7.6 vid Notes

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Ex 1) Multiply the matrices below

$$\begin{bmatrix} 1 & 2 \\ -3 & -5 \end{bmatrix} \cdot \begin{bmatrix} -5 & -2 \\ 3 & 1 \end{bmatrix} = \begin{bmatrix} -5+6 & -2+2 \\ 15-15 & 6-5 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

The identity matrix

$$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \quad \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$IX = X \qquad \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} a \\ b \end{bmatrix} = \begin{bmatrix} a+0b \\ 0a+b \end{bmatrix} = \begin{bmatrix} a \\ b \end{bmatrix}$$

What is the inverse of the matrix?

The matrix that is multiplied by the given matrix where the product is the identity matrix

$$A \cdot A^{-1} = I \longrightarrow \text{only square matrices}$$

Ex 2) Find the inverse of the matrix below

$$\begin{bmatrix} 2 & 3 \\ -4 & 1 \end{bmatrix} \begin{bmatrix} a & b \\ c & d \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \qquad A^{-1} = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$$

$$2a + 3c = 1 \qquad 2b + 3d = 0$$
$$-4a + c = 0 \qquad -4b + d = 1$$

$$\left[\begin{array}{cc|c} 2 & 3 & 1 \\ -4 & 1 & 0 \end{array}\right] a \qquad \left[\begin{array}{cc|c} 2 & 3 & 0 \\ -4 & 1 & 1 \end{array}\right]$$
$$c$$

$$\left[\begin{array}{cc|cc} 2 & 3 & 1 & 0 \\ -4 & 1 & 0 & 1 \end{array}\right]$$

$$A^{-1} = \begin{bmatrix} a & b \\ c & d \end{bmatrix} = \begin{bmatrix} 1/14 & -3/14 \\ 2/7 & 1/7 \end{bmatrix}$$

$2R_1 + R_2$

$$\left[\begin{array}{cc|c} 2 & 3 & 1 \\ 0 & 7 & 2 \end{array}\right] \qquad \left[\begin{array}{cc|c} 2 & 3 & 0 \\ 0 & -7 & 1 \end{array}\right]$$

$A^{-1} = \frac{1}{14} \begin{bmatrix} -1 & -3 \\ 4 & 2 \end{bmatrix}$

$\frac{1}{7} R_2$

$$\left[\begin{array}{cc|c} 2 & 3 & 1 \\ 0 & 1 & 2/7 \end{array}\right] \qquad \left[\begin{array}{cc|c} 2 & 3 & 0 \\ 0 & 1 & -1/7 \end{array}\right]$$

$-3R_2 + R_1$

$$\left[\begin{array}{cc|c} 2 & 0 & 1/7 \\ 0 & 1 & 2/7 \end{array}\right] \qquad \left[\begin{array}{cc|c} 2 & 0 & -3/7 \\ 0 & 1 & 1/7 \end{array}\right]$$

$-\frac{1}{2} R_1$

$$\left[\begin{array}{cc|c} 1 & 0 & 1/14 \\ 0 & 1 & 2/7 \end{array}\right] a \qquad \left[\begin{array}{cc|c} 1 & 0 & -3/14 \\ 0 & 1 & 1/7 \end{array}\right] b$$
$$c \qquad\qquad d$$

Ex 3) Find inverse...

$$\begin{bmatrix} 3 & -1 \\ -6 & 2 \end{bmatrix} \longrightarrow \left[\begin{array}{cc|cc} 3 & -1 & 1 & 0 \\ -6 & 2 & 0 & 1 \end{array}\right]$$

$2R_1 + 1R_2$

$$\left[\begin{array}{cc|cc} 3 & -1 & 1 & 0 \\ 0 & 0 & 2 & 1 \end{array}\right]$$

No inverse

Ex 4) $\begin{bmatrix} 3 & -2 \\ -9 & 5 \end{bmatrix}$ Find inverse ...

$$\left[\begin{array}{cc|cc} 3 & -2 & 1 & 0 \\ -9 & 5 & 0 & 1 \end{array}\right]$$

$3R_1 + R_2$

$-2R_2 + R_1$

$-R_2$

$\frac{1}{3}R_1$

$$A^{-1} = \begin{bmatrix} -5/3 & -2/3 \\ -3 & -1 \end{bmatrix} = -\frac{1}{3}\begin{bmatrix} 5 & 2 \\ 9 & 3 \end{bmatrix}$$

$$A = \begin{bmatrix} 3 & -2 \\ -9 & 5 \end{bmatrix} \quad A = \begin{bmatrix} a & b \\ c & d \end{bmatrix} \quad A^{-1} = \frac{1}{} \begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$$

Ex 7) $\begin{cases} 2x - 5y = 8 \\ 3x - 8y = 1 \end{cases}$

$$A^{-1}\begin{bmatrix} 2 & -5 \\ 3 & -8 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 8 \\ 1 \end{bmatrix}$$

$$I\begin{bmatrix} x \\ y \end{bmatrix} = A^{-1}\begin{bmatrix} 8 \\ 1 \end{bmatrix}$$

$$A^{-1} = \frac{1}{-16 - (-15)}\begin{bmatrix} -8 & 5 \\ -3 & 2 \end{bmatrix} = -1\begin{bmatrix} -8 & 5 \\ -3 & 2 \end{bmatrix} = \begin{bmatrix} 8 & -5 \\ 3 & -2 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 8 & -5 \\ 3 & -2 \end{bmatrix}\begin{bmatrix} 8 \\ 1 \end{bmatrix} = \begin{bmatrix} 64 - 5 \\ 24 - 2 \end{bmatrix} = \begin{bmatrix} 59 \\ 22 \end{bmatrix}$$

$(59, 22)$