Portfolio optimizer

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1 Introduction

This project uses Markowitz portfolio theory to optimize a portfolio for given assets. According to Markowitz theory the optimal portfolio is the one with highest sharpe ratio.

$$S = \frac{w^T R - R_f}{\sqrt{w^T \Sigma w}}$$

Where w is the weights, R and R_f return and risk-free rate and Σ the covariance matrix. Notice that that

$$S(kw) = S(w), \forall k > 0 \tag{1}$$

Meaning that the optimal portfolio using leverage will in fact result in the same shape ratio as not.

2 Optimization

The optimal portfolio will be the weights that result in a $\nabla S = 0$. Using a modified version of Newton's metod we get that:

$$\nabla S(w + s) = \nabla S(w) + \nabla^2 S(w) s = 0$$

$$\implies \nabla^2 S(w) s = -\nabla S(w)$$

We keep iterating this while $\nabla S > tol$ with any w_0.

$$\begin{cases} \nabla^2 S(w_k) s_k = -\nabla S(w_k) \\ w_{k+1} = w_k + s_k \end{cases}$$

Notice that we don't need to normalize the weights for each iteration see (1).

w^* is not necessarily long only. Meaning that we have to make adjustments depending on what we allow.

$$w_{opt} = \begin{cases} \dfrac{w^*}{\sum\limits_{i:w_i^*>0} w_i^*}, & \text{short allowed} \\ \dfrac{proj_{\mathbb{R}+} w^*}{\sum\limits_{i:w_i^*>0} w_i^*}, & \text{long only} \end{cases}$$

Where the projection means $w : w_i = max(w_i^*, 0), \forall i$ and the sum is the normalization.