} = \frac{A_{ij}}{E_i} \qquad i, j \in \mathcal{B} \,, \quad V_{ij} \in [0, \infty) \,,$$

is used to describes the stress levels of each of the banks.
The entry $A_{ij}$ denotes the exposure of bank $i$ towards $j$ in the interbank network and $E_i$ indicates the available resources or capital buffer of bank $i$. Whenever $V_{i,j} \geq 1$, the default of financial institution $j$ leads $i$ into default as well. Intermediate values inside the interval $(0, 1)$ lead $i$ into distress but not into default.

The drawback of DebtRank is that it excludes that banks diffuse second- and high-order rounds of stress.

## Differential DebtRank

We now present an improved version of DeptRank: at each iteration, banks are only allowed to propagate the stress increment that they receive from the previous iteration. This idea suggest to replace (2) by (Bardoscia et al. (2015)): for $t \geq 0$,

$$h_i(t) = \min\Big\{1, h_i(t-1) + \sum_{j\in\mathcal{B}} V_{ij} \underbrace{\big[h_j(t-1) - h_j(t-2)\big]}_{=\Delta h_j(t-1)}\Big\}. \quad (4)$$

Here $h(0)$ denotes the initial stress scenario that the user supplies, $h(t) = 0$ for $t < 0$, and $\Delta h_j(t-1)$ is the stress differential of the bank $j$ at the previous iteration $t-1$.

## Brazilian literature on the systemic risk:

- Silva, Guerra, da Silva & Tabak, Interconnectedness, Firm Resilience and Monetary Policy, Central Bank of Brazil Working Papers 478, July 2018.
- Silva, Tabak & Guerra, Why do vulnerability cycles matter in financial networks?, Central Bank of Brazil Working Papers 442, June 2016.
- Silva, Guerra, Tabak and Miranda, Financial Networks, Bank Efficiency and Risk-Taking Central Bank of Brazil Working Papers 428, April 2016.

# Part III: Random graphs

In the sequel, we will discuss default cascade models. Financial networks are often only partially visible and financial dependencies may change substantially within hours. Thus there is a good argument for using random graphs.

## Random Graph Models

Graphs consist of nodes and edges, if the edges are directed, the links are orders pairs between the nodes. The in-/out- degree counts the number of links associated to a node. Random graphs are probability distributions on the set of graphs. They are characterised by the node-type and edge-type distribution.

## Assortative Configuration Graphs

Graphs representing the financial system have a number of peculiar properties: small banks have a tendency to choose large banks as counterparties. Therefore we wish to construct a class of assortative configuration graphs (ACG).

## Inhomogeneous Random Graphs

Nodes have types other than degree: we can have commercial banks, investment banks, hedge funds, and so on, each with additional continuous characteristics such as geographical location or size.

## Pareto tails

In addition to measuring moments of the degree distribution $P_{jk}$, such as the mean (in- and out-) degree

$$z = \sum_{j,k} j\,P_{jk} = \sum_{j,k} k\,P_{jk} \ ,$$

network practitioners also focus on the tail properties. Tail exponents are defined by large graph limits:

$$\alpha^{\pm} = -\limsup_{j\to\infty} \frac{\log P^{\pm}_j}{\log j}$$

Finite tail exponents are indicators of Pareto tails, and signal the existence of non-negligible numbers of hubs, that can be significant focal points for systemic risk.

## Clustering Coefficients

Clustering in social networks refers to the propensity of the friends of our friends to be our friends. In a general setting it means the likelihood that a connected triple of nodes forms a triangle. It is measured by the clustering coefficient,

$$C(g) = \frac{3 \times (\text{number of triangles}).}{(\text{number of connected triples})} .$$

In directed networks, an even more basic notion of clustering is reflexivity, that refers to the fraction of the number of node pairs that have a reflexive pair of edges (*i.e.*, an edge pointing in both directions) to the total number of directed edges.

## Connectivity and connected components

Contagion can only propagate across connected components. Hence, the sizes of connected subgraphs of a network are additional measures of its susceptibility to large scale contagion. A strongly connected component (SCC) of a network is a subgraph each of whose nodes are connected to any other node by a path of downstream edges, and which is maximal in the sense that it is not properly contained in a larger SCC.

A critical question is whether there is a SCC that is infinite or even a positive fraction of the entire network. When this happens in a random graph model, the infinite SCC is typically unique, and we call it the giant strongly connected component (GSCC).

In summary: since the actual financial dependencies are mostly unknown, and also may change rapidly, it is best to develop systemic risk models, which are able to adapt to a wide range of financial networks.

However, the random graphs have to take characteristic aspects of the financial network they are intended to represent into account. The aim of this research project is to come up with numerical simulations of the default cascades models which include the shadow banking system, and this involves selection of the appropriate random graphs.

# Part IV: Default cascade models

We will now present a list of models, which aim to describe chain defaults of banks. It is one of our research objective, to come up with a model, which takes the current nature of the financial network into account. Currently, the shadow banking is a 52 trillion USD industry, a number which dwarfs the US Banks total assets of 18 trillion USD (at the end of 2018). It is widely believed that the currently the systemic risk has accumulated in the shadow banking sector.

Before we can describe the default cascade models, we have to explain the variables which are used to describe the financial situation of banks (and shadow banks).

Equity is the value of the firm to the share owners, it is defined to be the difference

$$E = A - L$$

between asset value $A$ and liability value $L$. The most fundamental characteristic of a bank's balance sheet is its accounting ratio, leverage $A/E$.

A defaulted bank is a bank with $E = 0$. A solvent bank is a bank with $E > 0$. A stressed bank is a bank with external liquid assets $Y^L = 0$.

Nominal values, denoted by upper case letters with bars, give the aggregated values of assets and liabilities, valued as if all banks are solvent. The nominal value of assets of bank $v$ at any time consists of nominal external assets $\overline{Y}_v$, both fixed and liquid, denoted by

$$\overline{Y}_v = \overline{Y}_v^F + \overline{Y}_v^L ,$$

plus nominal interbank assets $\overline{Z}_v$. The nominal value of liabilities of the bank includes nominal external debt $\overline{D}_v$ and nominal interbank debt $\overline{X}_v$. The bank's nominal equity is defined by

$$\overline{E}_v = \overline{Y}_v + \overline{Z}_v - \overline{D}_v - \overline{X}_v .$$

The nominal exposure of bank $w$ to bank $v$, that is the amount $v$ owes $w$, is denoted by $\Omega_{vw}$.

## The Eisenberg-Noe Model (2001)

Assumptions: External debt is senior to interbank debt and all interbank debt is of equal seniority; there are no losses due to bankruptcy charges. Set

$$q_v^{(n)} = \min\Big\{ \overline{Y}_v + \sum_w \overline{\Pi}_{wv} p_w^{(n-1)} \,,\, \overline{D}_v + \overline{X}_v \Big\} \,,$$

$$p_v^{(n)} = \big( q_v^{(n)} - \overline{D}_v \big)^+ \,.$$

Here $p_v^{(n)}$ denote the amount available to pay $v$'s internal debt at the end of the $n$-th step of the cascade, and let $q_v^{(n)}$ denote the amount available to pay $v$'s total debt. The value $p_v^{(n)}$ is split amongst the creditor banks of $v$ in proportion to the fractions $\overline{\Pi}_{wv} = \overline{\Omega}_{vw} / \overline{X}_v$ (when $\overline{X}_v = 0$, we define $\overline{\Pi}_{wv} = 0$ for all $w$).

The Gai-Kapadia Default Model (2010) Assumptions:

- At step 0 of the cascade, one or more banks experience asset shocks that make their default buffers

$$\overline{\Delta}_v \doteq \overline{Y}_v + \sum_w \overline{\Omega}_{vw} - \overline{D}_v - \overline{X}_v \ ,$$

go negative.

- A defaulted bank $v$'s interbank liabilities recover zero value and thus a default shock of magnitude $\overline{\Omega}_{vw}$ is sent to each of $v$'s creditor banks $w$.
- At each step $n \geq 0$, bank $v$ marks to zero any interbank asset $\overline{\Omega}_{wv}$ from a newly defaulted counterparty bank $w$.

Gai-Kapadia 2010 Liquidity Cascade Model Assumptions:

- At step 0 of the cascade, one or more banks experience funding liquidity shocks or stress shocks that make their stress buffers $\Sigma(0) = \bar{\Sigma}$ go negative. (When non-zero, the liquid assets $\bar{Y}_v^L$ are used as a stress buffer $\bar{\Sigma}_v$ from which to pay liabilities as they arise.)
- Banks respond at the moment they become stressed by preemptively hoarding a fixed fraction $\lambda \leq 1$ of interbank lending. This sends a stress shock of magnitude $\lambda\bar{\Omega}_{wv}$ to each of the debtor banks $w$ of $v$.
- At each step $n \geq 0$, bank $v$ pays any interbank liabilities $\lambda\bar{\Omega}_{vw}$ that have been recalled by newly stressed banks $w$.

Under stress, banks act preemptively to shrink their balance sheets!

Fire Sales of One Asset Assumptions: we include a capital adequacy ratio (CAR) as a regulatory constraint:

- For some fixed regulatory value $r^*$ (say 7%), the bank must maintain the lower bound
$$\frac{\Delta_v}{Y_v^F + Z_v} \geq r^* \, .$$
- A bank with
$$r^* Z_v \leq \Delta_v < r^*(Y_v^F + Z_v)$$
is called non-compliant but solvent, and must sell fixed assets, but not interbank assets, to restore the capital adequacy ratio (CAR) condition.
- A bank with $r^* Z_v > \Delta_v$ is insolvent, and must be fully liquidated. The picture is that even by selling all of $Y^F$, the bank $v$ cannot achieve the CAR condition, and hence must be terminated, even if its default buffer is still positive.

– In the event of insolvency, the defaulted interbank assets are distributed at face value proportionally among the bank's creditors, and the bank ceases to function. External deposits have equal seniority to interbank debt and thus defaulted liabilities are valued in proportion to

$$\overline{\Pi}_{vw} = \frac{\overline{\Omega}_{vw}}{(\overline{X}_v + \overline{D}_v)} .$$

These models just provide some examples, of how financial regulations like Basel III are determining the response of individual players in a default cascade. It is our intention to provide an improved model, which takes new developments like a fast growing shadow banking sector into account.

## Literature on default cascades


- T.R. Hurd, Contagion! Systemic Risk in Financial Networks, Springer (2016).
- L. Eisenberg, T.H. Noe, Systemic risk infinancial systems. Manag. Sci. 47(2), 236–249 (2001).
- P. Gai, S. Kapadia, Liquidity hoarding, network externalities, and interbank market collapse. Proc. R. Soc. A 466, 2401–2423 (2010).
- P. Gai, S. Kapadia, Contagion in financial networks. Proc. R. Soc. A 466 (2120), 2401–2423 (2010).


Of course, there are many more interesting models in the literature.