Numerical Analysis

Testing Methods 2

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Testing Data

$$A = \begin{pmatrix} 4 & -1 & 0 & 3 \\ 1 & 15.5 & 3 & 8 \\ 0 & -1.3 & -4 & 1.1 \\ 14 & 5 & -2 & 30 \end{pmatrix}, \qquad b = \begin{pmatrix} 1 \\ 1 \\ 1 \\ 1 \end{pmatrix}, \qquad x_0 = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \end{pmatrix}, \qquad Tol = 1e-7,$$

$$Nmax = 100$$

$$\text{Tabla} = \begin{array}{|c|c|c|c|c|} \hline \text{x} & -1 & 0 & 3 & 4 \\ \hline \text{y} & 15.5 & 3 & 8 & 1 \\ \hline \end{array}$$

1. LU with Simple Gaussian

```
run:
Original matrix
4.0                     -1.0                    0.0                     3.0                     1.0
1.0                     15.5                    3.0                     8.0                     1.0
0.0                     -1.3                    -4.0                    1.1                     1.0
14.0                    5.0                     -2.0                    30.0                    1.0

Stage 1
Goal: Fill with zeros under the element A1,1= 4.0

Multipliers:
Multipliers2,1 : 0.25
Multipliers3,1 : 0.0
Multipliers4,1 : 3.5
A:
4.0                     -1.0                    0.0                     3.0                     1.0
0.0                     15.75                   3.0                     7.25                    0.75
0.0                     -1.3                    -4.0                    1.1                     1.0
0.0                     8.5                     -2.0                    19.5                    -2.5

L:
1.0                     0.0                     0.0                     0.0
0.25                    1.0                     0.0                     0.0
0.0                     0.0                     1.0                     0.0
3.5                     0.0                     0.0                     1.0
```

```
Stage 2
Goal: Fill with zeros under the element A2,2= 15.75

Multipliers:
Multipliers3,2 : -0.08253968253968254
Multipliers4,2 : 0.5396825396825397
A:
4.0                     -1.0                    0.0                     3.0                     1.0
0.0                     15.75                   3.0                     7.25                    0.75
0.0                     0.0                     -3.7523809523809524     1.6984126984126986      1.061904761904762
0.0                     0.0                     -3.619047619047619      15.587301587301587      -2.9047619047619047

L:
1.0                     0.0                     0.0                     0.0
0.25                    1.0                     0.0                     0.0
0.0                     -0.08253968253968254    1.0                     0.0
3.5                     0.5396825396825397      0.0                     1.0
```

```
Stage 3
Goal: Fill with zeros under the element A3,3= -3.7523809523809524

Multipliers:
Multipliers4,3 : 0.9644670050761421
A:
4.0                     -1.0                    0.0                     3.0                     1.0
0.0                     15.75                   3.0                     7.25                    0.75
0.0                     0.0                     -3.7523809523809524     1.6984126984126986      1.061904761904762
0.0                     0.0                     0.0                     13.949238578680202      -3.9289340101522843

L:
1.0                     0.0                     0.0                     0.0
0.25                    1.0                     0.0                     0.0
0.0                     -0.08253968253968254    1.0                     0.0
3.5                     0.5396825396825397      0.9644670050761421      1.0

1.0                     0.0                     0.0                     0.0                     1.0
0.25                    1.0                     0.0                     0.0                     1.0
0.0                     -0.08253968253968254    1.0                     0.0                     1.0
3.5                     0.5396825396825397      0.9644670050761421      1.0                     1.0

4.0                     -1.0                    0.0                     3.0                     1.0
0.0                     15.75                   3.0                     7.25                    0.75
0.0                     0.0                     -3.7523809523809524     1.6984126984126986      1.061904761904762
0.0                     0.0                     0.0                     13.949238578680202      -3.9289340101522843

x4 = -0.28165938864628826
x3 = -0.41048034934497823
x2 = 0.2554585152838428
x1 = 0.5251091703056769
```

2. LU with Partial Pivoting

```
Original matrix
4.0                     -1.0                    0.0                     3.0                     1.0
1.0                     15.5                    3.0                     8.0                     1.0
0.0                     -1.3                    -4.0                    1.1                     1.0
14.0                    5.0                     -2.0                    30.0                    1.0

Stage 1
Goal: Fill with zeros under the element A1,1= 4.0

Multipliers:
Multipliers2,1 : 0.07142857142857142
Multipliers3,1 : 0.0
Multipliers4,1 : 0.2857142857142857
```

```
A:
14.0                    5.0                     -2.0                    30.0                    1.0
0.0                     15.142857142857142      3.142857142857143       5.857142857142858       0.9285714285714286
0.0                     -1.3                    -4.0                    1.1                     1.0
0.0                     -2.4285714285714284     0.5714285714285714      -5.571428571428571      0.7142857142857143

L:
1.0                     0.0                     0.0                     0.0
0.07142857142857142     1.0                     0.0                     0.0
0.0                     0.0                     1.0                     0.0
0.2857142857142857      0.0                     0.0                     1.0

Stage 2
Goal: Fill with zeros under the element A2,2= 15.142857142857142

Multipliers:
Multipliers3,2 : -0.0858490566037736
Multipliers4,2 : -0.16037735849056603
```

```
Stage 3
Goal: Fill with zeros under the element A3,3= -3.730188679245283

Multipliers:
Multipliers4,3 : -0.28831562974203334
A:
14.0                    5.0                     -2.0                    30.0                    1.0
0.0                     15.142857142857142      3.142857142857143       5.857142857142858       0.9285714285714286
0.0                     0.0                     -3.730188679245283      1.6028301886792455      1.0797169811320755
0.0                     0.0                     0.0                     -4.169954476479514      1.1745068285280729

L:
1.0                     0.0                     0.0                     0.0
0.07142857142857142     1.0                     0.0                     0.0
0.0                     -0.0858490566037736     1.0                     0.0
0.2857142857142857      -0.16037735849056603    -0.28831562974203334    1.0

x4 = -0.28165938864628826
x3 = -0.41048034934497823
x2 = 0.25545851528384284
x1 = 0.5251091703056769
```

3. Crout

```
Stage 0:
A:
4.0     -1.0     0.0     3.0
1.0     15.5     3.0     8.0
0.0     -1.3     -4.0     1.1
14.0     5.0     -2.0     30.0

Stage 1:
L:
4.0     0.0     0.0     0.0
1.0     1.0     0.0     0.0
0.0     0.0     1.0     0.0
14.0     0.0     0.0     1.0

U:
1.0     -0.25     0.0     0.75
0.0     1.0     0.0     0.0
0.0     0.0     1.0     0.0
0.0     0.0     0.0     1.0

Stage 2:
L:
4.0     0.0     0.0     0.0
1.0     15.75     0.0     0.0
0.0     -1.3     1.0     0.0
14.0     8.5     0.0     1.0

U:
1.0     -0.25     0.0     0.75
0.0     1.0     0.19047619047619047     0.4603174603174603
0.0     0.0     1.0     0.0
0.0     0.0     0.0     1.0
```

```
Stage 3:
L:
4.0     0.0     0.0     0.0
1.0     15.75     0.0     0.0
0.0     -1.3     -3.7523809523809524     0.0
14.0     8.5     -3.619047619047619     1.0

U:
1.0     -0.25     0.0     0.75
0.0     1.0     0.19047619047619047     0.4603174603174603
0.0     0.0     1.0     -0.45262267343485624
0.0     0.0     0.0     1.0

Stage 4:
L:
4.0     0.0     0.0     0.0
1.0     15.75     0.0     0.0
0.0     -1.3     -3.7523809523809524     0.0
14.0     8.5     -3.619047619047619     13.949238578680202

U:
1.0     -0.25     0.0     0.75
0.0     1.0     0.19047619047619047     0.4603174603174603
0.0     0.0     1.0     -0.45262267343485624
0.0     0.0     0.0     1.0


Progressive Substitution and Regressive Substitution
x1 = 0.5251091703056769
x2 = 0.2554585152838428
x3 = -0.4104803493449782
x4 = -0.28165938864628826
```

4. **Doolittle**

```
Stage 0:
A:
4.0     -1.0     0.0     3.0
1.0     15.5     3.0     8.0
0.0     -1.3     -4.0     1.1
14.0     5.0     -2.0     30.0

Stage 1:
L:
1.0     0.0     0.0     0.0
0.0     1.0     0.0     0.0
0.0     0.0     1.0     0.0
0.0     0.0     0.0     1.0

U:
4.0     -1.0     0.0     3.0
0.0     0.0     0.0     0.0
0.0     0.0     0.0     0.0
0.0     0.0     0.0     0.0

Stage 2:
L:
1.0     0.0     0.0     0.0
0.25     1.0     0.0     0.0
0.0     0.0     1.0     0.0
3.5     0.0     0.0     1.0

U:
4.0     -1.0     0.0     3.0
0.0     15.75     3.0     7.25
0.0     0.0     0.0     0.0
0.0     0.0     0.0     0.0
```

```
Stage 3:
L:
1.0     0.0     0.0     0.0
0.25     1.0     0.0     0.0
0.0     -0.08253968253968254     1.0     0.0
3.5     0.5396825396825397     0.0     1.0

U:
4.0     -1.0     0.0     3.0
0.0     15.75     3.0     7.25
0.0     0.0     -3.7523809523809524     1.6984126984126986
0.0     0.0     0.0     0.0

Stage 4:
L:
1.0     0.0     0.0     0.0
0.25     1.0     0.0     0.0
0.0     -0.08253968253968254     1.0     0.0
3.5     0.5396825396825397     0.9644670050761421     1.0

U:
4.0     -1.0     0.0     3.0
0.0     15.75     3.0     7.25
0.0     0.0     -3.7523809523809524     1.6984126984126986
0.0     0.0     0.0     13.949238578680202


Progressive Substitution and Regressive Substitution
x1 = 0.5251091703056769
x2 = 0.2554585152838428
x3 = -0.41048034934497823
x4 = -0.28165938864628826
```

5. **Cholesky**

```
Stage 0:
A:
4.0     -1.0     0.0     3.0
1.0     15.5     3.0     8.0
0.0     -1.3     -4.0     1.1
14.0     5.0     -2.0     30.0

Stage 1:
L:
1.0     0.0     0.0     0.0
0.0     1.0     0.0     0.0
0.0     0.0     1.0     0.0
0.0     0.0     0.0     1.0

U:
4.0     -1.0     0.0     3.0
0.0     0.0     0.0     0.0
0.0     0.0     0.0     0.0
0.0     0.0     0.0     0.0

Stage 2:
L:
1.0     0.0     0.0     0.0
0.25     1.0     0.0     0.0
0.0     0.0     1.0     0.0
3.5     0.0     0.0     1.0

U:
4.0     -1.0     0.0     3.0
0.0     15.75     3.0     7.25
0.0     0.0     0.0     0.0
0.0     0.0     0.0     0.0
```

```
Stage 3:
L:
1.0     0.0     0.0     0.0
0.25     1.0     0.0     0.0
0.0     -0.08253968253968254     1.0     0.0
3.5     0.5396825396825397     0.0     1.0

U:
4.0     -1.0     0.0     3.0
0.0     15.75     3.0     7.25
0.0     0.0     -3.7523809523809524     1.6984126984126986
0.0     0.0     0.0     0.0

Stage 4:
L:
1.0     0.0     0.0     0.0
0.25     1.0     0.0     0.0
0.0     -0.08253968253968254     1.0     0.0
3.5     0.5396825396825397     0.9644670050761421     1.0

U:
4.0     -1.0     0.0     3.0
0.0     15.75     3.0     7.25
0.0     0.0     -3.7523809523809524     1.6984126984126986
0.0     0.0     0.0     13.949238578680202


Progressive Substitution and Regressive Substitution
x1 = 0.5251091703056769
x2 = 0.2554585152838428
x3 = -0.41048034934497823
x4 = -0.28165938864628826
```

6. **Jacobi**

```
iter:  1
E =  0.3609341242060913
x 1 :  0.25
x 2 :  0.06451612903225806
x 3 :  -0.25
x 4 :  0.03333333333333333


iter:  2
E =  0.14562059206430902
x 1 :  0.2411290322580645
x 2 :  0.07956989247311828
x 3 :  -0.2618010752688172
x 4 :  -0.110752688172043


iter:  3
E =  0.14300818286037656
x 1 :  0.35295698924731184
x 2 :  0.15679327089836975
x 3 :  -0.3063172043010753
x 4 :  -0.10990860215053762
```

… the rest of the iterations…

```
iter:  51
E =  1.1940504535335952e-07
x 1 :  0.5251089306169734
x 2 :  0.2554583699130022
x 3 :  -0.4104802164436999
x 4 :  -0.28165919629051855


iter:  52
E =  8.997367394699865e-08
x 1 :  0.5251089896961394
x 2 :  0.2554584057444048
x 3 :  -0.4104802492016183
x 4 :  -0.28165924370300127


T:
0      0.25      0.0      -0.75
-0.06451612903225806      0      -0.1935483870967742      -0.5161290322580645
-0.0      -0.325      0      0.275
-0.4666666666666667      -0.16666666666666666      0.06666666666666667      0

C:
0.25      0.06451612903225806      -0.25      0.03333333333333333

Spectral Radius:  0.7535169428701507
```

7. Gauss-seidel

```
iter:  1
x 1 :  0.25
x 2 :  0.04838709677419355
x 3 :  -0.2657258064516129
x 4 :  -0.10911290322580645
E =  0.38387125576026065

iter:  2
x 1 :  0.3439314516129032
x 2 :  0.15007414151925078
x 3 :  -0.3287801443808533
x 4 :  -0.1740990439646202
E =  0.16541590217059

iter:  3
x 1 :  0.41809281835327783
x 2 :  0.1910348365487899
x 3 :  -0.3599635589686273
x 4 :  -0.2176133585875698
E =  0.1002177019121936
```

…the rest of the iterations…

```
iter:  28
x 1 :  0.5251088724331644
x 2 :  0.25545833579037724
x 3 :  -0.41048020923573025
x 4 :  -0.2816592103829217
E =  2.773619133558324e-07

iter:  29
x 1 :  0.5251089917347855
x 2 :  0.25545840767972766
x 3 :  -0.41048026535121496
x 4 :  -0.28165928177960226
E =  1.6627504060748336e-07

iter:  30
x 1 :  0.5251090632546336
x 2 :  0.25545845077650514
x 3 :  -0.4104802989917548
x 4 :  -0.28165932458103016
E =  9.967983269187507e-08

T:
[[ 0.          0.25        0.         -0.75      ]
 [ 0.         -0.01612903 -0.19354839 -0.46774194]
 [ 0.          0.00524194  0.06290323  0.42701613]
 [ 0.         -0.11362903  0.03645161  0.45642473]]
C:
[ 0.25        0.0483871  -0.26572581 -0.1091129 ]

Spectral Radius:  0.5994876461601171
```

8. SOR

```
iter: 1
x1: 0.375
x2: 0.06048387096774194
x3: -0.4044858870967742
x4: -0.2680695564516129
E: 0.6162413639182875

iter: 2
x1: 0.5117597026209677
x2: 0.34197623325312176
x3: -0.4500491621988
x4: -0.3046959881420314
E: 0.3183687701744234

iter: 3
x1: 0.5901442228192222
x2: 0.23522844810699672
x3: -0.3903363824613489
x4: -0.30859371217532394
E: 0.1453273235043118
```

... the rest of the iterations...

```
iter: 33
x1: 0.5251091913602488
x2: 0.2554584875672992
x3: -0.41048028830434424
x4: -0.28165940102782727
E: 1.8071483197842902e-07

iter: 34
x1: 0.5251091633139185
x2: 0.2554585216829697
x3: -0.41048038809225434
x4: -0.2816593830357972
E: 1.1059753957804027e-07

iter: 35
x1: 0.5251091698894261
x2: 0.2554585190301649
x3: -0.41048032948334456
x4: -0.2816593901105754
E: 5.9458653410847457e-08

T:
[[-5.00000000e-01  3.75000000e-01  2.41217055e-17 -1.12500000e+00]
 [ 4.83870968e-02 -5.36290323e-01 -2.90322581e-01 -6.65322581e-01]
 [-2.35887097e-02  2.61441532e-01 -3.58467742e-01  7.36844758e-01]
 [ 3.35544355e-01 -1.02283266e-01  3.67338710e-02  5.27515121e-01]]
C:
[ 0.375       0.06048387 -0.40448589 -0.26806956]

Spectral Radius:  0.6312081938144991
```

9. **Vandermonde**

```
Vandermonde Matrix:
-1.0   1.0   -1.0   1.0   15.5
0.0   0.0   0.0   1.0   3
27.0   9.0   3.0   1.0   8
64.0   16.0   4.0   1.0   1

Polynomial coefficients:
-1.1416666666666666  5.824999999999999  -5.533333333333332  3.0

Polynomial:
-1.1416666666666666 x^ 3  + 5.824999999999999 x^ 2 -5.533333333333332 x^ 1  + 3.0
```

10. **Newton**

```
Table of divided differences:
15.5     0     0     0
3     -12.5     0     0
8     1.6666666666666667     3.5416666666666665     0
1     -7.0     -2.1666666666666665     -1.1416666666666666

Newton's Polynomial coefficients:
15.5     -12.5     3.5416666666666665     -1.1416666666666666

Newton's Polynomial:
15.5 -  12.5(x +  1 ) +  3.5416666666666665(x +  1 )(x +  0 ) -  1.1416666666666666(x +  1 )(x +  0 )(x -  3 )
```

11. **Lagrange**

```
L 0 =  (x -  -1 ) (x -  2 ) (x -  3 )   -0.6067667640000001

L 1 =  (x -  -2 ) (x -  2 ) (x -  3 )   0.5306566200833334

L 2 =  (x -  -2 ) (x -  -1 ) (x -  3 )   0.3842453250833333

L 3 =  (x -  -2 ) (x -  -1 ) (x -  2 )   0.10427684614999999
```

12. **Lineal Spline**

Generated matrix:

```
{ -1.0, 1.0, 0.0, 0.0, 0.0, 0.0, 15.5},
{ 0.0, 1.0, 0.0, 0.0, 0.0, 0.0, 3.0},
{ 0.0, 0.0, 3.0, 1.0, 0.0, 0.0, 8.0},
{ 0.0, 0.0, 0.0, 0.0, 4.0, 1.0, 1.0},
{ 0.0, 1.0, -0.0, -1.0, 0.0, 0.0, 0.0},
{ 0.0, 0.0, 3.0, 1.0, -3.0, -1.0, 0.0},
```

Coefficients:

```
x6 = 29.0
x5 = -7.0
x4 = 3.0
x3 = 1.6666666666666667
x2 = 3.0
x1 = -12.5
```

13. **Square Spline**

Generated matrix:

```
{ 1.0, -1.0, 1.0, 0.0, 0.0, 0.0, 0.0, 0.0, 0.0, 15.5},
{ 0.0, 0.0, 1.0, 0.0, 0.0, 0.0, 0.0, 0.0, 0.0, 3.0},
{ 0.0, 0.0, 0.0, 9.0, 3.0, 1.0, 0.0, 0.0, 0.0, 8.0},
{ 0.0, 0.0, 0.0, 0.0, 0.0, 0.0, 16.0, 4.0, 1.0, 1.0},
{ 0.0, 0.0, 1.0, -0.0, -0.0, -1.0, 0.0, 0.0, 0.0, 0.0},
{ 0.0, 0.0, 0.0, 9.0, 3.0, 1.0, -9.0, -3.0, -1.0, 0.0},
{ 0.0, 1.0, 0.0, -0.0, -1.0, 0.0, 0.0, 0.0, 0.0, 0.0},
{ 0.0, 0.0, 0.0, 6.0, 1.0, 0.0, -6.0, -1.0, 0.0, 0.0},
{ 2.0, 0.0, 0.0, 0.0, 0.0, 0.0, 0.0, 0.0, 0.0, 0.0},
```

Coefficients:

```
x9 = -245.0000000000001
x8 = 152.8333333333334
x7 = -22.833333333333343
x6 = 2.999999999999994
x5 = -12.500000000000001
x4 = 4.722222222222226
x3 = 2.999999999999994
x2 = -12.500000000000001
x1 = 0.0
```

14. **Cubic Spline**

Generated matrix:

```
{ -1.0, 1.0, -1.0, 1.0, 0.0, 0.0, 0.0, 0.0, 0.0, 0.0, 0.0, 0.0, 15.5},
{ 0.0, 0.0, 0.0, 1.0, 0.0, 0.0, 0.0, 0.0, 0.0, 0.0, 0.0, 0.0, 3.0},
{ 0.0, 0.0, 0.0, 0.0, 27.0, 9.0, 3.0, 1.0, 0.0, 0.0, 0.0, 0.0, 8.0},
{ 0.0, 0.0, 0.0, 0.0, 0.0, 0.0, 0.0, 0.0, 64.0, 16.0, 4.0, 1.0, 1.0},
{ 0.0, 0.0, 0.0, 1.0, -0.0, -0.0, -0.0, -1.0, 0.0, 0.0, 0.0, 0.0, 0.0},
{ 0.0, 0.0, 0.0, 0.0, 27.0, 9.0, 3.0, 1.0, -27.0, -9.0, -3.0, -1.0, 0.0},
{ 0.0, 0.0, 1.0, 0.0, -0.0, -0.0, -1.0, 0.0, 0.0, 0.0, 0.0, 0.0, 0.0},
{ 0.0, 0.0, 0.0, 0.0, 27.0, 6.0, 1.0, 0.0, -27.0, -6.0, -1.0, 0.0, 0.0},
{ 0.0, 2.0, 0.0, 0.0, -0.0, -2.0, 0.0, 0.0, 0.0, 0.0, 0.0, 0.0, 0.0},
{ 0.0, 0.0, 0.0, 0.0, 18.0, 2.0, 0.0, 0.0, -18.0, -2.0, 0.0, 0.0, 0.0},
{ -6.0, 2.0, 0.0, 0.0, 0.0, 0.0, 0.0, 0.0, 0.0, 0.0, 0.0, 0.0, 0.0},
{ 0.0, 0.0, 0.0, 0.0, 0.0, 0.0, 0.0, 0.0, 24.0, 2.0, 0.0, 0.0, 0.0},
```

Coefficients:

```
x12 = -93.00000000000001
x11 = 88.56666666666668
x10 = -24.400000000000006
x9 = 2.033333333333334
x8 = 3.0
x7 = -7.4333333333333345
x6 = 7.6
x5 = -1.5222222222222215
x4 = 3.0
x3 = -7.4333333333333345
x2 = 7.6
x1 = 2.533333333333333
```

15. **Neville Method**

Input:

```
double[] nxs = {1, 1.2, 1.4, 1.6, 1.8, 2};
double[] nys = {0.674732261, 0.849196173, 1.121407696, 1.492135973, 1.960735619, 2.525897371};
double xConst = 1.45;
```

Result:

```
{ 1.067276063, 0.0, 0.0, 0.0, 0.0},
{ 1.18946057675, 1.20473364096875, 0.0, 0.0, 0.0},
{ 1.21408976525, 1.2048538195625, 1.2048237749140625, 0.0, 0.0},
{ 1.1406862384999996, 1.20491432440625, 1.2048790299140626, 1.2048548558515626, 0.0},
{ 0.9717025529999997, 1.2040551205624996, 1.2048427240859374, 1.2048676843427735, 1.2048606286726073},
```

16. **Gaussian Elimination for tridiagonal matrixes**

Input:

```
double[][] tridiag = {
        {5, 2, 0, 0, 0, 0, 12},
        {1, 4, 2, 0, 0, 0, 24},
        {0, -4, 10, 3, 0, 0, -8},
        {0, 0, 3, 12, -8, 0, 13},
        {0, 0, 0, 5, -25, 4, -30},
        {0, 0, 0, 0, 7, 12, 9},
};
```

Result:

```
x6 = 0.6159795407485242
x5 = 1.5135576520771286
x4 = 2.2291612056722143
x3 = -1.2017032050286345
x2 = 7.334279558349241
x1 = -0.5337118233396965
{ 5.0, 2.0, 0.0, 0.0, 0.0, 0.0, 12.0},
{ 0.0, 3.6, 2.0, 0.0, 0.0, 0.0, 24.0},
{ 0.0, 0.0, 12.222222222222221, 3.0, 0.0, 0.0, -8.0},
{ 0.0, 0.0, 0.0, 11.263636363636364, -8.0, 0.0, 13.0},
{ 0.0, 0.0, 0.0, 0.0, -21.44874899112187, 4.0, -30.0},
{ 0.0, 0.0, 0.0, 0.0, 0.0, 14.610874882408279, 9.0},
```