Distributed Systems

Logical Clocks

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Logical clocks

Assign sequence numbers to messages

- All cooperating processes can agree on order of events
- vs. **physical clocks**: time of day

Assume no central time source

- Each system maintains its own local clock
- No total ordering of events
  - No concept of *happened-when*

Happened-before

Lamport's "happened-before" notation

$a \rightarrow b$ event a happened before event b

e.g.: a: message being sent, b: message receipt

Transitive:

if $a \rightarrow b$ and $b \rightarrow c$ then $a \rightarrow c$

Logical clocks & concurrency

Assign "clock" value to each event

- if $a \rightarrow b$ then **clock(a) < clock(b)**
- since time cannot run backwards

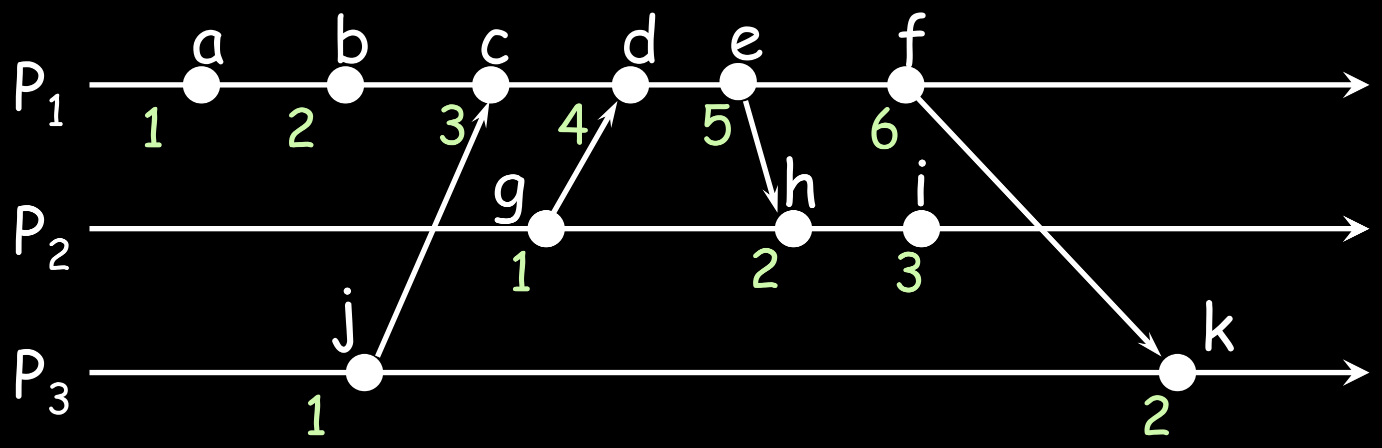
If a and b occur on different processes that do not exchange messages, then neither $a \rightarrow b$ nor $b \rightarrow a$ are true

- These events are **concurrent**

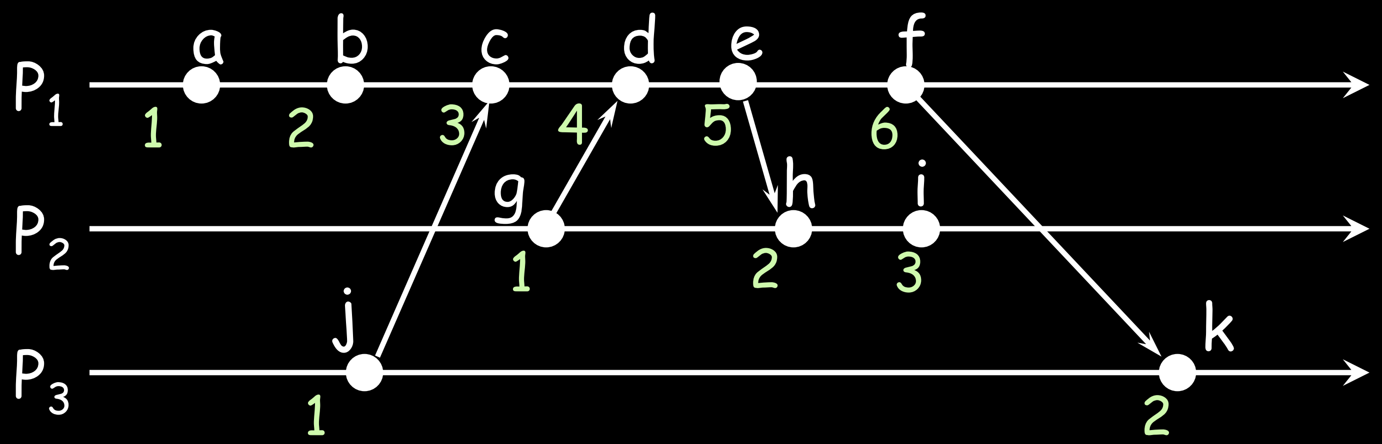
Event counting example

- Three systems: P_0, P_1, P_2
- Events *a*, *b*, *c*, …
- Local event counter on each system
- Systems occasionally communicate

Event counting example



Event counting example



Bad ordering:

e → h

f → k

Lamport's algorithm

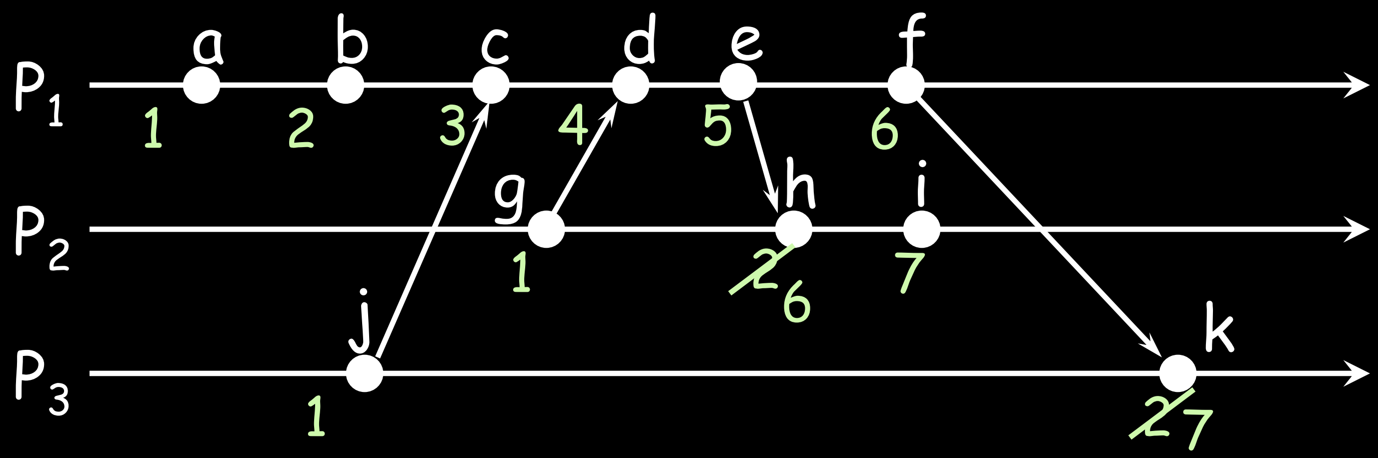
- Each message carries a timestamp of the sender's clock
- When a message arrives:
  - if receiver's clock < message timestamp
    set system clock to (message timestamp + 1)
  - else do nothing
- Clock must be advanced between any two events in the same process

Lamport's algorithm

Algorithm allows us to maintain time ordering among related events

- *Partial ordering*

Event counting example

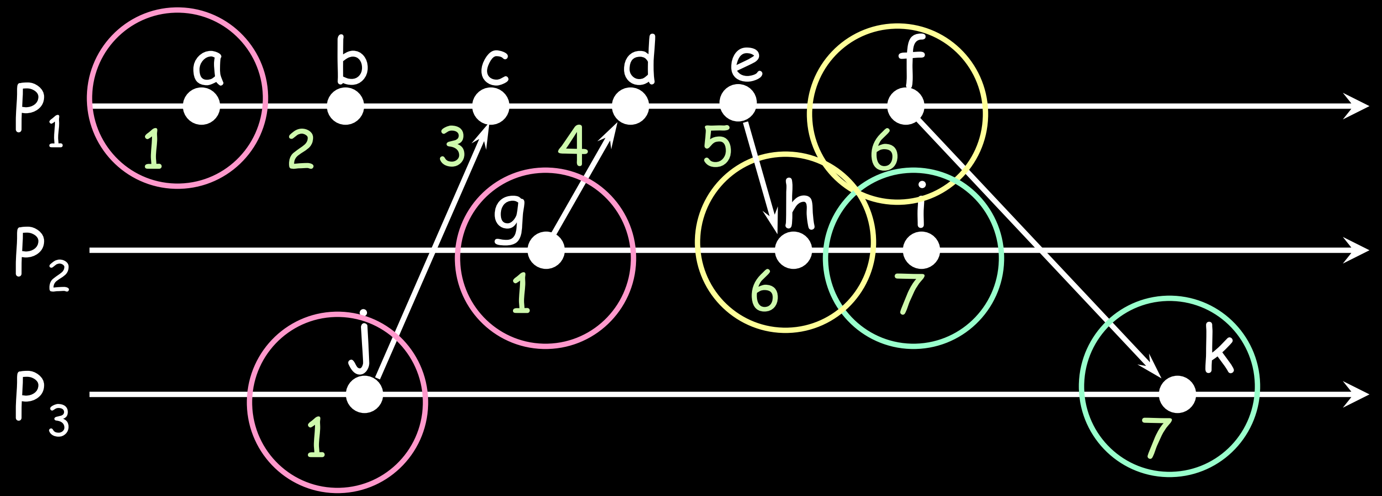


Summary

- Algorithm needs monotonically increasing software counter
- Incremented at least when events that need to be timestamped occur
- Each event has a **Lamport timestamp** attached to it
- For any two events, where $a \rightarrow b$:

$$L(a) < L(b)$$

Problem: Identical timestamps



a→b, b→c, …: local events sequenced

i→c, f→d , d→g, … : Lamport imposes a *send*→*receive* relationship

Concurrent events (e.g., a & i) *may* have the same timestamp … or not

Unique timestamps (total ordering)

We can force each timestamp to be unique

- Define global logical timestamp (T_i, i)
  - T_i represents local Lamport timestamp
  - i represents process number (globally unique)
    - E.g. (host address, process ID)
- Compare timestamps:

  $(T_i, i) < (T_j, j)$

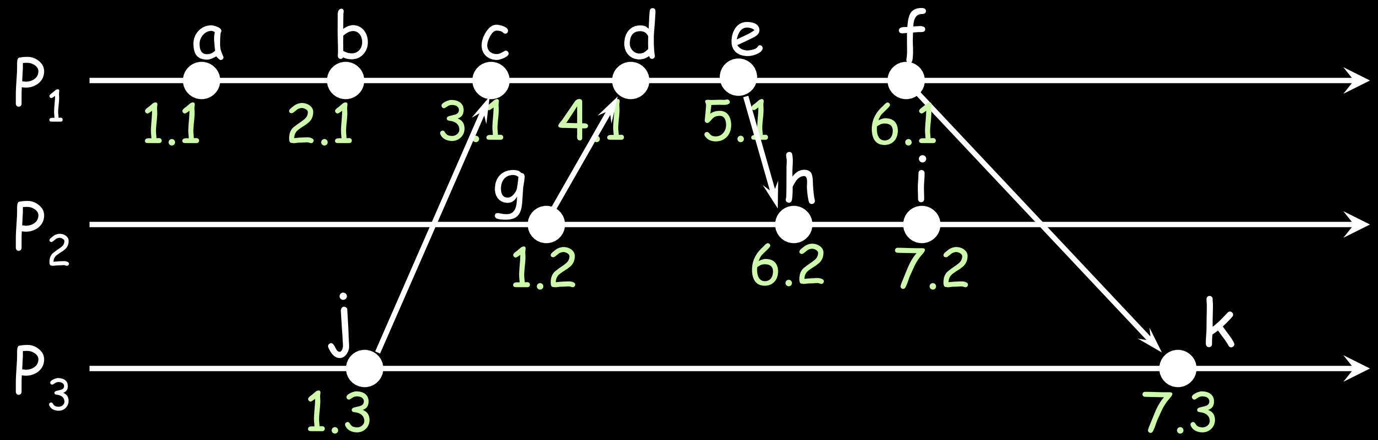
  if and only if

  $T_i < T_j$ or

  $T_i = T_j$ and $i < j$

Does not relate to event ordering

Unique (totally ordered) timestamps



Problem: Detecting causal relations

If $L(e) < L(e')$

- Cannot conclude that $e \rightarrow e'$

Looking at Lamport timestamps

- Cannot conclude which events are causally related

Solution: use a **vector clock**

Vector clocks

Rules:

1. Vector initialized to 0 at each process

   $V_i[j] = 0$ for $i, j = 1, \ldots, N$

2. Process increments its element of the vector in local vector before timestamping event:

   $V_i[i] = V_i[i] + 1$

3. Message is sent from process P_i with V_i attached to it
4. When P_j receives message, compares vectors element by element and sets local vector to higher of two values

   $V_j[i] = \max(V_i[i], V_j[i])$ for $i = 1, \ldots, N$

Comparing vector timestamps

Define

$V = V'$ iff $V[i] = V'[i]$ for $i = 1 \ldots N$

$V \leq V'$ iff $V[i] \leq V'[i]$ for $i = 1 \ldots N$

For any two events e, e'

if $e \rightarrow e'$ then $V(e) < V(e')$

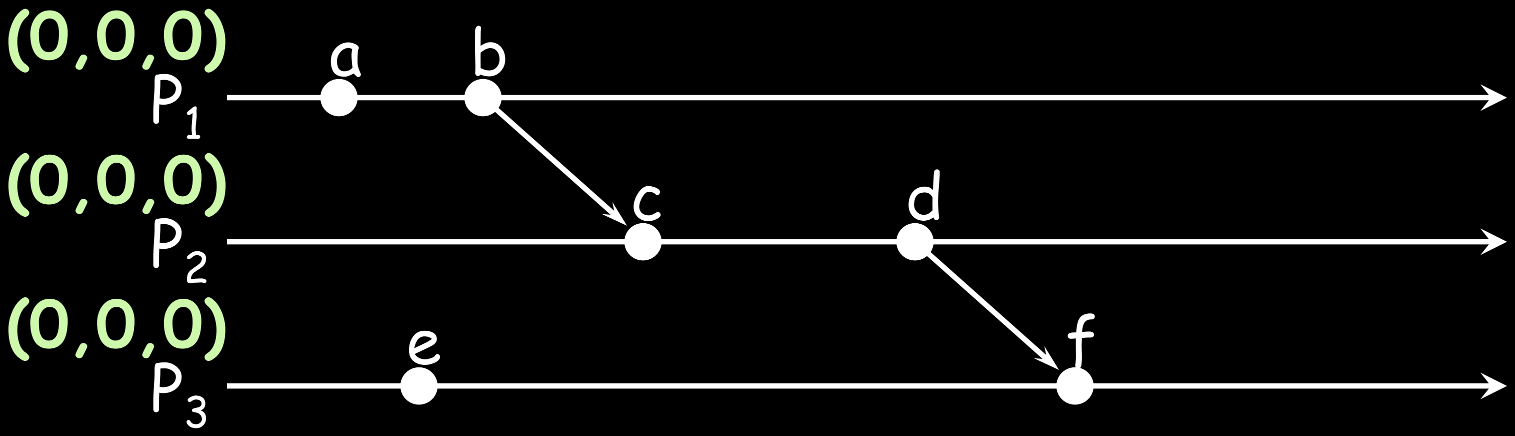
- Just like Lamport's algorithm

if $V(e) < V(e')$ then $e \rightarrow e'$

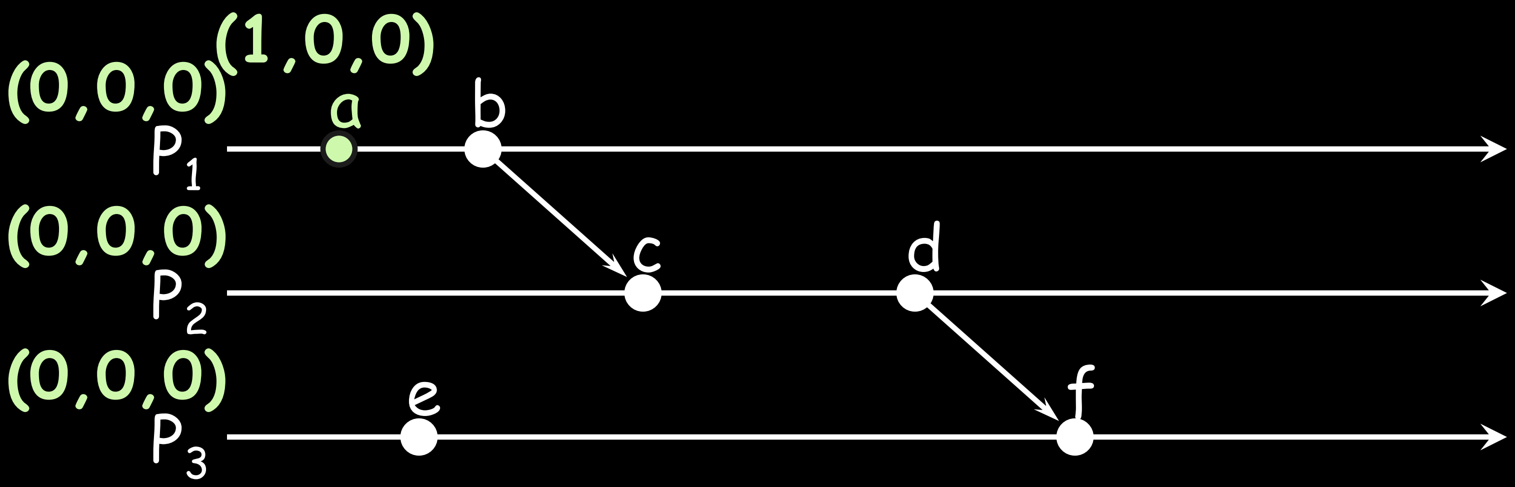
Two events are **concurrent** if **neither**

$V(e) \leq V(e')$ nor $V(e') \leq V(e)$

Vector timestamps

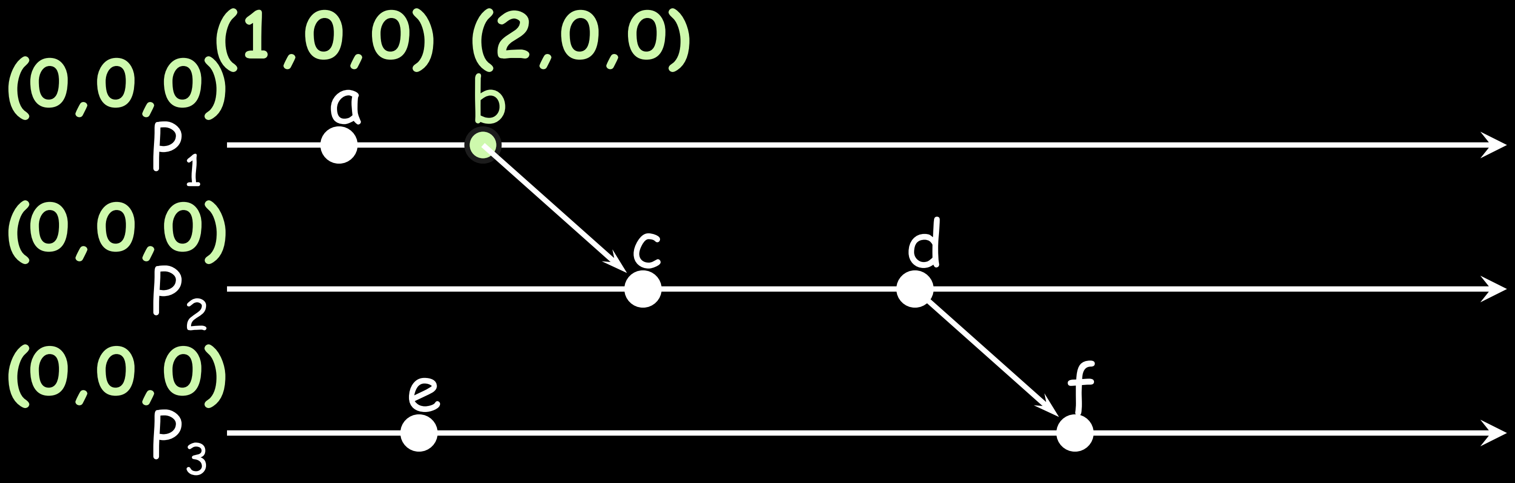


Vector timestamps



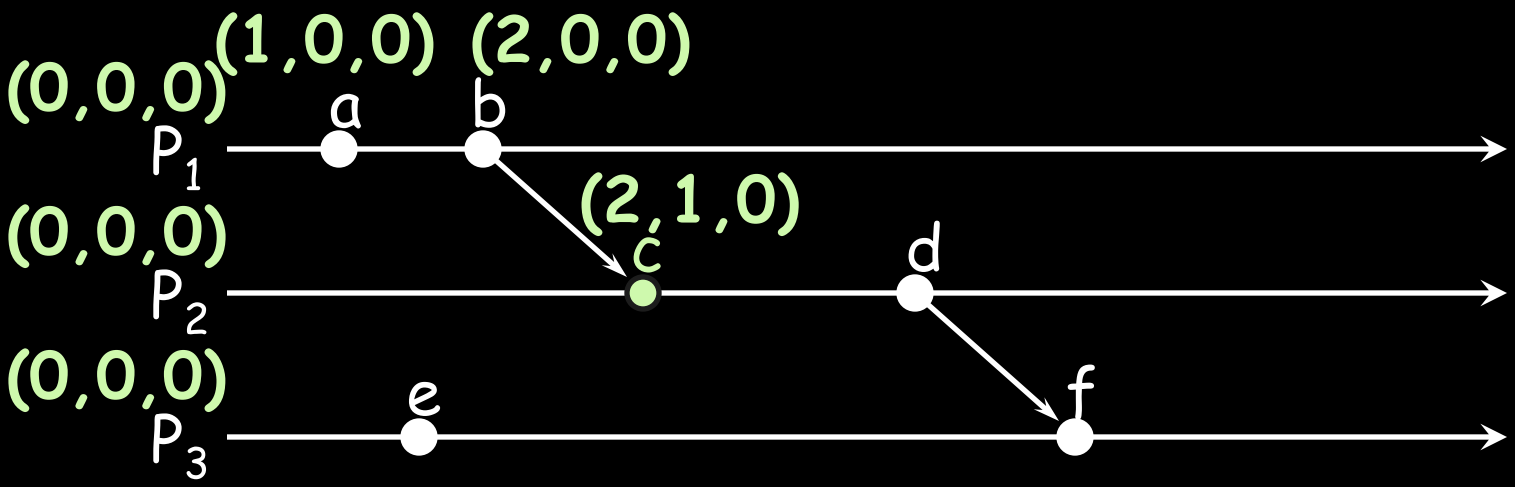
Event	timestamp
a	(1,0,0)

Vector timestamps



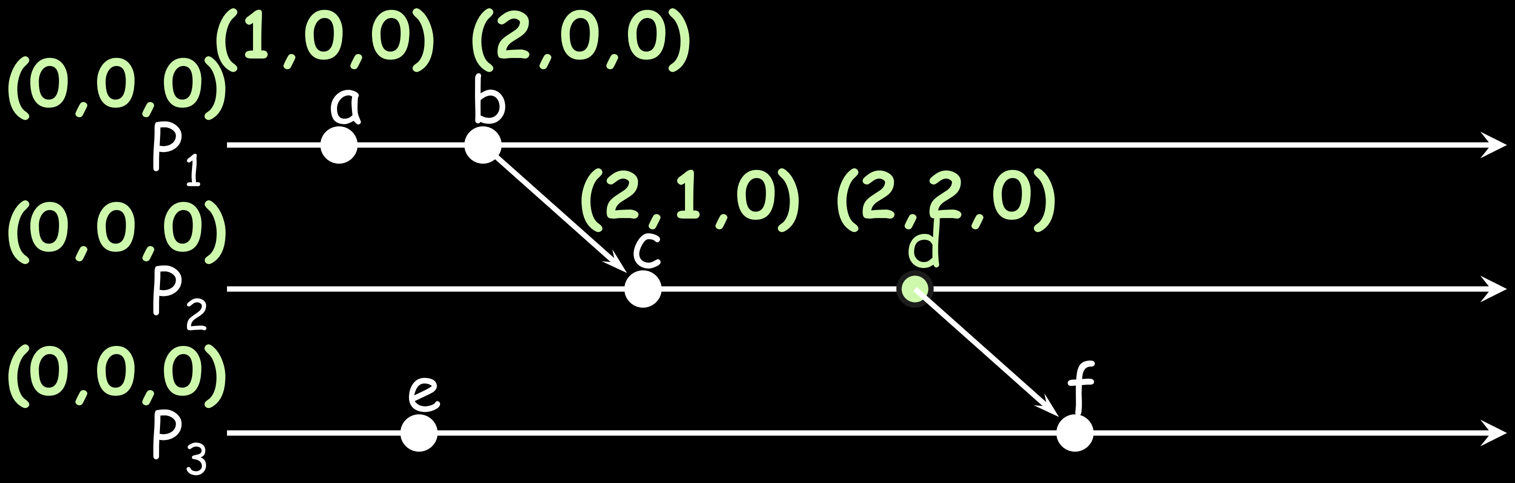
Event	timestamp
a	(1,0,0)
b	(2,0,0)

Vector timestamps



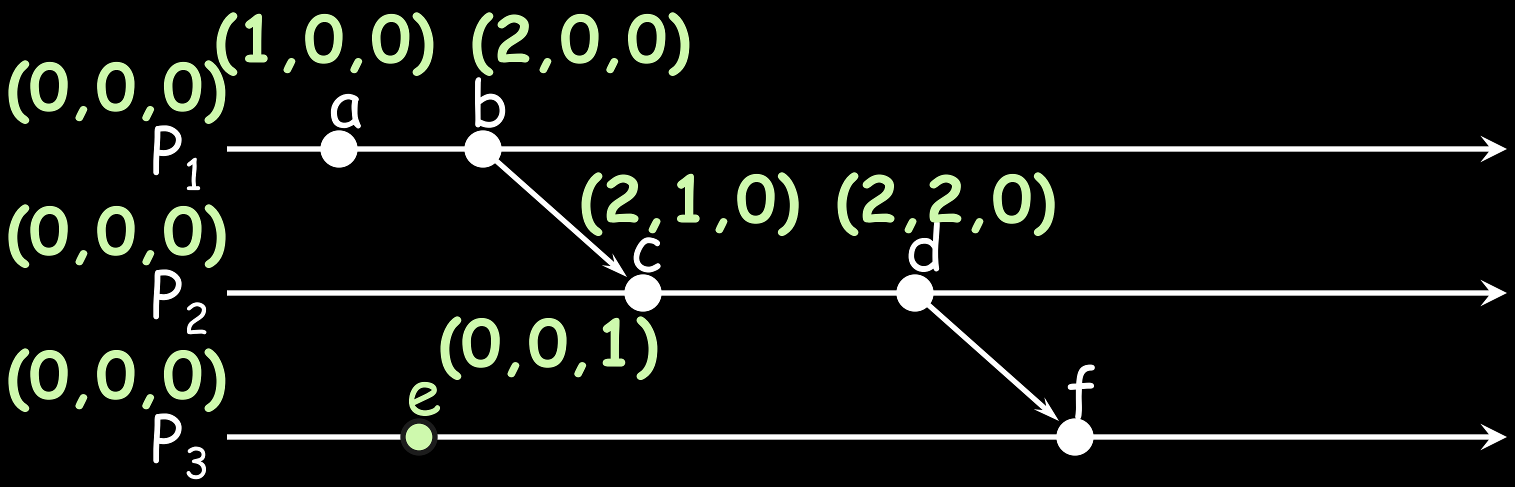
Event	timestamp
a	(1,0,0)
b	(2,0,0)
c	(2,1,0)

Vector timestamps



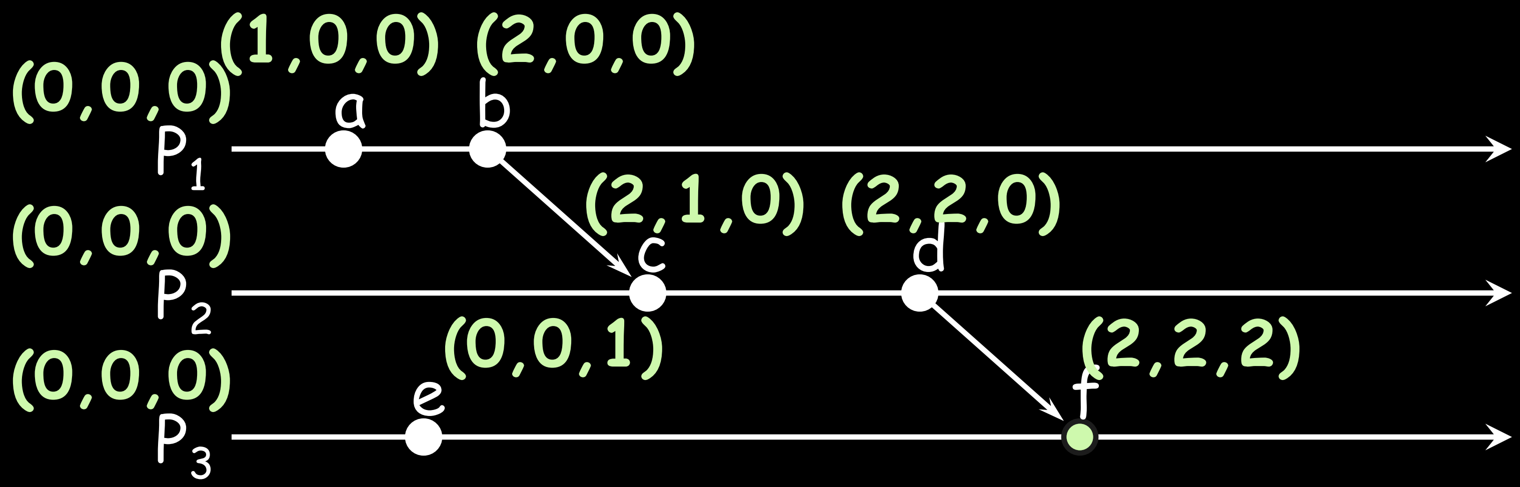
Event	timestamp
a	(1,0,0)
b	(2,0,0)
c	(2,1,0)
d	(2,2,0)

Vector timestamps



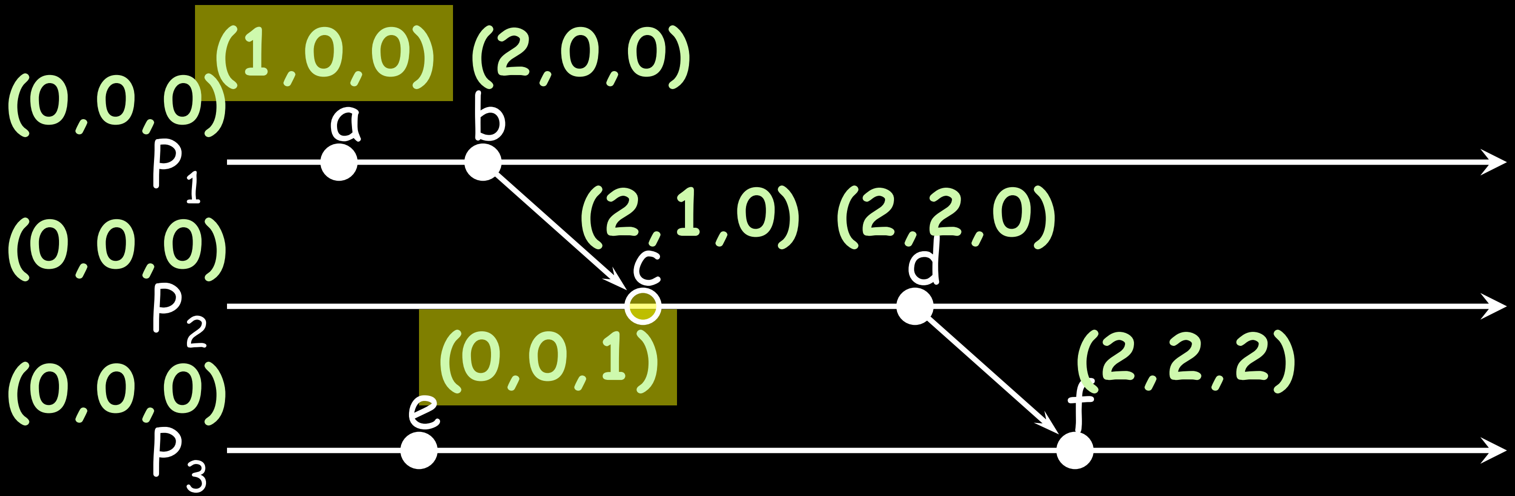
Event	timestamp
a	(1,0,0)
b	(2,0,0)
c	(2,1,0)
d	(2,2,0)
e	(0,0,1)

Vector timestamps



Event	timestamp
a	(1,0,0)
b	(2,0,0)
c	(2,1,0)
d	(2,2,0)
e	(0,0,1)
f	(2,2,2)

Vector timestamps



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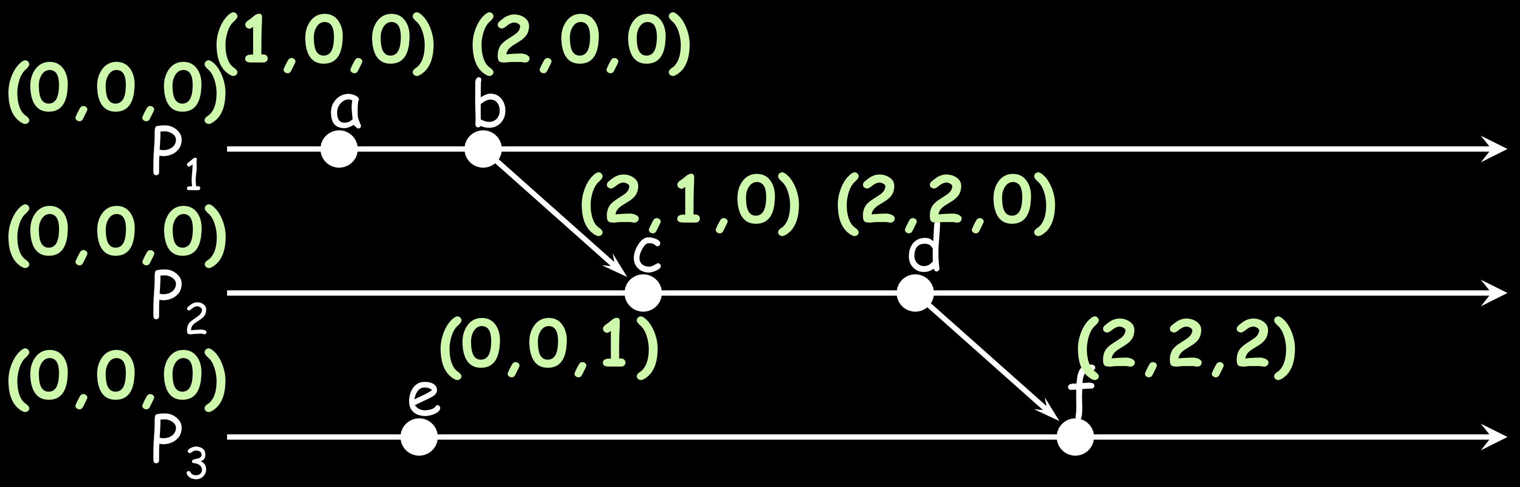
concurrent events

Vector timestamps



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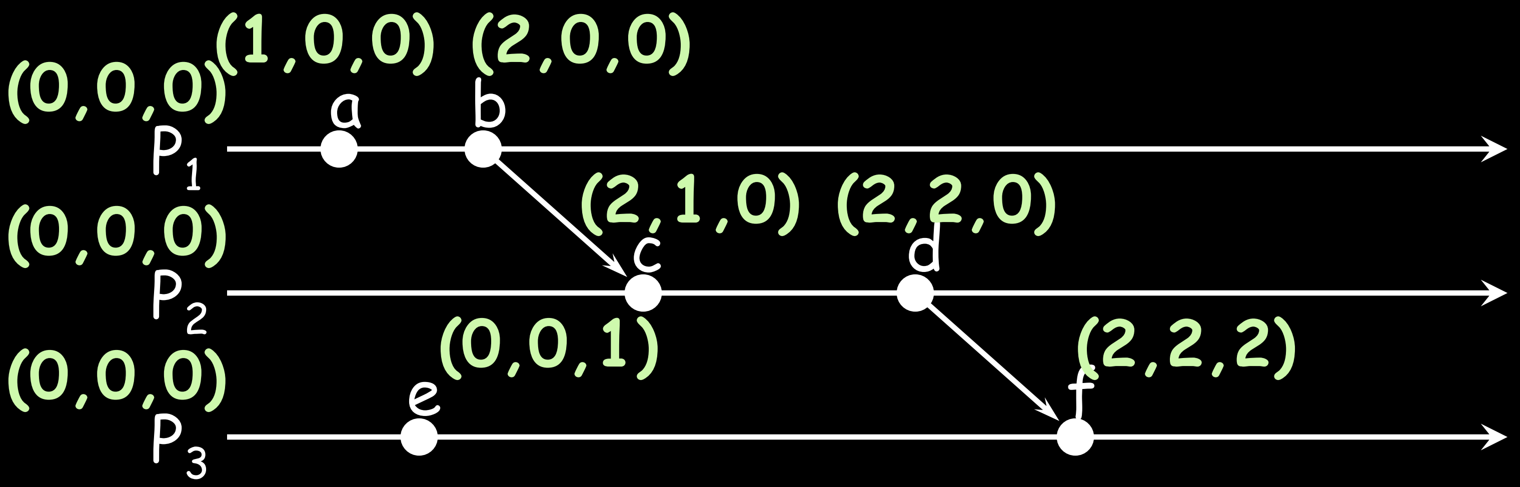
concurrent events

Vector timestamps



Event	timestamp
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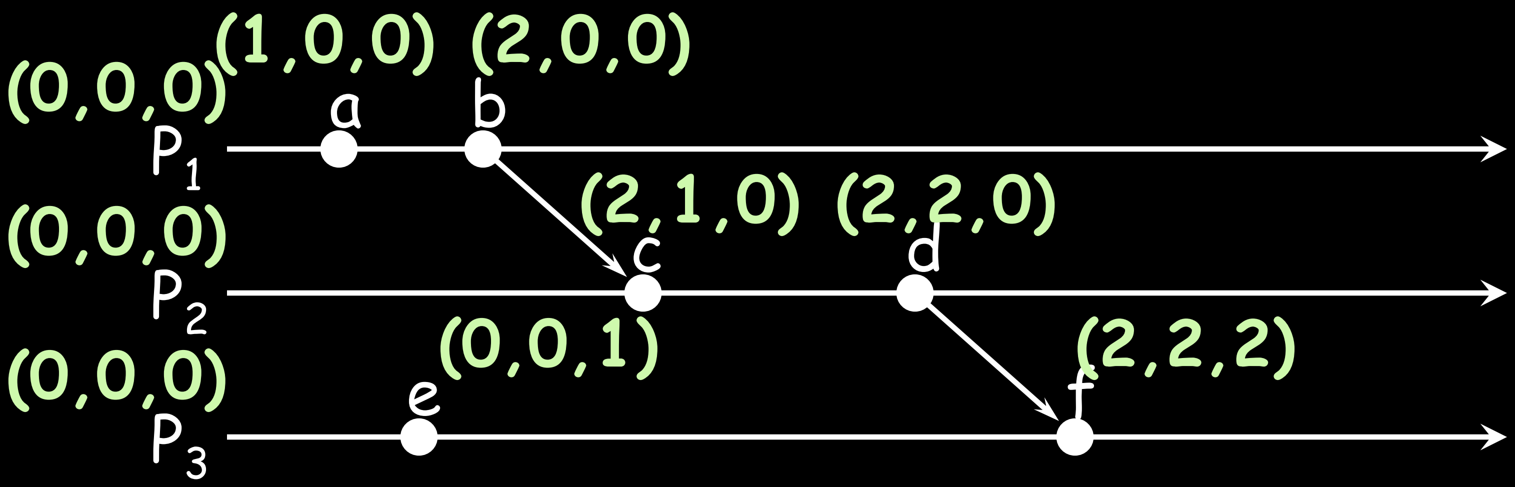
concurrent events

Vector timestamps



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concurrent events

Summary: Logical Clocks & Partial Ordering

- Causality
  - If *a->b* then event *a* can affect event *b*
- Concurrency
  - If neither *a->b* nor *b->a* then one event cannot affect the other
- Partial Ordering
  - Causal events are sequenced
- Total Ordering
  - All events are sequenced

The end.