Multivariate Analysis: Assessment 1

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Setup

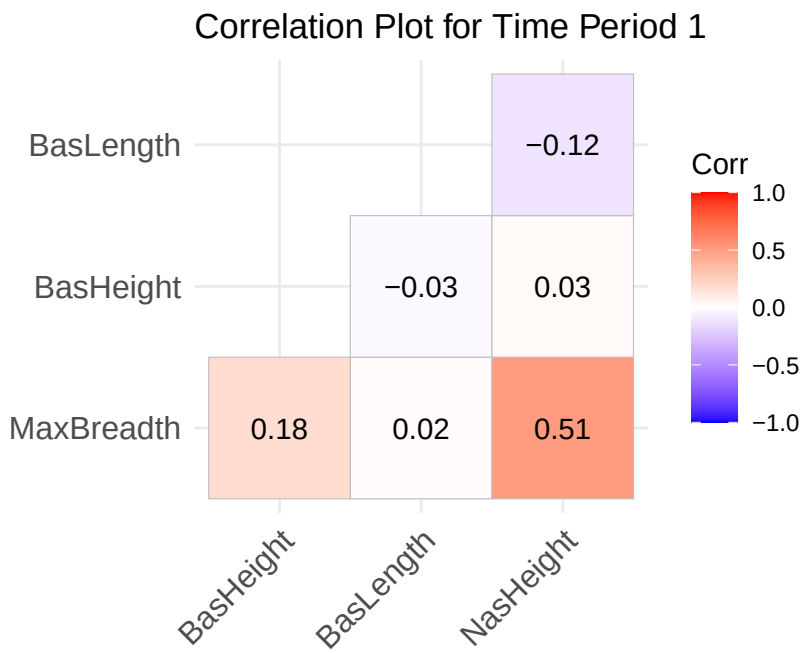
Load data and required packages into R.

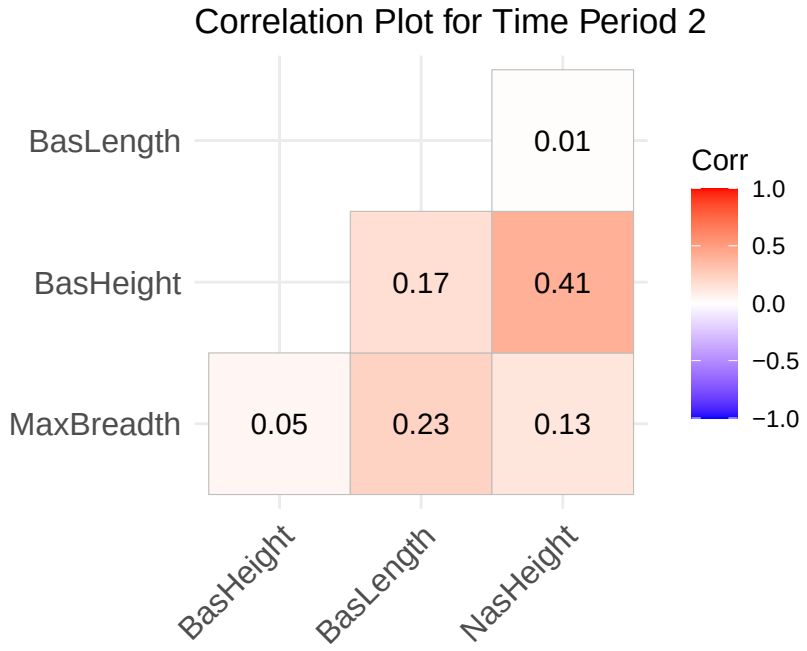
Question 1

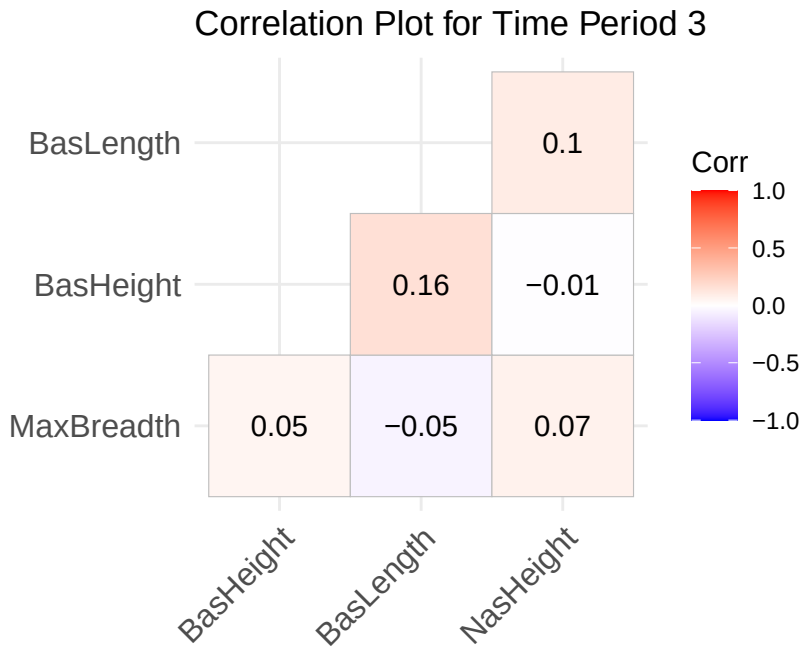
Table 1: Sample Means for Time Periods.
X1 = Maximal Breadth of skull (mm), X2 = Basilbregmatic Height of Skull (mm),
X3 = Basialveolar Length of Skull (mm), X4 = Nasal Height of Skull (mm)

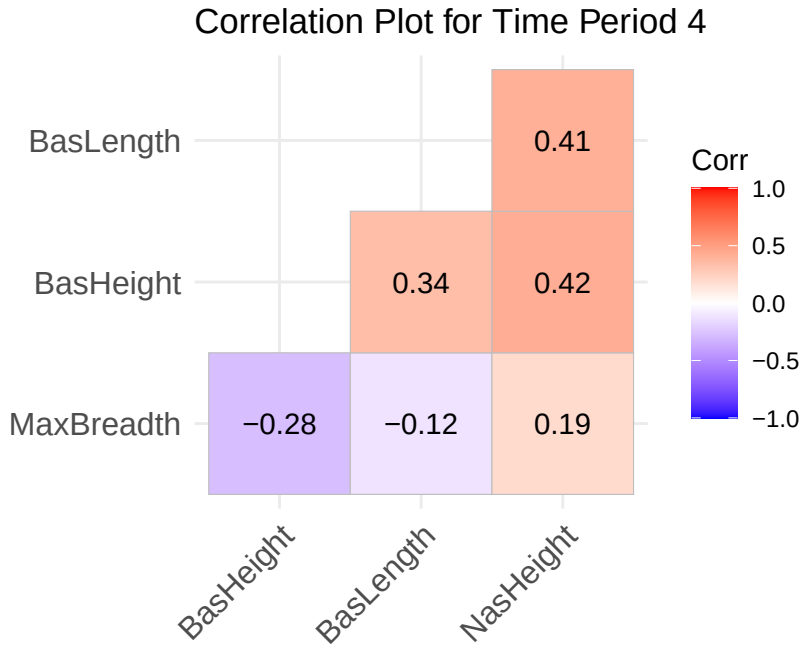
Time Period	Average X1	Average X2	Average X3	Average X4
1	131.3667	133.6000	99.16667	50.53333
2	132.3667	132.7000	99.06667	50.23333
3	134.4667	133.8000	96.03333	50.56667
4	135.5000	132.3000	94.53333	51.96667
5	136.1667	130.3333	93.50000	51.36667

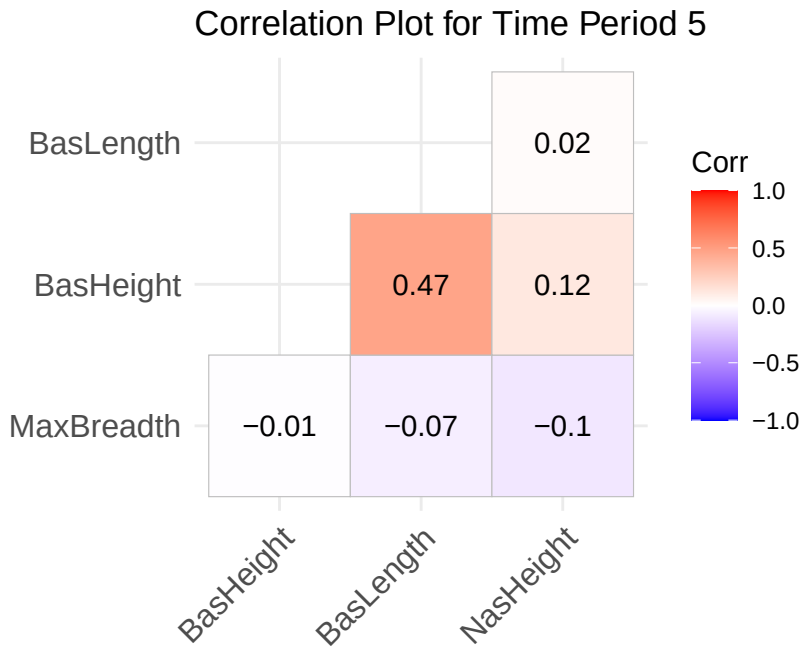
Question 2


Correlation Plot for Time Period 1
BasLength
BasHeight
MaxBreadth
−0.12
−0.03
0.03
0.18
0.02
0.51
BasHeight
BasLength
NasHeight
Corr
1.0
0.5
0.0
−0.5
−1.0


Correlation Plot for Time Period 2
BasLength
BasHeight
MaxBreadth
0.01
0.17
0.41
0.05
0.23
0.13
BasHeight
BasLength
NasHeight
Corr
1.0
0.5
0.0
−0.5
−1.0


Correlation Plot for Time Period 3
BasLength
BasHeight
MaxBreadth
0.1
0.16
−0.01
0.05
−0.05
0.07
Corr
1.0
0.5
0.0
−0.5
−1.0
BasHeight
BasLength
NasHeight


Correlation Plot for Time Period 4
BasLength
BasHeight
MaxBreadth
0.41
0.34
0.42
−0.28
−0.12
0.19
Corr
1.0
0.5
0.0
−0.5
−1.0
BasHeight
BasLength
NasHeight



Across the time periods, the correlation between basialveolar length of skull and nasal height of skull change from being weakly negatively correlated to weakly positively correlated. A similar trend is observed in the correlation between maximal breadth of skull and basilbregmatic height of skull, except these variables change from a weak, positive correlation to a weak negative correlation. Most variable observed a change in correlation from positive to negative or negative to positive over the observed time periods. This could be due to multicollinearity in the explanatory variables.

Question 3

$\cos(\theta) = 0.0150425$
This values is expected to be the same as the correlation coefficient between X1 and X3 for Time period 1. As seen in Question 2, this value is 0.02. This is the same as cosine of the angle between the deviation vectors, D1 and D3, 0.02 (rounded to 2 decimal places).

Question 4

Coefficient vector:

$$\mathbf{b} = \begin{bmatrix} -1 \\ 0 \\ 0 \\ 3 \end{bmatrix}$$

Sample means:

$$\bar{\mathbf{y}}_\mathbf{i} = \begin{bmatrix} 20.23333 \\ 18.3333 \\ 17.2333 \\ 20.4 \\ 17.9333 \end{bmatrix}$$

Covariance matrix:

$$\mathbf{Y} = \begin{bmatrix} 51.56437 & 0 & 0 & 0 & 0 \\ 0 & 90.71264 & 0 & 0 & 0 \\ 0 & 0 & 120.1161 & 0 & 0 \\ 0 & 0 & 0 & 74.73103 & 0 \\ 0 & 0 & 0 & 0 & 165.0299 \end{bmatrix}$$