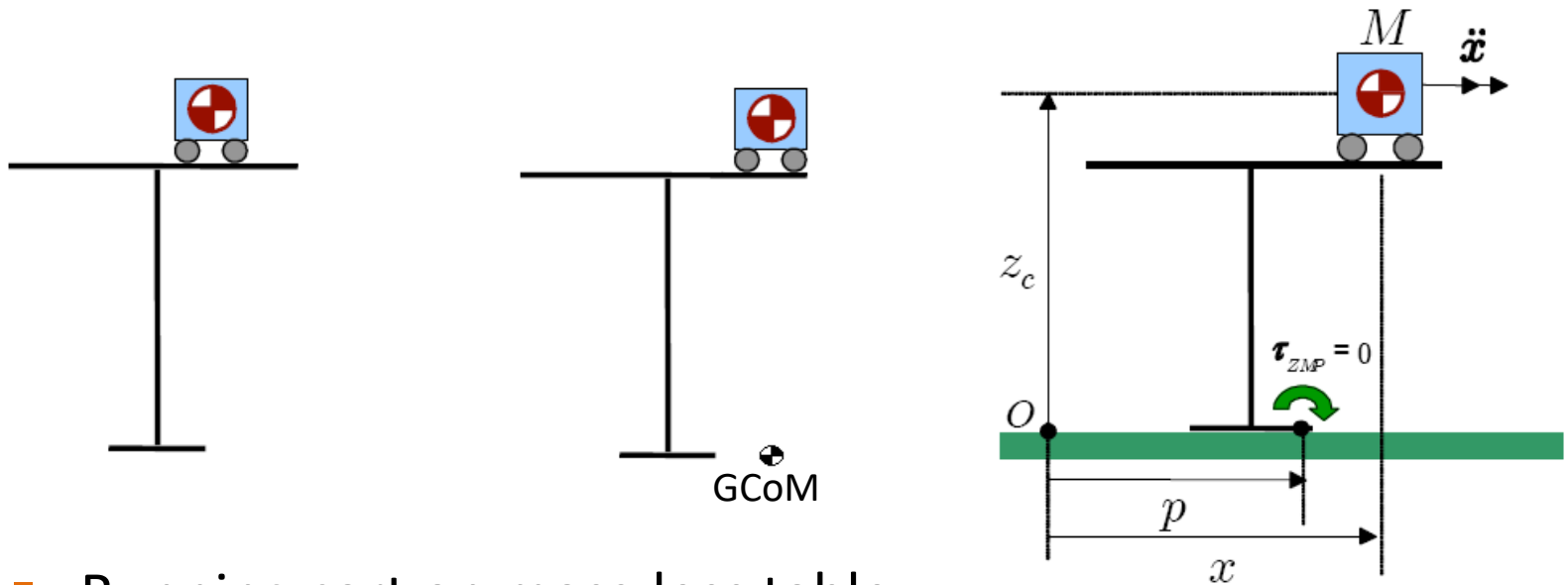


Zero Moment Point

ME 193B / 292B

- GRF: Ground Reaction Force
- CoM: Center of Mass
- GCoM: Ground Projection of Center of Mass
- CoP: Center of Pressure
- ZMP: Zero Moment Point
- FRI: Foot Rotation Indicator

Simplified Biped Model

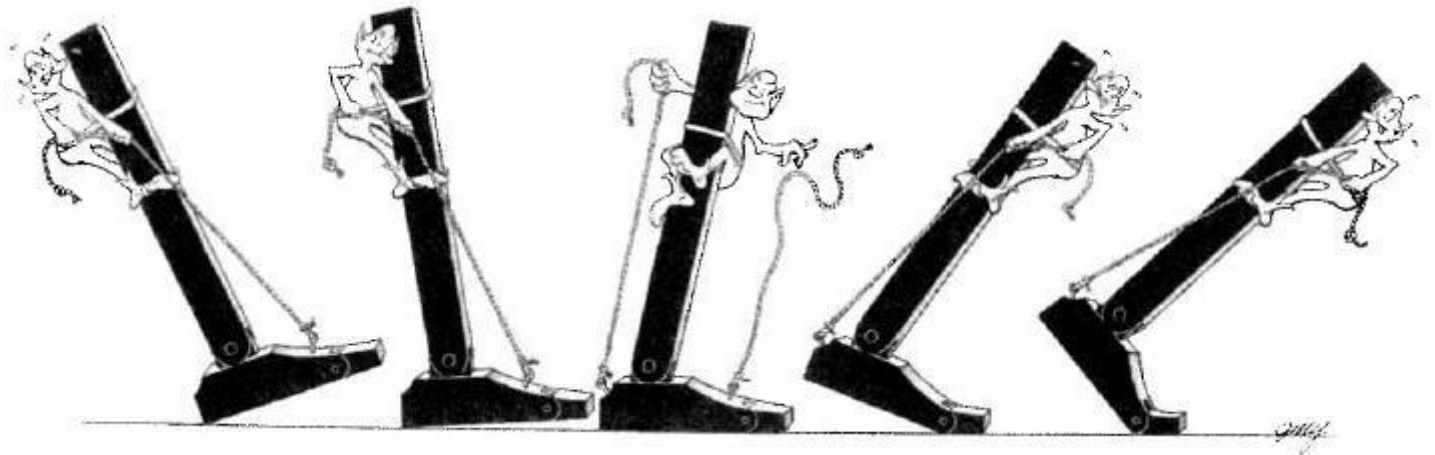


- Running cart on mass-less table
- Cart represents the CoM motion
- Table represents supporting foot

$$\tau_{ZMP} = Mg(x - p_x) - M\ddot{x}z_h \quad \Rightarrow \quad p_x = x - \frac{z_h}{g}\ddot{x}$$

- Similar set of equations for y motion

Foot Action



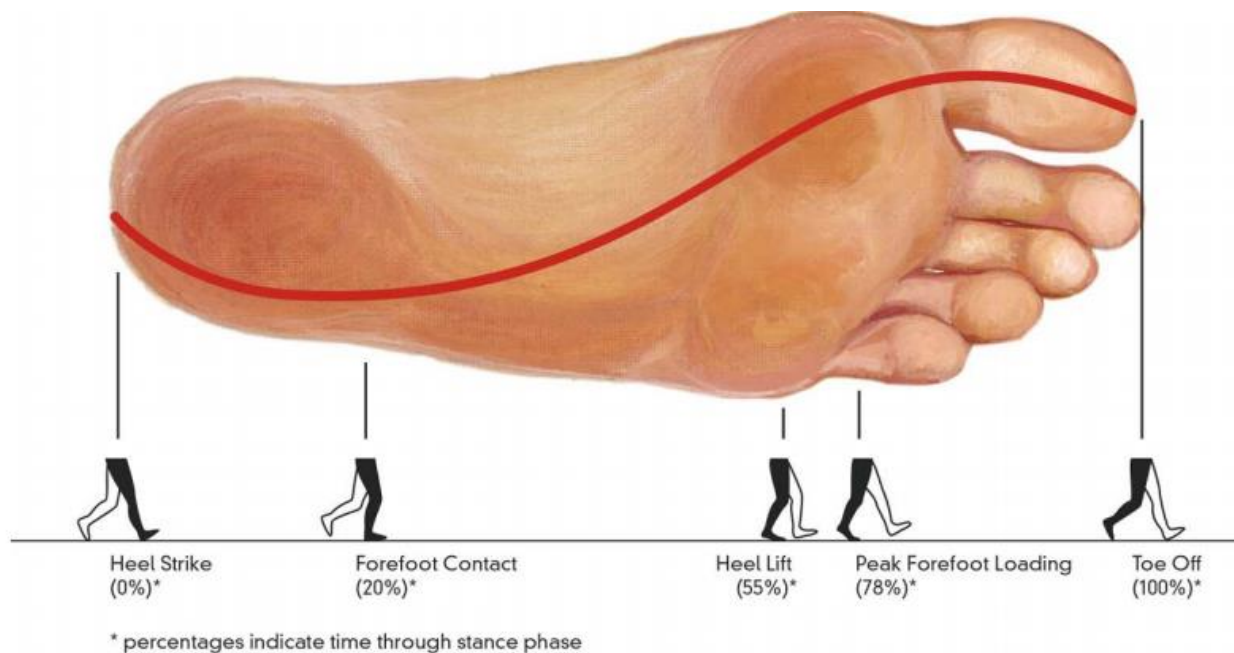
Heel Strike

Flat Foot

Toe Off

CoP

- Point where the net GRF acts.
- CoP can never leave support polygon



Zero Moment Point

- Foot Ground Joint is “Unilateral” and “Underactuated”

ZMP

[Vukobratovich]



ZMP Walking Videos

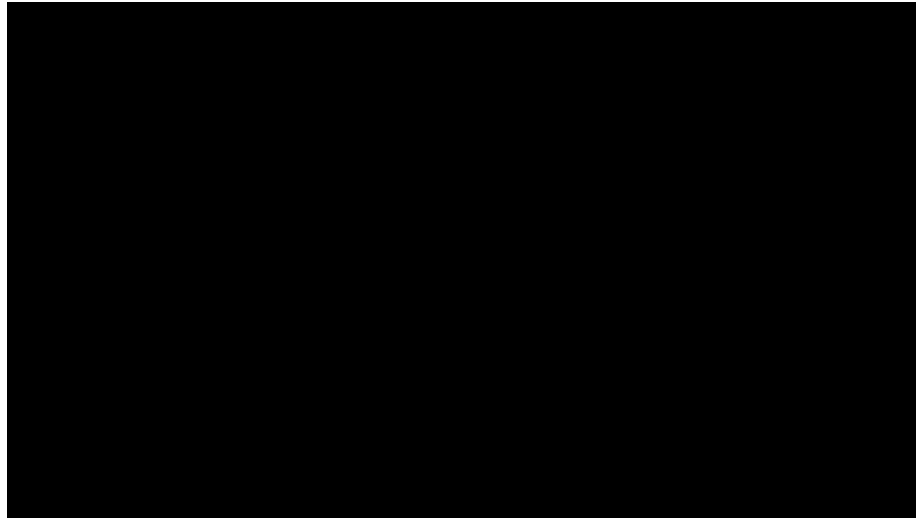


JOHNNIE

The TUM Biped Walking Robot

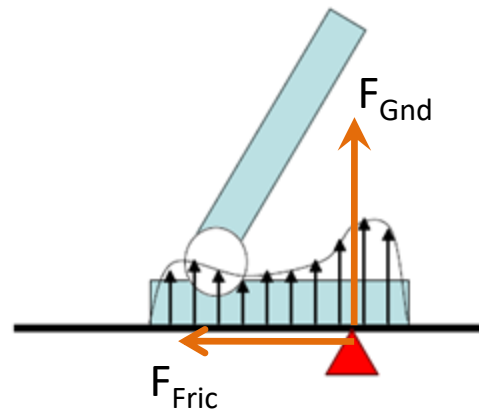
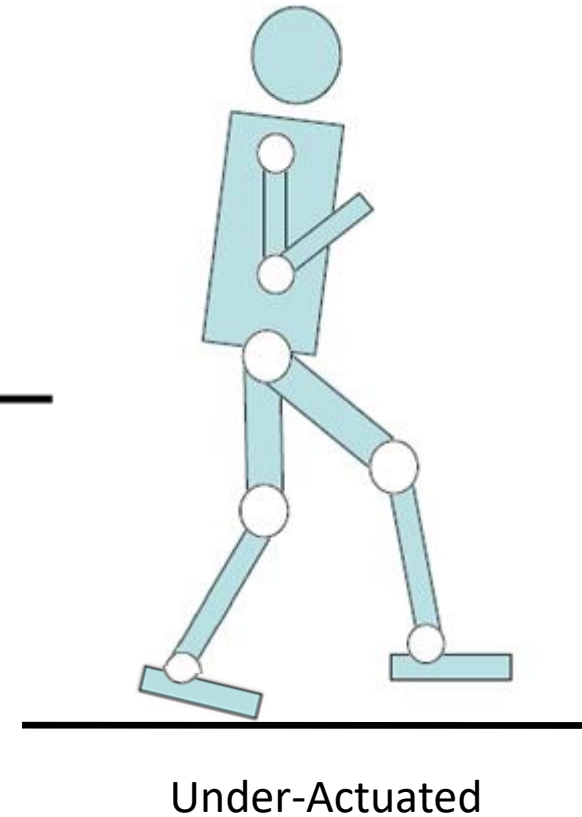
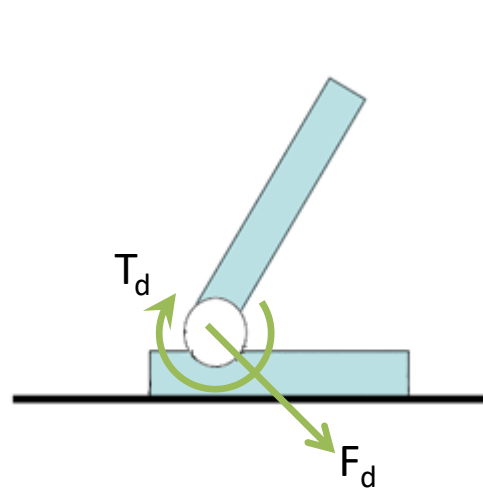
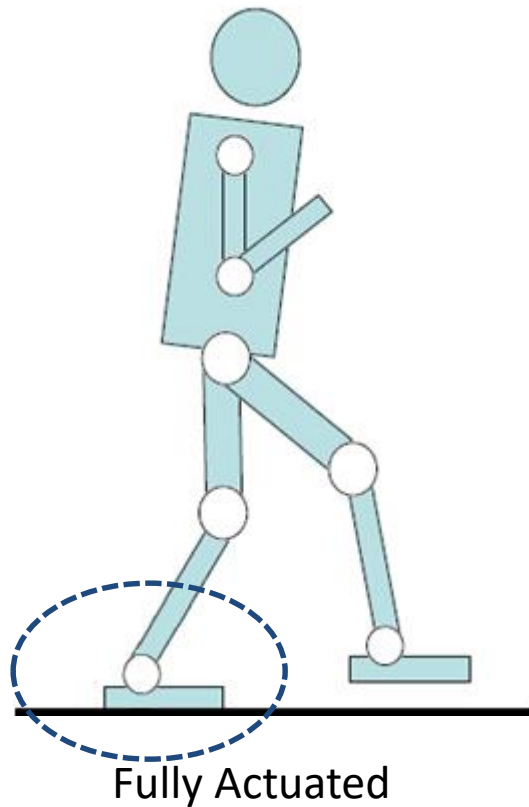
Technical University of Munich
Institute for Applied Mechanics
Prof. Dr.-Ing. Friedrich Pfeiffer

Design: M. Glenger
Control: K. Löffler

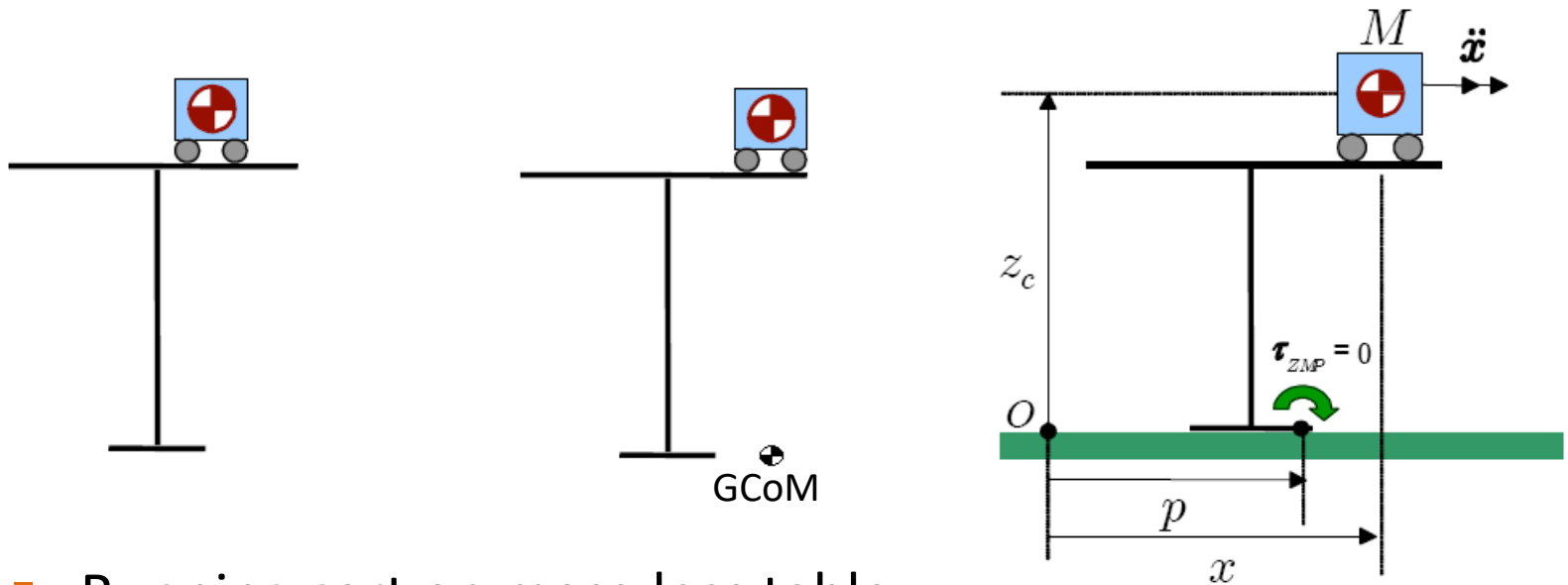


<https://youtu.be/27UdIAegBdk>

ZMP = Flat-Footed Walking



Simplified Biped Model



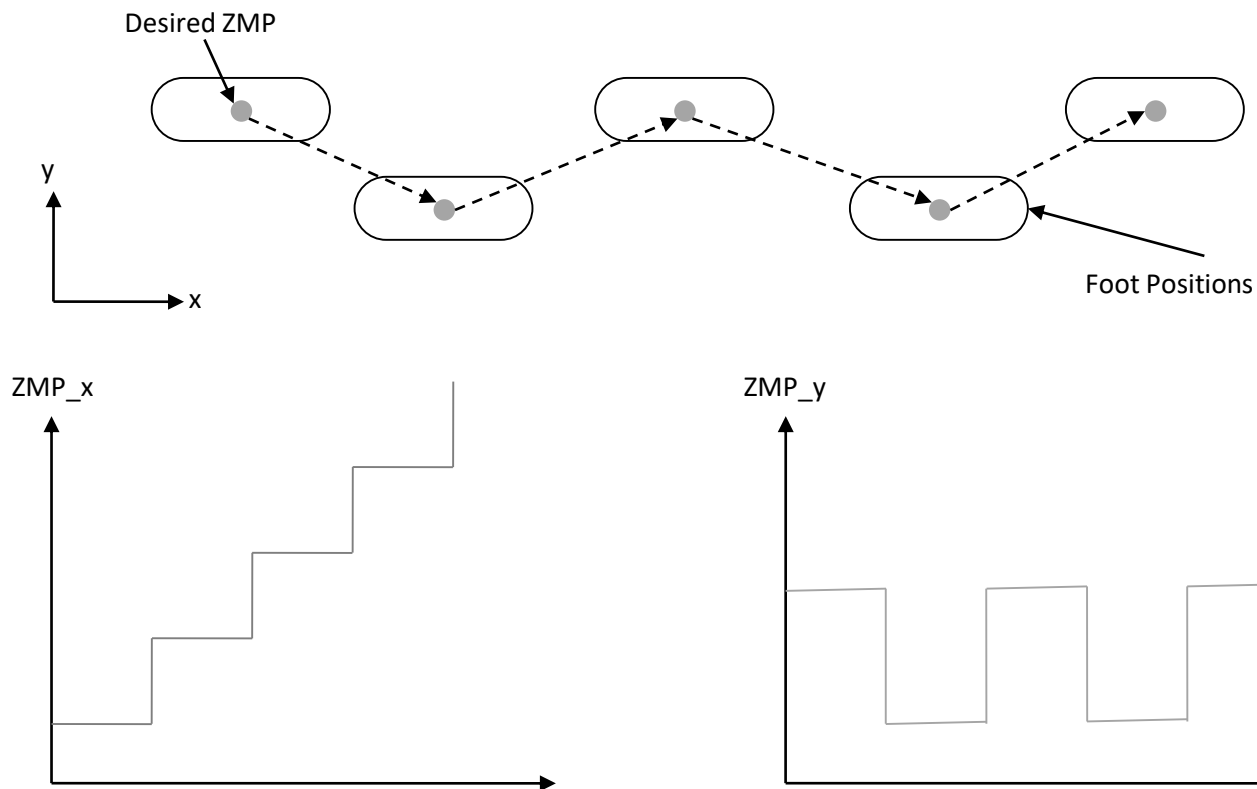
- Running cart on mass-less table
- Cart represents the CoM motion
- Table represents supporting foot

$$\tau_{ZMP} = Mg(x - p_x) - M\ddot{x}z_h \quad \Rightarrow \quad p_x = x - \frac{z_h}{g}\ddot{x}$$

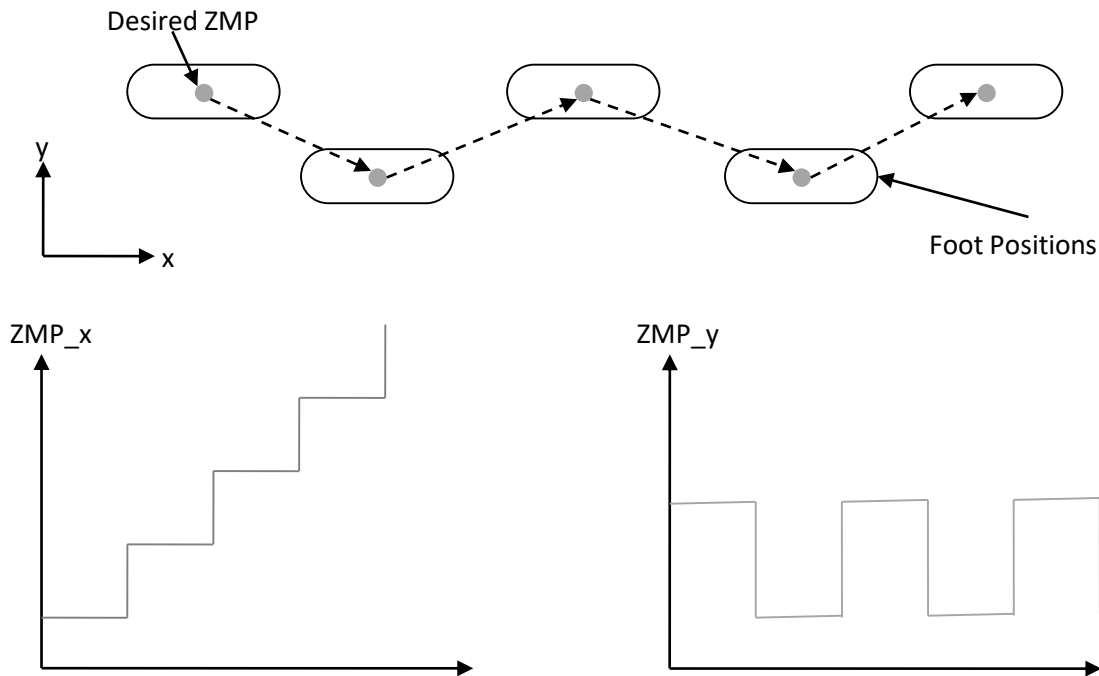
- Similar set of equations for y motion

Bipedal Walking based on ZMP

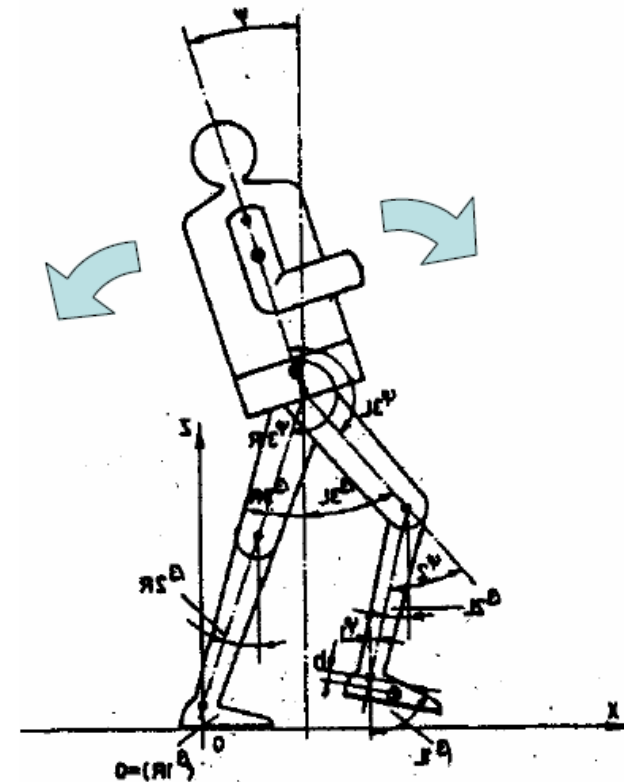
- Feet Positions - Top View



Walking Based on ZMP



(a) Leg motion prepared in advance.



(b) Calculate upper body motion so that ZMP is at desired location.

FRI

- Point on the foot/ground surface where the net GRF would have to act to keep foot stationary.
- FRI could be inside / outside support polygon
- FRI is dynamics-based criterion – reduces to GCoM for a stationary robot.
- FRI point indicates direction of foot rotation
- FRI point indicates stability margin of the robot.