Numerical Analysis

Testing methods

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1. Incremental searches

Parameters:

X0 = -3

$\Delta x = 0.5$

N = 50

$f(x) = \ln(sen(x)^2 + 1) - \frac{1}{2}$

```
Insert a start value
-3
Insert an increase value
0.5
There is a root between [-2.5 , -2.0]
There is a root between [-1.0 , -0.5]
There is a root between [0.5 , 1.0]
There is a root between [2.0 , 2.5]
There is a root between [4.0 , 4.5]
There is a root between [5.0 , 5.5]
There is a root between [7.0 , 7.5]
There is a root between [8.0 , 8.5]
There is a root between [10.0 , 10.5]
There is a root between [11.5 , 12.0]
There is a root between [13.5 , 14.0]
There is a root between [14.5 , 15.0]
There is a root between [16.5 , 17.0]
There is a root between [17.5 , 18.0]
There is a root between [19.5 , 20.0]
There is a root between [21.0 , 21.5]
There is a root between [22.5 , 23.0]
There is a root between [24.0 , 24.5]
There is a root between [26.0 , 26.5]
There is a root between [27.0 , 27.5]
There is a root between [29.0 , 29.5]
There is a root between [30.0 , 30.5]
There is a root between [32.0 , 32.5]
There is a root between [33.5 , 34.0]
There is a root between [35.0 , 35.5]
```

```
There is a root between [33.5 , 34.0]
There is a root between [35.0 , 35.5]
There is a root between [36.5 , 37.0]
There is a root between [38.5 , 39.0]
There is a root between [39.5 , 40.0]
There is a root between [41.5 , 42.0]
There is a root between [43.0 , 43.5]
There is a root between [44.5 , 45.0]
There is a root between [46.0 , 46.5]
There is a root between [48.0 , 48.5]
There is a root between [49.0 , 49.5]
```

2. Bisection

Parameters:

a = 0

b = 1

iterations = 100

tolerance = 10^{-7}

$$f(x) = \ln(sen(x)^2 + 1) - \frac{1}{2}$$

The result is not in table form for aesthetic reasons

```
Iter: 1
a: 0.0
xm: 0.5
b: 1.0
f(xm): -0.2931087267313766
E:

Iter: 2
a: 0.5
xm: 0.75
b: 1.0
f(xm): -0.11839639385347844
E: 0.25

Iter: 3
a: 0.75
xm: 0.875
b: 1.0
f(xm): -0.036817690757380395
E: 0.125

Iter: 4
a: 0.875
xm: 0.9375
b: 1.0
f(xm): 6.339161592386899E-4
E: 0.0625
```

...

… *the other iterations*

…

```
Iter: 22
a: 0.9364042282104492
xm: 0.9364044666290283
b: 0.9364047050476074
f(xm): -6.616007947046754E-8
E: 2.384185791015625E-7

Iter: 23
a: 0.9364044666290283
xm: 0.9364045858383179
b: 0.9364047050476074
f(xm): 2.8715108069121698E-9
E: 1.1920928955078125E-7

Iter: 24
a: 0.9364044666290283
xm: 0.9364045262336731
b: 0.9364045858383179
f(xm): -3.164428308277678E-8
E: 5.9604644775390625E-8

The maximum tolerance permitted 1.0E-7. In the iteration 25 the maximum tolerance was reached 5.9604644775390625E-8

Data in the last iteration
Iteration 24
xm= 0.9364045262336731
f(xm)= -3.164428308277678E-8
E= 5.9604644775390625E-8
```

3. **Fake Rule**
   iterations = 100
   tolerance = 10^{-7}

```
3
to change the equation please change the code of the method evaluateF and re-run the program
Enter the right end of the interval
0
Enter the left end of the interval
1
| iter|      a          |      xm        |      b          |      f(Xm)   |      E     |
| 0 | 0.0 | 0.9339403807182157 | 1.0 | -0.0014290767036854723 | 0.9339403807182157 |
| 1 | 0.9339403807182157 | 0.9365060516656253 | 1.0 | 5.8756008358140654E-5 | 0.06349394833437472 |
| 2 | 0.9339403807182157 | 0.9364047307426411 | 0.9365060516656253 | 8.678254082017389E-8 | 1.0132092298420492E-4 |
| 3 | 0.9339403807182157 | 0.936404581100869 | 0.9364047307426411 | 1.2815393191090152E-10 | 1.4964177208476315E-7 |
| 4 | 0.9339403807182157 | 0.9364045808798893 | 0.936404581100869 | 1.894040480010517E-13 | 2.2097967899981086E-10 |
The root reached was 0.9364045808798893
```

4. **Fixed Point**
   iterations = 100

tolerance = 10^{-7}

```
4
to change the equation please change the code of the methods evaluateF and evaluateG and re-run the program
Enter the initial approach
-0.5
| iter|     xi          |     g(xi)       |   f(xi)   |     E     |
| 0 |  -0.5  |  -0.2931087267313766  |  0.2068912732686234  |  0.2068912732686234  |
| 1 |  -0.2931087267313766  |  -0.41982154360625734  |  -0.12671281687488073  |  0.12671281687488073  |
| 2 |  -0.41982154360625734  |  -0.346304519177665 1  |  0.07351702442859226  |  0.07351702442859226  |
| 3 |  -0.346304519177665 1  |  -0.3909584565423095  |  -0.0446539373646444  |  0.0446539373646444  |
| 4 |  -0.3909584565423095  |  -0.3644050348941392  |  0.02655342164817026  |  0.02655342164817026  |
| 5 |  -0.3644050348941392  |  -0.3804263031679563  |  -0.016021268273817058  |  0.016021268273817058  |
| 6 |  -0.3804263031679563  |  -0.37083679528020885  |  0.009589507887747428  |  0.009589507887747428  |
| 7 |  -0.37083679528020885  |  -0.3766056453635812  |  -0.005768850083372357  |  0.005768850083372357  |
| 8 |  -0.3766056453635812  |  -0.373145417607189  |  0.003460227756392209  |  0.003460227756392209  |
| 9 |  -0.373145417607189  |  -0.3752246411870562  |  -0.002079223579867173  |  0.002079223579867173  |
| 10 |  -0.3752246411870562  |  -0.37397658604830963  |  0.00124805513874654  |  0.00124805513874654  |
| 11 |  -0.37397658604830963  |  -0.3747262157084321  |  -7.496296601224861E-4  |  7.496296601224861E-4  |
| 12 |  -0.3747262157084321  |  -0.37427613331045395  |  4.5008239797816874E-4  |  4.5008239797816874E-4  |
| 13 |  -0.37427613331045395  |  -0.3745464284580923  |  -2.7029514763832196E-4  |  2.7029514763832196E-4  |
| 14 |  -0.3745464284580923  |  -0.374384126434844 7  |  1.623020232475736E-4  |  1.623020232475736E-4  |
| 15 |  -0.374384126434844 7  |  -0.3744815908319551  |  -9.746439711039168E-5  |  9.746439711039168E-5  |
| 16 |  -0.3744815908319551  |  -0.37442306518389706  |  5.8525648058027624E-5  |  5.8525648058027624E-5  |
| 17 |  -0.37442306518389706  |  -0.37445820986270584  |  -3.514467880877392E-5  |  3.514467880877392E-5  |
| 18 |  -0.37445820986270584  |  -0.3744371058494556  |  2.110401325022826E-5  |  2.110401325022826E-5  |
| 19 |  -0.3744371058494556  |  -0.37444977872741303  |  -1.2672877957420337E-5  |  1.2672877957420337E-5  |
| 20 |  -0.37444977872741303  |  -0.37444216876320036  |  7.609964212673681E-6  |  7.609964212673681E-6  |
| 21 |  -0.37444216876320036  |  -0.3744467385052047  |  -4.5697420043566694E-6  |  4.5697420043566694E-6  |
| 22 |  -0.3744467385052047  |  -0.37444399440652526  |  2.744098679452467E-6  |  2.744098679452467E-6  |
| 23 |  -0.37444399440652526  |  -0.37444564222126353  |  -1.647814738270359E-6  |  1.647814738270359E-6  |
| 24 |  -0.37444564222126353  |  -0.37444465271927385  |  9.895019896788426E-7  |  9.895019896788426E-7  |
| 24 |  -0.37444564222126353  |  -0.37444465271927385  |  9.895019896788426E-7  |  9.895019896788426E-7  |
| 25 |  -0.37444465271927385  |  -0.3744452469090602  |  -5.941897863737111E-7  |  5.941897863737111E-7  |
| 26 |  -0.3744452469090602  |  -0.37444489010190096  |  3.568071592630062E-7  |  3.568071592630062E-7  |
| 27 |  -0.37444489010190096  |  -0.37444510436235334  |  -2.1426045238026603E-7  |  2.1426045238026603E-7  |
| 28 |  -0.37444510436235334  |  -0.3744449757003151  |  1.28662038245686E-7  |  1.28662038245686E-7  |
| 29 |  -0.3744449757003151  |  -0.37444505296105535  |  -7.726074024994034E-8  |  7.726074024994034E-8  |
The root reached was -0.37444505296105535
```

5. **Newton**
   iterations = 100
   tolerance = 10^{-7}

```
5
to change the equation please change the code of the methods evaluateF and evaluateFDerived and re-run the program
Enter the initial approach
0.5
| iter|     xi          |   f(xi)   |     E     |
| 0 |  0.5  |  -0.2931087267313766  |  0.4283919899125719  |
| 1 |  0.9283919899125719  |  -0.004662157097372055  |  0.007974751354759446  |
| 2 |  0.9363667412673313  |  -2.1912619882713535E-5  |  3.783875165885853E-5  |
| 3 |  0.9364045808795621  |  -1.1102230246251565E-16  |  8.605719470367035E-10  |
The root reached was 0.9364045808795621
```

6. **Secant**
   iterations = 100
   tolerance = 10^{-7}

```
6
to change the equation please change the code of the methods evaluateF and evaluateFDerived and re-run the program
Enter the initial approach #1
0.5
Enter the initial approach #2
1
| iter|     xi         |   f(xi)   |     E     |
| 0 |  0.5  |  -0.2931087267313766  |  0.44616622230652503  |
| 1 |  0.946166222306525  |  0.005619392737863826  |  0.008392859815018472  |
| 2 |  0.9377733624915066  |  7.919798731234051E-4  |  0.0013769230512663544  |
| 3 |  0.9363964394402402  |  -4.714558902108035E-6  |  8.14814886096471E-6  |
| 4 |  0.9364045808795949  |  1.887379141862766E-14  |  6.709506283897326E-9  |
The root reached was 0.9364045808795949
```

7. **Multiple Roots**

   Parameters:

   x0 = 1

   iterations = 100

   tolerance = 10^{-7}

   $h(x) = e^{x} - x - 1$

   $h(x)' = e^{x} - 1$

   $h(x)'' = e^{x}$

The result is not in table form for aesthetic reasons

```
Iter: 0
x0: 1.0
f(xi): 0.7182818284590451
E:

Luego de hacer la super operacion: 1.2342106135535142
Iter: 1
xi: -0.23421061355351425
f(xi): 0.025405775475345838
E: 1.2342106135535142

Iter: 2
xi: -0.00845827991076109
f(xi): 3.567060801401567E-5
E: 0.22575233364275316

Iter: 3
xi: -1.1890183808588653E-5
f(xi): 7.068789997788372E-11
E: 0.008446389726952502

Iter: 4
xi: -4.218590698935789E-11
f(xi): 0.0
E: 1.1890141622681664E-5

An approximation of the root was found in -4.218590698935789E-11
```

8. Gaussian Elimination

Parameters:

$$A = \begin{pmatrix} 2 & -1 & 0 & 3 \\ 1 & 0.5 & 3 & 8 \\ 0 & 13 & -2 & 11 \\ 14 & 5 & -2 & 3 \end{pmatrix}, \qquad b = \begin{pmatrix} 1 \\ 1 \\ 1 \\ 1 \end{pmatrix}$$

Results:

```
Original matrix
2.0                 -1.0                0.0                 3.0                 1.0

1.0                 0.5                 3.0                 8.0                 1.0

0.0                 13.0                -2.0                11.0                1.0

14.0                5.0                 -2.0                3.0                 1.0


Stage 1
Goal: Fill with zeros under the element A 1,1= 2.0

Multipliers:
Multipliers 2 , 1: 0.5
Multipliers 3 , 1: 0.0
Multipliers 4 , 1: 7.0

2.0                 -1.0                0.0                 3.0                 1.0

0.0                 1.0                 3.0                 6.5                 0.5

0.0                 13.0                -2.0                11.0                1.0

0.0                 12.0                -2.0                -18.0               -6.0
```

```
Stage 2
Goal: Fill with zeros under the element A 2,2= 1.0

Multipliers:
Multipliers 3 , 2: 13.0
Multipliers 4 , 2: 12.0

2.0                 -1.0                0.0                 3.0                 1.0

0.0                 1.0                 3.0                 6.5                 0.5

0.0                 0.0                 -41.0               -73.5               -5.5

0.0                 0.0                 -38.0               -96.0               -12.0


Stage 3
Goal: Fill with zeros under the element A 3,3= -41.0

Multipliers:
Multipliers 4 , 3: 0.926829268292683

2.0                 -1.0                0.0                 3.0                 1.0

0.0                 1.0                 3.0                 6.5                 0.5

0.0                 0.0                 -41.0               -73.5               -5.5

0.0                 0.0                 0.0                 -27.878048780487802 -6.902439024390244
```

```
Backward substitution
X4 = 0.24759405074365706
X3 = -0.30971128608923887
X2 = -0.18022747156605434
X1 = 0.03849518810148722
```

9. Partial Pivot

Parameters:

$$A = \begin{pmatrix} 2 & -1 & 0 & 3 \\ 1 & 0.5 & 3 & 8 \\ 0 & 13 & -2 & 11 \\ 14 & 5 & -2 & 3 \end{pmatrix}, \qquad b = \begin{pmatrix} 1 \\ 1 \\ 1 \\ 1 \end{pmatrix}$$

Results:

```
14.0 5.0 -2.0 3.0 1.0
0.0 0.1428571428571429 3.142857142857143 7.785714285714286 0.9285714285714286
0.0 13.0 -2.0 11.0 1.0
0.0 -1.7142857142857142 0.2857142857142857 2.5714285714285716 0.8571428571428572


14.0 5.0 -2.0 3.0 1.0
0.0 13.0 -2.0 11.0 1.0
0.0 0.0 3.1648351648351647 7.664835164835164 0.9175824175824177
0.0 2.220446049250313E-16 0.021978021978021955 4.021978021978022 0.989010989010989


14.0 5.0 -2.0 3.0 1.0
0.0 13.0 -2.0 11.0 1.0
0.0 0.0 3.1648351648351647 7.664835164835164 0.9175824175824177
0.0 2.220446049250313E-16 0.0 3.96875 0.982638888888889




14.0 5.0 -2.0 3.0 1.0
0.0 13.0 -2.0 11.0 1.0
0.0 0.0 3.1648351648351647 7.664835164835164 0.9175824175824177
0.0 2.220446049250313E-16 0.0 3.96875 0.982638888888889


x4 = 0.24759405074365706
x3 = -0.30971128608923887
x2 = -0.18022747156605426
x1 = 0.03849518810148731
```

10. Total Pivot

Parameters:

$$A = \begin{pmatrix} 2 & -1 & 0 & 3 \\ 1 & 0.5 & 3 & 8 \\ 0 & 13 & -2 & 11 \\ 14 & 5 & -2 & 3 \end{pmatrix}, \qquad b = \begin{pmatrix} 1 \\ 1 \\ 1 \\ 1 \end{pmatrix}$$

Results:

```
14.0 5.0 -2.0 3.0 1.0
0.0 0.1428571428571429 3.142857142857143 7.785714285714286 0.9285714285714286
0.0 13.0 -2.0 11.0 1.0
0.0 -1.7142857142857142 0.2857142857142857 2.5714285714285716 0.8571428571428572


14.0 5.0 -2.0 3.0 1.0
0.0 13.0 -2.0 11.0 1.0
0.0 0.0 3.1648351648351647 7.664835164835164 0.9175824175824177
0.0 2.220446049250313E-16 0.021978021978021955 4.021978021978022 0.989010989010989


14.0 5.0 3.0 -2.0 1.0
0.0 13.0 11.0 -2.0 1.0
0.0 0.0 7.664835164835164 3.1648351648351647 0.9175824175824177
0.0 2.220446049250313E-16 0.0 -1.638709677419355 0.5075268817204301


14.0 5.0 3.0 -2.0 1.0
0.0 13.0 11.0 -2.0 1.0
0.0 0.0 7.664835164835164 3.1648351648351647 0.9175824175824177
0.0 2.220446049250313E-16 0.0 -1.638709677419355 0.5075268817204301


x3 = -0.3097112860892388
x4 = 0.24759405074365703
x2 = -0.18022747156605423
x1 = 0.03849518810148732
```

11. Aitken

Parameters:

X0 = 1

Tolerance = 10^{-7}

```
PS C:\Proyectos en java> java Aitken
Insert X1:
1
The value of the root is: 0.7320508070759111
```

12. Steffensen

Parameters:

x0 = -0.5

iterations = 100

tolerance = 10^{-7}

```
 |ite|          xi              |            f(xi)           |            E            |
 | 0 |  -0.5   |  0.2068912732686234  |  0.1283077762514827  |
 | 1 |  -0.3716922237485173  |  -0.004402295146406721  |  0.00275147625374883  |
 | 2 |  -0.3744437000022661  |  -2.119006566103643e-06  |  1.3239708119283655e-06  |
 | 3 |  -0.37444502397338436  |  -1.1102230246251565e-16  |  3.063105324940807e-13  |
 The root reached was -0.37444502397338436
```

13. Müller

Parameters:

$$f(x) = x^3 + 2x^2 + 10x - 20$$

A = 0

B = 1

C = 2

Results:

```
run:
The value of the root is 1.3688080368924294
BUILD SUCCESSFUL (total time: 0 seconds)
```