Intelligent Robots (CS4278/CS5478) Homework 1

(Autumn 2024) Due: **11:59pm September 04**

Name ____________________________

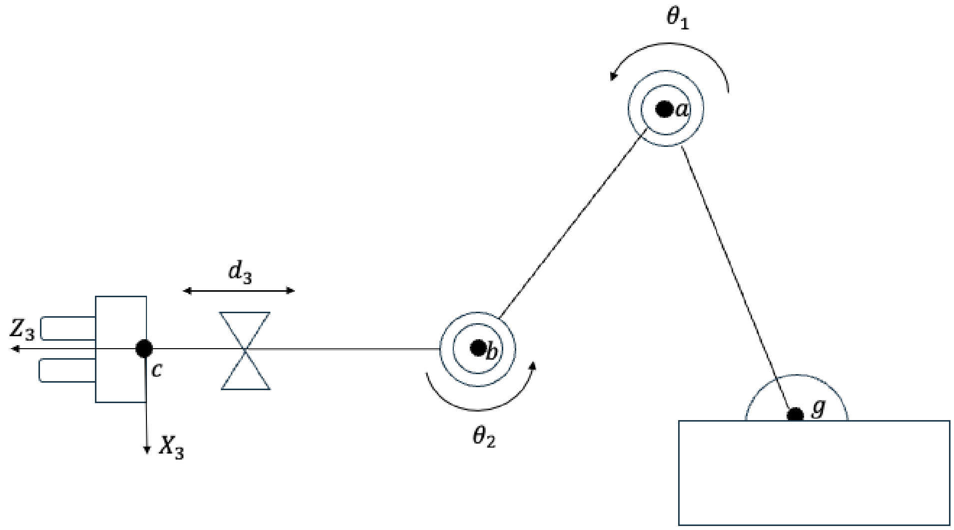
1. Given two frames {B} and {C} that are initially coincident with each other. First, we rotate {C} about $\hat{Z}_C$ by θ_1 degrees. Then, we rotate the resulting frame {C} about the new $\hat{Y}_C$ by θ_2.

   (a) (3 points) Determine the 3×3 rotation matrix, ${}^{B}_{C}R$, that will change the description of a vector P in frame {C}, ${}^{C}\mathbf{P}$, to frame {B}, ${}^{B}\mathbf{P}$.

   (b) (2 Points) What is the value of ${}^{B}_{C}R$, if $\theta_1 = 45°$, $\theta_2 = 60°$?

   (c) (3 Points) We then define a new frame A which translates from the frame B along the vector of ${}^{B}\mathbf{q} = [q_1, q_2, q_3]^T$. Write down the homogeneous transformation ${}^{A}_{C}T$ from frame C to frame A.

2. Consider the following manipulator with two revolute joint and one prismatic joint.



(a) (3 Points) Draw the frames of this manipulator. Define l_1 to the length connecting points g and a, and l_2 to be the length connecting points a and b. Note that frame 3 has been done for you, and your solution needs to be consistent with the given frame 3.

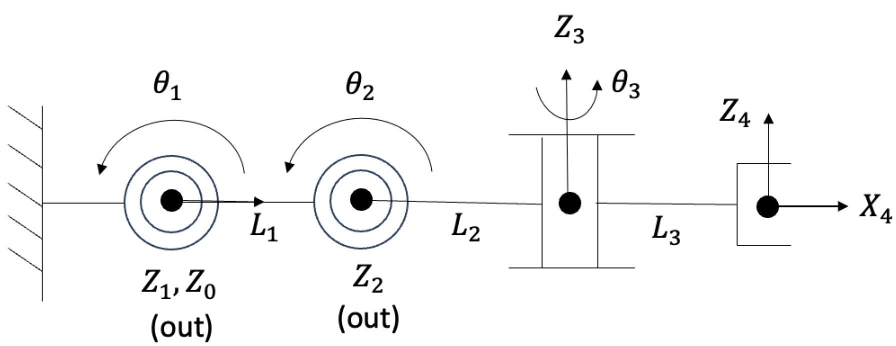
Hint: Frame 0 is not located at point g

(b) (4 Points) Find the Denavit-Hartenberg parameters for this manipulator and fill in the entries of the following table

i	a_{i-1}	α_{i-1}	d_i	θ_i
1				
2				
3				

(c) (3 Points) Given $\theta_1 = 225°$, $\theta_2 = 45°$, $l_1 = 0.5$, $l_2 = 0.4$, and $d_3 = 0.25$, find the matrix ${}^0_3T$ at the configuration from part (a). You may write down the answer as a product of matrices.

3. You are presented with the RRR manipulator below. L_1, L_2, and L_3 are strictly positive.



(a) (3 Points) Find the Denavit-Hartenberg parameters for this manipulator. Assign the frames such that all your a_i are positive.

i	a_{i-1}	α_{i-1}	d_i	θ_i
1				
2				
3				
4				

(b) (3 Points) The position of the end-effector is:

$$^{0}P_4 = \begin{bmatrix} L_1 c_1 + L_2 c_{12} + L_3 c_{12} c_3 \\ L_1 s_1 + L_2 s_{12} + L_3 s_{12} c_3 \\ -L_3 s_3 \end{bmatrix},$$

where $c_{12} = cos(\theta_1 + \theta_2)$.

Derive the linear Jacobian $^{0}J_v$.

(c) (6 Points) Find the singular configurations of this manipulator. For each singularity, draw the robot configuration and clearly state how the movement is restricted (in terms of frame axes).

Hint: The linear Jacobian in frame {2} is given to you here:

$$^{2}J_v = \begin{bmatrix} -L_1 s_2 & 0 & -L_3 s_3 \\ L_1 c_2 + L_2 + L_3 c_3 & L_2 + L_3 c_3 & 0 \\ 0 & 0 & -L_3 c_3 \end{bmatrix}$$

4. (10 Points) Let us consider the manipulator RPRP shown below, find the linear jacobian ${}^0J_v$ and the angular jacobian ${}^0J_\omega$ for the end effector point (origin of frame $\{4\}$), expressed in frame $\{0\}$.