INSTITUTE OF QUALITY & TECHNOLOGY MANAGEMENT
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Class: BSc Industrial Engineering & Management **Subject:** Operations Research – 1
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Lab Activity: Mixed Integer Linear Programming Problems

Problem: Capacitated Facility Location Problem

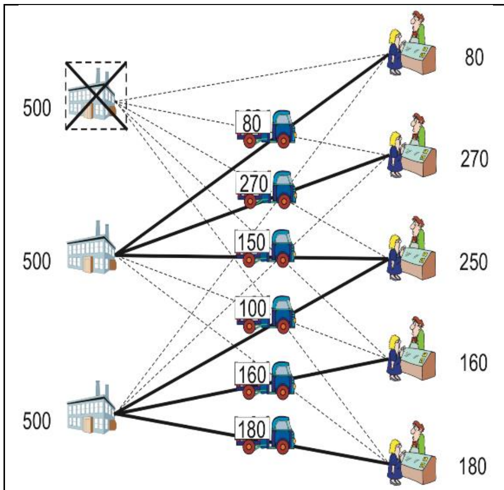
Consider a company with three potential sites for installing its facilities/warehouses and five demand points. Each site j has a yearly *activation cost (fixed cost)* f_j, i.e., an annual leasing expense that is incurred for using it, independently of the volume it services. This volume is limited to a given maximum amount that may be handled yearly, M_j. Additionally, there is a transportation cost c_{ij} per unit serviced from facility j to the demand point i. These data are shown in Table Data for the facility location problem: demand, transportation costs, fixed costs, and capacities.

Data for the facility location problem: demand, transportation costs, fixed costs, and capacities

Customer i	1	2	3	4	5		
Annual demand d_j	80	270	250	160	180		
Facility j	c_{ij}					f_j	M_j
1	4	5	6	8	10	1000	500
2	6	4	3	5	8	1000	500
3	9	7	4	3	4	1000	500

Model for the Problem:

Consider n customers $i = 1,2,\dots,n$ and m sites for facilities $j = 1,2,\dots,m$.
Define continuous variables $x_{ij} \geq 0$ as the amount serviced from facility j to demand point i, and binary variables $y_j = 1$ if a facility is established at location j, $y_j = 0$ otherwise. An integer-optimization model for the capacitated facility location problem can now be specified as follows:



$$\text{minimize} \quad \sum_{j=1}^{m} f_j y_j + \sum_{i=1}^{n}\sum_{j=1}^{m} c_{ij} x_{ij}$$

$$\begin{aligned}
\text{subject to:} \quad & \sum_{j=1}^{m} x_{ij} = d_i && \text{for } i = 1,\cdots,n \\
& \sum_{i=1}^{n} x_{ij} \leq M_j y_j && \text{for } j = 1,\cdots,m \\
& x_{ij} \leq d_i y_j && \text{for } i = 1,\cdots,n;\ j = 1,\cdots,m \\
& x_{ij} \geq 0 && \text{for } i = 1,\cdots,n;\ j = 1,\cdots,m \\
& y_j \in \{0,1\} && \text{for } j = 1,\cdots,m
\end{aligned}$$

- The **objective of the problem** is to minimize the sum of facility activation costs and transportation costs.
- The **first constraints** require that each customer's demand must be satisfied.
- The capacity of each facility j is limited by the **second constraints**: if facility j is activated, its capacity restriction is observed; if it is not activated, the demand satisfied by j is zero.
- **Third constraints** provide variable upper bounds; even though they are redundant, they yield a much tighter linear programming relaxation than the equivalent.
- **Fourth constraints** are non-negativity constraints.
- **Fifth** indicates that variables are binary.