Gibbs Sampling

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```
# gerando dados artificiais.
n = 100 # tamanho amostral.
mu_real = 10 # mu real.
phi_real = 2 # phi real.
y = rnorm(n,mu_real,sqrt(1/phi_real))
# especificacoes a priori.
m = 5; v = 100; # mu ~ N(m,v).
a = 0.01; b = 0.01; # phi ~ Ga(a,b).
# semente de inicializacao da cadeia MCMC.
mu = 50; phi = 10;
# numero total de iteracoes.
N = 10000
# objetos auxiliares para salvar as cadeias.
save_mu = mu; save_phi = phi;

for(j in 1:N){
  # concidional completa de mu.
  vs = 1/(n*phi + 1/v)
  ms = vs * (phi*sum(y) + m/v)
  mu = rnorm(1,ms,sqrt(vs))
  # concidional completa de phi.
  as = a + n/2
  bs = b + 0.5*(sum(y^2) -2*mu*sum(y) +n*mu^2)
  phi = rgamma(1,as,rate=bs)
  # salvando cadeia.
  save_mu = c(save_mu, mu)
  save_phi = c(save_phi, phi)
}

df_mu <- data.frame(Mu = save_mu, Iteracao = 1:(N+1))
df_phi <- data.frame(Phi = save_phi, Iteracao = 1:(N+1))

grafico1 <- ggplot(df_mu, aes(x = Iteracao, y = Mu)) +
  geom_line() +
  geom_hline(yintercept = mu_real, color = "red",
             linetype = "solid", linewidth = 0.6) +
  theme_light()

grafico2 <- ggplot(df_phi, aes(x = Iteracao, y = Phi)) +
  geom_line() +
  geom_hline(yintercept = phi_real, color = "red",
             linetype = "solid", linewidth = 0.6) +
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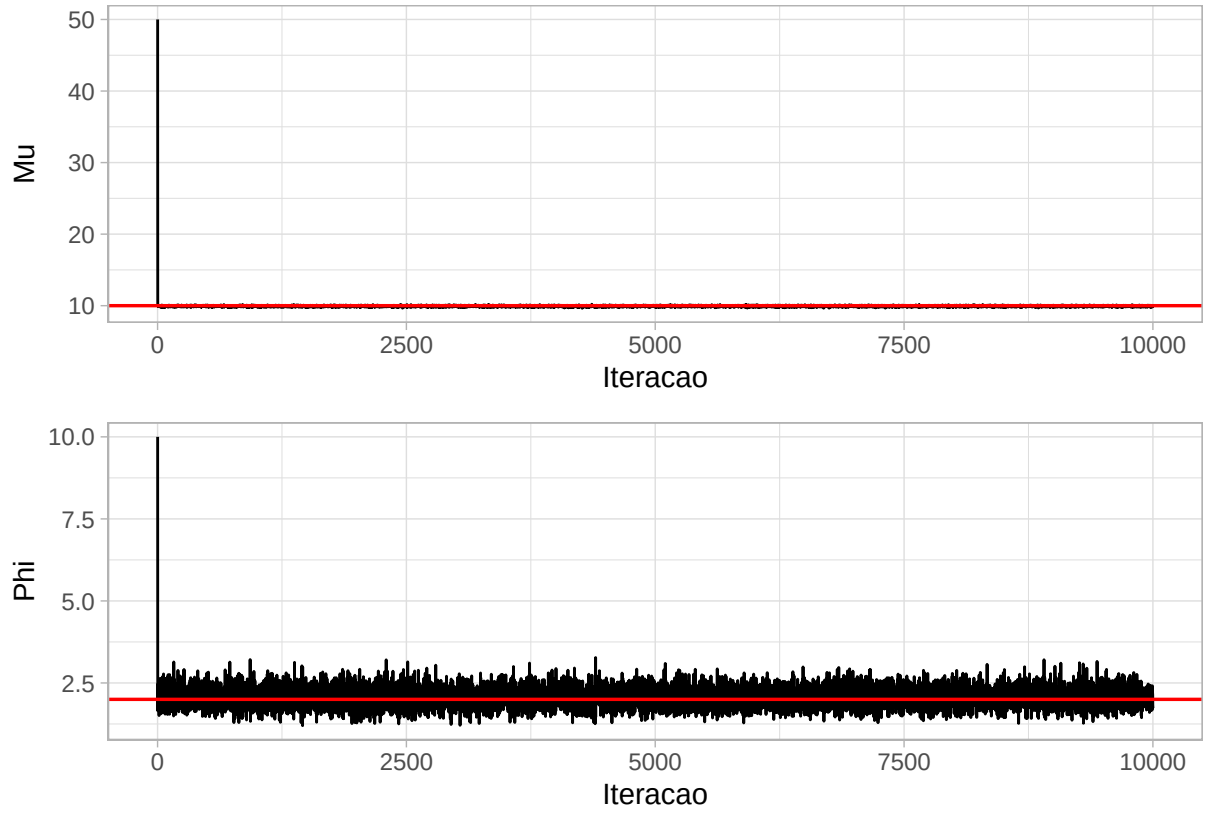
  theme_light()

grafico1 / grafico2
```




```
# Remover burn-in
save_mu <- save_mu[10:10000]
save_phi <- save_phi[10:10000]
save_var <- 1 / save_phi
save_dp <- sqrt(save_var)

calc_moda <- function(x) {
  d <- density(x)
  d$x[which.max(d$y)]
}

# Calcular estatísticas para cada parâmetro
tabela <- tibble::tibble(
  Parâmetro = c("mu", "phi", "sigma^2", "sigma"),
  `Valor real` = c(mu_real, phi_real, 1/phi_real, sqrt(1/phi_real)),
  Média = c(mean(save_mu), mean(save_phi), mean(save_var), mean(save_dp)),
  Moda = c(calc_moda(save_mu), calc_moda(save_phi), calc_moda(save_var), calc_moda(save_dp)),
  Mediana = c(median(save_mu), median(save_phi), median(save_var), median(save_dp)),
  `Desvio padrão` = c(sd(save_mu), sd(save_phi), sd(save_var), sd(save_dp)),
  `HPD 2.5%` = c(
    HPDinterval(as.mcmc(save_mu), prob = 0.95)[1],
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```
    HPDinterval(as.mcmc(save_phi), prob = 0.95)[1],
    HPDinterval(as.mcmc(save_var), prob = 0.95)[1],
    HPDinterval(as.mcmc(save_dp), prob = 0.95)[1]
  ),
  `HPD 97.5%` = c(
    HPDinterval(as.mcmc(save_mu), prob = 0.95)[2],
    HPDinterval(as.mcmc(save_phi), prob = 0.95)[2],
    HPDinterval(as.mcmc(save_var), prob = 0.95)[2],
    HPDinterval(as.mcmc(save_dp), prob = 0.95)[2]
  )
)

kable(tabela)
```

Parâmetro	Valor real	Média	Moda	Mediana	Desvio padrão	HPD 2.5%	HPD 97.5%
mu	10.0000000	9.9307301	9.9198674	9.9297203	0.0705816	9.7871676	10.0624146
phi	2.0000000	2.0560631	2.0475117	2.0428106	0.2886174	1.5117966	2.6277762
sigma^2	0.5000000	0.4961537	0.4806793	0.4895216	0.0711467	0.3687474	0.6412529
sigma	0.7071068	0.7026112	0.6940070	0.6996582	0.0499147	0.6121294	0.8053489

```
df_mu <- data.frame(Mu = save_mu)
df_phi <- data.frame(Phi = save_phi)
df_var <- data.frame(Sigma2 = save_var)

vlines_mu <- data.frame(
  valor = c(mu_real, mean(df_mu$Mu)),
  tipo = c("Valor real", "Média GS")
)
grafico1 <- ggplot(df_mu, aes(x = Mu)) +
  geom_histogram(bins = 50, fill = "steelblue", color = "white") +
  geom_vline(data = vlines_mu, aes(xintercept = valor, color = tipo), linewidth = 0.7) +
  scale_color_manual(values = c("Valor real" = "red", "Média GS" = "darkorange")) +
  labs(x = expression(mu),
       y = "Frequência",
       color = "Legenda") +
  theme_light()


vlines_phi <- data.frame(
  valor = c(phi_real, mean(df_phi$Phi)),
  tipo = c("Valor real", "Média GS")
)
grafico2 <- ggplot(df_phi, aes(x = Phi)) +
  geom_histogram(bins = 50, fill = "steelblue", color = "white") +
  geom_vline(data = vlines_phi, aes(xintercept = valor, color = tipo), linewidth = 0.7) +
  scale_color_manual(values = c("Valor real" = "red", "Média GS" = "darkorange")) +
  labs(x = expression(phi),
       y = "Frequência",
       color = "Legenda") +
  theme_light()
```

```r
vlines_sigma <- data.frame(
  valor = c(1/phi_real, mean(df_var$Sigma2)),
  tipo = c("Valor real", "Média GS")
)
grafico3 <- ggplot(df_var, aes(x = Sigma2)) +
  geom_histogram(bins = 50, fill = "steelblue", color = "white") +
  geom_vline(data = vlines_sigma, aes(xintercept = valor, color = tipo), linewidth = 0.7) +
  scale_color_manual(values = c("Valor real" = "red", "Média GS" = "darkorange")) +
  labs(x = expression(sigma^2),
       y = "Frequência",
       color = "Legenda") +
  theme_light()

grafico1 / grafico2 / grafico3
```