Optimization under uncertainty 2025 - assignment

Organizational instructions.

1. Due date: December 8, 2025, 23:59. The assignment brings 10% of the course grade.
2. You can do the assignment in a group of at most two people. Place yourself in Canvas → People in one of the groups under the group set "Assignment".
3. The minimal knowledge needed to do the assignment is the information from weeks 1 – 4 of the course. Feel free to use additional knowledge to, but mention which and why.
4. Any solution approach is acceptable if it uses the course methods, makes sense, you justify your choices and answer the questions listed in the problem description below.
5. Explain everything you do in a report, write clearly but briefly (do not write the same thing many times in different words), around 5 pages should be enough for such a report.
6. Submission: Hand in *via Canvas* (click on the Assignment there) the report in a pdf-file and your code in a zip-file.
7. The questions of the assignment can bring max 100 points, the grade will be the number of points divided by 10.
8. Have fun and good luck!

Problem description.

In this assignment we will consider *the day-ahead electricity market optimization problem.* Every day, energy generating companies submit their technical information to the energy market operator, and the operator uses this data to plan electricity production for each hour of the next day. Let's consider a simple day-ahead model with J generators. The generators provide to the energy market operator their costs, the rate showing how fast the generation can be adjusted within one hour (the so-called ramp rates), the upper bounds on generation, and estimations of emission rates. All problem parameters and variables are described in the table below.

Notation	**Indices**
j	Index of the conventional energy technology ($j \in \{1, \ldots, J\}$)
t	Index of hour ($t \in \{1, \ldots, 24\}$)
	Fixed parameters
c_j	Marginal cost of generator j (1000 euro per MWh)
k_j	Maximal capacity of generator j (MW)
γ_j^u	Ramp-up rate of generator j ($\gamma_j^u \in [0, 1]$)
γ_j^d	Ramp-down rate of generator j ($\gamma_j^d \in [-1, 0]$)
	Uncertain parameters
d_t	Residual electricity demand for hour t (MWh)
e_j	Marginal emission of generator j (ton CO2 per MWh)
	Decision variables
$\mathbf{q}_{t,j}$	Production of generator j in hour t (MWh)

The market operator wants the electricity to be cheap but not too carbon intense, so it prefers to avoid dirty energy sources, such as coal, when possible. Thus, the operator minimizes the weighted sum of the total cost and CO2 emission obtaining the following problem.

$$
\min_{\mathbf{q}\in\mathbb{R}^{T\times J}} \; 0.65\sum_{t=1}^{24}\sum_{j=1}^{J} c_j \mathbf{q}_{t,j} + 0.35\sum_{t=1}^{24}\sum_{j=1}^{J} e_j \mathbf{q}_{t,j} \tag{1a}
$$

$$
\text{s.t. } 0 \le \mathbf{q}_{t,j} \le k_j \qquad t=1,\dots,24,\; j=1,\dots,J \tag{1b}
$$

$$
\gamma_j^d k_j \le \mathbf{q}_{t+1,j} - \mathbf{q}_{t,j} \le \gamma_j^u k_j \qquad t=1,\dots,23,\; j=1,\dots,J \tag{1c}
$$

$$
\sum_{j=1}^{J} \mathbf{q}_{t,j} \ge d_t \qquad t=1,\dots,24. \tag{1d}
$$

Constraints (1b) indicate that electricity generation is non-negative and bounded by its maximum capacity. Constraints (1c) show how much the generation can vary between hours because of the ramp-up and -down limits. Constraints (1d) require the electricity demand to be met.

The generation cost, maximal capacity and ramp rates are known and contain little uncertainty. On the contrary, the residual demand and emission rates can be rather volatile. The residual demand is computed as $d_t = d_t^0 - r_t$, where d_t^0 is the total electricity demand for hour t, and r_t is the amount of renewable energy produced by solar and wind generators. These indicators are highly uncertain and estimated one day before their realization using specialized forecasts. The emission rate e_j of each generator j is uncertain too, due to measurement errors and varying characteristics of coal and gas. Contracts on the day-ahead markets are binding, so the contracted amounts of energy will be produced on the next day independently of the realizations of the uncertain parameters. If the demand is higher than the resulting generation, additional energy is purchased via real-time energy markets. The prices and participants pool on those markets are more volatile than on the day-ahead market, which imposes higher risks.

You have characteristics of $J = 22$ largest generating facilities on the Dutch electricity market for the end of the year 2022 and some observations of the uncertain parameters. In particular, there are about three years (01.01.2020 – 07.12.2022) of hourly residual demand forecasts and actual residual demand realizations, as well as measurements of each generating facility's marginal emissions for the same days. You also have the residual demand forecast for 09.12.2022. Assume you are in 08.12.2022 and need to solve the day-ahead market optimization for 09.12.2022 using all the available information and approaching the uncertainty with some of the models studied in the course. Answer the following questions.

1. (15p): Specify the uncertainty structure you want to use in the formulation (i.e., explain if you want to use the worst-case or probabilistic setup, how and why). Make assumptions if needed to justify your modeling choices, and mention these assumptions.
2. (5p): Specify the objective you want to optimize (average, worst-case, achieved with some probability, something else that sounds reasonable to you), motivate your choice.
3. (15p): Write a mathematical formulation of the full uncertain problem you are going to solve. Explain/derive a solution approach for your mathematical formulation (specify the uncertainty structure in such a way that the resulting "uncertain" problem is feasible).
4. (10p): Implement your solution approach using a programming language of your choice, save the optimal solution, we will call it *the OUU solution* further.
5. (5p): Find the so-called *nominal solution* defined as the solution obtained by solving (1) with the residual demand forecast for 09.12.2022 and the sample mean inserted as the value of the marginal emissions. Save this solution.
6. (10p): The day-ahead energy price set by the market operator is equal to the marginal cost of the most expensive generator invoked. Compare the solutions, objective values,

and the day-ahead energy prices from your OUU solution with the nominal one, report and explain what you observe. How does each uncertainty type (emission and demand) separately contribute to the difference between the nominal and OUU solution and why?

7. (10p): Next you will need to simulate new uncertain parameters realizations to see how your solution performs out-of-sample. Choose the distributions from which you will sample the new parameters realizations, they should reflect well the samples provided with this assignment. Choose the sample size you need to simulate in order to evaluate your solution's quality. Motivate both choices. Finally, simulate and save your new vectors of residual demand and marginal emission, this will be your test sample.

8. (10p): Evaluate the performance of the OUU solution *out-of-sample*. Namely, for every simulated vector of returns from the test sample, substitute your solution into the problem and report what you observe about the objective and feasibility of the constraints. Choose yourself which indicators to use in the report. Make sure you consider not only the average performance but also some descriptive statistics reflecting the distribution of the results.

9. (5p): In the same out-of-sample setting, evaluate the nominal solution.

10. (5p): Implement and evaluate in the same out-of-sample setting the so-called *perfect hindsight* solution approach. That is, for each realization of residual demand and emission rates from the test sample, you solve problem (1) separately, which gives you the best possible objective you could achieve if you knew the residual demand and emission rates in advance (here you should obtain as many solutions as the observations in the test sample).

11. (10p): Compare the out-of-sample performance of all three solution approaches (OUU, nominal, perfect hindsight), describe what you observe and explain your results.